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# The impact of carbon footprinting aggregation on realizing emission reduction targets

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## Abstract

A variety of activity-based methods exist for estimating the carbon footprint in transportation. For instance, the Greenhouse Gas protocol suggests a more aggregate estimation method than the Network for Transport and Environment (NTM) method. In this study, we implement a detailed estimation method based on NTM and different aggregate approaches for transportation carbon emissions in the Dynamic Lot Sizing Model. Analytical results show the limitations of aggregate models for both accurate estimation of real emissions and risks of compliance with carbon constraints (e.g. carbon caps). Extensive numerical experimentation shows that the magnitude of errors can be substantial. We provide insights under which limited conditions aggregate estimations can be used safely and when more detailed estimates are appropriate.

## Key words

Transport carbon footprint; dynamic lot sizing; GHG emission factors; carbon caps; supply chain carbon footprint

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A variety of activity-based methods exist for estimating the carbon footprint in transportation. For instance, the Greenhouse Gas protocol suggests a more aggregate estimation method than the Network for Transport and Environment (NTM) method. In this study, we implement a detailed estimation method based on NTM and different aggregate approaches for transportation carbon emissions in the Dynamic Lot Sizing Model. Analytical results show the limitations of aggregate models for both accurate estimation of real emissions and risks of compliance with carbon constraints (e.g. carbon caps). Extensive numerical experimentation shows that the magnitude of errors can be substantial. We provide insights under which limited conditions aggregate estimations can be used safely and when more detailed estimates are appropriate.

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## **1. Introduction.**

There is a widespread belief that anthropogenic carbon emissions contribute to global warming, based on a vast number of studies by climate scientists (IPCC, 2007). The three main contributing sectors to emissions in the developed world are electricity production, energy-intensive manufacturing, and transportation. While technology developments are expected to contribute significantly to curtailing emissions in electricity production and in energy-intensive manufacturing, prospects for the transportation sector are grimmer, and a substantial contribution is needed by more efficient operation of the world's supply chains (European Commission, 2011).

Decision makers in industry are increasingly taking the consequences of their decisions for climate change into account. Hoffman and Woody (2008) argue that regardless of whether executives believe in the results of climate change research, a market shift is occurring that drives many companies towards measuring and disclosing their carbon emissions, and implementing policies to reduce those emissions. For instance, the number of companies around the world reporting their emissions to the Carbon Disclosure Project, a not-for-profit foundation, has increased from 687 in 2007 to 1525 in 2010 (CDP, 2011). Moreover, a substantial number of companies publicly state carbon emission reduction targets. For instance, in the 2011 Carbon Disclose Project annual report, 926 companies publicly

commit to a self-imposed carbon target such as FedEx, UPS, Wal-Mart, AstraZeneca, Pepsico, Coca-Cola, Danone, Volkswagen, Campbell, and Ericsson.

In order to report emissions, and as a first step towards emissions reductions, actual emissions need to be estimated. The most common approach to measure emissions is the Greenhouse Gas Protocol (GHG, 2011). The protocol distinguishes emissions in three so-called “scopes”. Scope 1 emissions include all emissions by assets owned by the reporting company. For a manufacturing company, this typically includes on-site fuel consumption for production or heating. For a transportation company, this would include the fuel consumption of its trucks. Scope 2 emissions are emissions caused by the production of electricity that those assets consume. For a company in the service industry, typically scope 1 emissions are substantially smaller than scope 2 emissions, while for a steel plant the reverse would be true. Finally, scope 3 emissions include all remaining emissions by other companies from which products or services are bought, directly or indirectly. Scope 3 reporting is generally underdeveloped due to measurement complexity but those companies that do report often include here the emissions from their transportation service providers.

Within each of the scopes, the methods to actually measure or estimate the emissions may vary widely. While for scope 1 and scope 2 estimates or actual measurements based on fuel consumption may be easy to obtain, this is substantially more challenging for scope 3 emissions. Hence, emissions in scope 3 are typically not estimated based on energy (or fuel consumption), but on reference parameters for industry activities. For instance, the US Environmental Protection Agency (EPA, 2011) provides estimates for emissions per ton-mile for an average “heavy duty truck”. Based on shipment data, a shipper can then estimate its scope 3 transportation emissions using such an estimate, which is called an emission factor. In addition to the EPA data, other more detailed models are available that estimate emissions at a much more granular level. An example of such a model is the one developed by NTM (NTM, 2008), which provides alternative calculation approaches for transportation. More detailed estimates however also require more detailed data collection on behalf of the shipper. For instance, rather than using an “average heavy-duty truck” the shipper would need to know what type of truck was used for a particular shipment to apply

a more specific NTM parameter. Another parameter that has a substantial impact on emissions is the load factor of a truck. Whereas the EPA data uses an average load factor across the United States, the NTM model allows for using more specific load factors for a particular network. Also here, more detailed information on actual shipments would need to be collected.

For managers that are determining their transportation carbon footprint, it is not obvious whether aggregate estimates are sufficient, and whether the decisions based on an aggregate estimation model are effectively identical to those based on a more detailed transportation carbon footprint. It can be argued that an aggregate estimate should be sufficient, since the underlying driver - reducing the number of shipments - is identical and hence it does not pay off investing time and effort in developing a detailed estimate. On the other hand, it can be argued that by estimating an aggregate footprint, any subsequent decision making or optimization would take advantage of the poor estimation and hence drive the firm towards increasing its footprint rather than reducing it.

Recent work explores the inclusion of carbon emissions in lot-sizing problems. Absi et al. (2011) present different types of constraints in lot-sizing problems including a complexity analysis. Bouchery et al. (2010) present the sustainable economic order quantity (EOQ) problem in a multi-objective setting. Chen et al. (2011) modify the EOQ by adding a carbon constraint. Mooij et al. (2011) include a carbon constraint for the dynamic lot-sizing problem, similar to Benjaafar et al. (2010), and develop an algorithm based on lagrangean relaxation to find a lower bound for the problem. All these authors use a single emission factor per unit (supplied or produced), i.e., their structures are based on aggregate estimations of carbon emissions (see section 4 for a more in depth discussion of the differences).

In this paper, we study the impact of the choice of carbon footprint aggregation level in transportation decision making. We do not assume a cost per unit of CO<sub>2</sub> emitted since this parameter is very specific to each company: companies may be subject to a cap-and-trade system, or may use carbon emission reductions as a driver for brand management, product

differentiation or employee motivation (CDP, 2011). We are interested in learning whether (1) using aggregate carbon footprint drives decisions so self-imposed carbon reduction targets are reached, and (2) whether actual lot-sizing decisions are different under different aggregation levels of carbon footprinting. Cost aggregation has been studied before in the operations research literature in Jalil et al., (2011) who found that errors can be significant in real-life spare parts planning. Unlike prices or costs, carbon emission information is not currently shared among supply chain members. Thus, a company faces data aggregation choices based on activity-based methods that change the underlying problem.

To study this, we add transportation carbon footprint to the Dynamic Lot Sizing (DLS) Problem. The DLS Problem is a well-known production planning problem that has been extensively studied. The problem was introduced by Wagner and Whitin (1958). The model considers a firm that faces deterministic dynamic lot planning decisions for a single item over a finite planning horizon, and minimizes the setup, inventory holding and production/transportation cost, deciding when and how much to order in each period so that the demand is satisfied at minimum cost. We build on the formulation by Zangwill (1969) which includes the cost of backordering. The dynamic lot size model is applicable in different contexts such as material requirement planning and outbound distribution (Lee et al., 2001) and it has been received interest for its simplicity and importance as a sub-problem for more difficult lot sizing problems (Brahimi, et al., 2006).

We provide both analytical and numerical results, showing that under a wide range of conditions the use of an aggregate model will lead to decisions by which the carbon reduction targets are not obtained while a detailed model is able to reach such targets at limited additional cost.

The remainder of this paper is organized as follows. Section 2 includes a review of the GHG and NTM methodologies to estimate the carbon footprint in transport activities. The detailed carbon footprint formulation and various aggregate models are explained in Section 3. Section 4 includes the formulation for the DLS problem and models with different carbon emission constraints based on different levels of aggregation. That section

also includes a set of properties that help to understand the behavior of the models under different conditions. We conduct an experimental analysis for all models based on the data of a retail company. These results are shown in Section 5. We summarize our conclusions in Section 6.

## **2. Methodologies to estimate carbon emissions in transportation activities.**

Generally, transportation modes are classified into four different types: air, water, rail and road. In this study, we limit our modeling to road transportation. Road transportation accounts for a large share of freight transport emissions. In the European Union (EU), for instance, road transport accounts for more than 65% of EU transport-related greenhouse gas emissions and over 20% of the EU's total emissions of CO<sub>2</sub>, the main greenhouse gas (EU Transport GHG, 2007). In the United States, road accounts for approximately 30% of greenhouse gas emissions and it is the fastest-growing major source of greenhouse gases (EPA Road, 2011).

Actual emissions in transportation can be calculated if information is available on the fuel consumption of a vehicle. In practice, however, transportation is outsourced and this information is either difficult to obtain or - if available to the carrier - the carrier has no incentive to share this information as it serves as a proxy for the cost price. Consequently, emissions cannot be measured but need to be estimated.

In this paper, we use estimation methods that are *activity-based*, i.e., the emission factors are estimated based on a reference model and database that relates a particular transportation activity to the emissions caused by that activity. One such method is the transportation estimation model of the GHG Protocol.

In the GHG Protocol methodology, the required parameters to estimate the carbon emissions if the fuel consumption information is not available are freight distance traveled

(tonmile or tonkilometer) and type of vehicle. The carbon emissions are then estimated according to:

(2.1)

where,

- TE* total emissions in grams of CO<sub>2</sub>
- EF* emission factor in grams of CO<sub>2</sub> per kilogram-kilometer
- D* distance in kilometers
- W* cargo weight in kilograms

The most common database used by the GHG protocol to define the value of  $EF$  is the data provided by the United States Environmental Protection Agency (EPA) (GHG Protocol calculation tools, 2011). The EPA uses the same emission factor independent of the type of vehicle or type of road by developing an average of the trucks that use diesel as fuel (GHG Protocol core module guidance, 2008). This methodology does not include a specific load factor, and therefore it is based on an average utilization per truck. This means that the total emissions are not related to the number of shipments but only to the amount of units.

An alternative activity-based methodology is the one developed by NTM (2008). In the NTM methodology, at the highest level of aggregation, the estimation is also calculated using Equation 2.1. At the next level of detail, however, the NTM methodology requires more detailed parameters: fuel consumption, distance traveled and weight per shipment (NTM Road, 2008). The fuel consumption is a function of the type of truck/trailer, the load factor and the type of road. NTM uses the European ARTEMIS (Assessment and Reliability of Transport Emission Models and Inventory Systems) database which developed a detailed emission model for all transport modes to provide consistent emission estimates at the national, international and regional level (TRL, 2010). At this level of aggregation, the NTM estimation model then is:

(2.2)

where



$TE$	total emissions in grams of CO <sub>2</sub>
$FC$	fuel consumption in liters per kilometer
$CE$	constant emission factor (2,621 grams of CO <sub>2</sub> /liter)
$D$	distance in kilometers
	fuel consumption of the empty vehicle liters/km
	fuel consumption of the fully loaded vehicle liters/km.
	load factor, defined as $w/W$ ( $w$ =cargo weight, $W$ =truck capacity).

Note that in this estimation model, the emissions consist of a fixed amount per shipment and a variable amount related to the number of units shipped. Hence, Eq. 2.2 can be reduced to

$$(2.3)$$

such that  $E_{fixed}$  are the fixed emissions due to the number of trips (shipments) required and  $E_{marginal}$  is the marginal emission factor per unit to be transported.

Since we recognize the fixed and variable components of transport emissions, we use the NTM methodology to include these emissions into the formulation of the dynamic lot sizing problem. We define the formulation based on this methodology as the detailed approach.

Table 1 provides the fuel consumption for six different types of trucks that we will use in this paper (adapted from NTM Road, 2008). The parameters  $W$  (Capacity),  $FC_{empty}$  and  $FC_{loaded}$  are values related to the type of trailer and type of road.

Table 1- Fuel consumption and maximum truck capacity per type of trailer. Adapted from NTM Road (2008)

Type of Trailer	W (tons)	Motorway	Rural	Urban			
1	7.5	0.122	0.137	0.107	0.126	0.11	0.134
2	14	0.165	0.201	0.152	0.197	0.171	0.228
3	26	0.204	0.273	0.199	0.284	0.244	0.352
4	28	0.201	0.294	0.205	0.318	0.255	0.402
5	40	0.226	0.36	0.23	0.396	0.288	0.504
6	50	0.246	0.445	0.251	0.495	0.317	0.634

From Table 1, we notice that for a constant distance, and are larger as the truck capacity increases. We will use this characteristic to demonstrate some of the properties in this paper.

### 3. Detailed carbon footprint formulation and aggregate models.

As mentioned earlier, more detailed estimates also require more detailed data collection on behalf of the shipper. However, this needs to be traded off against the effort needed to collect the required data. NTM provides an analytical framework for a very detailed estimation of the carbon footprint in transportation when data is available. Some of the required data to compute those estimations are obtained from the transportation supplier, e.g., type of trailer, fuel consumption, distance, utilization of trucks. A common practice in companies is to estimate the parameters based on average information, similar to the information published by the EPA (2011). When using this approach, all emission factors are identical, regardless of the type of trailer or road. Other companies use the total demand of units divided by the total number of shipments to estimate a common load factor per truck. Thus, tactical and operational decisions that incorporate these estimation models could be sensitive to the specific aggregation level chosen.

In this paper, we implement six different levels of aggregation in the DLS problem in order to compare the performance of the inventory policies and the magnitude of the error. We consider two dimensions in the level of detail to distinguish the six different approaches: two levels of *emission factor* (EF) and three levels of *load factor* (LF).

In the EF dimension we define two levels of detail: *Standard EF*, when a single number (a standard truck) is used to estimate the carbon emissions, and *EF per truck*, when a truck specific EF is used to estimate carbon emissions. In the LF dimension we define three levels of detail: *FTL* (Full Truck Load), when the load factor is estimated assuming full truck utilization; *Average demand*, when the LF is estimated based on the average per period shipment size (total demand divided by the time horizon); and *Lot size per shipment*, when the load factor is calculated using the actual number of units on each shipment. When combining these two dimensions

Table 2 – Problem names for the six aggregation approaches

	Lot size per shipment	Average demand	FTL
EF per truck	Model A	Model C	Model E
Standard EF	Model B	Model D	Model F

Table 2 helps to identify the following six different models:

- A. This model corresponds to the highest level of detail. The model uses specific EF for each type of truck, and the truck load factor (LF) is based on the actual units shipped. We will also refer to this model as the detailed approach.
- B. This model uses a standard EF, and a LF is based on the actual units shipped.
- C. This model uses specific EF for each type of truck, but the LF is estimated by dividing the total demand by the total number of shipments.
- D. This approach assumes a standard EF, and average demand to calculate an average LF. This approach is one of the most commonly used in practice to estimate carbon emissions.

- E. This model uses a standard EF, and a LF assuming a full-truck-load, which implies 100% utilization.
- F. This final model corresponds to the highest level of aggregation (or lowest resolution). The model uses a standard EF and assumes all shipments are FTL.

The remainder of this paper will compare the six modeling approaches and analyze the magnitude of error per model compared to the true carbon emissions. This will provide insight into the impact of aggregation on the performance in the DLS problem. The next section shows the different formulation of the DLS with carbon footprint constraints based on each of the models described in this section.

## **4. Formulation.**

This section presents the formulation of the standard DLS model and the model variations introduced in Section 3. In Section 4.1 we present the basic DLS problem of Wagner and Within (1958). Section 4.2 includes the formulation of the carbon footprint constraint for the detailed and aggregate approaches, and in Section 4.3 we present the formal comparison between the detailed model and the aggregate approaches.

### **4.1.A Single Item Dynamic Lot Size Model with Backlogging.**

In the Dynamic Lot Sizing problem an entity must determine, over a specified period planning horizon with known demand, when and how much to order (produce). We will base the discussion on the formulation proposed by Zangwill (1969) where backlogs are allowed. The other key assumptions of the model are as follows:

- Demand is dynamic and known in advance
- Planning horizon is finite
- Supply lead times are zero
- Unfulfilled demand is backordered
- All the demand should be fulfilled during the planning horizon

The parameters are defined as follows:

fixed cost per order per period  $t$   
variable cost per unit per period  $t$   
cost per unit for inventory carried at the end of period  $t$   
cost per unit backordered carried at the end of period  $t$   
demand per period  $t$

The decision variables associated with the problem are denoted by

{

order quantity per period  $t$   
amount of inventory at the end of period  $t$   
amount of inventory shortage at the end of period  $t$

The problem (P1) is formulated as follows:

Minimize  
Subject to

This problem has been proven to be NP-hard even under simplifying conditions in the cost structure (Florian, et al., 1980). However, if the costs are concave, a set of “zero-switch”

properties for the optimal solution defined by Zangwill (1969) allows solving the problem in polynomial time by using dynamic programming.

The next section will add carbon emissions to this formulation.

## **4.2. Carbon emissions formulation**

In order to include carbon emissions in the DLS model, we make the following additional assumptions:

- The supply chain (SC) consists of one supplier and one buyer.
- The lot size decision is made by the buyer.
- The buyer includes transportation carbon emissions in his GHG inventory via Scope 1 or Scope 3 guidelines (GHG Protocol Standard, 2011).
- Transportation is conducted by truck, with different kinds of trailers of known capacity.
- The distance is known and constant between the two SC entities.
- The warehouse carbon emissions are fixed.
- Production emissions are a linear function of production quantities.

The total carbon emissions in this supply chain include those related to storage, production and transportation. Since emissions due to storage do not depend on the number of units in the warehouse, they are not a function of the lot size and can be ignored from the DLS formulation. Furthermore, there is evidence showing that storage emissions are substantially smaller than transportation emissions, by a factor of 10 for some products (Cholette and Venkat, 2009). In the base DLS model we assume that all demand is satisfied at the end of the planning horizon (equations 4.2-4.3). Hence, total emissions due to production are constant and can also be ignored in the base DLS model. Consequently, all variations of carbon emissions are obtained by various options of shipment frequency and lot sizes, and not a function of time-phased production decisions, unsatisfied demand or

varying levels of inventory. From now on, the carbon constraint is assumed to be adjusted to exclusively reflect carbon emissions due to the transport activity.

#### 4.2.1. Model A: Detailed approach

Since we consider a single item, we can replace  $Q_{it}$  in Eq. 2.3 by its equivalent units. Since  $\alpha_{it}$  and  $\beta_{it}$  could be different per period according to the type of trailer (see Table 2), the expression of transport carbon constraint can be reduced to Equation 4.7a:

where:

The emissions consist of a fixed amount per shipment and a variable amount related to the number of units shipped. Notice that expression 4.1 has a different structure compared to 4.7a, specifically decision variables  $Q_{it}$  and  $Q_{it}^*$  are not included in 4.7a. Therefore, a reduction in cost does not necessarily imply a reduction in carbon emissions and vice versa. A reduction of fuel for the DLS problem is driven by reductions in the number of shipments (associated with transport cost) within the planning horizon ( $T$ ), while this reduction could imply an increase in inventory or backorder cost.

We define Model A as the DLS formulation presented in section 4.1 plus Expression 4.7a. This constraint has similarities to a capacity restriction, but it is different from the capacitated lot-sizing problem since it accumulates the lot sizes for all periods. Since it has been proven that the capacitated lot-sizing problem without backlogging may not satisfy the “zero-switch” property (Xie and Dong, 2002), we conjecture that when emission factors  $\alpha_{it}$  are different per period, the zero-switch property will not hold in this model and therefore heuristic approaches need to be developed.

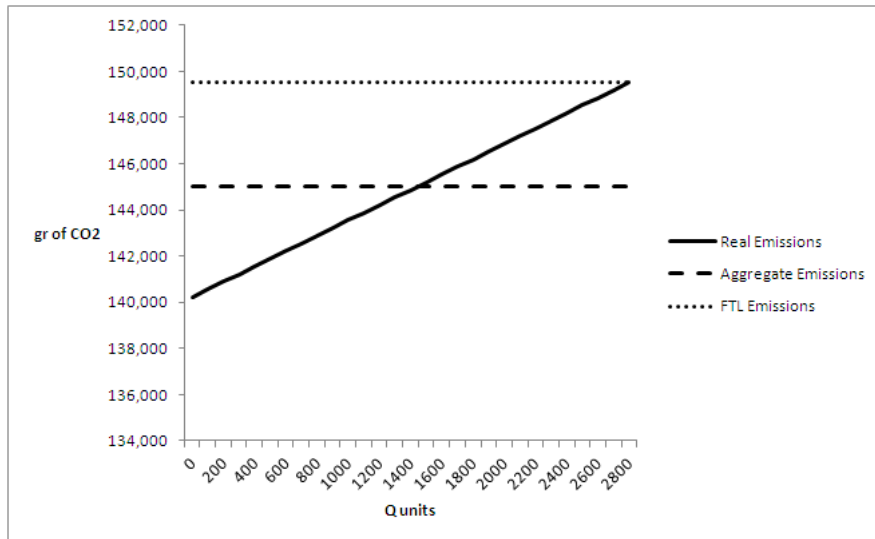


Figure 1- CO2 emissions per unit under different modeling assumptions

Figure 1 compares the emissions of a vehicle based on the number of units it carries. The solid line represents the behavior of Eq. (4.7a): a fixed emission per truck and variable emission related to the number of units shipped in the truck. We will refer to this approach as “Real Emissions” for the purpose of this paper, since this is the most detailed level of analysis.

#### 4.2.2. Model B: Lot size per shipment and standard EF per period

When operations are contracted with a transportation service or third party logistics provider, information of the specific type of trucks is not readily available. Thus, a common practice is to assume a standard truck for all transportation movements. In this case, constraint (4.7a) is modified to the following expression (4.7b):

Where  $\alpha$  and  $\beta$  are the emission factors related to a standard truck with known capacity across all time periods. On this formulation, the decision variable  $Q$  is still part of the



carbon constraint and therefore the utilization estimation is based on the amount of units shipped as in model A.

#### **4.2.3. Model C: Average demand utilization and flexible EF per period**

Since information on actual load factor may not be available, a common approach is to assume an average load factor across the planning horizon. Thus, any carbon emission calculations will not include a variable emission but only a fixed emission every time an order is delivered. Figure 1 (dash line) shows the carbon emissions for model C compared to Models A and E.

For model C we use an average demand to estimate the truck utilization while keeping the flexible assumption of different emission factors per period. Since the total demand for the item will be satisfied at the end of the horizon (4.7), an estimation of the average number of units shipped per truck can be calculated by using the total demand. In this case, the carbon constraint is defined as follows (4.7c):

With ———. This approach has the advantage of considering different emission factors per period. However, since the model is based on the assumption of delivering the average demand every time an order is released, this implies that carbon emissions could be overestimated or underestimated.

#### **4.2.4. Model D: Standard EF and average demand of utilization**

Model D is similar to model C but assuming a standard truck for the entire planning horizon . This implies constant emission factors . This modeling approach does not

require any detailed operational information, and it is very common in practical industrial initiatives. The carbon constraint for this model is defined as follows (4.7d):

With  $\alpha$  — such that  $\alpha$  correspond to the emission factors related to a standard truck. In this approach the truck is assumed to be loaded with an average demand every time an order is released.

#### **4.2.5. Model E: FTL utilization and different EF**

In some instances, in order to avoid a solution that exceeds the carbon constraint, a conservative approach is to assume FTL on all shipments. This curve is shown in Figure 1 (dot line). Model E assumes different EFs per shipment but an utilization fixed to the capacity of the truck. The following expression (4.7e) shows the formulation of model E:

With  $\alpha$  . When the shipment sizes are close to the maximum truck utilization, assuming that if the capacity of the trucks is more or less equal to the total demand, then the estimated emissions are closer to the real emissions.

#### **4.2.6. Model F: FTL utilization and standard EF**

Finally, Model F requires just the information related to one standard truck and a conservative assumption of FTL for all shipments This model uses the highest level of aggregation to estimate carbon emissions. The following expression (4.7f) shows the formulation:

where

As in model E, this approach is based on the assumption that if the capacity of the trucks is close to the total demand, then the estimated emissions are a good approximation to real emissions.

Benjaafar et al, (2010) developed some approaches that include the formulation of carbon emissions subject to a mandatory external carbon emission constraint in a series of lot sizing models. Their study defined emissions associated with transportation, ordering (production) and per unit in storage. Even though their proposed formulation structure is general (fixed and variable factors), in their interpretation they assumed a fixed emission factor due to transportation activity (as in models C, D, E and F), while common methodologies also include a variable transportation emission factor (see section 2). The analysis in the following sections will give insight into the impact of this assumption, including an experimental study with estimated parameters based on real data from a major retail company in North America. We will show that the differences in estimations are not only directly due to the differences in parameters between the methods, but are aggravated by the optimization, which is also consistent with Wagelmans (1990) who demonstrated, through post optimality analysis of parametric variations, that perturbations of data can highly affect the optimal solution in mixed integer problems.

The next section focuses on the analytical comparison and properties for the proposed various models.

### **4.3. Analytical comparison between model formulations**

In this Section, we provide theoretical properties for the six aggregate approaches defined in Section 4.2, to highlight the conditions under which these formulations can lead to significant differences in optimal inventory policies. The goal is to understand performance of various levels of modeling strategies.

Undesirable performance of aggregate approaches occurs when solutions have one of these two properties:

- a. The solution exceeds the carbon cap, which means that the solution is feasible in the aggregate model but infeasible in the detailed model.
- b. Excludes an optimal solution, which means that the solution is infeasible in the aggregate model even if optimal in the detailed model.

In this section, we prove that solutions to models C and D that use average demand to estimate the load factor, can exceed the carbon cap. We also prove that Models E and F based on FTL estimation cannot reach the optimal solution. In Section 4.3.1, the properties related to model C are explained. In Section 4.3.2 the property related to model D is explained and Section 4.3.3 explains the properties related to models E and F. The formal proofs are shown in the Appendix section.

#### 4.3.1. Properties of using average utilization and flexible EF per period (Model C)

We define as follows:

- Let  $\mathbf{C}$  be the carbon emissions calculated by model C, such that  $\mathbf{C} = \mathbf{E} \mathbf{X}$  and  $\mathbf{E}$  are two matrices related to emission factors and decision variables respectively, where  $\mathbf{E} = [\mathbf{E}_1, \mathbf{E}_2, \dots, \mathbf{E}_n]$  and  $\mathbf{X} = [X_1, X_2, \dots, X_n]$ .
- Let  $\mathbf{C}^*$  be the real total emissions for  $\mathbf{C}$  such that  $\mathbf{C}^* = \mathbf{C} + \mathbf{C}_0$  and  $\mathbf{C}_0$ .
- Let  $\mathbf{C}^*$ .
- If  $\mathbf{C} > \mathbf{C}^*$  then  $\mathbf{C}$  is an inventory policy that is exceeding the carbon cap.

Then, the following property can be defined:

- Property 1.** Let  $\pi^*$  and  $\pi^*$  be the optimal inventory policy obtained by model C. Suppose  $\pi^*$  and  $\pi^*$  such that  $\pi^*$  and  $\pi^*$ , if  $\pi^*$  and  $\pi^*$   $\neg$ , then  $\pi^*$  exceeds the carbon cap.

Property 1 shows that whenever there are optimal solutions for Model C that place orders in every period and have larger lot sizes at the beginning of the time horizon (due to lower emissions), the solutions will exceed the true carbon cap. Clearly, not all optimal solutions will have this property. However, property 1, shows that the modeling approach used in Model C is unreliable.

Property 1 also provides strong evidence that if an estimation of carbon emissions is based on average demand instead of the specific amount requested per period, even when the total demand and the total number of lots remains the same, if the largest number of orders are located in the smallest trucks (with the lowest capacity), the aggregate approach faces an underestimation of carbon emissions.

#### 4.3.2. Properties of using standard EF and average demand of utilization (Model D)

We define as follows:

- Let  $E^D$  be the carbon emissions calculated by Model D, such that  $E^D = \sum_{i=1}^n \sum_{j=1}^n x_{ij} \cdot e_{ij}$  and  $x$  is the matrix of decision variables such that  $x_{ij} \geq 0$  for  $i, j = 1, \dots, n$ .
- Let  $E^R$  be the real total emissions for  $\pi$  such that  $E^R = \sum_{i=1}^n \sum_{j=1}^n \pi_{ij} \cdot e_{ij}$  and  $\pi_{ij} = \sum_{k=1}^n x_{ik} \cdot \frac{e_{kj}}{e_{ij}}$ .
- If  $E^D < E^R$ , then  $\pi$  is an inventory policy that is exceeding the carbon emissions cap.

Then, the following property and corollary can be defined:

- **Property 2.** Let  $\pi^*$  and  $\pi$  be the optimal inventory policy obtained by model D. Suppose  $\pi^* - \pi$  such that  $\pi$  is the number of  $\pi$  for any  $\pi$ , then  $\pi$  exceeds the carbon cap.
  - **Corollary 2.1** Let  $\pi^*$  and  $\pi$  be the optimal inventory policy obtained by model D. Suppose  $\pi^* - \pi$ , if  $\pi^* - \pi$  and  $\pi^* - \pi$ , then  $\pi$  exceeds the carbon cap.

Corollary 2.1 is a particular case of property 2. Since  $\pi^* - \pi$ , the only way not to exceed the carbon cap is when  $\pi^* - \pi$ . This relation shows that if the emissions calculated by Model D are strictly equal to the cap target, the only case in which  $\pi$  cannot exceed the cap is when the optimal solution is to release orders in each period. Thus, an optimal solution in which an order is not loaded in one of the periods, implies that the solution will exceed the carbon cap constraint. Moreover, property 2 provides the range for which the solution obtained by model D will exceed the cap if  $\pi^* - \pi > 0$  and an optimal solution such that  $\pi^* - \pi$ . It is intuitively clear to expect optimal solutions that try to decrease the difference of the constraint and the resource as much as possible. Therefore, the difference between the carbon cap and the carbon emissions calculated by Model D tends to zero in most cases. Thus,  $\pi^* - \pi$  is in the range of exceeding the carbon emissions cap.

This analysis was based on the assumption of having the same truck capacity in all periods. Since this cannot always be guaranteed, we can expect that the approach generates solutions that exceed the carbon cap in a wide range of situations.

#### 4.3.3. Properties of using FTL utilization with standard or different EF (Models E and F)

In order to avoid the underestimation of carbon emissions, models E and F assume that every time an order is placed, the carbon emissions associated to that order are equal to the full truck emissions. In that case, the LF is not estimated with the average demand, but by assuming that the full truck capacity was used in that period. Clearly, this approach does not underestimate carbon emissions. However, we show that this approach, even though guarantees that the carbon cap is never exceeded, cannot reach an optimal solution. In order to show this, we use the following definitions:

- Let  $E_{i,t}$  be the carbon emissions calculated by model F, such that  $E_{i,t} = \sum_{j \in N} x_{i,j,t} \cdot e_j$  and  $x_{i,j,t}$  is the matrix of decision variables where  $x_{i,j,t} \geq 0$  for  $i \in N, j \in N, t \in T$ .
- Let  $E_{i,t}$  be the real total emissions for  $i \in N$  such that  $E_{i,t} = \sum_{j \in N} x_{i,j,t} \cdot e_j$  and  $E_{i,t} \leq C_i$ .
- Let  $\pi_i$  and  $\pi_j$  be an optimal inventory policy obtained by the detailed approach and let  $\pi_i$  and  $\pi_j$  be an optimal inventory policy obtained by model F, then the solution of model F is the *true optimal* if  $\pi_i = \pi_j$  and  $\pi_j = \pi_i$ .

The following property and corollary can be defined:

- **Property 3.** Let  $\pi_i$  be the optimal inventory policy obtained by model F. Suppose  $\pi_j$ , if  $\pi_j$  such that  $\pi_j$  is the number of  $\pi_j$  for any  $\pi_j$ , then  $\pi_j$  is not the true optimal.
- **Corollary 3.1** Let  $\pi_i$  be the optimal inventory policy obtained by model F. Suppose  $\pi_j$ , if  $\pi_j$  and  $\pi_j$  then the optimal solution of model F is not the true optimal solution.

Property 3 provides the range of solutions of model F that are not true optimal solutions: when the difference between the total demand and the capacity of the trucks is close to zero, i.e.,  $\epsilon$ . If the capacity of the trucks is equal to the total demand, corollary 3.1 shows that the solution is not the true optimal solution for any inventory policy when an order is not placed in at least one period, i.e., some  $t \in \{1, \dots, T\}$ . However, since a key assumption of the DLS problem without capacity constraint is that the capacity of the trucks should be much larger than the lot size,  $C \gg Q$  implies that the truck capacity per period should be exactly equal to the average demand. Thus, this condition could lead to an infeasible solution for some  $t$ . In order to prevent such an infeasible solution, we define the following condition  $C \geq Q$  and therefore  $C \geq Q$  for all  $t$ . Since that condition implies  $C \geq Q$ , we notice that an optimal solution obtained by the detailed approach could be infeasible for the FTL approach. The following property specifies this overestimation problem for model E (different trucks per period), and therefore the results are also applicable to model F (standard truck for the planning horizon). In order to show this relation, we use the following definitions:

- Let  $E$  be the carbon emissions calculated by model E, such that  $E = \sum_{t=1}^T \sum_{i=1}^n e_{it} x_{it}$  and  $E = \sum_{t=1}^T \sum_{i=1}^n e_{it} x_{it}$  are two matrices related to emission factors and decision variables respectively, where  $e_{it}$  and  $x_{it}$  for  $i=1, \dots, n$  and  $t=1, \dots, T$ .
- Let  $E$  be the real total emissions for  $t=1, \dots, T$  such that  $E = \sum_{t=1}^T E_t$  and  $E_t = \sum_{i=1}^n e_{it} x_{it}$ .
- Let  $x$  and  $y$  be an optimal inventory policy obtained by model E and let  $x$  and  $y$  be an optimal inventory policy obtained by model F.
- The optimal solution is the *true optimal* if  $E = \sum_{t=1}^T E_t$  and  $x = y$  or  $E = \sum_{t=1}^T E_t$  and  $x \neq y$  for model E and F respectively.

We define the following property and corollary:



- **Property 4.** *Let  $\mathcal{C}$  and  $\mathcal{C}^*$  be the optimal inventory policy obtained by model E. If  $\mathcal{C} \neq \mathcal{C}^*$  then  $\mathcal{C}$  is not the true optimal.*

- **Corollary 4.1.** *Let  $\mathcal{C}$  and  $\mathcal{C}^*$  be the optimal inventory policy obtained by model F. If  $\mathcal{C} \neq \mathcal{C}^*$  then  $\mathcal{C}$  is not the true optimal.*

Property 4 provides a range for  $\mathcal{C}$  where the optimal solutions of model E are not true optimal solutions. It can be noticed that as the number of  $\mathcal{C}$  increases, the range increases as well. This implies that if the carbon cap increases, the number of possible optimal solutions obtained by model E that are not perfect are larger. Corollary 4.1 shows similar results for model F. Thus, it is clear that the FTL approaches (models E and F) overestimate the carbon emissions and therefore they result in worse solutions. The following Section shows the experimental results of all the models.

## 5. Experiments

In order to study the difference in the decisions obtained by the detailed and aggregate approaches, seven MIP formulations were implemented in Mathematica 7 for models A, B, C, D, E, F and the original DLS without carbon constraint (see Section 3). For all the experiments we used urban road fuel consumption parameters and distance  $D$  equal to 492 kilometers. Since the cost of producing  $\mathcal{C}$  is constant, we do not include this parameter in the experimental study. The information related to truck capacities, demand and emission factor parameters used on the experiments are shown in Table 3.

Table 3- Parameters for the models.

Period	Demand			W
1	4451	328830.66	3.39	56000
2	4922	371385.22	3.48	80000
3	4003	408781.65	4.09	100000
4	4365	328830.66	3.39	56000
5	4139	371385.22	3.48	80000
6	4287	408781.64	4.09	100000
7	4665	328830.66	3.39	56000
8	4459	371385.22	3.48	80000
9	4311	408781.64	4.09	100000
10	4467	328830.66	3.39	56000
11	4183	371385.22	3.48	80000
12	4745	408781.64	4.09	100000

We ran 5000 experiments with 100 different carbon caps each with 50 different setup, holding and backorder costs in order to evaluate the differences in cost, carbon emissions and inventory policies for the various DLS models. We generated these parameters randomly using ranges from operations of a North American retail company, as follows:

$\alpha$ ,  $\beta$  and  $\gamma$ ,  $\tau$  tons of CO<sub>2</sub>, where U denotes the uniform distribution. The parameters of the Uniform distribution for the carbon cap  $C$  were determined by the Lower Bound and Upper Bound, respectively of model A. In our data, 2,000 units weigh one ton of truck capacity. Finally, we assume that the truck with the lowest capacity is capable of carrying the total demand.

Figure 2 shows the average total cost for each of the carbon caps (tons of CO<sub>2</sub>), based on the EF dimension: a) EF per shipment (models A, C and D) and b) Standard EF (models B, D and F). All the models are compared against the original DLS without carbon cap. In Figure 2a, we notice that the average cost of the detailed approach (model A) is always between the average cost of models C and E. This occurs because models C and E face the underestimation and overestimation problem respectively described in Section 4.3. Model

C has lower cost than model A, because its emissions are larger, and therefore a lower cost solution can be found. Model E overestimates carbon emissions and therefore its cost is larger. The three approaches achieve the same solution (the original DLS solution) if the carbon cap increases and the carbon constraint becomes redundant.

Figure 2b is based on the assumption of considering an identical EF for all periods. This situation implies discrete jumps in the cost function. The general behavior observed in Figure 2a is similar to 2b in the sense that the average cost of model B is always between the average cost of model D and F, because these two models face underestimation and overestimation of carbon emissions respectively. However, an important difference between the models in Figure 2a versus those in Figure 2b is that the latter approaches could exceed the lower bound of the carbon constraint and therefore no solution to the problem can be found. In Figure 2b, this phenomenon can be observed for models B and F.

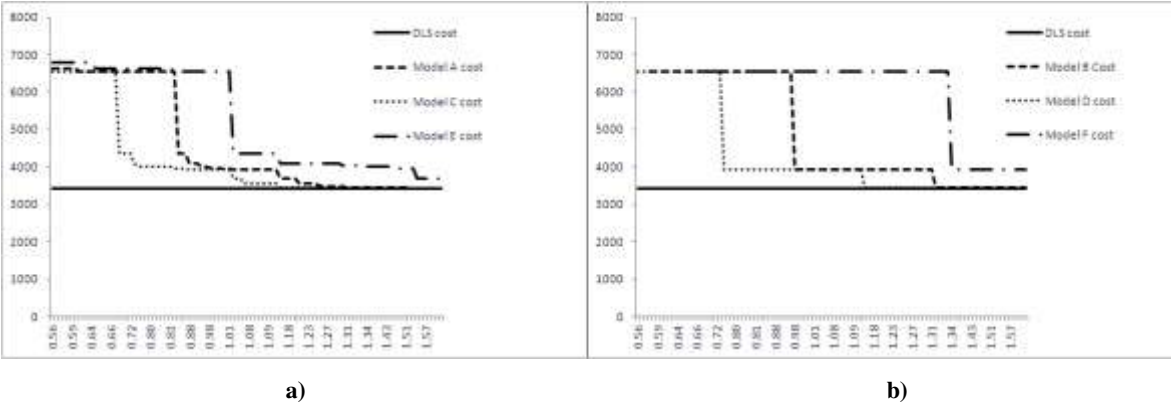


Figure 2- Total average cost per carbon cap, a) EF per shipment b) Standard EF

Figure 3 shows the carbon emissions for the DLS without carbon constraint per different experiment (the experiments are sorted in ascending order of carbon cap). Obviously, the carbon cap (solid line) does not impact the solution. Note that the dotted line shows the average emissions across the different parameter settings, while the dashed and dash-dotted lines show the minimum and the maximum emissions obtained, respectively. We use these carbon emissions from the original as a baseline to make the comparison among different approaches.

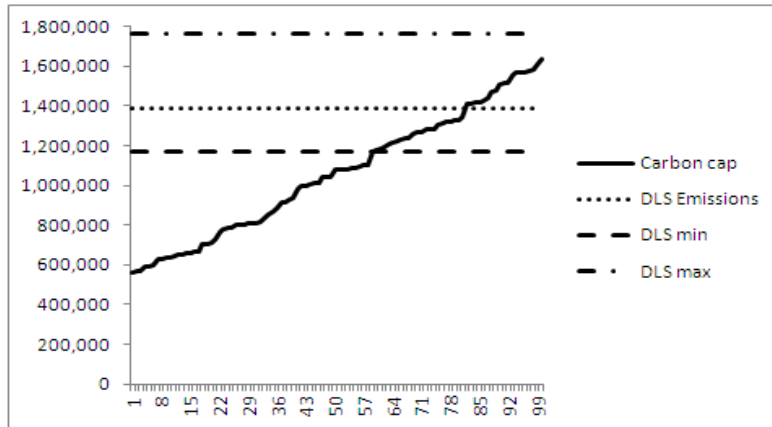
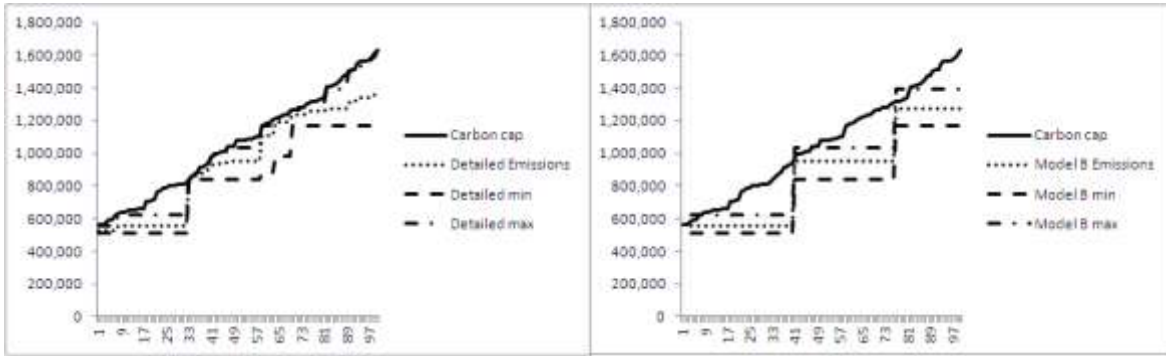


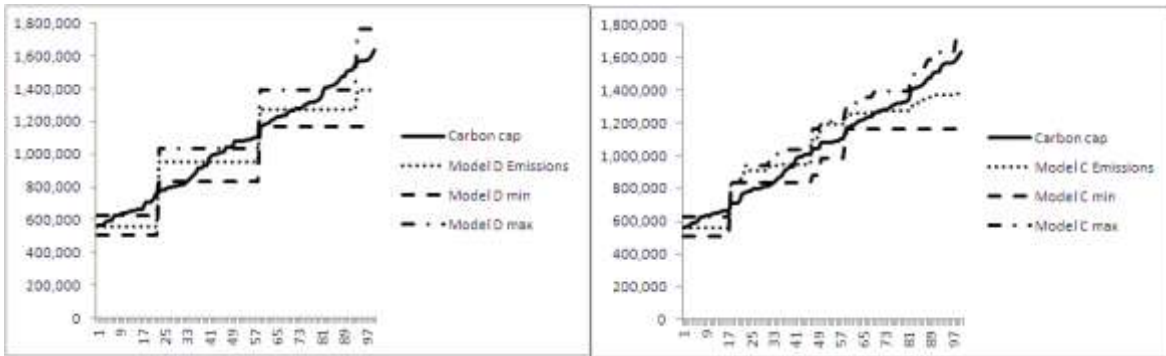
Figure 3- Carbon emissions (grams of CO<sub>2</sub>) for the DLS.

Figure 4 shows the average carbon emissions per inventory policy obtained by the approaches, including the maximum and the minimum value. Figure 4a shows the detailed approach. The average emissions are very close to the carbon cap. Besides, we notice that the maximum emission value is always below the carbon cap. A more aggregate approach is model B. Figure 4b shows that the average emission is always below the carbon cap, but the maximum emissions across all experiments could exceed the carbon cap by a small amount. Similar to the cost chart in Figure 2b, the discrete jumps can be explained by the EF remaining the same across the entire planning horizon. Figures 4c and 4d show the emissions of models C and D respectively. Both approaches have the unreliable behavior explained in Section 4.3, i.e., sometimes above the carbon cap and sometimes below. Finally, in Figures 4e and 4f, models E and F provide inventory policies with carbon emissions far below the carbon cap (overestimation) and therefore they require a high carbon cap in order to achieve lower cost solutions. Model F does not generate solutions if the carbon cap is less than 660 thousands of grams of CO<sub>2</sub>.



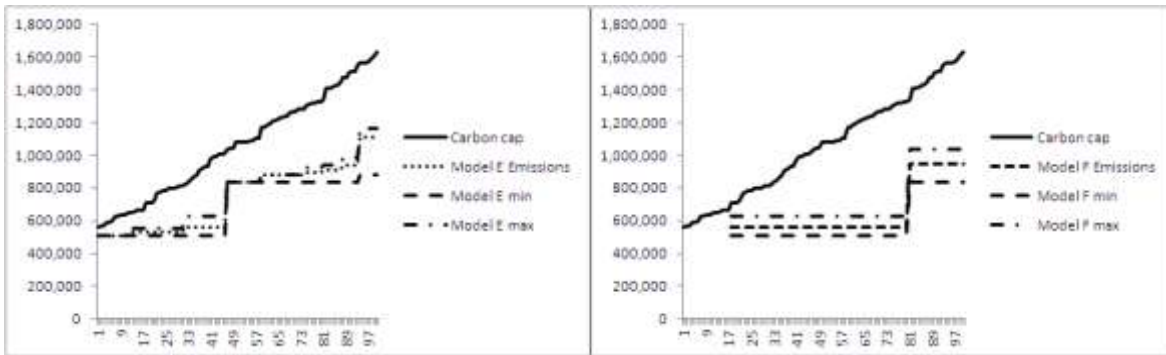
a)

b)



c)

d)



e)

f)

Figure 4- Carbon emissions (grams of CO<sub>2</sub>) per model. a) Model A, b) Model B, c) Model C, d) Model D, e) Model E, f) Model F

In order to quantify the magnitude of error for the aggregate approaches, a mean percentage error is calculated as follows:

where:

- Emissions estimated by aggregate approach.
- Real emissions of aggregate approach.
- Number of carbon caps.
- Number of different sets of cost parameters.

For a given inventory policy obtained by the aggregate approaches, the error is measured as the absolute difference between the carbon emissions estimated by these approaches compared to their real emissions. Figure 5 shows the mean percentage error for each of the approaches. We define the formulation of the detailed approach as the real emissions. We notice that as the level of aggregation increases, the magnitude of the error increases as well. Furthermore, this chart allows showing the points where the big changes in error occur. If a company is interested in increasing the level of detail in measuring carbon emissions, a movement between models C and D is clearly not significant (0.1% of difference). However, the break points from model F to D (~9%) and from model C to A (~16.4%) are related to a larger change in the error.

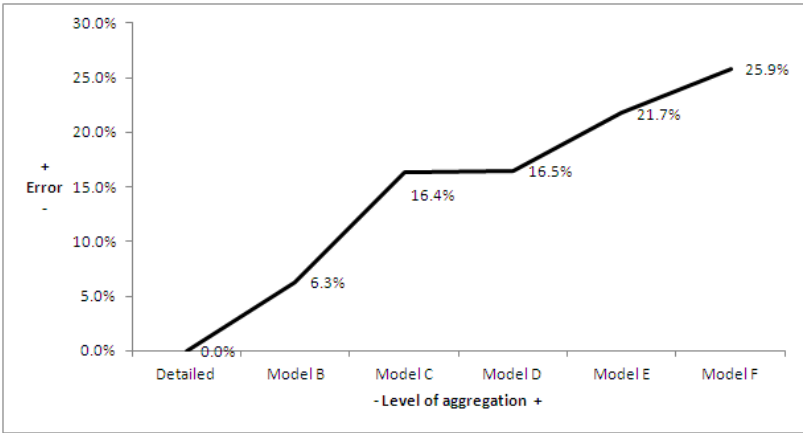


Figure 5- Mean percentage of error for aggregate carbon emissions versus carbon cap

Alternatively, the experimental results show significant differences in the inventory policies. Figure 6 shows the average inventory and backorder cost for different values of the carbon cap. Since the carbon emissions are related to the activity of ordering, we notice that including a carbon cap implies a higher inventory and backorder cost than a cost minimization approach.

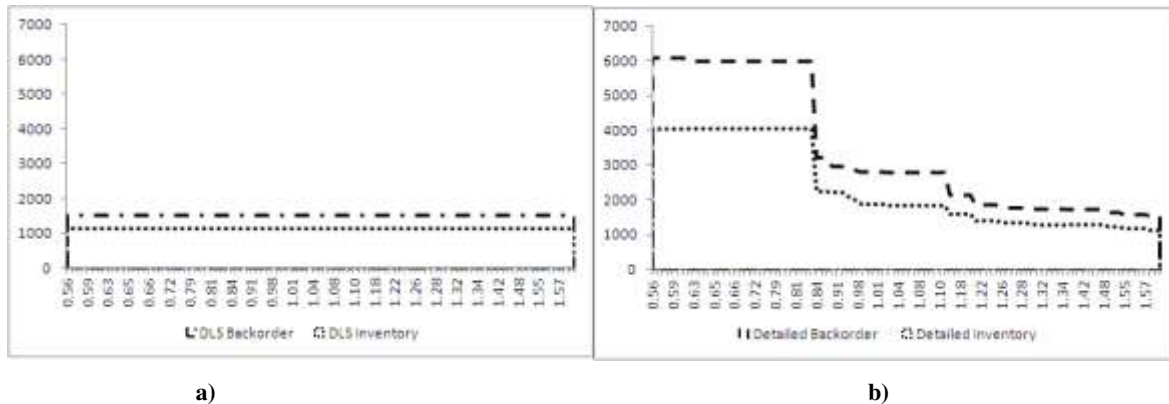


Figure 6- Average inventory and backorder cost per carbon cap (tons of CO2)  
a) DLS, b) Detailed approach.

## 6. Conclusions.

The importance of including carbon emissions in planning decisions is becoming an important element in supply chain management, considering the relevance of greenhouse gases as the main cause of climate change, and also the risks associated with the potential implementation of regulatory policies. This paper addresses the relevance of including a more detailed formulation to measure carbon emissions due to transportation activities in the dynamic lot sizing model as a self-imposed constraint. We have defined a lot-sizing model that can provide both an optimal inventory policy under a carbon constraint and the total carbon emissions under such a policy, in accordance with the NTM methodology. We developed a comparison between aggregate approaches to include carbon emissions, versus a more detailed approach. The formal proofs show that under different conditions, aggregate approaches can cause a significant distortion in the lot sizing decisions such that,

by defining an inventory policy, the carbon cap is exceeded or a cost-worse solution is obtained. The numerical experiments confirm this conclusion and show that the magnitude of error could be up to 25% for a very aggregate approach (with a standard emission factor and a full truck load utilization assumption) and 16% for models that assume an average load factor rather than a specific one. Furthermore, we show formally that some of the models systematically overestimate the carbon emissions, while other models underestimate emissions under some conditions. Errors associated with aggregation tend to be substantial and systematic.

Our results show the importance of selecting a sufficient level of detail in estimating carbon emissions in transportation, based on the significant differences in the decisions obtained by more aggregate models through the optimization process. This paper may motivate future research related to the implementation of detailed carbon constraints in more complex lot-sizing problems such as Capacitated Lot-Sizing, Lot sizing with Multi-setup costs, Multi-item Lot Sizing, among others. Our work poses an open issue with regard to the complexity of the lot sizing problem under a carbon constraint. We conjectured that the detailed approach does not keep the properties of Zangwill (1969), but the formal proof of this and the study of solution methods through heuristics remain as future research.

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## Appendix.

- **Property 1.** Let  $\{x_t^C\}_{t=0}^{\infty}$  and  $\{x_t^D\}_{t=0}^{\infty}$  be the optimal inventory policy obtained by model C. Suppose  $\{x_t^C\}_{t=0}^{\infty}$  and  $\{x_t^D\}_{t=0}^{\infty}$  such that  $x_t^C > x_t^D$  and  $x_t^C > x_t^D$ , if  $x_t^C > x_t^D$  and  $x_t^C > x_t^D$ , then  $x_t^C$  exceeds the carbon cap.

Proof.

Since  $x_t^C > x_t^D$ , for some  $t$ . If we assume that  $x_t^C$  is not exceeding the carbon cap, the following relation should always hold:

$x_t^C > x_t^D$ , which implies

such that  $x_t^C > x_t^D$ . Thus,

. Using the nomenclature the condition is reduced to  $x_t^C > x_t^D$

such that  $x_t^C$  and  $x_t^D$  are the emission factors related to  $x_t^C$  and  $x_t^D$  respectively. Clearly

, and therefore it is possible to define a factor  $\alpha$  such that

and  $x_t^C = \alpha x_t^D$ . Therefore,

Since condition  $x_t^C > x_t^D$  implies that  $x_t^C > x_t^D$ , thus

- **Property 2.** Let  $\{x_t^C\}_{t=0}^{\infty}$  and  $\{x_t^D\}_{t=0}^{\infty}$  be the optimal inventory policy obtained by model D. Suppose  $\{x_t^C\}_{t=0}^{\infty}$ , if  $x_t^C > x_t^D$  such that  $x_t^C$  is the number of  $x_t^C$  for any  $t$ , then  $x_t^C$  exceeds the carbon cap.

Proof.

Since  $\dots$ , and we can define  $\dots$  for some  $\dots$ . If we assume that  $\dots$  is not exceeding the carbon cap, the following relation should always hold:  $\dots$ , which implies the following condition  $\dots$ . Thus,

- **Corollary 2.1.** *Let  $\dots$  and  $\dots$  be the optimal inventory policy obtained by model D. Suppose  $\dots$ , if  $\dots$  and  $\dots$ , then  $\dots$  exceeds the carbon cap.*

Proof.

If  $\dots$  thus  $\dots$ . However, since  $\dots$  it is clear that the previous condition is just valid for  $\dots$ , but we know that

- **Property 3.** *Let  $\dots$  be the optimal inventory policy obtained by model F. Suppose  $\dots$ , if  $\dots$  such that  $\dots$  is the number of  $\dots$  for any  $\dots$ , then  $\dots$  is not the true optimal.*

Proof.

Since  $\dots$ , we can define  $\dots$  for some  $\dots$ . Besides, we notice that  $\dots$  and therefore we define  $\dots$  for some  $\dots$ . If we assume that the optimal solution of model F is indeed the true optimal solution, the following relation should always hold:  $\dots$ , thus

, which implies

. Thus and therefore the following condition should hold

- **Corollary 3.1.** *Let be the optimal inventory policy obtained by model F. Suppose , if and then the optimal solution of model F is not the true optimal solution.*

Proof.

If thus . However, since it is clear that the previous condition is just valid for , but we know that

- **Property 4.** *Let and be the optimal inventory policy obtained by model E. If then is not the true optimal.*

Proof.

Since we define for some . Furthermore, we notice that . If we assume that model E is a true optimal solution the following relation should always hold: , thus , which implies

- **Corollary 4.1.** *Let and be the optimal inventory policy obtained by*

*model F. If*  $\dots$  *then*  $\dots$  *is not*  
*the true optimal.*

Proof.

Since  $\dots$ , property 2 is reduced to