KEEPING UP WITH THE JONESES IN A NON-SYMMETRIC EQUILIBRIUM*

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Abstract
We prove the existence of a representative agent in an economy populated with investors who keep up with the Joneses and have heterogeneous portfolio endowments. This result is independent of the endowment distribution and robust to a more general definition of the Joneses. The implications for the home bias puzzle are discussed.

Keywords
Keeping up with the Joneses, non-symmetric equilibrium, representative agent.

JEL Classification Numbers
G12, D71

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Introduction

Galí (1994) studies the portfolio choice and asset pricing implications of "keeping up with the Joneses" preferences whereby a representative agent evaluates her consumption relative to her peers' *contemporaneous* average or "per capita" consumption. Recent papers - see Shore and White (2002), Lauterbach and Reisman (2004) and Gómez, Priestley and Zapatero (2004)- use this type of preferences as a possible explanation of the home bias puzzle: why investors overinvest in domestic assets inspite of the documented gains of international risk-diversification.

An important and more basic question to ask is whether Joneses preferences *per se* can generate biased portfolios in equilibrium. Galí (1994) showed that, in a symmetric equilibrium, they cannot. In this paper we extend this "negative result" by showing that Joneses preferences cannot solve, *on their own*, the home bias puzzle *even if agents have heterogenous portfolio endowments.*

In Section 1, we prove the existence of a representative agent in an economy populated with investors who keep up with the Joneses and have heterogenous portfolio endowments. This result is independent of the portfolio endowment distribution.

In Section 2, we check the robustness of our result to the Joneses definition. Arguably, agents with too low or too high consumption (below or above a given consumption threshold) might be considered as outliers and "fall out" of the Joneses average consumption. Moreover, the Joneses do not necessarily have to be unique. Consumers living in different "clusters" or communities (defined in terms of geographical proximity, language, age or any other affinity attribute) might have "different Joneses." We show that the representative agent result is robust to these refinements.

The paper concludes with Section 3.

1 The existence of a representative agent

Assume a frictionless economy with J agents. Each agent j has an utility function

$$u(c_j, X) = \frac{c_j^{(1-\alpha)}}{1-\alpha} X^{\gamma\alpha},$$

with $\alpha > 0$ the risk aversion coefficient and $0 < \gamma < 1$ the "Joneses parameter." X represents the economy's *average* consumption, $X = (1/J)\sum_j c_j$.

There is one only good in this economy. Financial assets are represented by shares of $K-1$ "firms" (*Lucas' trees*). At time $t=1$, firm k has random payoffs $Y_k = (y_k(1), ..., y_k(s), ..., y_k(S))'$. Payoffs are expressed in units of the consumption good. We assume the number of states $S = K$. Let asset K be a riskless bond with price $1/(1+r)$. Call $Y = (Y_1, ..., Y_k, ..., Y_K)$ the payoff matrix. We assume that financial markets are complete, i.e., Y is non-singular. Let p_k denote the price of a share of firm k. Hence, $p = (p_1, ..., p_k, ..., p_K)$ is the price vector, with $p_K = 1/(1+r)$. We denote π_s the probability of state s.

At time $t = 0$, each investor j is endowed with a portfolio of shares $\overline{\theta}_j = (\overline{\theta}_j^1, ..., \overline{\theta}_j^k, ..., \overline{\theta}_j^K)'$. Let $\overline{\theta} = \sum_j \overline{\theta}_j$ be the aggregate endowment. We assume $\overline{\theta}^k > 0$ for all $k < K$. The bond is in zero net supply ($\overline{\theta}^K = 0$).

At time $t = 0$ the, investor j chooses the portfolio that maximizes her expected utility of future consumption $c_j(\theta_j) = Y\theta_j$ given prices p:

$$\begin{aligned} \theta_j^*(p, \bar{\theta}_j) = \quad & \arg\max_\theta \quad Eu(c_j(\theta), X) \\ & \text{s.t.} \qquad p(\theta - \overline{\theta}_j) \leq 0 \end{aligned}$$

From the FOC and given the price vector p we obtain:[1]

$$\lambda_j^* p_k = \sum_s \pi_s u'(c_j^*(s|p)) y_k(s), \tag{1}$$

for each agent j and asset k; λ_j^* denotes agent j wealth constrain multiplier; $c_j^*(s|p) = \sum_k y_k(s)\theta_j^{*k}(p, \bar{\theta}_j)$, the optimal state s contingent consumption at prices p.

In this setting we **define the equilibrium** as a collection of portfolio choices and prices $\{(\theta_1^*, ..., \theta_J^*); p^*\}$ such that (i) $\theta_j^* = \theta_j^*(p^*, \bar{\theta}_j)$ for all j and (ii) financial markets clear: $\sum_j \theta_j^* = \bar{\theta}$.

In equilibrium, for each Arrow-Debreu *pure* security with price ψ_s and payoffs $y_s(s) = 1$, $y_s(s') = 0$ for all $s' \neq s$, equation (1) becomes:

$$\lambda_j^* \psi_s = \pi_s u'(c_j^*(s)), \tag{2}$$

with $c_j^*(s) = c_j^*(s|p^*)$ the state s contingent consumption in equilibrium. For asset K (the bond), $\lambda_j^* = (1+r)\sum_s \pi_s u'(c_j^*(s)) = (1+r)E_\pi(u'(c_j^*))$. Hence, under the assumption of positive marginal utility of consumption, $\lambda_j^* > 0$ for all j. Equation (2) can be rewritten as:

[1] To simplify the notation, we drop hereafter the X from the utility functions.

$$\psi_s = \frac{1}{1+r} \frac{\pi_s \, u'(c_j^*(s))}{E_\pi(u'(c_j^*))}. \tag{3}$$

Proposition 1 *Given the equilibrium* $\{(\theta_1^*, ..., \theta_J^*); p^*\}$*, if we replace the* J *agents with utility function* $u(c_j, X)$*, with* $X = (1/J)\sum_j c_j$*, by a representative agent with utility function*

$$U(C) = \frac{C^{(1-\alpha(1-\gamma))}}{1-\alpha(1-\gamma)} \tag{4}$$

endowed with the economy's aggregate endowment, the equilibrium prices p^* *will not change.*

More concretely, for any endowment distribution $(\bar{\theta}_1, ..., \bar{\theta}_J)$ *and prices* p *the aggregate portfolio demand* $\sum_j \theta_j^*(p, \bar{\theta}_j) = \theta^*(p, \bar{\theta})$*, with* $\theta^*(p, \bar{\theta})$ *the representative agent's portfolio and* $\bar{\theta} = \sum_j \bar{\theta}_j$ *the aggregate endowment supply.*

In equilibrium, $\theta^*(p^*, \bar{\theta}) = \bar{\theta}$ *with* $p_k^* = \sum_s \psi_s y_k(s)$*. State prices are a function of the aggregate endowment,* $\psi_s = \frac{\pi_s}{1+r} \frac{U'(\bar{C}(s))}{E_\pi(U'(\bar{C}))}$*, with* $\bar{C} = Y\bar{\theta}$*.*

Proof: Let's define a *Bergson-Samuelson social welfare function* as a function $W : \Re^J \to \Re$ that assigns a utility value to each possible vector $(u_1, ..., u_J) \in \Re^J$ of utility levels for the J consumers in the economy. We assume that the social welfare function is monotonous non-decreasing and differentiable.

For each level of prices p and aggregate endowment $\bar{\theta}$, let $(\bar{\theta}_1(p, \bar{\theta}), ..., \bar{\theta}_J(p, \bar{\theta}))$ be the endowments distribution with $\sum_j \bar{\theta}_j(p, \bar{\theta}) = \bar{\theta}$ such that the portfolios $(\theta_1^*(p, \bar{\theta}_1), ..., \theta_J^*(p, \bar{\theta}_J))$ solve

$$\begin{aligned} V(p, (\bar{\theta}_1, ..., \bar{\theta}_J)) = \quad & \max_{\theta_1, ... \theta_J} \quad W\left(u(c_1(\theta_1)), ..., u(c_J(\theta_J))\right) \\ & \text{s.t.} \qquad p\left(\sum_j \theta_j - \bar{\theta}\right) \leq 0. \end{aligned}$$

Then we know[2] that the value function $V(p, (\bar{\theta}_1, ..., \bar{\theta}_J))$ is an indirect utility function of a representative consumer with portfolio $\theta^*(p, (\bar{\theta}_1, ..., \bar{\theta}_J)) = \sum_j \theta_j^*(p, \bar{\theta}_j(p, \bar{\theta}))$, the aggregate portfolio demand. Notice that, in general, the representative agent's portfolio depends on the endowments distribution.

Since the payoff matrix Y is non-singular, there is a one-to-one mapping between state-contingent consumption and portfolios. Thus, we can workout

[2] Proposition 4.D.1 in Mas-Colell, Whiston and Green (1995), page 117

directly the representative agent's utility function as a function of aggregate consumption. For any state $s = 1, 2, ..., S$, and aggregate consumption $C(s)$ let us rewrite the social welfare function problem as

$$\begin{aligned} U(C(s)) = \quad & \max_{c_1(s),...,c_J(s)} \quad W\left(u(c_1(s)), ..., u(c_J(s))\right) \\ & \text{s.t.} \quad \sum_j c_j(s) - C(s) \leq 0, \\ & \qquad\quad c_j(s) \geq 0 \text{ for all } j. \end{aligned}$$

The Lagrangian function for the later problem will be:

$$\Phi(c_1(s), ..., c_J(s), \lambda_s) = W\left(u(c_1(s)), ..., u(c_J(s))\right) - \lambda_s \left(\sum_j c_j(s) - C(s) \right).$$

By Khun-Tucker's theorem the global optimal optimum $c_1^*, ..., c_J^*, \lambda_s^* \geq 0$ satisfies, for every agent j and state s:[3]

$$\frac{\partial}{\partial c_j} W\left(u(c_1^*(s)), ..., u(c_J^*(s))\right) - \lambda_s^* \leq 0, \tag{5}$$

$$\left(\frac{\partial}{\partial c_j} W\left(u(c_1^*(s)), ..., u(c_J^*(s))\right) - \lambda_s^* \right) c_j^*(s) = 0, \tag{6}$$

$$\sum_j c_j^*(s) - C(s) \leq 0, \tag{7}$$

$$\left(\sum_j c_j^*(s) - C(s) \right) \lambda_s^* = 0. \tag{8}$$

Assume the consumption is strictly positive for all agents in all states. By condition (6), this implies that the optimality condition (5) is binding and $\lambda_s^* = \frac{\partial}{\partial c_j(s)} W(\cdot) > 0$ for all s. Applying the chain rule to the right-hand side:

$$\lambda_s^* = W^j \, u'(c_j^*(s)), \tag{9}$$

with λ_s^* the multiplier of the aggregate wealth constrain and $W^j = \frac{\partial}{\partial u(c_j(\theta_j))} W(\cdot)$, constant across states.

[3] The function $W(\cdot)$ is monotonous non-decreasing. This plus the concavity of the utility function guarantees that the objective function is quasi-concave.

Replacing $u'(c_j(s)) = \left(\frac{X^\gamma(s)}{c_j(s)}\right)^\alpha$ in (9) and solving for $c_j(s)$ we obtain:

$$c_j^*(s) = X(s)^\gamma \left(\frac{W^j}{\lambda_s^*}\right)^{1/\alpha}. \tag{10}$$

After aggregating over j, we solve for λ_s^*:

$$\lambda_s^* = C(s)^{-\alpha} X(s)^{\alpha\gamma} \left(\sum_j (W^j)^{1/\alpha}\right)^\alpha, \tag{11}$$

where we have used the binding[4] aggregate consumption constraint (7). By the envelope theorem:

$$\frac{\partial}{\partial C(s)} W\left(u(c_1^*(s)), ..., u(c_J^*(s))\right) = \lambda_s^*. \tag{12}$$

Replacing (11) for λ_s^* in (12) and integrating over $C(s)$ we obtain the problem's value function $U(C(s))$:

$$U(C(s)) = \frac{C(s)^{(1-\alpha)}}{1-\alpha} X(s)^{\alpha\gamma} \left(\sum_j (W^j)^{1/\alpha}\right)^\alpha. \tag{13}$$

Take now the definition of $X(s) = (1/J)\sum_j c_j(s)$. By the binding condition (7), $X(s) = (1/J)C(s)$. Therefore, the value function (13) becomes

$$U(C(s)) = \frac{C(s)^{(1-\alpha(1-\gamma))}}{1-\alpha} (1/J)^{\alpha\gamma} \left(\sum_j (W^j)^{1/\alpha}\right)^\alpha.$$

The aggregate agent's utility function (4) is just a monotonous non-decreasing, affine transformation of the later function, hence representing the same preferences. Call $\theta(p, (\bar{\theta}_1, ..., \bar{\theta}_J)) = \sum_j \theta_j(p, \bar{\theta}_j)$ the demand function of the representative agent. The utility function (4) is independent of the endowment distribution; it only depends on the aggregate endowment. Hence, $\theta(p, (\bar{\theta}_1, ..., \bar{\theta}_J)) = \theta(p, \sum_j \bar{\theta}_j) = \theta(p, \bar{\theta})$. In equilibrium, by market clearing, $\theta^* = \theta(p^*, \bar{\theta}) = \bar{\theta}$. Applying equilibrium condition (3) to the representative agent we obtain the state prices as a function of aggregate endowment. *QED*

[4]Since $\lambda_s^* > 0$ and given the optimality condition (8).

2 Robustness of the result: Refining the Joneses

In this section we study whether our representative agent derivation is robust to the inclusion of some *measure of dispersion* in the Joneses definition. Notice that this question cannot be addressed in a symmetric equilibrium where every agent has the same endowment. However, in our setting, portfolio endowments (hence consumption) differ across agents. To see this, just solve for $u'(c_j^*(s))$ in (9) and replace it in (2). We obtain:

$$\frac{W^j}{\lambda_s^*} = \frac{\pi_s}{\psi_s}\frac{1}{\lambda_j^*}.$$

Replacing the later in (10):

$$c_j^*(s) = X(s)^\gamma \left(\frac{\pi_s}{\psi_s}\frac{1}{\lambda_j^*}\right)^{1/\alpha}. \tag{14}$$

λ_j^* is the Lagrange multiplier for the budget constraint in the agent's optimal portfolio problem. Hence, as long as the market value (at the equilibrium prices p^*) of the portfolio endowments is different across agents, λ_j^* (and consumption) will be also different.

Let $\bar{J} \subseteq J$ be a subset (as well as the cardinal) of agents that satisfy certain condition (in terms of portfolio endowment, and hence, consumption) to belong to the Joneses. For instance, $\bar{J} = \{j \in J \text{ such that } \bar{\theta}_l < \bar{\theta}_j < \bar{\theta}_u\}$, for a given lower ($\bar{\theta}_l$) and upper ($\bar{\theta}_u$) bound on portfolio endowments. The following proposition shows that as long as the Joneses are defined as an *average* consumption, in equilibrium, the resulting representative investor has a "Joneses-free" utility function.

Proposition 2 *Let $\bar{X}(s) = (1/\bar{J})\sum_{j\in\bar{J}} c_j^*(s)$, $\bar{J} \subseteq J$, represent the Joneses average consumption under a given dispersion measure. Assume $\bar{J} \neq \emptyset$. Then, the representative agent's utility function in equilibrium is an affine transformation of the "Joneses-free" utility function (4) in Proposition 1.*

Proof: From the (binding) optimality condition (7), $C(s) = \sum_j c_j^*(s)$. Replacing $c_j^*(s)$ from (14) in the later equation we obtain that $C(s) = \sum_{j\in J} \bar{X}(s)^\gamma \left(\frac{\pi_s}{\psi_s}\frac{1}{\lambda_j^*}\right)^{1/\alpha}$. From this equation it follows that

$$\left(\frac{\pi_s}{\psi_s}\right)^{1/\alpha} = C(s)\bar{X}(s)^{-\gamma}\left(\sum_{j\in J}\left(1/\lambda_j^*\right)^{1/\alpha}\right)^{-1}.$$

By definition, $\bar{X}(s) = (1/\bar{J}) \sum_{j\in\bar{J}} \bar{X}(s)^{\gamma} \left(\frac{\pi_s}{\psi_s}\right)^{1/\alpha} \left(1/\lambda_j^*\right)^{1/\alpha}$. Replacing $\left(\frac{\pi_s}{\psi_s}\right)^{1/\alpha}$ from the former equation into the later we obtain:

$$\bar{X}(s) = (1/\bar{J})\, C(s) \sum_{j\in\bar{J}} \left(1/\lambda_j^*\right)^{1/\alpha} \left(\sum_{j\in J} \left(1/\lambda_j^*\right)^{1/\alpha}\right)^{-1}.$$

Therefore, the newly defined Joneses are also proportional, in any state, to the aggregate consumption $C(s)$. Notice that, in the later equation, if $\bar{J} \equiv J$, then $\bar{X}(s) \equiv X(s)$. Replacing $\bar{X}(s)$ into equation (13) the proof is complete. *Q.E.D.*

After this proposition, it is very simple to show that the representative agent's utility function in Proposition 2 is robust to the existence of several Joneses. Assume now that agents belong to $q = \{1, ..., Q\}$ disjoint communities ($Q \geq 2$). Each community has J_q members, so that $\bigcup_q J_q = J$ is the total set of consumers. The preferences of agent j that belongs to community q are represented by the utility function

$$u(c_j, X_q) = \frac{c_j^{(1-\alpha)}}{1-\alpha} X_q^{\gamma\alpha},$$

with $X_q = (1/J_q) \sum_{j\in J_q} c_j$. Let $\theta_j^q(p, \bar{\theta}_j)$ denote the optimal portfolio for agent j in community q.

Corollary 1 *Let* $\{[(\theta_1^q(p^*, \bar{\theta}_1), .., \theta_{J_q}^q(p^*, \bar{\theta}_{J_q})]_{q=1,..,Q}; p^*\}$ *be the equilibrium. If we replace all members in community q with utilities* $u(c_j, X_q)$, *with* $X_q = (1/J_q) \sum_{j\in J_q} q_j$, *by a representative agent with utility function*

$$U(C) = \frac{C^{(1-\alpha(1-\gamma))}}{1-\alpha(1-\gamma)}$$

endowed with all the community's aggregate endowment $\bar{\theta}_q = \sum_{j\in J_q} \bar{\theta}_j$, *the equilibrium prices* p^* *will not change. Additionally, the same equilibrium prices will prevail if all community representative agents are replaced by a single representative agent, with the same CRRA, "Joneses-free" utility function endowed with the overall aggregate endowment* $\bar{\theta} = \sum_q \bar{\theta}_q$.

The proof of this corollary follows trivially after applying Proposition 2 first to all members within each community and then to the representative agents across communities.

3 Conclusion

In this paper, we prove the existence of a representative agent in an economy populated with investors who keep up with the Joneses and have heterogenous portfolio endowments. The representative investor has a "Joneses-free" utility function with lower risk aversion coefficient. This result is independent of the portfolio endowment distribution and robust to the inclusion of a dispersion measure in the Joneses definition.

These results have implications for portfolio holdings and asset pricing. Concretely: (i) no aggregate portfolio bias can be explained *exclusively* on the basis of Joneses behaviour; and (ii) absent any additional friction, the equilibrium asset pricing model in the presence of Joneses behaviour is equivalent to the single factor International CAPM with a (proper) lower global market risk-premium.

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