The Identification of Fixed Costs from Consumer Behaviour*

by

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Abstract

Some household expenditures, such as those for subsistence or basic needs, are fixed. Using the methodology of equivalence scales, we develop a model in which differences in fixed costs of characteristics across households can be identified from household behaviour.

Equivalent expenditure for a household is the expenditure needed to bring a reference household, such as a single childless adult, to the level of well-being of household members. The equivalence scale for the household is the ratio of expenditure to equivalent expenditure. Only two types of equivalent-expenditure functions are used in practice: those in which the ratio of household expenditure and equivalent expenditure is independent of expenditure, and those in which their difference is independent of expenditure. We propose a class of equivalent-expenditure functions that allows for differences in fixed costs and generalizes both relationships.

Using Canadian consumer-demand micro-data, we estimate equivalent-expenditure functions and equivalence scales. We find that, for large households, fixed costs of characteristics are large and positive, resulting in equivalence scales that decline with expenditure.

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1. Introduction

Some household expenditures, such as those for subsistence or basic needs, are fixed. Because different households have different needs, there is variation in fixed costs across households. That variation may matter in welfare analysis. In addition, accounting for fixed costs may help to explain patterns of saving behaviour. In this paper, we present a model in which differences in fixed costs of characteristics can be identified, using the methodology of equivalent expenditures and equivalence scales.

Equivalence scales and equivalent expenditures are used to make interpersonal comparisons of well-being for purposes such as indexing social transfers and measuring poverty, inequality and social welfare. Equivalent expenditure for a household is the expenditure level needed to bring the well-being of a reference household, such as a single childless adult, to the level of well-being of household members. The relative (standard) equivalence scale for the household is equal to the ratio of household expenditure and equivalent expenditure. If, for example, a family of four has an expenditure level of \$50,000 and the equivalence scale for its type is equal to 2.5 at the prices it faces, the household is equivalent, for welfare purposes, to four (reference) single adults who spend $\$50,000/2.5 = \$20,000$ each and face the same prices. $\$20,000$ is the equivalent expenditure of the household. Equivalent expenditures and equivalence scales carry the same information: if one is known, the other can be computed. These functions account for differences in needs and differences in the economies of household size and composition. Because equivalent-expenditure functions convert an economy of different household types into one in which each household consists of a single identical person, social welfare measurement practices that treat individuals symmetrically may use equivalent expenditures.[1]

Although equivalent expenditure may have any increasing relationship with household expenditure, only two types of equivalent-expenditure functions are used in practice: those in which the ratio of equivalent expenditure and household expenditure is independent of expenditure, and those in which their difference is independent of expenditure. In the first case, equivalence scale exactness (ESE) is satisfied, a condition that is implicitly used in all empirical work on poverty, inequality and social welfare measurement.[2] In the second, absolute equivalence-scale exactness (AESE) is satisfied,[3] a condition that seems to underlie

[1] Differences in needs and differences in economies of household size and composition are in principle distinct. Chiappori (1993) and Browning, Chiappori and Lewbel (2004) propose "collective household models" in which scale economies may be estimated independently of the needs of individuals. These models neither inform nor structure interpersonal comparisons of well-being. Thus, even if we use collective household approaches to identify scale economies, the measurement of poverty, inequality or social welfare still require something like an equivalent-expenditure function to deal with heterogeneous needs across individuals.

[2] A possible interpretation of Ebert and Moyes (2003) is that the indirect social-evaluation function should not necessarily be independent of the choice of a reference household.

[3] See Blackorby and Donaldson (1991, 1993, 1994) and Lewbel (1989, 1991).

the logic of many social transfer systems. For example, in Canada, the tax benefits for elderly and disabled people are tax credits in fixed dollar amounts (rather than credits which are related to expenditure levels). Both types of equivalent-expenditure functions are used in practice, and both have some intuitive appeal.

We examine a class of equivalent-expenditure functions which generalizes both relationships, and show that if the equivalent-expenditure function is assumed to lie in this class, identification of equivalent expenditure from demand behaviour is possible. Members of the new class of equivalent-expenditure functions satisfy a condition we call generalised absolute equivalence-scale exactness (GAESE). GAESE is based on a shared property of ESE and AESE: the response of equivalent expenditure to a marginal change in expenditure is independent of expenditure. GAESE exhausts all possible equivalent-expenditure functions satisfying ESE and AESE, and introduces other possibilities. Given GAESE, the ratio of the marginal cost of utility for a household with characteristics z to the marginal cost of utility for a reference household is independent of expenditure.

The (absolute) cost of characteristics for a household type is the expenditure needed to provide a given standard of living for the household's members less the cost of providing the same standard of living for a reference household. GAESE allows the costs of characteristics for non-reference households to have a fixed component as well as one that varies with the utility level.

Various features of equivalent expenditure can be revealed from demand behaviour. Blundell and Lewbel (1991) note that, in contrast to Pollak and Wales (1979) pessimistic view that behaviour and equivalence scales are essentially disjoint, the price responses of equivalent-expenditure functions are always identified from behaviour alone. If, for example, the equivalent-expenditure function at one price vector is known, demand behaviour can be used to adjust the function for an increase in the price of food. Blundell and Lewbel (1991) argue persuasively that this is the limit for identification without further restrictions.

Blackorby and Donaldson (1993) and Donaldson and Pendakur (2004) show that more can be identified from behaviour if certain assumptions about the functional form of equivalent expenditure are made.[4] Blackorby and Donaldson (1993) show that if ESE and a condition on reference preferences are satisfied, the equivalent-expenditure function is uniquely identified from behaviour. Although infinitely many equivalent-expenditure functions are consistent with behaviour, only one of them is consistent with ESE. Blackorby and Donaldson (1994) make a similar case for identification when AESE is satisfied. Donaldson and Pendakur (2004) examine equivalent-expenditure functions in which equivalent expenditure is isoelastic with respect to expenditure, a condition they call generalized equivalence-scale exactness (GESE). They show that if GESE and the same condition on reference preferences

[4] These assumptions cannot be tested by demand behaviour alone.

used by Blackorby and Donaldson (1993) are satisfied, the equivalent-expenditure function is uniquely identified by demand behaviour.

Equivalent-expenditure functions that satisfy GAESE are affine in household expenditure—they have a fixed component which is independent of household expenditure and a component which is proportional to household expenditure. If the proportional component is equal to expenditure, the model collapses to AESE and, if the fixed component is zero, the model collapses to ESE. If neither condition holds, then the equivalence scale—defined as the ratio of household expenditure to equivalent expenditure—will be different at different expenditure levels. The fixed component of the equivalent-expenditure function captures differences in fixed costs of characteristics faced by different types of households and people. For example, if the minimum cost of housing is greater for disabled people than for others, the fixed component for disabled people is positive. In that case, the equivalence scale declines with household expenditure. This property was found in the equivalence scales of large households by Koulovatianos et al. (2004) in their survey-based study of equivalent-expenditure functions. If the fixed component is negative, then the equivalence scale rises with household expenditure.

We show that, if GAESE is accepted as a maintained hypothesis, the equivalent-expenditure function is identifiable from behaviour as long as the reference expenditure function is neither affine nor log-affine in transformed utility.[5] A wealth of empirical work shows that expenditure is neither affine nor log-affine in expenditure (see especially Banks, Blundell and Lewbel (1997)), so this asks little of the data. GAESE implies testable restrictions on preferences, which allow the researcher to reject GAESE if it is false. However, GAESE also implies untestable restrictions, so such tests are partial in the sense that GAESE may not be rejected even if it is false.

This paper is not the first to offer a functional-form restriction on the equivalent-expenditure function which permits identifiable equivalence scales that vary with expenditure. Donaldson and Pendakur (2004) show that if the equivalent-expenditure function is isoelastic in expenditure, then the equivalence scale is identifiable and expenditure-dependent. Because prior commitment to that functional form is essential to identification, the model employed should fit prior intuitions. In our view, GAESE fits those intuitions better than the isoelastic model does.

Functional restrictions on equivalent-expenditure functions imply restrictions on the the relationships among the demand behaviours of different household types. Because GAESE generalizes both ESE or AESE, the restrictions it implies are weaker than those implied by either condition. In particular, ESE implies that the demand for any good not consumed by

[5] The function $f\colon \mathcal{R}_+ \to \mathcal{R}$ is affine if and only if there exist $a, b \in \mathcal{R}$ such that $f(x) = ax + b$ and the function $g\colon \mathcal{R}_{++} \to \mathcal{R}_{++}$ is log-affine if and only if there exist $c, d \in \mathcal{R}$ such that $\ln g(x) = c \ln x + d$.

the reference household type has an expenditure elasticity of one, and AESE implies that the elasticity is zero. In contrast, the demand for such goods may be affine in expenditure given GAESE, which allows for expenditure elasticities that are neither zero nor one, and which vary with expenditure. Many goods fall into this class, including children's goods and goods consumed exclusively by the medically infirm and disabled.

Using Canadian consumer demand micro-data, we estimate the equivalent-expenditure function given GAESE. We find that equivalent expenditure has important fixed and varying components, and that neither alone is sufficient to accommodate behaviour. The estimated fixed components for multiple-member households are large and positive. This implies that the equivalence scales for these households decline with expenditure. The expenditure dependence of equivalence scales is substantial—for example, for dual parents with one child, the equivalence scale for poor households is 2.11 and for rich households only 1.98. Allowing for expenditure-dependent equivalence scales affects measured inequality. In Canada, inequality measured without allowing for expenditure dependence declined slightly between 1969 and 1999, but inequality measured allowing for expenditure dependence shows a moderate increase over the same period.

In the next two sections, we define equivalent-expenditure functions and equivalence scales formally and examine generalised equivalence-scale exactness. In section 3.1, we find the conditions under which the equivalent-expenditure function is identified from behaviour, and in section 4, we develop an empirical model. Section 5 presents results using Canadian price, expenditure and demographic data, and Section 7 concludes.

2. Equivalent-Expenditure Functions and Equivalence Scales

$\mathcal{Z}$ is a set of vectors of possible household characteristics whose elements may describe characteristics exhaustively, including the number and ages of household members, special needs and so on, or it may include only a subset of all possible characteristics. We restrict attention to the consumption of m private goods only and measure the well-being of each household member with the indirect utility function $V\colon \mathcal{R}^m_{++} \times \mathcal{R}_+ \times \mathcal{Z}$, $m \geq 2$.[6] Thus, the utility level of each member of a household with characteristics $z \in \mathcal{Z}$ facing prices $p \in \mathcal{R}^m_{++}$ with expenditure $x \in \mathcal{R}_+$ is $u = V(p, x, z)$. We assume that V is continuous, increasing in x and homogeneous of degree zero in (p, x) for each $z \in \mathcal{Z}$. The expenditure function corresponding to V is E where $E(u, p, z)$ is the minimum expenditure needed by a household with characteristics z facing prices p to attain utility level u for each of its members. E is continuous in (u, p) for each z, increasing in u and homogeneous of degree one in p.

[6] See Blackorby and Donaldson (1993) for a short discussion of more complex formulations.

To define equivalent-expenditure functions and equivalence scales, we use a childless single adult with characteristics z^r as the reference household. For a household with characteristics z, equivalent expenditure x^e is that expenditure which, if enjoyed by a reference household facing the same prices, would result in a utility level equal to that of each household member. Equivalent expenditure is implicitly defined by

$$V(p, x, z) = V(p, x^e, z^r) = V^r(p, x^e), \tag{2.1}$$

where $V^r = V(\cdot, \cdot, z^r)$ is the indirect utility function of the reference household. We assume (2.1) can be solved for $x^e \in \mathcal{R}_+$ for every (p, x, z) in the domain of V, so

$$x^e = X(p, x, z). \tag{2.2}$$

Regularity of V implies that X is homogeneous of degree one in (p, x), increasing in x, and $X(p, x, z^r) = x$ for all (p, x). Because $V(p, x, z) = u \Leftrightarrow E(u, p, x) = x$,

$$X(p, x, z) = E^r\big(V(p, x, z), p\big), \tag{2.3}$$

where $E^r = E(\cdot, \cdot, z^r)$ is the expenditure function of the reference household. We do not assume that V and E are globally regular. In particular, we allow V to be sensitive to prices when expenditure is zero, with a corresponding condition on E.

A relative equivalence scale S_R is equal to the ratio of expenditure to equivalent expenditure (x/x^e), and

$$S_R(p, x, z) = \frac{x}{X(p, x, z)}. \tag{2.4}$$

An absolute equivalence scale S_A is equal to the difference between expenditure and equivalent expenditure $(x - x^e)$, and

$$S_A(p, x, z) = x - X(p, x, z). \tag{2.5}$$

In general, equivalence scales depend on expenditure, but those that do not depend on expenditure are called exact. A relative scale is exact ($S_R(p, x, z) = \bar{S}_R(p, z)$) if and only if the expenditure function is multiplicatively decomposable, with $E(u, p, z) = \bar{S}_R(p, z)E^r(u, p)$ (Blackorby and Donaldson (1991, 1993), Lewbel (1991)), a condition we call equivalence-scale exactness (ESE). An absolute equivalence scale is exact ($S_A(p, x, z) = \bar{S}_A(p, z)$) if and only if the expenditure function is additively decomposable, with $E(u, p, z) = \bar{S}_A(p, z) + E^r(u, p)$ (Blackorby and Donaldson (1994)). We call this condition absolute equivalence-scale exactness (AESE).

Without additional assumptions, neither equivalent-expenditure functions nor equivalence scales are identified by household demand behaviour alone.[7] Blackorby and Donaldson (1993, 1994) have investigated theoretical identification when ESE or AESE is accepted as

[7] See Donaldson and Pendakur (2004) for a discussion.

a maintained hypothesis. They find that, given ESE and a technical condition, estimation from demand behaviour is possible if and only if the reference expenditure function is *not* log-affine, that is, if it does not satisfy

$$\ln E^r(u,p) = C(p)f(u) + \ln D(p). \tag{2.6}$$

In contrast, if AESE and a similar technical condition are maintained hypotheses, estimation from demand behaviour is possible if and only if the reference expenditure is *not* affine, that is, if it does not satisfy

$$E^r(u,p) = B(p)g(u) + F(p). \tag{2.7}$$

3. Generalized Absolute Equivalence-Scale Exactness

Equivalence-scale exactness and absolute equivalence-scale exactness share a property that we use to define a class of equivalent-expenditure functions. Suppose a household's expenditure is increased by one unit. The equivalent-expenditure function allows us to calculate the change in the reference household's expenditure that preserves equality of well-being, and it is $\partial X(p,x,z)/\partial x$. In general, this value depends on all three independent variables but, given ESE or AESE, it is independent of expenditure. Thus, we assume that there is a function $\rho\colon \mathcal{R}^m_{++} \times \mathcal{Z}$ such that, for all (p,x,z),

$$\frac{\partial X(p,x,z)}{\partial x} = \rho(p,z). \tag{3.1}$$

This condition implies

$$x^e = X(p,x,z) = \rho(p,z)x + \alpha(p,z), \tag{3.2}$$

for some function α. Because X is increasing in x and homogeneous of degree one in (p,x), $\rho(p,z) > 0$ for all (p,z), ρ is homogeneous of degree zero and α is homogeneous of degree one in p. In addition, because $X(p,x,z^r) = x$ for all (p,x), $\rho(p,z^r) = 1$ and $\alpha(p,z^r) = 0$ for all p. If $\alpha(p,z) = 0$ for all (p,z), ESE is satisfied and, if $\rho(p,z) = 1$ for all (p,z), AESE is satisfied.

Defining $R(p,z) = 1/\rho(p,z)$ and $A(p,z) = -\alpha(p,z)/\rho(p,z)$, (3.2) can be rewritten as

$$x^e = X(p,x,z) = \frac{x - A(p,z)}{R(p,z)}. \tag{3.3}$$

Because $V(p,x,z) = V^r(p,x)$, the indirect utility function can be written as

$$V(p,x,z) = V^r\left(p, \frac{x - A(p,z)}{R(p,z)}\right) \tag{3.4}$$

and the expenditure function E can be written as

$$E(u,p,z) = R(p,z)E^r(u,p) + A(p,z). \tag{3.5}$$

Because $\rho(p,z^r) = 1$ and $\alpha(p,z^r) = 0$, $R(p,z^r) = 1$ and $A(p,z^r) = 0$. We call the condition expressed in (3.3)–(3.5) generalized absolute equivalence-scale exactness (GAESE). (3.5) implies that the ratio of the marginal cost of utility for a household with characteristics z to the marginal cost of utility for a reference household is independent of expenditure.

The costs of characteristics is the cost of maintaining a household with characteristics z at a particular utility level less the cost of maintaining a reference household at the same utility level, and is given by $E(u,p,z) - E^r(u,p)$. Given GAESE, this becomes

$$E(u,p,z) - E^r(u,p) = \big(R(p,z) - 1\big)E^r(u,p) + A(p,z). \tag{3.6}$$

The last term is a fixed cost which is the same at all utility levels. If ESE holds, it is zero and, if AESE holds, it is the only cost of characteristics. GAESE allows this fixed cost to be positive or negative.

Given GAESE, the equivalence scale is

$$S_R(p,x,z) = \frac{R(p,z)x}{x - A(p,z)} \tag{3.7}$$

and the absolute equivalence scale is

$$S_A(p,z) = \frac{\big(R(p,z) - 1\big)x + A(p,z)}{R(p,z)}. \tag{3.8}$$

ESE holds, therefore, if and only if $A(p,z) = 0$ and AESE holds if and only if $R(p,z) = 1$. S_R is increasing (decreasing) in x if and only if $A(p,z) < 0$ $(A(p,z) > 0)$, and S_A is increasing (decreasing) in x if and only if $R(p,z) > 1$ $(R(p,z) < 1)$.

If $V(\cdot,\cdot,z)$ is differentiable for all z and $\partial V(p,x,z)/\partial x > 0$ for all (p,x,z), the ordinary commodity demand function $D_j(\cdot,\cdot,z)$ is related to the reference demand function $D_j^r := D_j(\cdot,\cdot,z^r)$ by

$$D_j(p,x,z) = R(p,z)D_j^r\left(p, \frac{x - A(p,z)}{R(p,z)}\right) + \frac{\partial R(p,z)}{\partial p_j}\left(\frac{x - A(p,z)}{R(p,z)}\right) + \frac{\partial A(p,z)}{\partial p_j}, \tag{3.9}$$

$j = 1,\dots,m$.

If good c is a good not consumed by the reference household such as a children's good, (3.9) implies

$$D_c(p,x,z) = \frac{\partial R(p,z)}{\partial p_c}\left(\frac{x - A(p,z)}{R(p,z)}\right) + \frac{\partial A(p,z)}{\partial p_c} \tag{3.10}$$

which is affine in expenditure. If ESE holds, $A(p,z) = 0$ and the second term of (3.10) vanishes with the implication that the elasticity of demand is one. On the other hand, if

AESE holds, the first term of (3.10) vanishes and the elasticity is zero. GAESE generalizes both AESE and ESE; the expenditure elasticities are not forced to be zero or one.[8] Because there is no good reason to believe the demand for goods such as those consumed exclusively by children or the handicapped is zero- or unit-elastic, such a generalization may be important.

3.1. Theoretical Identification

The behaviour implied by the indirect utility function V or the expenditure function E has no implications, by itself, for equivalent-expenditure functions. Without additional restriction, these functions are not identified. In this section, we investigate the relationship of behaviour and equivalent-expenditure functions when GAESE is satisfied and show that the results of Blackorby and Donaldson (1991, 1993, 1994) concerning ESE and AESE can be generalized. Assuming that there are at least two characteristics, such as age, that are continuous variables and that V is a continuous function of them, we show that the equivalent-expenditure function and equivalence scales are uniquely identified by behaviour if GAESE holds and the reference expenditure function is neither affine (quasi-homothetic preferences) nor log-affine (almost ideal preferences).

The range condition used for the theorem in this section are global: it applies to all $p \in \mathcal{R}^m_{++}$. It need not hold globally, however. The theorem needs only two price vectors at which the range condition holds and, if the condition holds for one price vector, continuity ensures that it holds in a neighborhood. Consequently, the theorem can be applied locally.

Theorem 1 and the remarks that follow show that, if an expenditure function satisfies GAESE and the reference expenditure function is neither affine nor log-affine, then any other expenditure function that represents the same preferences necessarily does not satisfy GAESE. Consequently, if the reference expenditure function is neither affine nor log-affine and GAESE is assumed as a maintained hypothesis, the equivalent-expenditure function and equivalence scales are uniquely determined by behaviour.

Two indirect utility functions $\hat{V}$ and $\tilde{V}$ represent the same behaviour if and only if there exists a function ϕ, increasing in its first argument, such that $\hat{V}(p,x,z) = \phi(\tilde{V}(p,x,z),z)$ for all (p,x,z). Equivalently, the expenditure functions $\hat{E}$ and $\tilde{E}$ represent the same behaviour if and only if there exists a function ψ, increasing in its first argument, such that, for all (u,p,z), $\widehat{E}(u,p,z) = \tilde{E}(\psi(u,z),p,z)$. Because $\hat{V}(\cdot,\cdot,z)$, $\tilde{V}(\cdot,\cdot,z)$, $\widehat{E}(\cdot,\cdot,z)$ and $\widetilde{E}(\cdot,\cdot,z)$ are continuous by assumption, $\phi(\cdot,z)$ and $\psi(\cdot,z)$ are continuous for all $z \in \mathcal{Z}$.

If the functions $\hat{E}$ and $\tilde{E}$ satisfy GAESE, then $\hat{E}(u,p,z) = \hat{R}(p,z)\hat{E}^r(u,p) + \hat{A}(p,z)$ and $\tilde{E}(u,p,z) = \tilde{R}(p,z)\tilde{E}^r(u,p,z) + \tilde{A}(p,z)$. Consequently,

$$\hat{R}(p,z)\hat{E}^r(u,p) + \hat{A}(p,z) = \tilde{R}(p,z)\tilde{E}^r(\psi(u,z),p) + \tilde{A}(p,z). \tag{3.11}$$

[8] See Donaldson and Pendakur (2005) for a correction to Donaldson and Pendakur (2004).

If $\tilde{E}^r$ and $\hat{E}^r$ represent the same (reference) preferences, there exists a continuous and increasing function λ such that $\tilde{E}^r(u,p) = \hat{E}^r(\lambda(u),p)$.[9] Defining $\sigma = \lambda \circ \psi$, it follows that, if two indirect utility functions $\tilde{V}$ and $\hat{V}$ represent the same preferences,

$$\hat{R}(p,z)\hat{E}^r(u,p) + \hat{A}(p,z) = \tilde{R}(p,z)\hat{E}^r(\sigma(u,z),p) + \tilde{A}(p,z) \tag{3.12}$$

for all (u,p,z). Rearranging terms,

$$\hat{E}^r\big(\sigma(u,z),p\big) = \frac{\hat{R}(p,z)}{\tilde{R}(p,z)}\hat{E}^r(u,p) + \frac{\hat{A}(p,z) - \tilde{A}(p,z)}{\tilde{R}(p,z)}. \tag{3.13}$$

Defining $J(p,z) = \hat{R}(p,z)/\tilde{R}(p,z)$ and $T(p,z) = (\hat{A}(p,z) - \tilde{A}(p,z))/\tilde{R}(p,z)$, (3.13) becomes

$$\hat{E}^r(\sigma(u,z),p) = J(p,z)\hat{E}^r(u,p) + T(p,z). \tag{3.14}$$

We employ a range condition for Theorem 1. We assume that $\mathcal{Z}$ contains at least two continuous variables such as age and, assuming that $\hat{R}$ is different from $\tilde{R}$ and $\hat{A}$ is different from $\tilde{A}$, $J(p,z)$ can be moved independently of $T(p,z)$.

Range Condition: *$\mathcal{Z}$ contains a subset $\mathring{\mathcal{Z}}$ of at least two continuous variables such as age and, for every $p \in \mathcal{R}^m_{++}$, the functions $\hat{R}$, $\tilde{R}$, $\hat{A}$ and $\tilde{A}$ are continuous in and sensitive to $\mathring{z} \in \mathring{\mathcal{Z}}$. In addition, $J(p,z)$ can be moved independently of $T(p,z)$ by changing $\mathring{z}$.*

Note that $J(p,z^r) = 1$ and $T(p,z^r) = 0$. If $\hat{R} \neq \tilde{R}$, then $J(p,z) \neq 1$ for some z. The range condition implies that $J(p,z)$ can be moved through an interval by changing z. A similar consideration applies to $T(p,z)$.

Theorem 1 applies to the case in which $\hat{R} \neq \tilde{R}$ and $\hat{A} \neq \tilde{A}$ and it proves that that the equivalent-expenditure function and equivalence scales are uniquely identified by behaviour if GAESE holds and the reference expenditure function is not affine (quasi-homothetic preferences).

Theorem 1: *Two indirect utility functions $\hat{V}$ and $\tilde{V}$ represent the same preferences and satisfy GAESE and the range condition with $\hat{R} \neq \tilde{R}$ and $\hat{A} \neq \tilde{A}$ if and only if there exist functions $B\colon \mathcal{R}^m_{++} \longrightarrow \mathcal{R}_{++}$, $F\colon \mathcal{R}^m_{++} \longrightarrow \mathcal{R}_{++}$, $\hat{g}\colon \mathcal{R} \longrightarrow \mathcal{R}$, $\tilde{g}\colon \mathcal{R} \longrightarrow \mathcal{R}$, $j\colon \mathcal{Z} \longrightarrow \mathcal{R}_{++}$ and $t\colon \mathcal{Z} \longrightarrow \mathcal{R}$ such that, for all (u,p,z),*

$$\hat{E}^r(u,p) = B(p)\hat{g}(u) + F(p), \tag{3.15}$$

$$\hat{E}^r(u,p) = B(p)\tilde{g}(u) + F(p), \tag{3.16}$$

$$\hat{R}(p,z) = j(z)\tilde{R}(p,z) \tag{3.17}$$

[9] λ is the inverse of $\psi(\cdot, z^r)$.

and

$$\hat{A}(p,z) = \tilde{A}(p,z) + \tilde{R}(p,z)\Big[B(p)t(z) + F(p)\big(1 - j(z)\big)\Big], \tag{3.18}$$

where B *and* F *are homogeneous of degree one,* $\hat{g}$ *and* $\tilde{g}$ *are increasing and continuous,* $j(z^r) = 1$ *and* $t(z^r) = 0$*.*

Proof. Equation (3.14) is analogous to equation (4.7) in Donaldson and Pendakur (2004) with $\ln \hat{E}^r(\sigma(u,z),p)$ and $\ln \hat{E}^r(u,p)$ replaced by $\hat{E}^r(\sigma(u,p),p)$ and $\hat{E}^r(u,p)$. The result, including sufficiency, follows from the proof of Theorem 2. ■

Theorem 1 applies to one of three possible cases. In the other two, either $\hat{R} = \tilde{R}$ or $\hat{A} = \tilde{A}$. In the first of these, (3.14) reduces to the functional equation considered by Blackorby and Donaldson (1994) in their investigation of AESE. In that case, the equivalence scale and equivalent-expenditure function are identified from behaviour if the reference expenditure function is not affine ((3.15) is not satisfied), which is the condition given by Theorem 1 above.

A different possibility arises in the case in which $\hat{R} \neq \tilde{R}$ and $\hat{A} = \tilde{A}$. The second term disappears from equation (3.14) and the functional equation investigated by Blackorby and Donaldson (1993, Theorem 5.1) results. This case resembles ESE, and the condition on reference preferences is the same: the reference expenditure function must not be log-affine. That is, it must not be true that, for all (u,p,z),

$$\ln E^r(u,p) = \ln D(p) + C(p) \ln g(u). \tag{3.19}$$

It follows that, in order to be able to identify equivalent-expenditure functions and equivalence scales from behaviour alone, given GAESE, the reference expenditure function must be neither affine nor log-affine. If this condition is met, there is a unique equivalent-expenditure function.

4. Empirical Model

Given GAESE, the equivalent-expenditure function can be estimated using a demand system. Parametric estimation of equivalent-expenditure functions requires specification of the demand system and parametric expressions of the restrictions embodying GAESE. The econometric strategy we employ exploits the following convenient characteristic of GAESE.

Recent work in the parametric estimation of consumer demand systems has focussed on expenditure share equations which are quadratic in the natural logarithm of expenditure rather than on demands which are quadratic in expenditure (see Banks, Blundell and Lewbel (1997)). The quadratic almost ideal (QAI) demand system has an indirect utility function which is typically specified as $V(p,x,z) = f(p, x/a(p,z))$. If GAESE holds, $V(p,x,z) = \bar{f}(p,(x - d(p,z))/a(p,z))$.

Recently, Lewbel (2003) has proposed a translated QAI demand system. Here, the QAI is transformed so that indirect utility is a function of an affine function of expenditure for each household type which accommodates GAESE. Given GAESE, if any household has translated QAI preferences, then all households have translated QAI preferences. Thus, we use the translated QAI demand system under the maintained assumption of GAESE to identify the equivalent-expenditure function. In this context, testing the observable implications of GAESE is relatively easy: GAESE implies that the functions d and a capture the whole of the effect of z on demand. In addition, ESE implies $d(p,z) = 0$ and AESE implies $a(p,z) = 1$.

4.1. The Data

We use expenditure data from the 1969, 1974, 1978, 1982, 1984, 1986, 1990, 1992 and 1996 Canadian Family Expenditure Surveys and the 1997, 1998 and 1999 Surveys of Household Spending (Statistics Canada) to estimate demand systems given GESE and recover equivalent- expenditure functions. These data contain annual expenditures in approximately one hundred categories for five to fifteen thousand households per year. We use only: (1) households in cities with 30,000 or more residents (to match commodity price data and to minimize the effects of home production); (2) households with rental tenure (to avoid rent imputation); (3) households whose members are all full-year members under the age of sixty-five; and (4) households whose heads are aged twenty-five to sixty-four.

We estimate a demand system composed of the following nine expenditure categories: (1) food purchased from stores; (2) food from restaurants; (3) total rent, including utilities; (4) household operation (including child care); (5) household furnishing and equipment; (6) clothing; (7) private transportation operation (does not include capital expenditures); (8) public transportation; and (9) personal care. Personal care is the left-out equation in all estimation. Price data for all these commodity groups except rent are available from Browning and Thomas (1999) for 1969 to 1996 in five regions of Canada: (1) Atlantic Canada; (2) Quebec; (3) Ontario; (4) the Prairies; and (5) British Columbia. Prices for 1997 to 1999 are taken from Pendakur's (2001) update of these price series. Rent prices are from Pendakur (2001, 2002). Prices are normalized so that residents of Ontario in 1986, who form the largest population subgroup in the sample, face the prices $(100, ..., 100)$. The data are multi-stage stratified sampled and come with inverse-probability weights which are used in the estimation. Since cluster and strata information are not publically available, such information is not used in the estimation, and estimated parameters are presented with hetero-robust standard errors.

We use several household demographic characteristics in our estimation, including household size, age of household head and the presence of children. Summary statistics on the data are given in Table 1.

Table 1: The Data

No. of households: 19276	**mean**	**std dev**	**min**	**max**
Total Expenditure	14283	8658	799	110555
Food From Stores	0.217	0.103	0	0.809
Restaurant Food	0.061	0.063	0	0.643
Rent	0.33433	0.127	0.001	0.949
Household Operation	0.082	0.049	0	0.636
Household Furnishing and Equipment	0.049	0.057	0	0.646
Clothing	0.101	0.063	0	0.585
Private Transportation Operation	0.093	0.083	0	0.591
Public Transportation	0.030	0.039	0	0.452
Personal Care	0.034	0.021	0	0.219
Childless Single Adults	0.361	0.480	0	1
Childless Adult Couples	0.216	0.412	0	1
Log Household Size (Children not Present)	0.213	0.368	0	1.946
Log Household Size (Children Present)	0.464	0.636	0	2.398
Single Parent Indicator	0.055	0.228	0	1
Age of Head Less 40 (Children not Present)	1.111	9.731	−15	24
Age of Head Less 40 (Children Present)	−1.417	5.047	−15	24
Food From Stores Price	4.426	0.539	3.193	4.942
Restaurant Food Price	4.499	0.602	3.147	5.139
Rent Price	4.356	0.498	3.289	4.971
Household Operation Price	4.424	0.54006	3.208	4.924
Household Furnishing and Equipment Price	4.470	0.378	3.666	4.803
Clothing Price	4.599	0.38649	3.737	5.032
Private Transportation Operation Price	4.371	0.595	3.072	5.133
Public Transportation Price	4.452	0.71128	3.025	5.292
Personal Care Price	4.479	0.463	3.491	4.942

4.2. Parametric Demand-System Specifications

For estimation given GAESE, we use the translated quadratic almost ideal (QAI) demand system described in Lewbel (2003) in which

$$V(p,x,z) = \left(\left(\frac{ln \frac{x-d(p,z)}{a(p,z)}}{b(p,z)} \right)^{-1} - q(p,z) \right)^{-1}, \tag{4.1}$$

where d and a are homogeneous of degree one in p and b and q are homogeneous of degree zero in p. Increasingness of V in x implies $a(p,z) > 0$ for all (p,z).

The translated QAI is a rank 4 demand system which is almost polynomial in log-expenditure (see Lewbel 2003). As expenditure gets large relative to d, the translated QAI has expenditure share equations that approach a function that is quadratic in the natural logarithm of expenditure. However, as expenditure gets small relative to d, expenditure share equations asymptote to plus or minus infinity, and the equations are undefined for expenditure less than d.

Assuming that the reference indirect utility function is translated QAI, GAESE implies that all households have translated QAI preferences and

$$d(p,z) = R(p,z)d^r(p) + A(p,z), \tag{4.2}$$

$$a(p,z) = R(p,z)a^r(p), \tag{4.3}$$

$$b(p,z) = b^r(p), \tag{4.4}$$

and

$$q(p,z) = q^r(p). \tag{4.5}$$

Thus, we can estimate equivalent-expenditure functions given GAESE by restricting $b(p,z) = b^r(p)$, $q(p,z) = q^r(p)$ and calculating

$$R(p,z) = \frac{a(p,z)}{a^r(p)} \tag{4.6}$$

and

$$A(p,z) = d(p,z) - R(p,z)d^r(p). \tag{4.7}$$

With the these specifications for d, a, b and q, A can be either positive or negative but R is positive.

In addition to estimating equivalent-expenditure functions under GAESE, use of the translated QAI allows a simple parametric test of GAESE against an unrestricted translated QAI alternative. In particular, if preferences do not satisfy $q(p,z) = q^r(p)$ and $b(p,z) = b^r(p)$, then GAESE cannot hold. It is also possible to test down from GAESE in this framework. We can test AESE against GAESE by asking whether $R(p,z) = 1$ or, equivalently, $a(p,z) =$

$a^r(p)$. Further, we can test ESE against GAESE by asking whether $A(p,z) = 0$, which is true only if $d(p,z) = 0$.

To estimate the demand systems, we specify the functions d, a, b and q as follows:

$$d(p,z) = \sum_{k=1}^{m} d_k(z) p_k, \tag{4.8}$$

$$\ln a(p,z) = a_0(z) + \sum_{k=1}^{m} a_k(z) \ln p_k + \frac{1}{2} \sum_{k=1}^{m} \sum_{l=1}^{m} a_{kl} \ln p_k \ln p_l, \tag{4.9}$$

where $\sum_{k=1}^{m} a_k(z) = 1$, $\sum_{l=1}^{m} a_{kl} = 0$ for all k, and $a_{kl} = a_{lk}$ for all k, l;

$$\ln b(p,z) = \sum_{k=1}^{m} b_k(z) \ln p_k, \tag{4.10}$$

where $\sum_{k=1}^{m} b_k(z) = 1$; and

$$q(p,z) = \sum_{k=1}^{m} q_k(z) \ln p_k, \tag{4.11}$$

where $\sum_{k=1}^{m} q_k(z) = 0$.

It is convenient to denote

$$d_0(z) = \sum_{k=1}^{m} d_k(z). \tag{4.12}$$

The functions d_k, a_k, b_k and q_k depend on z, and we assume that

$$\begin{aligned} d_k(z) = d_k^r &+ d_k^{couple} couple + d_k^{nlhsize} nlhsize + d_k^{clhsize} clhsize \\ &+ d_k^{nage} nheadage + d_k^{cage} cheadage + d_k^{single} singlepar, \end{aligned} \tag{4.13}$$

$$\begin{aligned} a_k(z) = a_k^r &+ a_k^{couple} couple + a_k^{nlhsize} nlhsize + a_k^{clhsize} clhsize \\ &+ a_k^{nage} nheadage + a_k^{cage} cheadage + a_k^{single} singlepar, \end{aligned} \tag{4.14}$$

$$\begin{aligned} b_k(z) = b_k^r &+ b_k^{couple} couple + b_k^{nlhsize} nlhsize + b_k^{clhsize} clhsize \\ &+ b_k^{nage} nheadage + b_k^{cage} cheadage + b_k^{single} singlepar, \end{aligned} \tag{4.15}$$

and

$$\begin{aligned} q_k(z) = q_k^r &+ q_k^{couple} couple + q_k^{nlhsize} nlhsize + q_k^{clhsize} clhsize \\ &+ q_k^{nage} nheadage + q_k^{cage} cheadage + q_k^{single} singlepar, \end{aligned} \tag{4.16}$$

where *couple* indicates a childless married couple, *nlhsize* is the natural logarithm of household size for households with no children, *clhsize* is the natural logarithm of household size for households with children, *singlepar* is a dummy indicating that the household has children present but only has one adult aged 18 or greater, *nheadage* is the age of the household head less 40 for households with no children, and *cheadage* is the age of the household head less 40 for households with children. Children are defined as persons less than 18 years of age. For households consisting of a single childless adult aged forty (the reference household type), $a_k(z) = a_k^r$, $b_k(z) = b_k^r$ and $q_k(z) = q_k^r$.

We note that, with the translated QAI specification, the GAESE functions A and R take on relatively simple forms in terms of the parameters if evaluated at an m-vector of equal prices $\hat{p}\mathbf{1_m} = (\hat{p}, ..., \hat{p})$, with

$$R(\hat{p}\mathbf{1}_m, z) = \exp(a_0(z) - a_0(z^r)) \tag{4.17}$$

and

$$A(\hat{p}\mathbf{1}_m, z) = d_0(z) - \exp(a_0(z) - a_0(z^r))d_0(z^r). \tag{4.18}$$

Defining, $w_j(p, x, z)$ as the function giving the share of expenditure commanded by the jth commodity, application of Roy's Theorem in its logarithmic form to (4.1) generates the expenditure-share equations

$$w_j(p, x, z) = \frac{x - d(p, z)}{x} \left(a_j(z) + \sum_{k=1}^{m} a_{jk} \ln p_k + b_j(z)\hat{x} + \frac{q_j(z)}{b(p, z)}\hat{x}^2 \right) + \frac{p_j d_j(z)}{x} \tag{4.19}$$

where

$$\hat{x} = \ln \frac{x - d(p, z)}{a(p, z)} \tag{4.20}$$

$j = 1, \ldots, m$. Equation (4.19) does not necessarily satisfy GAESE, which requires $b_k(z) = b_k^r$ and $q_k(z) = q_k^r$.

The translated QAI demand system corresponds to an affine expenditure function if and only if $b(p, z)$ and $q(p, z)$ are both zero, and corresponds to a log-affine expenditure function if and only if $d(p, z)$ and $q(p, z)$ are both zero. Since equivalent-expenditure functions given GAESE are uniquely identifiable if the expenditure function is neither affine nor log-affine, identification is possible given GAESE if either q is nonzero or if both d and b are nonzero.

5. Results

Selected coefficients are presented in the tables; complete and detailed parameter estimates are available from the authors on request. The reference household type for all estimation is a childless single adult aged 40. Because the parameter a_0^r is hard to identify, and because it does not directly affect equivalence scale parameters (which are all expressed as differences from this parameter), we set rather than estimate a_0^r at the average log total expenditure of single-member (reference) households in the base price regime, so that $a_0^r = 4.717$.

Table 2 presents model statistics for seven models of translated QAI that are: (1) unrestricted; (2) GAESE-restricted; (3) ESE-restricted; (4) AESE-restricted; (5) ESE-restricted log-affine; (6) GAESE-restricted affine; and (7) AESE- restricted affine.

Table 2: Model Statistics

Model	**Restriction**	**df**	**Log-Likelihood**
Affine	AESE	107	248583
Affine	GAESE	155	250377
Log-affine	ESE/GAESE	100	250520
QAI	ESE	114	251081
Translated QAI	AESE	123	250100
Translated QAI	GAESE	177	251576
Translated QAI	Unrestricted	273	252019

Table 2 can be summarized simply: every testable hypothesis of interest is rejected at conventional levels of significance. We reject GAESE against an unrestricted translated QAI alternative. Given an unrestricted translated QAI model, GAESE requires that $q(p,z) = q(p)$ and that $b(p,z) = b(p)$. The likelihood ratio test statistic for this hypothesis is 938 and is distributed as a χ^2_{96} which has a one-sided 1 percent critical value of 67. However, given the maintained assumption of GAESE, we reject the hypothesis that the GAESE equivalent-expenditure function is not identified. Non-identification requires that the demand system is either affine or log-affine against a GAESE-restricted translated QAI alternative, which requires that either $\ln b(p,z) = q(p,z) = 0$ or $d(p,z) = q(p,z) = 0$. The likelihood ratio test statistics for these hypotheses are 2398 and 2112, respectively, and are distributed as a χ^2_{22} and a χ^2_{77}, respectively, so that non-identification is decisively rejected. These two results do not combine well. They say that the observable restrictions imposed by GAESE are not satisfied by behaviour, but if we assume that the GAESE restrictions are true, then behaviour is sufficiently nonlinear to allow estimation of the GAESE equivalent-expenditure function.

We also test down from GAESE to AESE and ESE to ask whether both the fixed and marginal cost components of the equivalent-expenditure function are important, or whether just one of these components adequately summarizes equivalent-expenditure. The hypothesis that ESE is true against a GAESE alternative requires that $d(p,z) = 0$, which removes the translation parameters from the translated QAI and turns it into a QAI model. The likelihood ratio test statistic for this hypothesis 990 and is distributed as a χ^2_{63} with a one-sided 1 percent critical value of 40. The hypothesis that AESE is true against a GAESE alternative requires that $a(p, z) = a(p)$. The likelihood ratio test statistic for this hypothesis is 2952 which is distributed as a χ^2_{54} with a one-sided 1 percent critical value of 33. These two results suggest that GAESE fits the data much better than do either ESE or AESE. This implies that, given GAESE, the expenditure-dependence of the equivalent-expenditure function is statistically significant. It remains to be shown that it is economically important.

5.1. Equivalent-Expenditure Functions Given GAESE

Table 3 shows selected parameter estimates from GAESE-restricted translated QAI demand systems. These parameters are sufficient to estimate equivalent-expenditure functions and equivalence scales at a vector of equal prices. We consider the vector of equal prices $p = 100(\mathbf{1_9}) = (100, 100, 100, 100, 100, 100, 100)$ which is the price vector for Ontario in 1986. Table 4 gives the values of the functions A and R which determine the equivalent-expenditure function for several household types.

Table 3: Estimated Parameters Given Translated QAI and GAESE

	ESE			**GAESE**		
Parameter	**Est.**	**Std. Err.**	**P-val.**	**Est.**	**Std. Err.**	**P-val.**
a_0^{couple}	-0.120	0.041	0.003	-0.177	0.083	0.034
$a_0^{nlfsize}$	0.596	0.047	< 0.001	0.452	0.085	<0.001
$a_0^{clfsize}$	0.238	0.026	< 0.001	0.602	0.035	<0.001
a_0^{single}	-0.428	0.048	< 0.001	0.176	0.066	0.007
a_0^{nage}	0.004	0.001	0.007	0.005	0.002	0.045
a_0^{cage}	0.034	0.002	< 0.001	0.005	0.004	0.212
d_0^{r}				8.346	2.87	0.004
d_0^{couple}				4.103	5.21	0.431
$d_0^{nlfsize}$				8.02	6.39	0.210
$d_0^{clfsize}$				9.74	2.273	<0.001
d_0^{single}				-0.046	3.81	0.990
d_0^{nage}				0.275	0.107	0.010
d_0^{cage}				0.946	0.194	<0.001

Table 4: Slopes and Intercepts of Equiv.-Expenditure Functions at Base Prices

	ESE	**GAESE**	
Household Type	$R(p, z)$ (Slope)	$R(p, z)$ (Slope)	$A(p, z)$ (Intercept)
Childless Couple	1.34	1.14	844
Three Adults	1.92	1.64	344
Single Parent, Two Children	0.85	2.31	529
Dual Parent, One Child	1.29	1.94	533
Dual Parent, Two Children	1.39	2.31	814
Dual Parent, Three Children	1.47	2.64	1031

The estimated equivalence scales given ESE do not vary with expenditure and are just equal to $R(p, z)$, the slope of the equivalent-expenditure function. The estimated equivalence scales given GAESE can be more easily understood graphically. Figure 1 shows equivalent-expenditures given GAESE for six household types: childless couples; three adults; single parent with two children; dual parents with one child; dual parents with two children; and dual parents with three children. The intercept term may be interpreted as differences in fixed costs for different household types, and the slopes may be interpreted as differences in the marginal costs for different household types.

Figure 2 shows expenditure-dependent equivalence scales given GAESE for the same six household types. These relative equivalence scales lie between one and the number of household members, satisfying our intuition that there are scale economies in household consumption. The econometric specification requires R to be positive but does not restrict A. Thus, relative equivalence scales are not restricted to any particular range. For example, for A large and negative, relative equivalence scales can be less than one or even negative. That estimated relative equivalence scales lie in the relatively narrow plausible range is encouraging.

If $A(p, z) = 0$, then equivalence scales given GAESE are independent of expenditure, and are equal to $R(p, z)$. For $A(p, z) > 0$, equivalence scales decline with expenditure, and for $A(p, z) < 0$, equivalence scales rise with expenditure. In our view, the most important feature of Figure 2 is not how equivalence scales vary with household type, but rather that these equivalence scales generally decrease with expenditure. For example, the scale for childless couples is 1.28 at an expenditure of \$8,000 (the bottom vigintile) and 1.18 at an expenditure of \$30,000 (the top vigintile). For dual parents with one child, the relative equivalence scale decreases substantially with expenditure—it is 2.11 at the bottom vigintile expenditure of \$6,000 and 1.98 at the top vigintile expenditure of \$28,000. For dual parents with three children, the expenditure-dependence is even stronger: the equivalence scale is 2.87 at the bottom vigintile expenditure of \$13,000 and 2.72 at the top vigintile expenditure of \$32,000.

That these scales decrease with expenditure suggests that scale economies in household consumption are smaller for poor households than for rich households. These results are similar to those reported by Donaldson and Pendakur (2004) and Koulovatianos et al. (2004).

5.2. *GAESE and Measured Inequality*

Table 5 shows the estimated Gini Coefficient for household consumption of urban residents in Canada over 1969 to 1999 where the equivalent-expenditure function is estimated either given ESE or given GAESE. The rightmost column shows results using the GAESE-restricted estimates presented in Table 3 and the leftmost column shows results using the ESE-restricted estimates. The middle column shows results using the GAESE estimates of R and setting A to zero for all people. This is equivalent to assigning the GAESE equivalence scale for a very rich household to all households, regardless of how rich they are. Thus, comparison of the middle and rightmost columns allows us to assess the effect of expenditure-dependent equivalence scales on measured inequality by holding the function R constant across the comparison. Figure 3 displays this information graphically.

Table 5: GAESE and Measured Inequality

Year	**ESE**	**GAESE** ($d(p,z)=0$)	**GAESE**
1969	0.230	0.208	0.204
1974	0.235	0.208	0.204
1978	0.236	0.213	0.211
1982	0.231	0.208	0.209
1984	0.231	0.215	0.216
1986	0.236	0.219	0.220
1990	0.223	0.207	0.208
1992	0.229	0.206	0.206
1996	0.228	0.205	0.207
1997	0.216	0.204	0.205
1998	0.226	0.205	0.208
1999	0.227	0.206	0.209

Looking first at the comparison between the leftmost and rightmost columns, it is clear that the relaxation of ESE to GAESE makes a substantial difference in the measured level of inequality. The Gini coefficient is about 3 percentage points lower if equivalence expenditure functions are allowed to have fixed costs. Apart from the level effect, there is also a noticeable difference in the trend over time. Measured inequality seems to be lower in the late 1990s than in the 1970s when using ESE scales, but the opposite is true when using GAESE scales.[10]

These differences in the level of and trend in measured inequality are due to two things: (1) the estimates of R are different for ESE and GAESE; and (2) the value of A is zero given ESE. To help distinguish the effect of these two elements, the middle column of Table 5 gives measured inequality where the equivalence scale is taken to equal the limiting value of the GAESE equivalence scale. That is, this column uses the estimated expenditure-dependent equivalence scale value for the very rich as an expenditure-independent equivalence scale. By comparing the results in the middle column to those in the rightmost column, we hold constant the first difference noted above, and focus on the second difference, which is the variation of the GAESE equivalence scale over expenditure.

Comparing the middle column to the rightmost column, we see that the overall level of measured inequality is not dramatically different between the GAESE estimates and the GAESE estimates with A 'frozen' at zero. Both sets are 2 to 3 percentage points lower than the ESE estimates over most of the period. This suggests that the difference in the level of measured inequality between ESE and GAESE estimates is primarily due to the differences in the values of equivalence scale, and not primarily due to differences in expenditure-dependence of equivalence scale.

In contrast, the trend in measured inequality is different between the middle and rightmost columns. In particular, the Gini Coefficient is about one-half of one percentage point higher in the early 1970s when the expenditure-dependence of the equivalence scale is frozen, but by the 1990s, their relative positions have reversed. In particular, while the middle column shows slightly lower inequality in the 1990s than in the 1970s, the rightmost column—which allows for expenditure-dependence in the equivalence scale—shows slightly higher inequality in the 1990s than in the 1970s. Thus, while expenditure-dependence of the equivalence scale does not much affect the overall level of inequality, much of the difference in the trend in measured inequality between the ESE and GAESE estimates can be explained by the expenditure-dependence of the GAESE scales.

[10] Asymptotic standard errors are not shown in the table, but have been calculated following Barrett and Pendakur (1995), and they range from 0.0012 to 0.0018 for the estimated values in Table 5. Comparing 1974 and 1978 to 1996-1999, the pairwise comparisons using the leftmost column of Table 5 (ESE) suggest that inequality in the late years is marginally or strongly significantly lower in late 1990s than in the 1970s. In contrast, pairwise comparisons using the rightmost column of Table 5 (GAESE) suggest that inequality was significantly higher in the late 1990s than in 1974, and not significantly lower in the late 1990s than in 1978.

Accounting for the expenditure-dependence of equivalence scales results in measured inequality which is more increasing over the decades. Using an expenditure-dependent equivalence scale in the calculation of Gini Coefficients shows an increase of 0.5 percentage points from 1969 to 1999, whereas ignoring expenditure dependence results in a small decrease of about 0.2 percentage points.

6. Concluding Remarks:

In practice, equivalent-expenditure functions used to index social transfers or to measure poverty, inequality or social welfare are of two types: those in which the ratio of household expenditure to equivalent expenditure is independent of expenditure, and those in which their difference is independent of expenditure. Our new class of equivalent-expenditure functions generalizes both types and, given several assumptions, permits the estimation of equivalent-expenditure functions from demand behaviour.

We find that, for large households, equivalent-expenditure functions and equivalence scales estimated from Canadian consumer micro-data exhibit large and positive fixed costs of characteristics, resulting in equivalence scales for those households that decline with expenditure. Trends in measured inequality from 1969 to 1999 are affected by the use of these scales: although inequality falls when expenditure-independent scales are used, inequality rises when expenditure-dependent scales are used.

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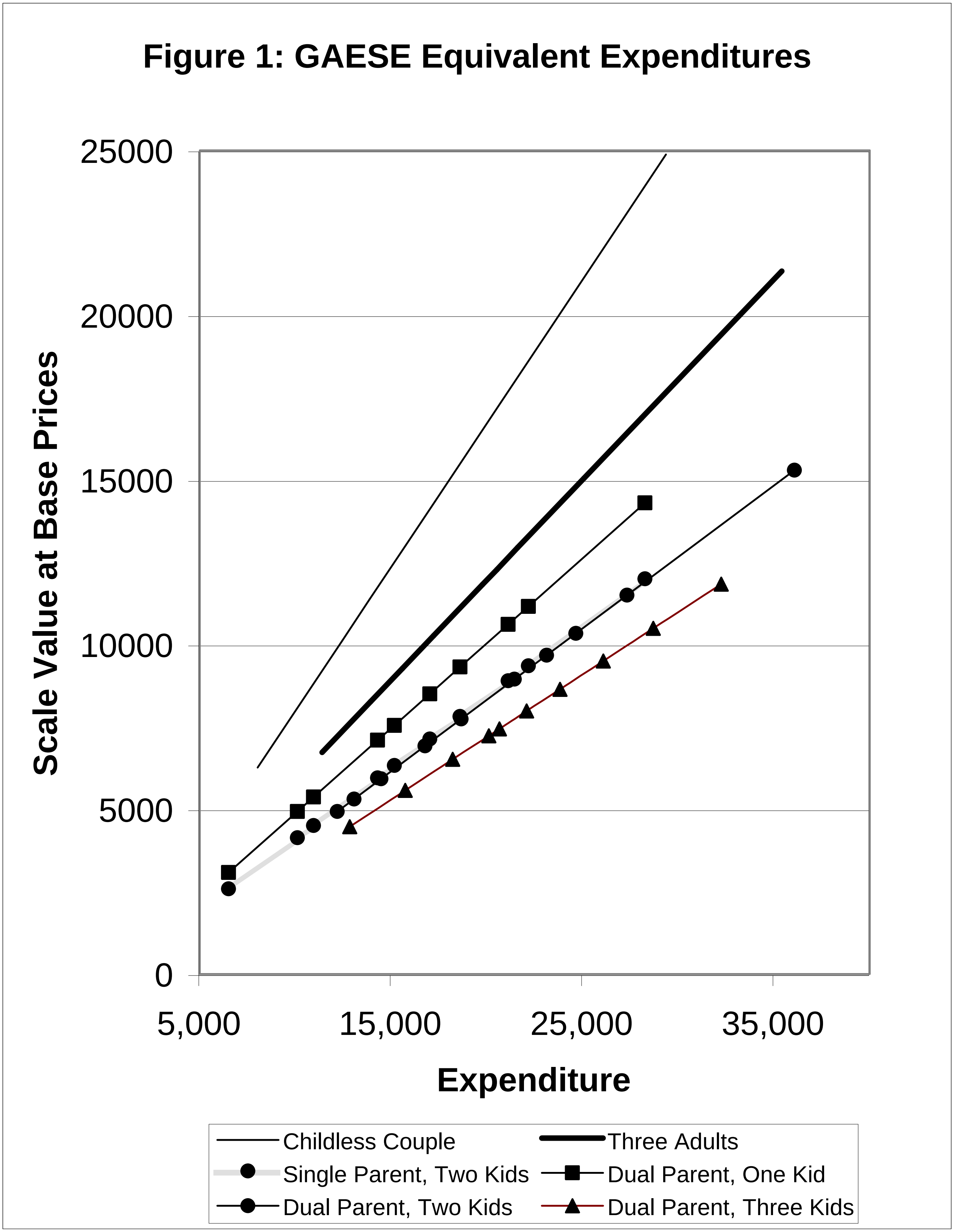
Figure 1: GAESE Equivalent Expenditures
25000
20000
15000
10000
5000
0
Scale Value at Base Prices
5,000
15,000
25,000
35,000
Expenditure
Childless Couple
Three Adults
Single Parent, Two Kids
Dual Parent, One Kid
Dual Parent, Two Kids
Dual Parent, Three Kids

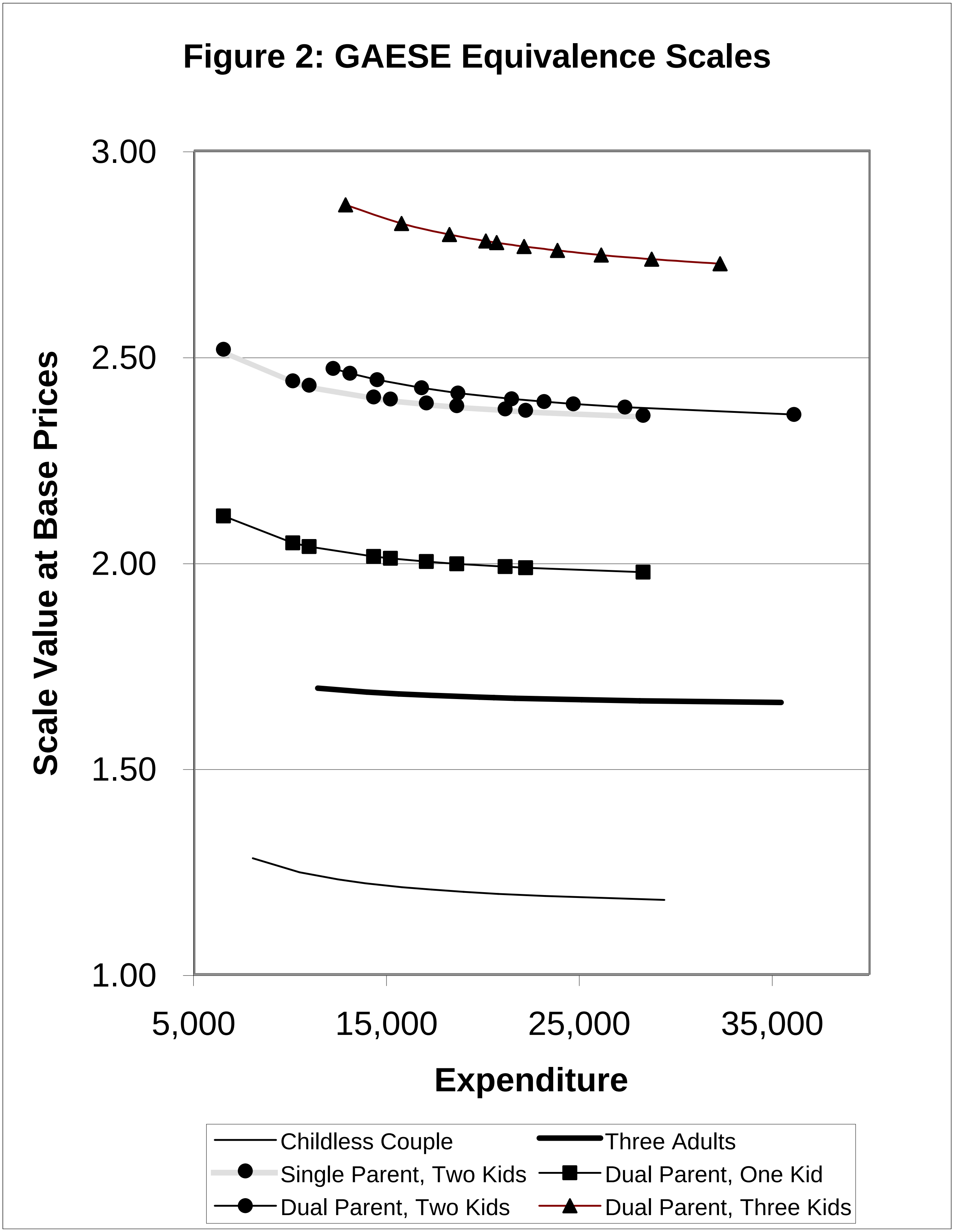
Figure 2: GAESE Equivalence Scales
Scale Value at Base Prices
3.00
2.50
2.00
1.50
1.00
5,000
15,000
25,000
35,000
Expenditure
Childless Couple
Three Adults
Single Parent, Two Kids
Dual Parent, One Kid
Dual Parent, Two Kids
Dual Parent, Three Kids

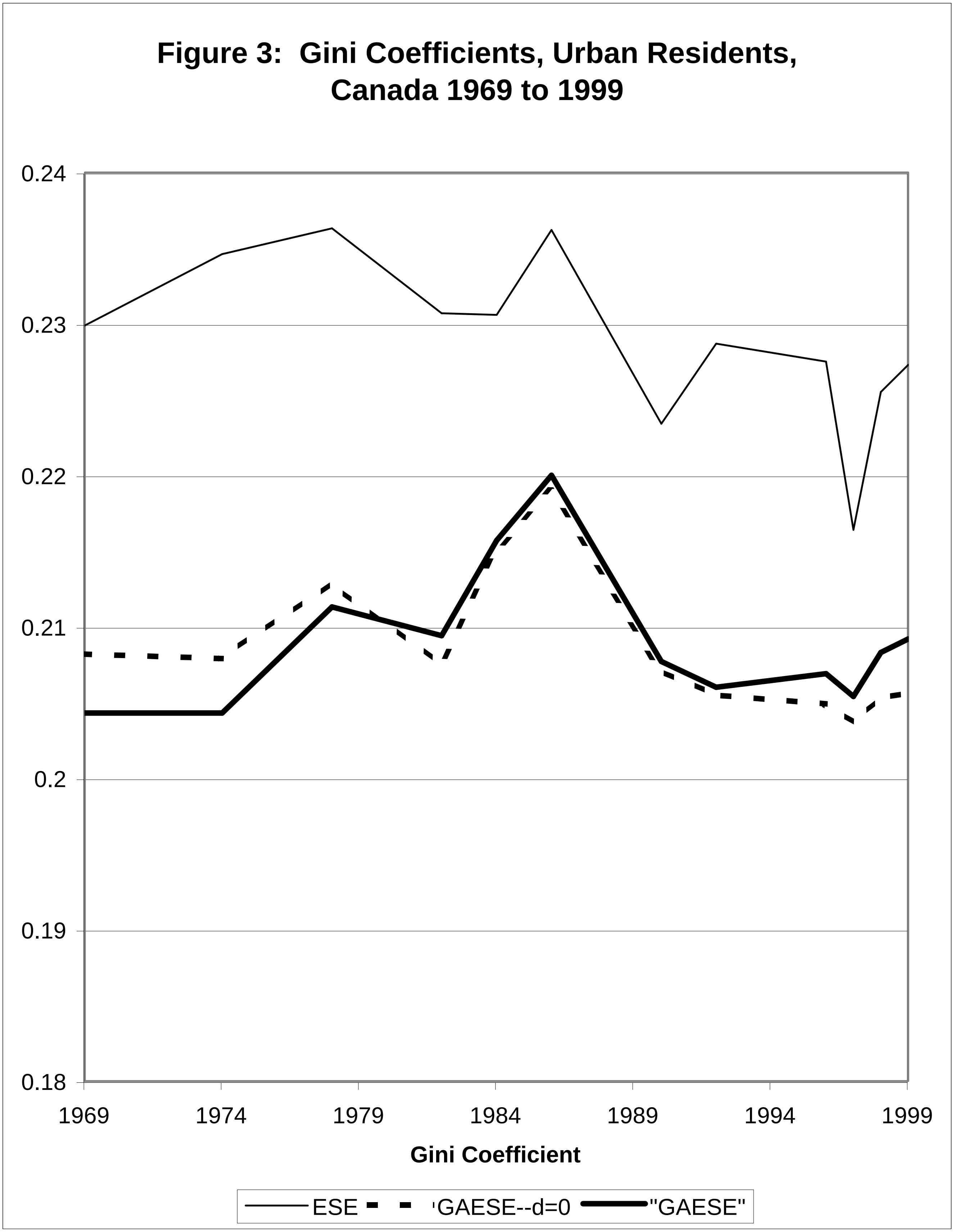
Figure 3: Gini Coefficients, Urban Residents, Canada 1969 to 1999
0.24
0.23
0.22
0.21
0.2
0.19
0.18
1969
1974
1979
1984
1989
1994
1999
Gini Coefficient
ESE
'GAESE--d=0
"GAESE"