POMS PRODUCTION AND OPERATIONS MANAGEMENT

PRODUCTION AND OPERATIONS MANAGEMENT POMS
Vol. 0, No. 0, xxxx–xxxx 2013, pp. 1–17
ISSN 1059-1478 | EISSN 1937-5956 | 13 | 00 | 0001
DOI 10.1111/poms.12182

The Impact of Modular Assembly on Supply Chain Efficiency

Tianjun Feng
School of Management, Fudan University, 670 Guoshun Road, 200433, Shanghai, China, tfeng@fudan.edu.cn

Fuqiang Zhang
Olin Business School, Washington University, Simon Hall 236, 1 Brookings Drive, St. Louis, Missouri 63130-4899, USA
fzhang22@wustl.edu

This article studies the impact of modular assembly on supply chain efficiency. In the modular assembly approach, a manufacturer acquires pre-assembled modules from its suppliers, rather than the individual components, as in the traditional assembly approach. We analyze the competitive behavior of a two-stage modular assembly system consisting of a manufacturer, and a supplier who pre-assembles two components into a module. The firms can choose their own inventory policies and we show the existence of Nash equilibrium in the inventory game. Moving from the traditional to the modular approach has a twofold effect on the supply chain. First, we investigate the effect of centralizing the component suppliers. It can be shown that when there is no production time shift, the module supplier always holds more component inventories than suppliers do in the traditional approach, which yields a lower cost for the manufacturer. However, the suppliers, and therefore the supply chain may incur a higher cost in the modular approach. Second, we study the effect of a shift in production time from the manufacturing stage to the supplier stage. From numerical studies, it has been found that such a lead time shift always benefits a centralized supply chain, but not necessarily so for a decentralized system. Combining the two effects, we find that the modular approach generally reduces the cost to the manufacturer and the supply chain, which explains the prevalence of modular assembly from the perspective of inventory management. These results also provide some insight into how firms can improve supply chain efficiency by choosing the right decision structure and lead time configuration.

Key words: modular assembly; supply chains; inventory management; game theory
History: Received: March 2012; Accepted: August 2013 by Jayashankar Swaminathan, after 3 revisions.

1. Introduction

A practice known as modular assembly has recently become a trend in the automobile industry and has attracted attention from both potential practitioners and academics (Bernstein and DeCroix 2004). In the modular approach, manufacturers obtain pre-assembled modules from a reduced base of suppliers, as opposed to the traditional approach in which individual components are procured and assembled by the manufacturer. For example, at the truck and bus plant located in Resende, Brazil, Volkswagen splits a vehicle into several modules and outsources the production and installation of these modules to suppliers (Green 1998). In the modular assembly model used by Nissan for its Mississippi-based truck factory, a handful of outside suppliers deliver assembled vehicle sections for the final assembly line (Chappell 2001). Such a practice has been observed in other manufacturing sectors too. As Boeing prepares to build its proposed fuel-efficient 787 Dreamliner jet, it wants to make the plane so modular that the last stage of assembly can be achieved as quickly as possible (Michaels and Lunsford 2005). In the semiconductor industry, a modular approach is often used when multi-chip models or hybrid is assembled into the final product (McClintick 2003).

Due to the prevalence of such a shift, there have been many discussions in the business press about the benefits of taking the modularization path. A frequently cited cause for the shift by the media is the search for cost reduction. For instance, the initial reaction of many executives to the new system at Volkswagen's Brazilian plant was a suspicion that Volkswagen's primary objective was to cut assembly labor costs, since supplier firms typically pay lower wages than those earned by the automotive assembly line workers (Sheridan 1997). Other quoted reasons for such a shift include reliable quality, reduced assembly time, and improved logistics and inventory management (Benko and McFarlan 2003, Green 1998, Sheridan 1997).

Although some of the above reasons (e.g., reduced labor cost and improved quality) are understandable, the impact of the modular approach on logistics and inventory management remains unclear. From the supply chain's point of view, the modular approach differs from the traditional approach mainly in two aspects: First, the operational decisions regarding multiple components are centralized in the hands of a single supplier; second, a portion of the assembly job is shifted from the final stage to the upstream stage, therefore, the production time at the final stage is reduced. On the basis of this observation, we pose the following research questions in this study.

How does the centralization of decision rights affect the performance of a decentralized assembly system? Compared with the traditional approach, the modular approach actually integrates a portion of the supply chain by allowing a single supplier to manage many different components. In a traditional assembly system, individual component suppliers manage their own inventories while in the modular assembly system a single supplier controls the inventories of a number of component sets. Does a partial integration improve the supply chain performance? If the manufacturer has served as the major driver of the modularization trend, then how does the modular approach affect the manufacturer in terms of logistics and inventory management? This study aims to investigate the impact of the centralization of the suppliers' stocking decisions on the assembly system.

How does the change in lead time configuration affect supply chain efficiency? By taking the modularization direction, the manufacturer actually outsources an increasing part of the assembly job to its suppliers. The modular approach used by Ford has been reported to save money due to reduced production time at the final assembly stage. Boeing plans to modularize its assembly system so that the production time at the last stage is minimized. One may conjecture that the production lead time at the final stage is critical in terms of supply chain efficiency. An equivalent question to ask is, if a firm has the opportunity to re-configure the lead times in its supply chain, which stage should the firm focus on? In particular, does a lead time shift from the downstream stage to the upstream stage always improve supply chain efficiency? Most of the literature on supply chain management searches for optimal policies or coordination contracts given exogenous supply chain configurations. This study tries to shed some light on how to improve supply chain efficiency by optimizing its configuration.

The above questions will be addressed in the rest of the article, which is organized as follows. The next section reviews the literature. Section 3 describes the model setting. Section 4 analyzes the inventory game under the modular approach. The comparison of equilibria between the traditional and modular approaches is given in section 5. Section 6 presents the numerical studies. Section 7 concludes the article. All proofs are given in the Appendix.

2. Literature Review

The literature on multi-echelon inventory models dates back to Clark and Scarf (1960). Their classic work was then followed by Federgruen and Zipkin (1984), on serial systems; Schmidt and Nahmias (1985) and Rosling (1989), on assembly systems; Roundy (1985), on distribution systems; and Chen and Zheng (1994), on general systems. These articles study centralized inventory systems and characterize the optimal policies. The method we use to evaluate a firm's costs in this study is based on the accounting scheme proposed by Chen and Zheng (1994).

There has been a growing interest in studying decentralized supply chains over the past decade. Cachon and Zipkin (1999) study a two-stage inventory system under periodic review. Cachon (2001) considers a distribution system with one supplier and multiple retailers under continuous review. Parker and Kapuscinski (2011) extend this line of research by considering a decentralized serial supply chain with capacity limits at each stage. Several articles have studied decentralized assembly systems, which are closely related to this study. Bernstein and DeCroix (2004) consider a three-tier assembly system in which a manufacturer procures from first-tier subassemblers, who in turn replenish from the second-tier suppliers. They focus on a pricing and capacity game and aim at providing insight into structural decisions such as how to group components to form a subassembly line. Wang and Gerchak (2003) investigate the competitive behavior of assembly systems in a single-period setting. Wang (2006) studies joint pricing and production decisions in supply chains with complementary products and uncertain demand. Jiang and Wang (2010) further explore the effect of competition among component suppliers on a decentralized assembly system with price-sensitive and uncertain demand. Bernstein and DeCroix (2006) and Zhang (2006) consider decentralized assembly systems under periodic review. These two articles essentially study the traditional approach, in which all supply chain members are independent and act in their own interests. Their results can therefore be used as a benchmark for comparison with the modular approach.

There is a stream of research on assemble-to-order systems in which multiple products are assembled from multiple sets of components. See Song and Zipkin (2003) for a review. Hsu et al. (2006) introduce lead time—dependent pricing in the assemble-to-order systems. They develop and analyze an

Please Cite this article in press as: Feng, T., et al. The Impact of Modular Assembly on Supply Chain Efficiency. *Production and Operations Management* (2013), doi:10.1111/poms.12182

optimization model to determine the optimal stocking levels for components in an environment where demand is uncertain and the prices for both components and final product depend on their delivery lead times. Fang et al. (2008) consider a similar but decentralized model and propose a revenue-sharing scheme to coordinate the decisions of independent suppliers.

An important issue in multi-product assembly system is production line design with component commonality. The seminal study by Swaminathan and Tayur (1998) proposes the so-called "vanilla boxes" (i.e., semi-finished products) to exploit component commonality in an assembly system. Based on a two-stage integer program with recourse, Swaminathan and Tayur (1998) analyze the optimal configurations and inventory levels of vanilla boxes, and discuss the impact of various parameters on the performance of the assembly system. Notable follow-up studies along this line include Van Mieghem (2004), Heese and Swaminathan (2006), and Bernstein et al. (2007, 2011). The concept of modular assembly is similar to that of vanilla boxes where common modules can be used to customize different products. However, there is only a single product in our study, so the focus is not on component commonality. Instead, we adopt a game-theoretic model to obtain managerial implications about the modular assembly approach.

Several articles examine the impact of lead time configuration in supply chains. Bollapragada et al. (2004) study a multi-echelon assembly system with random supply capacity. They investigate the impact of reducing component lead times on system efficiency. DeCroix (2013) studies the impact of supplier lead times in an assembly system subject to random supply disruptions. Shang and Song (2007) and Shang (2008) study serial inventory systems under centralized control and find that a lead time reduction at the downstream stage is more beneficial than a lead time reduction at the upstream stage. Our study examines the impact of lead time shift, that is, reducing the downstream's lead time while simultaneously increasing the upstream's lead time, on a decentralized system. It has been shown that the lead time shift may hurt supply chain efficiency.

3. Model Description

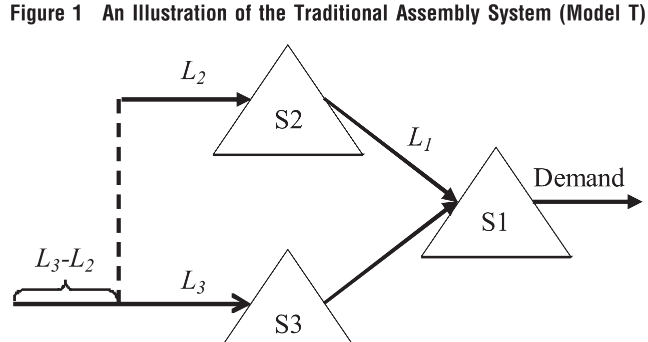
We consider a decentralized assembly system consisting of a manufacturer and two suppliers. The manufacturer assembles a final product using the components provided by the suppliers. Without loss of generality, assume that each final product requires one unit of each component. The suppliers have positive replenishment lead times, which may represent either transportation or production times. There is also a lead time between the suppliers and the manufacturer. This lead time could include the shipping and assembly time. For ease of exposition, we assume that the shipping time is zero so that the manufacturer's lead time is just the assembly time (the results will not change if there is a positive shipping time). Let L_i be the lead time for stage i, i = 1,2,3. Define $\theta(i)$ as the set containing i and all the successors of stage i. Define the total lead time for stage i by $M_i = \sum_{j \in \theta(i)} L_j$. The stages are indexed in such a way that M_i increases in i. That is, denote Stage 1 by S1, Stage 2 by S2, etc. In our problem, S1 is the manufacturer while the others are suppliers.

The system is under periodic review with an infinite horizon. Each stage uses a stationary base stock policy. This assumption is natural for two reasons. The system optimal solution involves stationary base stock policy as it is widely accepted in practice for ease of use. See Zhang (2006) for a brief discussion of inventory games involving non-stationary policies. Denote stage i's local base stock level by $\bar{s}_i$.

Two decision-right structures are compared in this study. Figure 1 depicts the first decision-right structure, or Model T. In this model structure, the two suppliers independently choose their inventory policies. Next, we describe the model setting based on the first decision-right structure (Model T). The modification for the second decision-right structure (Model M) then follows.

For each unit of stock that belongs to stage i, there is an echelon inventory holding cost h_i per period. A standard assumption $h_3 > 0$ and $h_1, h_2 \geq 0$ is used. As a result, S1's installation holding cost is $h = \sum_i h_i$, and S2 and S3's installation holding costs are h_2 and h_3, respectively. That is, a lower stage incurs a higher holding cost due to value added through the process. The suppliers are responsible not only for the inventory on hand but also for the inventory in assembly at the manufacturer. The manufacturer incurs holding cost only for the finished products. Under this assumption, the suppliers only ship matched components (unmatched components cannot enter assembly and the suppliers still incur a holding cost).

Figure 1 An Illustration of the Traditional Assembly System (Model T)



Please Cite this article in press as: Feng, T., et al. The Impact of Modular Assembly on Supply Chain Efficiency. *Production and Operations Management* (2013), doi:10.1111/poms.12182

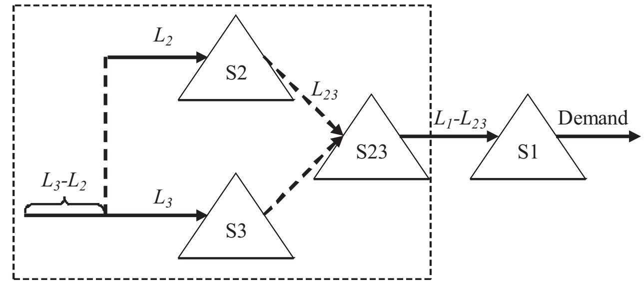
This is equivalent to assume that the manufacturer only orders the minimum of the intended quantity and the number of matched components at the suppliers. There is no setup cost in the system for order placing and processing.

Demand is random and occurs only at S1. Unsatisfied demand is backlogged and there is a backorder cost per unit of backorder per period. In industrial settings where customers are more likely to accept late deliveries, the backorder cost accounts for the loss of goodwill, which has a negative effect on future demand. Under situations where lost sales are more appropriate (e.g., in retail settings), the backorder assumption is mainly for tractability and the backorder cost estimates the impact of lost sales on profit. In both cases, all supply chain members dislike backorders: a lost sale of the final product (either in the current period or in the future) implies a lost sale of the components as well. Therefore, we assume that when a demand is backlogged at the manufacturer, stage i has to incur a backorder cost p_i. Let $p = \sum_i p_i$ and $\alpha_i = p_i/p$. Thus, α_i's can be viewed as the relative extent to which stage i dislikes backorders (or the relative magnitude of the profit margins each firm makes from selling a product). Note that $0 < \alpha_i < 1$ and $\sum_i \alpha_i = 1$. Such a backorder cost assumption has been widely used in the literature when studying decentralized production/inventory systems, including Cachon and Zipkin (1999), Caldentey and Wein (2003), Bernstein and DeCroix (2006), and Zhang (2006).

Zhang (2006) studies Model T under two different information schemes. In the first information scheme, the two suppliers do not share any information on their pipeline inventories. In the second, the suppliers can see each other's pipeline inventories and therefore (especially the supplier with a shorter lead time) can adopt a contingent stocking policy. Details about the contingent policy are provided in section 1. In our study, unless otherwise specified, we refer to Model T as the one with information sharing. This is because the objective of the study is to single out the impact of centralizing the suppliers. Numerical experiments show that if Model T without information sharing is used as the benchmark, then the benefit of centralizing the suppliers would be even greater for the manufacturer. This is consistent with the finding in Zhang (2006) that the manufacturer always benefits from information sharing between the suppliers in Model T. See section 5 for more discussion.

Figure 2 shows the structure for Model M, or the second decision-right structure. In this scenario, the two suppliers are integrated into a module supplier, which pre-assembles the module for the final product. It is assumed that by adopting the modular approach, there is a lead time shift (denoted by

Figure 2 An Illustration of the Modular Assembly System (Model M)



$L_{23} \geq 0$) from the manufacturer stage to the module supplier stage. As a result, the production time at the module supplier will be increased by L_{23}, that is, there is an assembly time L_{23} for the module at the supplier stage. The module supplier will use a base stock level $\bar{s}_{23}$ for the module component, that is, not all the matched components should enter the module assembly. The manufacturer's lead time in turn reduces to $L_1 - L_{23}$. The inventory holding cost for the module is h_{23}, which satisfies $h_2 + h_3 \leq h_{23} < h_1 + h_2 + h_3$. That is, due to the shift of some value-added activities from the manufacturer to the module supplier, a module's holding cost is greater than or equal to the sum of holding costs of individual components, but it is smaller than the inventory holding cost of the final product at the manufacturer. Finally, in Model M, the backorder cost at the module supplier stage is assumed to be $(1 - \alpha_1)p$.

The timing of events in each period is as follows (for both decision-right structures): (i) Inbound shipments are received at each stage; (ii) replenishment orders are submitted and outbound shipments are released; (iii) customer demand is realized and unsatisfied demand is backlogged; and finally, (iv) holding and backorder costs are evaluated and charged.

Demand is independently and identically distributed (i.i.d.) across periods. Let D^τ denote the demand over τ periods and define $\mu^\tau = E(D^\tau)$ as its mean (E denotes the expectation operation). Then, D^1 is the one-period demand and D^{L_i} represents the lead time demand for stage i. Let Φ^τ and ϕ^τ be the cumulative distribution function and density function, respectively, of demand over τ periods. Assume that $\Phi^\tau(x)$ is differentiable for all positive integers τ and $\Phi^1(0) = 0$, that is, only positive demand occurs in each period. Define the following mathematical notations for later use: $(x)^+ = \max(x,0)$, $(x)^- = \max(-x,0)$, $x \wedge y = \min(x, y)$, and $x \vee y = \max(x,y)$.

4. Analysis of Modular Assembly System

In a decentralized modular assembly system, the manufacturer and the module supplier are independent firms who act in their own interests. The competitive behavior of such a system can be analyzed as a non-cooperative inventory game. In this study we focus on the local inventory-tracking method, which is easy to use from a practical standpoint. The players are risk neutral and simultaneously choose their own inventory policies.

In the inventory game, the manufacturer chooses its local base stock level $\bar{s}_1$ while the module supplier makes decision on the local base stock levels for the module and the two components, $(\bar{s}_{23}, \bar{s}_2, \bar{s}_3)$. That is, the module supplier has a three-dimensional strategy space. This differs from the majority of the literature on inventory games, where each player controls a single decision variable. Suppose $\bar{s}_i \in \sigma = [0, M]$ for all i, where σ is the strategy space for the base stock levels and M is a large enough number so that the players are not constrained. Once the players have chosen their strategies, the system will run over an infinite horizon and the average cost criterion is used to evaluate players' costs. That is, the players' objective is to minimize their own average costs per period, including the inventory holding cost and backorder cost. All parameters are common knowledge in the inventory game.

Let $\bar{H}_1(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3)$ and $\bar{H}_{23}(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3)$ denote the expected costs of the manufacturer and module supplier per period given that they have chosen strategies $(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3)$ in Model M. Similarly, let $\bar{H}_i(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ denote the player i's (i = 1,2,3) expected cost per period given that the players have chosen strategies $(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ in Model T. In the following analysis we focus on Model M. The best reply mapping for the players is

$$\bar{r}_1(\bar{s}_{23}, \bar{s}_2, \bar{s}_3) = \{\bar{s}_1 \in \sigma \mid \bar{s}_1 \text{ minimizes } \bar{H}_1(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3)\},$$
$$\bar{r}_{23}(\bar{s}_1) = \{(\bar{s}_{23}, \bar{s}_2, \bar{s}_3) \in \sigma \times \sigma \times \sigma | (\bar{s}_{23}, \bar{s}_2, \bar{s}_3) \text{ minimizes } \bar{H}_{23}(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3)\}.$$

We search for Nash equilibrium in Model M. A Nash equilibrium is defined as a set of strategies chosen by the players under which no one has a unilateral incentive to deviate. Suppose $(\bar{s}_1^M, \bar{s}_{23}^M, \bar{s}_2^M, \bar{s}_3^M)$ is a Nash equilibrium in Model M. By definition, there is $\bar{s}_1^M \in \bar{r}_1(\bar{s}_{23}^M, \bar{s}_2^M, \bar{s}_3^M)$ and $(\bar{s}_{23}^M, \bar{s}_2^M, \bar{s}_3^M) \in \bar{r}_{23}(\bar{s}_1^M)$.

4.1. Cost Structure

As preparation for game analysis, we present the cost functions for the players in this section. The analysis is based on the cost evaluation approach in Chen and Zheng (1994). In period t, let $I_{i,t}$ denote the echelon inventory at stage i, $IL_{i,t}$ ($IL_{i,t}^-$) stands for the ending (beginning) echelon inventory level at stage i, and $IP_{i,t}$ denotes the echelon inventory position at stage i. At the end of each period, the players incur both inventory holding cost and backorder cost. Define

$$\hat{G}_1(x) = h(x)^+ + \alpha_1 p(x)^- = hx + (h + \alpha_1 p)(x)^-.$$

Let y be the manufacturer's inventory position in period t, then its expected cost in period t + (L_1–L_{23}) is

$$\begin{aligned} G_1(y) &= E[\hat{G}_1(y - D^{L_1 - L_{23}+1})] \\ &= h(y - \mu^{L_1 - L_{23}+1}) \\ &\quad + (h + \alpha_1 p) \int_y^{\infty} (x - y)\phi^{L_1 - L_{23}+1}(x)dx. \end{aligned}$$

It is straightforward to show that $G_1(y)$ is strictly convex in y. Note that the manufacturer's inventory position is constrained by the module supplier's on-hand inventory: In each period, the manufacturer would like to bring its inventory position to the base stock level, but the module supplier may fail to make immediate deliveries. Specifically, if the inventory positions for the two components are $\bar{s}_2$ and $\bar{s}_3$ in periods $t - L_2$–L_{23} and $t - L_3$–L_{23}, respectively, then the module's inventory position in period $t - L_{23}$ is

$$IP_{23,t-L_{23}} = \bar{s}_{23} \wedge (\bar{s}_{23} + \bar{s}_2 - D^{L_2}) \wedge (\bar{s}_{23} + \bar{s}_3 - D^{L_3}),$$

and the manufacturer's inventory position in period t is

$$\begin{aligned} IP_{1,t} = \bar{s}_1 \wedge (\bar{s}_1 + IP_{23,t-L_{23}} - D^{L_{23}}) = \bar{s}_1 \\ \wedge (\bar{s}_1 + \bar{s}_{23} - D^{L_{23}}) \wedge (\bar{s}_1 + \bar{s}_{23} + \bar{s}_2 - D^{L_2} - D^{L_{23}}) \\ \wedge (\bar{s}_1 + \bar{s}_{23} + \bar{s}_3 - D^{L_3} - D^{L_{23}}). \end{aligned}$$

As a result, the manufacturer's expected cost in period t + (L_1 – L_{23}) is given by

$$\begin{aligned} \bar{H}_1(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3) = E[G_1(\bar{s}_1 \wedge (\bar{s}_1 + \bar{s}_{23} - D^{L_{23}}) \\ \wedge (\bar{s}_1 + \bar{s}_{23} + \bar{s}_2 - D^{L_2} - D^{L_{23}}) \\ \wedge (\bar{s}_1 + \bar{s}_{23} + \bar{s}_3 - D^{L_3} - D^{L_{23}})]. \end{aligned}$$

Define $G(x) = pE(x - D^{L_1 - L_{23} + 1})^-$ as the system's backorder cost in period $t + L_1 - L_{23}$. It can be readily shown that $G' < 0$ and $G'' > 0$. The above cost function can be then re-written as

$$\begin{aligned}\bar{H}_1(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3) = &-h\mu^{L_1 - L_{23}+1}\\ &+ hE[\bar{s}_1 \wedge (\bar{s}_1 + \bar{s}_{23} - D^{L_{23}})\\ &\wedge (\bar{s}_1 + \bar{s}_{23} + \bar{s}_2 - D^{L_2} - D^{L_{23}})\\ &\wedge (\bar{s}_1 + \bar{s}_{23} + \bar{s}_3 - D^{L_3} - D^{L_{23}})]\\ &+ (h/p + \alpha_1)E[G(\bar{s}_1\\ &\wedge (\bar{s}_1 + \bar{s}_{23} - D^{L_{23}})\\ &\wedge (\bar{s}_1 + \bar{s}_{23} + \bar{s}_2 - D^{L_2} - D^{L_{23}})\\ &\wedge (\bar{s}_1 + \bar{s}_{23} + \bar{s}_3 - D^{L_3} - D^{L_{23}}))].\end{aligned}$$

Without loss of generality, we index the stages such that $L_2 \le L_3$. In this study, we present the analysis for the case $L_2 < L_3$, which is more important and interesting. When $L_2 = L_3$, the module supplier will choose $\bar{s}_2 = \bar{s}_3$, and the same analysis for $L_2 < L_3$ still applies with slight modification. For convenience, we refer to the component with lead time L_2 as a short lead time component and the component with L_3 as a long lead time component. Let $l_3 = L_3 - L_2$. Since there is a positive difference in the components' lead times, when the module supplier makes its ordering decision for the short lead time component in period $t - L_2 - L_{23}$, it has already observed the demand D^{l_3} that occurred during the time interval from period $t - L_3 - L_{23}$ to period $t - L_2 - L_{23} - 1$. So the module supplier knows that in period $t - L_{23}$, the number of pairs of components that can enter for assembly will be constrained by $\bar{s}_3 - D^{l_3}$. Therefore, in period $t - L_2 - L_{23}$, the module supplier should bring the inventory position to $\bar{s}_2 \wedge (\bar{s}_3 - D^{l_3})$ instead of $\bar{s}_2$. That is, the optimal ordering policy for the short lead time component is contingent on the pipeline inventory status of the long lead time component. Consequently, the module supplier's inventory on hand for the short lead time component in period $t - L_{23}$ is

$$I_{2,t-L_{23}} = [\bar{s}_2 \wedge (\bar{s}_3 - D^{l_3}) - D^{L_2}]^+.$$

Note that the module supplier may have to hold some of the long lead time component due to a lack of the short lead time component needed for matching. The inventory on hand for the long lead time component in period $t - L_{23}$ can be shown to be

$$\begin{aligned}I_{3,t-L_{23}} &= (\bar{s}_3 - D^{l_3}) - \bar{s}_2 \wedge (\bar{s}_3 - D^{l_3}) \wedge D^{L_2}\\ &= [(\bar{s}_3 - \bar{s}_2 - D^{l_3}) \vee (\bar{s}_3 - D^{l_3} - D^{L_2})]^+.\end{aligned}$$

The module supplier's inventory on hand for the module in period $t - L_{23}$ is

$$\begin{aligned}I_{23,t-L_{23}} &= [IP_{23,t-L_{23}} - D^{L_{23}}]^+\\ &= [\bar{s}_{23} \wedge (\bar{s}_{23} + \bar{s}_2 - D^{L_2}) \wedge (\bar{s}_{23} + \bar{s}_3 - D^{L_3})\\ &\quad - D^{L_{23}}]^+.\end{aligned}$$

The module supplier will also incur a backorder cost in period $t + L_1 - L_{23}$: $(1 - \alpha_1)E[G(IP_{1t})]$. Therefore, the module supplier's expected cost per period can be written as

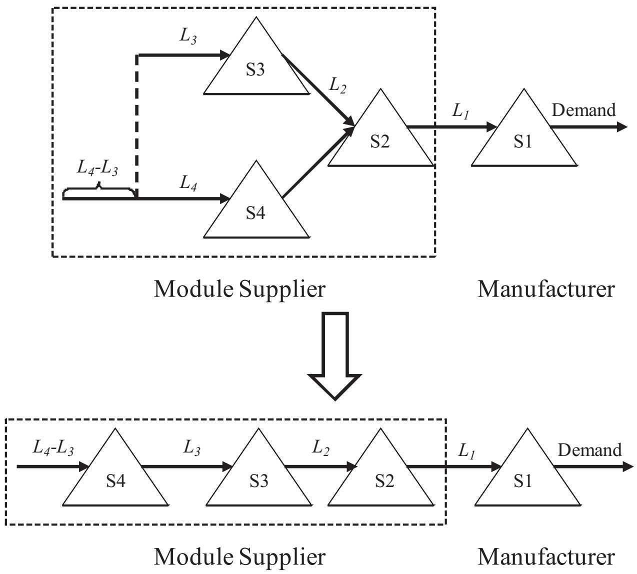
$$\begin{aligned}&\bar{H}_{23}(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3)\\ &= h_{23}\mu^{L_1 - L_{23}} + h_2\mu^{L_{23}} + h_3\mu^{L_{23}} + h_{23}E(I_{23,t}) + h_2E(I_{2,t}) + h_3E(I_{3,t}) + (1 - \alpha_1)E[G(IP_{1,t})]\\ &= h_{23}\mu^{L_1 - L_{23}} + h_2\mu^{L_{23}} + h_3\mu^{L_{23}} + h_{23}E[\bar{s}_{23} \wedge (\bar{s}_{23} + \bar{s}_2 - D^{L_2}) \wedge (\bar{s}_{23} + \bar{s}_3 - D^{L_3}) - D^{L_{23}}]^+\\ &\quad + h_2[\bar{s}_2 \wedge (\bar{s}_3 - D^{l_3}) - D^{L_2}]^+ + h_3E[(\bar{s}_3 - \bar{s}_2 - D^{l_3}) \vee (\bar{s}_3 - D^{l_3} - D^{L_2})]^+\\ &\quad + (1 - \alpha_1)E[G(\bar{s}_1 \wedge (\bar{s}_1 + \bar{s}_{23} - D^{L_{23}}) \wedge (\bar{s}_1 + \bar{s}_{23} + \bar{s}_2 - D^{L_2} - D^{L_{23}})\\ &\quad \wedge (\bar{s}_1 + \bar{s}_{23} + \bar{s}_3 - D^{L_3} - D^{L_{23}}))].\end{aligned} \tag{1}$$

4.2. Analysis of the Inventory Game

This section reports on the analysis of the inventory game. A standard approach for proving the existence of Nash equilibrium is to show that each player's cost function is convex in its strategy (or at least quasiconvex if the strategy is univariate). In our problem, the module supplier's decision consists of three variables and it is challenging to show that the cost function is jointly convex in $(\bar{s}_{23}, \bar{s}_2, \bar{s}_3)$. Instead of proving joint convexity, however, we discover that the complexity of the inventory game can be greatly reduced by using the following procedure. Rosling (1989) and Chen and Zheng (1994) show that an assembly system can be analyzed as a serial system with modified lead times. Specifically, our modular assembly system can be converted into a corresponding serial system as shown in Figure 3.

We will work with the modular supplier's cost in the serial system rather than in the original system. Without causing confusion, we index the four stages as $i = 1,2,3,4$, and use h_i and L_i to denote the echelon holding costs and lead times in Figure 3. Note there is a constant cost difference between the two systems ($h_4\mu^{L_3}$, which only exists in the serial

Figure 3 The Modular Assembly System and Its Equivalent Serial System



system); however, the constant difference does not affect our game analysis. Similar to Chen and Zheng (1994), let B_t denote the customer backorder level at the manufacturer stage in period t, and let $D[t_1,t_2]$ denote the demand during the interval from period t_1 to period t_2 (a closed interval $D(t_1,t_2)$ spans from $t_1 + 1$ to $t_2 - 1$). Following the same logic in Chen and Zheng (1994; section 3.1, p. 1429), the module supplier's per period cost function can be written as

$$\begin{aligned}\bar{H}_{23} &= \sum\nolimits_{i=1}^{N} h_i[I_{i,t} - I_{1,t}] + (1-\alpha_1)pB_t \\ &= \sum\nolimits_{i=1}^{N} h_i IL_{i,t} + (1-\alpha_1)pB_t - H_1 IL_{1,t},\end{aligned}$$

where $H_1 = \sum_{i=1}^{N} h_i$, $IL_{i,t} = I_{i,t} - B_t$, and $N = 4$. For simplicity, later we omit the subscript t and use the cost function $\bar{H}_{23} = \sum_{i=1}^{N} IL_i + (1-\alpha_1)pB_t - H_1 IL_1$.

Define

$$\begin{aligned}G_2(y) =& E[h_2 IL_2 + h_1 IL_1 + (1-\alpha_1)pB - H_1 IL_1] \\ =& E[h_2 IL_2 - (h_2 + h_3 + h_4) IL_1 + (1-\alpha_1)pB].\end{aligned}$$

We can readily show that $G_2(\cdot)$ is convex. Let Υ_2 be a minimum point of $G_2(\cdot)$ and $C_2 = G_2(\Upsilon_2)$. Define

$$\begin{aligned}G_2^2(y) &= \begin{cases} C_2 & \text{if } y \le \Upsilon_2 \\ G_2(y) & \text{otherwise} \end{cases}, \\ G_2^3(y) &= G_2(y) - G_2^2(y),\end{aligned}$$

and

$$G_3(y) = E[h_3 IL_3 + G_2^3(y)].$$

It can be readily shown that both $G_2^3(\cdot)$ and $G_3(\cdot)$ are convex. Let Υ_i be a minimum point of $G_i(\cdot)$ and $C_i = G_i(\Upsilon_i)$. Define

$$\begin{aligned}G_i^i(y) =& \begin{cases} C_i & \text{if } y \le \Upsilon_i \\ G_i(y) & \text{otherwise} \end{cases} \\ G_i^{i+1}(y) =& G_i(y) - G_i^i(y),\end{aligned}$$

and

$$G_{i+1}(y) = E[h_{i+1} IL_{i+1} + G_i^{i+1}(y)].$$

By following Chen and Zheng (1994), we can extend the above definitions inductively to $N = 4$ and show that all $G_i(\cdot)$'s (i = 2,3,4) are convex. Then, we have the following lemma:

LEMMA 1. *For the converted serial system, we have*

$$E\left[\sum\nolimits_{i=1}^{4} h_i IL_i + (1-\alpha_1)pB - H_1 IL_1\right] \ge E\left[\sum\nolimits_{i=2}^{3} G_i^i(IP_i) + G_4(IP_4)\right].$$

In addition, $\sum_{i=2}^{4} C_i$ is a lower bound on the module supplier's cost in the converted serial system, which can be achieve by the minimum points Υ_i (i = 2,3,4).

This lemma suggests that for any given $\bar{s}_1$, the module supplier's cost-minimizing base stock levels $(\bar{s}_2, \bar{s}_3, \bar{s}_4)$ in the serial system can be solved sequentially. Specifically, the module supplier first chooses the unique optimal $\bar{s}_2$ independent of $\bar{s}_3$ and $\bar{s}_4$; then, given $\bar{s}_2$, the supplier chooses the unique optimal $\bar{s}_3$ independent of $\bar{s}_4$; finally, the unique optimal $\bar{s}_4$ is determined. This means we may view the module supplier's cost as a function of $\bar{s}_2$ only (since the rest of the decisions can be uniquely solved given $\bar{s}_2$). Due to the equivalence between the serial and modular systems, this property applies to the modular system as well, that is, our inventory game in Model M reduces to a case where the module supplier only decides on $\bar{s}_{23}$. Now we are in a position to prove the existence of the Nash equilibrium in Model M. The equilibrium will be used to predict the outcome of the competitive supply chain.

THEOREM 2. *There exists a Nash equilibrium in the modular assembly system.*

Although it is challenging to show the uniqueness of equilibrium for Model M, numerical analysis in section 6 indicates that only one equilibrium exists for a wide range of parameter combinations. In addition,

Please Cite this article in press as: Feng, T., et al. The Impact of Modular Assembly on Supply Chain Efficiency. *Production and Operations Management* (2013), doi:10.1111/poms.12182

we will show in section 5 that there exists a unique Nash equilibrium in Model M when the lead time shift is zero ($L_{23} = 0$). Due to the complexity of the module assembly system, it is difficult to compare the equilibria derived from Model M and Model T. However, for the special case with a zero lead time shift, we can obtain analytical results for the equilibrium comparison between the two systems.

5. Comparison of Equilibria

In this section, we analytically compare the equilibria from Model M and Model T for a special case with $L_{23} = 0$. That is, the components can be instantly assembled into modules. In this case, there is $h_{23} = h_2 + h_3$ and the module supplier does not have incentives to hold modules, so the decision space reduces to $(\bar{s}_2, \bar{s}_3)$. The module supplier's expected cost per period in Equation (1) becomes

$$\begin{aligned}\bar{H}_{23}(\bar{s}_1, \bar{s}_2, \bar{s}_3) =& h_2\mu^{L_1} + h_3\mu^{L_1} + h_2E(I_{2,t}) + h_3E(I_{3,t}) \\ &+ (1-\alpha_1)E[G(IP_{1,t})] \\ =& \ h_2\mu^{L_1} + h_3\mu^{L_1} + h_2E[(\bar{s}_2 - D^{L_2}) \\ &\wedge (\bar{s}_3 - D^{l_3} - D^{L_2})]^+ \\ &+ h_3E[(\bar{s}_3 - \bar{s}_2 - D^{l_3}) \vee (\bar{s}_3 - D^{l_3} - D^{L_2})]^+ \\ &+ (1-\alpha_1)E[G(\bar{s}_1 \wedge (\bar{s}_1 + \bar{s}_2 - D^{L_2}) \wedge (\bar{s}_1 \\ &+ \bar{s}_3 - D^{L_3}))].\end{aligned}$$

Similarly, the manufacturer's cost function can be written as

$$\begin{aligned}\bar{H}_1(\bar{s}_1, \bar{s}_2, \bar{s}_3) =& -h\mu^{L_1+1} + hE[\bar{s}_1 \wedge (\bar{s}_1 + \bar{s}_2 - D^{L_2}) \\ &\wedge (\bar{s}_1 + \bar{s}_3 - D^{L_3})] \\ &+ (h/p + \alpha_1)E[G(\bar{s}_1 \wedge (\bar{s}_1 + \bar{s}_2 - D^{L_2}) \\ &\wedge (\bar{s}_1 + \bar{s}_3 - D^{L_3}))].\end{aligned}$$

Then, we have the following properties of the cost functions:

Lemma 3. *Suppose* $L_{23} = 0$. $\bar{H}_1(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ *is strictly convex in* $\bar{s}_1$ *and* $\bar{H}_{23}(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ *is quasiconvex in* $\bar{s}_2$.

Lemma 4. *Suppose* $L_{23} = 0$. *The module supplier's optimal* $\bar{s}_2$ *is independent of* $\bar{s}_3$.

The above lemmas imply that although the module supplier has a two-dimensional strategy space $(\bar{s}_2, \bar{s}_3)$, the stocking decisions can be sequentially made for the two components: Given the manufacturer's base stock level $\bar{s}_1$, the optimal base stock level $\bar{s}_2$ is first chosen; then, given $\bar{s}_1$ and $\bar{s}_2$, the optimal base stock level $\bar{s}_3$ is chosen. Based on Lemma 4, the supplier's best reply function can be written as

$$\bar{r}_2(\bar{s}_1) = (\bar{s}_2(\bar{s}_1), \bar{s}_3(\bar{s}_1, \bar{s}_2(\bar{s}_1))),$$

where $\bar{s}_2(\bar{s}_1)$ and $\bar{s}_3(\bar{s}_1, \bar{s}_2(\bar{s}_1))$ are the optimal stocking levels for the two components, respectively. The next lemma is regarding the firms' best reply functions.

Lemma 5. *Suppose* $L_{23} = 0$. *Then,* (1) *the manufacturer's best reply function* $\bar{s}_1(\bar{s}_2, \bar{s}_3)$ *satisfies* $-1 < \frac{\partial \bar{s}_1(\bar{s}_2, \bar{s}_3)}{\partial \bar{s}_2} < 0$, $-1 < \frac{\partial \bar{s}_1(\bar{s}_2, \bar{s}_3)}{\partial \bar{s}_3} < 0$, *and* $-1 < \frac{d\bar{s}_1(\bar{s}_2 + t, \bar{s}_3 + t)}{dt} < 0$; (2) *the module supplier's best reply functions satisfy* $-1 < \frac{\partial \bar{s}_2(\bar{s}_1, \bar{s}_3)}{\partial \bar{s}_1} < 0$ *and* $-1 < \frac{\partial \bar{s}_3(\bar{s}_1, \bar{s}_2(\bar{s}_1))}{\partial \bar{s}_1} < 0$; (3) *the module supplier's best reply function* $\bar{s}_3(\bar{s}_1, \bar{s}_2)$ *satisfies* $0 < \frac{\partial \bar{s}_3(\bar{s}_1, \bar{s}_2)}{\partial \bar{s}_2} < 1$ *for* $\bar{s}_2 < \bar{s}_2(\bar{s}_1)$; *and* (4) *for any* $\hat{s}_1 \in \sigma$, *the module supplier's best reply function* $\bar{s}_3(\bar{s}_1, \bar{s}_2)$ *satisfies* $-1 < \frac{\partial \bar{s}_3(\bar{s}_1, \bar{s}_2)}{\partial \bar{s}_1} < 0$ *for* $\bar{s}_1 < \hat{s}_1$ *and* $\bar{s}_2 < \bar{s}_2(\hat{s}_1)$.

The next theorem asserts the existence and uniqueness of the Nash equilibrium in this special case of Model M.

Theorem 6. *Suppose* $L_{23} = 0$. *There exists a unique Nash equilibrium in Model M.*

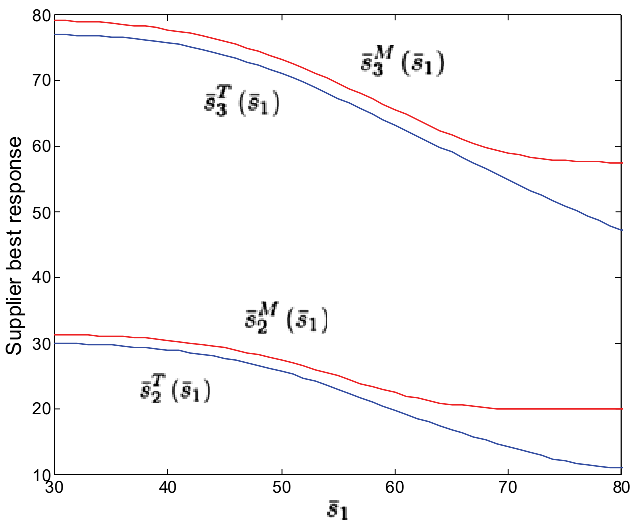
We have shown that there exists a unique Nash equilibrium in Model M with $L_{23} = 0$. The uniqueness of Nash equilibrium in the inventory games of Model T can be found in Zhang (2006). Hereafter, we use the superscripts M and T to denote equilibria in Model M and Model T, respectively. For example, $(\bar{s}_1^M, \bar{s}_2^M, \bar{s}_3^M)$ is the Nash equilibrium in Model M, while $(\bar{s}_1^T, \bar{s}_2^T, \bar{s}_3^T)$ denotes the Nash equilibrium in Model T. The rest of this section compares the Nash equilibria from both models and presents some of the key results of this study.

Theorem 7. *Suppose* $L_{23} = 0$. *In equilibrium, the suppliers will hold more inventory in Model M than in Model T, that is,* $\bar{s}_2^M > \bar{s}_2^T$ *and* $\bar{s}_3^M > \bar{s}_3^T$, *while the opposite is true for the manufacturer, that is,* $\bar{s}_1^M < \bar{s}_1^T$.

From Theorem 7, we know that the manufacturer will hold less, while the suppliers will hold more inventories in the modular approach than in the traditional approach. Under the modular approach, the module supplier can coordinate the inventories for both components, which helps reduce the risk of having mismatched individual components. In this sense, the module supplier is more capable of inventory management and thus has more incentives to hold inventory. To illustrate this finding, we depict the suppliers' best response curves in Figure 4 for a numerical example. In this example, we set $p = 1$, $h_i = 0.01$ $(i = 1,2,3)$, $\alpha_1 = 0.1$,

Please Cite this article in press as: Feng, T., et al. The Impact of Modular Assembly on Supply Chain Efficiency. *Production and Operations Management* (2013), doi:10.1111/poms.12182

Figure 4 Comparison of the Suppliers' Best Responses in Model M and Model T



$\alpha_2 = \alpha_3 = 0.45$, $L_1 = L_2 = 1$, $L_3 = 3$, and the demand in each period follows a normal distribution $N(20,5^2)$, so there is a negligible probability for negative demand to occur. We can see that the curves $\bar{s}_2^M(\bar{s}_1)$ and $\bar{s}_3^M(\bar{s}_1)$ are higher than their counterparts $\bar{s}_2^T(\bar{s}_1)$ and $\bar{s}_3^T(\bar{s}_1)$. This means that compared with the traditional approach, the supplier would use higher base stock levels in equilibrium, and, as a result, the manufacturer would use a lower inventory level.

It is clear that the more inventory the supplier holds, the better off the manufacturer will be. An immediate implication of the above two theorems is that the manufacturer incurs a lower cost in Model M than in Model T when $L_{23} = 0$.

THEOREM 8. *Suppose $L_{23} = 0$. The manufacturer's cost is lower in Model M than in Model T.*

Important managerial insight can be derived from Theorems 7 and 8 on how to manage suppliers efficiently. With $L_{23} = 0$, the benefit of modular assembly is basically from the centralization of the two suppliers. Theorem 7 indicates that the manufacturer will benefit from centralizing the suppliers because they will hold more component inventory in the modular system than in the traditional system. Note that in Model T, depending on whether the two suppliers share the pipeline inventory information, the unique Nash equilibrium can be different. Let $(\bar{s}_1^T, \bar{s}_2^T, \bar{s}_3^T)$ and $(\bar{s}_1^{Tw/o}, \bar{s}_2^{Tw/o}, \bar{s}_3^{Tw/o})$ denote the unique Nash equilibrium in Model T with and without information sharing, respectively. Zhang (2006) shows $\bar{s}_1^T < \bar{s}_1^{Tw/o}$, $\bar{s}_2^T > \bar{s}_2^{Tw/o}$, and $\bar{s}_3^T > \bar{s}_3^{Tw/o}$, which implies that the manufacturer can benefit from promoting information sharing on logistics among suppliers. In this study we suggest another strategy the manufacturer may use to potentially improve the performance relative to Model T with information sharing. That is, the manufacturer can be further better off from centralizing the suppliers' decisions by adopting the modular approach (since $\bar{s}_2^M > \bar{s}_2^T$ and $\bar{s}_3^M > \bar{s}_3^T$). In summary, Theorems 7 and 8 imply that it is a wise strategy for the manufacturer to pursue better coordination of the suppliers, which is consistent with the notion of the modular approach.

6. Numerical Studies

The previous section compares the equilibria from Model M and Model T for the special case $L_{23} = 0$. This section continues to compare the equilibrium outcomes for general assembly systems using numerical studies. The numerical studies in this section are designed to cover a wide range of parameter combinations that may arise in practice. The purpose is to obtain more insight into the effect of the modular approach on the supply chain. We first investigate the impact of centralizing the suppliers. Then, we examine the role of lead time shift. Finally, we combine these two effects to show the aggregate impact of the modular approach.

6.1. The Impact of Centralization of Suppliers

To investigate the impact of supplier centralization, we construct 960 scenarios from all combinations of the following parameters: $p = 1$, $h_i \in \{0.01, 0.05\}$ $(i = 1,2,3)$, $\alpha_1 \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, and

$$(\alpha_2, \alpha_3) \in \left\{ \left(\frac{1-\alpha_1}{3}, \frac{2(1-\alpha_1)}{3}\right), \left(\frac{1-\alpha_1}{2}, \frac{1-\alpha_1}{2}\right), \left(\frac{2(1-\alpha_1)}{3}, \frac{1-\alpha_1}{3}\right) \right\};$$

demand in a single period follows normal distribution $N(20,5^2)$; L_1, L_2 take values in $\{1,2\}$, L_3 takes values in $\{3,6\}$, and $L_{23} = 0$ (there is no lead time shift for now).

Define the benefit as the percentage cost reduction after the suppliers are centralized. Table 1 presents the benefits for the manufacturer, the suppliers, and the supply chain. In all the 960 scenarios, the manufacturer enjoys a lower cost in Model M than in Model T. This result is consistent with Theorems 7 and 8, which state that the suppliers will hold more inventories and the manufacturer will incur lower cost in the modular assembly system. Thus, the centralization of suppliers would help the manufacturer better serve demand at a lower cost. Such a finding seems to corroborate with anecdotal industry evidence. For example, it has been reported recently that Airbus wants some of its suppliers to consolidate. The company believes that such consolidation can create a more

Please Cite this article in press as: Feng, T., et al. The Impact of Modular Assembly on Supply Chain Efficiency. *Production and Operations Management* (2013), doi:10.1111/poms.12182

Table 1 The Impact of Centralization of Suppliers: Percentage Cost Reduction in the Modular Approach Relative to the Traditional Approach

	α_1	Minimum (%)	50th percentile (%)	Maximum (%)	Average (%)
Manufacturer	0.1	0.3	1.5	7.4	1.8
	0.3	1.0	4.3	18	5.0
	0.5	2.2	8.2	28	9.3
	0.7	4.3	10	26	11
	0.9	0.5	3.4	20	5.2
Suppliers	0.1	−2.5	−1.4	−0.5	−1.4
	0.3	−6.2	−3.0	−1.5	−3.1
	0.5	−20	−3.9	−1.7	−4.5
	0.7	−25	−6.0	−2.1	−7.0
	0.9	−14	−2.5	−0.7	−3.3
Supply chain	0.1	−1.9	−1.0	−0.4	−1.0
	0.3	−2.0	−0.9	2.1	−0.8
	0.5	−1.1	0.7	6.1	0.9
	0.7	0.0	1.7	9.8	2.2
	0.9	−0.4	0.8	13	1.9

capable supply chain to catch up with fast growing demand (Pearson and Michaels 2012). Table 1 also reveals that the average cost savings increases in α_1 unless α_1 is extremely large. This implies that centralization of suppliers tends to be more beneficial when the manufacturer incurs a relatively large portion of the system's backorder cost.

However, we can see from Table 1 that the suppliers are worse off in the modular assembly system since all cost reductions are negative. This result is not intuitive because one may think that a centralized supplier is more powerful and should be at least as well off as the decentralized suppliers. As explained in the previous section, the module supplier can coordinate the individual components to reduce mismatch risks; knowing the supplier's improved capabilities, the manufacturer may force the suppliers to hold more inventories by choosing a relatively low stocking level in equilibrium. In other words, the module supplier has to increase inventory levels because of internalized mismatch costs, and the manufacturer takes advantage of this by reducing its own stocking level. Consequently, the module supplier may have to incur a higher operating cost.

It is noteworthy that integrating the suppliers may either increase or decrease the supply chain's total cost. This means that a partial integration does not necessarily improve supply chain performance. In particular, the modular approach tends to decrease supply chain efficiency when α_1 is relatively small, that is, the manufacturer incurs a relatively small backorder cost.

6.2. The Impact of Lead Time Shift

In section 6.1, there is no lead time shift as we focus on the impact of centralizing the suppliers. This section investigates the impact of a lead time shift on the supply chain by using a second numerical study. The same set of parameters as in the previous section are used except the lead time combinations: now we have $L_1 = 6$, $L_2 = 3$, $L_3 \in \{4,6,8\}$, and the lead time shift L_{23} takes values in $\{1,2,3,4\}$. Generally speaking, the module holding cost h_{23} should be increasing in the lead time L_{23}. However, it is clear that a higher module holding cost will make the modular approach less attractive; so we focus on the benchmark scenario with $h_{23} = h_2 + h_3$ in the numerical study. There are 480 scenarios in total and 120 for each value of L_{23}.

Table 2 compares supply chain performance in Model M with and without the lead time shift. We emphasize two observations from the table. First, the performance of the manufacturer improves in the lead time shift L_{23}. This is because the more production time is shifted from the downstream stage to the upstream stage, the more quickly the manufacturer can respond to the market demand.

Second, while the manufacturer is better off, the supplier is worse off in the majority of cases. This is intuitive because the manufacturer's lead time is reduced while the supplier's is increased. As a result, with a lead time shift, the manufacturer holds less inventory and the supplier needs to carry more modules. This is confirmed from the numerical results. What is surprising is that the supply chain performance may deteriorate due to the lead time shift in the decentralized assembly system. How does the lead time shift affect a centralized supply chain? To answer this question, we extend the numerical study to centralized assembly systems. Table 3 presents the percentage cost reduction from the lead time shift for assembly systems under centralized control. We can see that shifting lead time from the downstream stage to the upstream stage always improves system

Table 2 The Impact of Lead Time Shift: Percentage Cost Reduction with a Lead Time Shift L_{23} Relative to without Lead Time Shift (both in Model M)

	L_{23}	Minimum (%)	50th percentile (%)	Maximum (%)	Average (%)
Manufacturer	1	4.4	7.8	21	8.8
	2	4.1	14	22	13
	3	4.8	19	24	18
	4	5.1	25	34	23
Module supplier	1	−7.3	−2.1	3.2	−2.1
	2	−6.8	−1.8	7.3	−1.5
	3	−7.1	−2.0	12	−1.0
	4	−7.4	−2.4	16	−0.7
Supply chain	1	−2.2	0.1	4.7	0.4
	2	−1.5	1.0	8.2	1.6
	3	−0.9	1.6	13	2.8
	4	−0.5	2.2	18	4.0

Please Cite this article in press as: Feng, T., et al. The Impact of Modular Assembly on Supply Chain Efficiency. *Production and Operations Management* (2013), doi:10.1111/poms.12182

Table 3 The Impact of Lead Time Shift: Percentage Cost Reduction with a Lead Time Shift L_{23} in a Centralized Assembly System

L_{23}	Minimum (%)	50th percentile (%)	Maximum (%)	Average (%)
1	0.0	0.3	1.7	0.5
2	0.1	0.7	3.7	1.0
3	0.2	1.2	6.0	1.7
4	0.3	1.8	8.7	2.5

efficiency in this numerical study. This indicates that a fast response time is indeed more important at the stage that is closer to customer demand. This result is consistent with the findings in Shang and Song (2007) and Shang (2008) for centralized serial systems: reducing lead time at the downstream stage is more effective than at the upstream stage. However, their setting is slightly different, that is, they consider a change in lead time at only one stage while we consider simultaneous lead time changes at both stages. Thus, Table 3 provides an important managerial implication for supply chain design: A beneficial change for a centralized system does not necessarily benefit a decentralized system. We also study the impact of lead time shift for serial supply chains and obtain similar results.

6.3. The Impact of Modular Assembly: Combined Results

We have investigated the two effects of the modular approach separately in the previous sections. We are also interested in the aggregate impact of modular assembly on supply chain efficiency. To this end, we numerically examine the percentage cost reduction by comparing Model M with Model T. The same parameter values have been used as in section 6.2. Table 4 presents the results for the comparison between Model M and Model T. We can see that the manufacturer always incurs a lower cost in the modular assembly, while the supplier is worse off in most cases. As to the supply chain, modular assembly generally improves supply chain efficiency except in a few cases. The magnitude of efficiency improvement increases with the length of the lead time shift. In conclusion, modular assembly should be preferred in general; however, the manufacturer may need to provide incentives to the suppliers to implement such an approach.

Table 4 The Impact of Modular Assembly: The Combined Results

	L_{23}	Minimum (%)	50th percentile (%)	Maximum (%)	Average (%)
Manufacturer	1	6.0	10	26	12
	2	5.2	16	29	16
	3	5.8	22	32	21
	4	6.1	29	37	26
Suppliers	1	−9.7	−3.4	3.2	−3.4
	2	−9.6	−3.2	6.7	−2.7
	3	−10	−3.2	11	−2.3
	4	−11	−3.5	16	−2.0
Supply chain	1	−2.7	0.0	7.7	0.1
	2	−1.9	0.9	8.7	1.4
	3	−1.3	1.5	13	2.6
	4	−0.7	2.2	18	3.7

Table 5 reports the aggregate impact of modular assembly on supply chain efficiency for different α_1 values. We make the following two observations. First, on average, when α_1 is not overwhelmingly large, the manufacturer's performance is quite flat in α_1, while the supplier's performance decreases in α_1. Second, the supply chain's performance first decreases in α_1 when α_1 is relatively small, and then increases in α_1 when α_1 is sufficiently large. Note that the reversed pattern is observed for supply chain efficiency when the lead time shift L_{23} is zero (see Table 1). This suggests that without a lead time shift, the manufacturer's cost plays a more important role in supply chain efficiency. By contrast, with $L_{23} > 0$, the module supplier has to hold significantly more inventories due to the combined effects of supplier integration and lead time shift, and thus tends to have a greater impact on supply chain efficiency.

7. Conclusion

The past decade has witnessed a shift in industry from the traditional assembly approach to the so-called modular assembly approach. This article studies the impact of such an increasingly common practice from the perspective of logistics and inventory management. We demonstrate that if there is no lead time shift from the manufacturer stage to the module supplier stage, then the module supplier will hold more component inventories in the modular approach than the suppliers in the traditional approach. Furthermore, the manufacturer's cost is always lower in the modular approach than in the traditional approach. Hence, the modular approach favors the manufacturer from the operations point of view. However, the assembly system may be worse off because the suppliers may incur a higher cost in the modular approach.

Another potential advantage of the modular approach is that it can shift part of the assembly lead time from the final stage to the supplier stage. Through numerical studies, we find that such a lead time shift always benefits a centralized system. By contrast, it may decrease the efficiency of a decentralized supply chain. Therefore, when designing the structure of a supply chain, we need to take into account whether it is under centralized or decentralized control. In particular, reducing the production lead time at the manufacturer by increasing the lead

Table 5 The Impact of Modular Assembly by Varying α_1: the Combined Results

	α_1	Minimum (%)	50th percentile (%)	Maximum (%)	Average (%)
Manufacturer	0.1	6.4	20	34	19.7
	0.3	6.9	20	35	19.7
	0.5	7.6	20	37	19.9
	0.7	8.1	19	34	19.2
	0.9	5.2	15	26	14.6
Suppliers	0.1	−1.3	3.4	16	4.0
	0.3	−5.9	−2.5	2.4	−2.5
	0.5	−9.4	−4.4	−1.9	−4.9
	0.7	−11	−5.4	−2.2	−5.6
	0.9	−8.2	−4.0	−1.3	−4.1
Supply chain	0.1	−0.5	4.5	18	5.3
	0.3	−2.7	0.6	11	1.2
	0.5	−2.7	0.0	8.5	0.6
	0.7	−1.5	0.5	8.8	1.0
	0.9	−0.3	1.2	9.2	1.8

times at the suppliers does not necessarily improve the performance of a decentralized assembly system.

This research can be extended in several directions. In this study, we assume that all players adopt the local base stock policies. Alternatively, they may use echelon base stock policies that require the manufacturer to share its information with the upstream suppliers. It would be interesting to study the value of vertical information sharing in a modular assembly system. In addition, we have focused on a two-tier modular assembly system. When switching to the modular approach, the manufacturer may select a major supplier to serve as the module supplier, who still needs to procure components from the other supplier. That is, the assembly system may consist of three tiers rather than two. The analysis of the three-tier system is more challenging and the impact of the additional tier is still an open question. Finally, in this study there are only two components in the assembly system. A more general setting may involve three or more components. In that case, how to group the components into different modules is also a promising topic for future research.

Acknowledgments

We would like to thank Jayashankar Swaminathan, the Senior Editor and two anonymous referees for their constructive comments who have helped improve the article. This work was supported in part by grants 70901022 and 71371057 from the National Natural Science Foundation of China.

Appendix

Proof of Lemma 1. Since $IL_2 = IP_2 - D[t - M_2, t - M_1]$ and $B = (IL_1)^-$, from the definition of $G_2(\cdot)$, we have

$$E[h_2IL_2 - (h_2 + h_3 + h_4)IL_1 + (1 - \alpha_1)pB] = E[G_2(IP_2)].$$

Since $IP_2 \le IL_3^-$ and $G_2^3(\cdot)$ is non-increasing, we have

$$G_2(IP_2) = G_2^2(IP_2) + G_2^3(IP_2) \ge G_2^2(IP_2) + G_2^3(IL_3^-).$$

Thus,

$$E[h_3IL_3 + h_2IL_2 - (h_2 + h_3 + h_4)IL_1 + (1 - \alpha_1)pB] \ge E[G_2^2(IP_2) + h_3IL_3 + G_2^3(IL_3^-)].$$

Note that $G_2^3(IL_3^-)$ corresponds to the induced penalty cost in Clark and Scarf (1960). Charging this induced penalty cost to stage 3, we have

$$\begin{aligned} E[h_3IL_3 + G_2^3(IL_3^-)] &= E[h_3(IP_3 - D[t - M_3, t - M_2]) \\ &\quad + G_2^3(IP_3 - D[t - M_3, t - M_2))] \\ &= E[G_3(IP_3)]. \end{aligned}$$

Therefore,

$$\begin{aligned} &E[h_3IL_3 + h_2IL_2 - (h_2 + h_3 + h_4)IL_1 \\ &+ (1 - \alpha_1)pB] \ge E[G_2^2(IP_2) + G_3(IP_3)], \end{aligned}$$

and the lemma holds for $N = 3$. A simple induction verifies the lemma for $N = 4$.

Recall that C_i is the minimum value of $G_i(\cdot)$ for $i < N$. It follows from Lemma 1 that

$$\begin{aligned} &E\left[\sum_{i=1}^{4} h_iIL_i + (1 - \alpha_1)pB - H_1IL_1\right] \\ &\ge \sum_{i=2}^{3} C_i + E[G_4(IP_4)]. \end{aligned}$$

In other words, given $IP_4 = y$, the expected system-wide holding and backorder costs charged to period $t - M_4$ under any policy are bounded below by $\sum_{i=2}^{3} C_i + EG_4(IP_4)$. By substituting the latter for the former, the original system reduces to a single-stage system with cost function $\sum_{i=2}^{3} C_i + G_4(y)$. Since $\sum_{i=2}^{3} C_i$ is a constant, the optimal policy for this single-stage system is a base stock policy with minimum cost $\sum_{i=2}^{4} C_i$. Clearly, this is a lower bound on the module supplier's cost in the serial system.

By construction, the minimum points Υ_i (i = 2,3,4) achieves the lower bound cost $\sum_{i=2}^{4} C_i$ for the module supplier. Therefore, they represent the optimal base stock levels that minimize the module supplier's cost. □

Proof of Theorem 2. Lemma 1 suggests that in the inventory game, for any given $\bar{s}_1$, the module supplier's optimal base stock levels $(\bar{s}_{23}, \bar{s}_2, \bar{s}_3)$ can be determined sequentially. Essentially, this means we may view the module supplier's cost as a function of $\bar{s}_{23}$ only (since the rest of the decisions can be uniquely solved given $\bar{s}_{23}$). Therefore, the game reduces to a case where the module supplier only decides on $\bar{s}_{23}$ while the manufacturer decides on $\bar{s}_1$. Next, we show the convexity properties of the players' cost functions.

Since

$$\begin{aligned}
&\bar{H}_1(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3) \\
&= \int_{\bar{s}_3-\bar{s}_2}^{\infty} \phi^{l_3}(x) \int_{\bar{s}_3-x}^{\infty} \phi^{L_2}(y) \int_{\bar{s}_{23}+\bar{s}_3-y-x}^{\infty} \phi^{L_{23}}(z) G_1(\bar{s}_1 + \bar{s}_{23} + \bar{s}_3 - z - y - x) dz dy dx \\
&\quad + \int_0^{\bar{s}_3-\bar{s}_2} \phi^{l_3}(x) \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) \int_{\bar{s}_{23}+\bar{s}_2-y}^{\infty} \phi^{L_{23}}(z) G_1(\bar{s}_1 + \bar{s}_{23} + \bar{s}_2 - z - y) dz dy dx \\
&\quad + \int_0^{\bar{s}_2} \phi^{L_2}(y) \int_0^{\bar{s}_3-y} \phi^{l_3}(x) \int_{\bar{s}_{23}}^{\infty} \phi^{L_{23}}(z) G_1(\bar{s}_1 + \bar{s}_{23} - z) dz dx dy \\
&\quad + \int_0^{\bar{s}_{23}} \phi^{L_{23}}(z) \int_0^{\bar{s}_{23}+\bar{s}_2-z} \phi^{L_2}(y) \int_0^{\bar{s}_{23}+\bar{s}_3-z-y} \phi^{l_3}(x) G_1(\bar{s}_1) dx dy dz
\end{aligned}$$

and $G_1(\cdot)$ is strictly convex, we know that $\bar{H}_1(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3)$ is strictly convex in $\bar{s}_1$.

Taking the second derivative of $\bar{H}_{23}(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3)$ with respect to $\bar{s}_{23}$ gives

$$\begin{aligned}
&\frac{\partial^2 \bar{H}_{23}}{\partial \bar{s}_{23}^2} \\
&= h_{23} \int_{\bar{s}_3-\bar{s}_2}^{\infty} \phi^{l_3}(x) \int_{\bar{s}_3-x}^{\infty} \phi^{L_2}(y) \phi^{L_{23}}(\bar{s}_{23} + \bar{s}_3 - y - x) dy dx \\
&\quad + h_{23} \int_0^{\bar{s}_3-\bar{s}_2} \phi^{l_3}(x) \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) \phi^{L_{23}}(\bar{s}_{23} + \bar{s}_2 - y) dy dx \\
&\quad + h_{23} \int_0^{\bar{s}_2} \phi^{L_2}(y) \int_0^{\bar{s}_3-y} \phi^{l_3}(x) \phi^{L_{23}}(\bar{s}_{23}) dx dy \\
&\quad + (1-\alpha_1) \int_{\bar{s}_3-\bar{s}_2}^{\infty} \phi^{l_3}(x) \int_{\bar{s}_3-x}^{\infty} \phi^{L_2}(y) \int_{\bar{s}_{23}+\bar{s}_3-y-x}^{\infty} \phi^{L_{23}}(z) G''(\bar{s}_1 + \bar{s}_{23} + \bar{s}_3 - z - y - x) dz dy dx \\
&\quad - (1-\alpha_1) \int_{\bar{s}_3-\bar{s}_2}^{\infty} \phi^{l_3}(x) \int_{\bar{s}_3-x}^{\infty} \phi^{L_2}(y) \phi^{L_{23}}(\bar{s}_{23} + \bar{s}_3 - y - x) G'(\bar{s}_1) dy dx \\
&\quad + (1-\alpha_1) \int_0^{\bar{s}_3-\bar{s}_2} \phi^{l_3}(x) \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) \int_{\bar{s}_{23}+\bar{s}_2-y}^{\infty} \phi^{L_{23}}(z) G''(\bar{s}_1 + \bar{s}_{23} + \bar{s}_2 - z - y) dz dy dx \\
&\quad - (1-\alpha_1) \int_0^{\bar{s}_3-\bar{s}_2} \phi^{l_3}(x) \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) \phi^{L_{23}}(\bar{s}_{23} + \bar{s}_2 - y) G'(\bar{s}_1) dy dx \\
&\quad + (1-\alpha_1) \int_0^{\bar{s}_2} \phi^{L_2}(y) \int_0^{\bar{s}_3-y} \phi^{l_3}(x) \int_{\bar{s}_{23}}^{\infty} \phi^{L_{23}}(z) G''(\bar{s}_1 + \bar{s}_{23} - z) dz dx dy \\
&\quad - (1-\alpha_1) \int_0^{\bar{s}_2} \phi^{L_2}(y) \int_0^{\bar{s}_3-y} \phi^{l_3}(x) \phi^{L_{23}}(\bar{s}_{23}) G'(\bar{s}_1) dx dy.
\end{aligned}$$

Since $G' < 0$ and $G'' > 0$, we know $\bar{H}_{23}(\bar{s}_1, \bar{s}_{23}, \bar{s}_2, \bar{s}_3)$ is strictly convex in $\bar{s}_{23}$.

Since both players' (the module supplier and the manufacturer) cost functions are convex, we know that there exists a Nash equilibrium in the modular assembly system by Theorem 1.2 in Fudenberg and Tirole (1991). □

Please Cite this article in press as: Feng, T., et al. The Impact of Modular Assembly on Supply Chain Efficiency. *Production and Operations Management* (2013), doi:10.1111/poms.12182

PROOF OF LEMMA 3. Since

$$\begin{aligned}\bar{H}_1(\bar{s}_1,\bar{s}_2,\bar{s}_3) = &\int_{\bar{s}_3-\bar{s}_2}^{\infty}\phi^{l_3}(x)\int_{\bar{s}_3-x}^{\infty}\phi^{L_2}(y)G_1(\bar{s}_1+\bar{s}_3-y-x)dydx\\ &+\int_{\bar{s}_3-\bar{s}_2}^{\infty}\phi^{l_3}(x)\int_{0}^{\bar{s}_3-x}\phi^{L_2}(y)G_1(\bar{s}_1)dydx\\ &+\int_{0}^{\bar{s}_3-\bar{s}_2}\phi^{l_3}(x)\int_{\bar{s}_2}^{\infty}\phi^{L_2}(y)G_1(\bar{s}_1+\bar{s}_2-y)dydx\\ &+\int_{0}^{\bar{s}_3-\bar{s}_2}\phi^{l_3}(x)\int_{0}^{\bar{s}_2}\phi^{L_2}(y)G_1(\bar{s}_1)dydx\end{aligned} \tag{2}$$

and $G_1(\cdot)$ is strictly convex, $\bar{H}_1(\bar{s}_1,\bar{s}_2,\bar{s}_3)$ is strictly convex in $\bar{s}_1$.

Taking the first derivative of $\bar{H}_{23}(\bar{s}_1,\bar{s}_2,\bar{s}_3)$ with respect to $\bar{s}_2$ gives

$$\begin{aligned}\frac{\partial\bar{H}_{23}}{\partial\bar{s}_2} =& h_2\Phi^{l_3}(\bar{s}_3-\bar{s}_2)\Phi^{L_2}(\bar{s}_2) - h_3\Phi^{l_3}(\bar{s}_3-\bar{s}_2)(1-\Phi^{L_2}(\bar{s}_2))\\ &+(1-\alpha_1)\int_0^{\bar{s}_3-\bar{s}_2}\phi^{l_3}(x)\int_{\bar{s}_2}^{\infty}\phi^{L_2}(y)G'(\bar{s}_1+\bar{s}_2-y)dydx\\ =&\left[\int_0^{\bar{s}_3-\bar{s}_2}\phi^{l_3}(x)dx\right]\left[\begin{array}{c}-h_3+(h_2+h_3)\Phi^{L_2}(\bar{s}_2)\\ +(1-\alpha_1)\int_{\bar{s}_2}^{\infty}\phi^{L_2}(y)G'(\bar{s}_1+\bar{s}_2-y)dy\end{array}\right]\end{aligned} \tag{3}$$

The first part $\int_0^{\bar{s}_3-\bar{s}_2}\phi^{l_3}(x)dx$ is positive for $\bar{s}_2<\bar{s}_3$ and zero for $\bar{s}_2\geq\bar{s}_3$. Let

$$f(\bar{s}_2) = -h_3+(h_2+h_3)\Phi^{L_2}(\bar{s}_2) + (1-\alpha_1)\int_{\bar{s}_2}^{\infty}\phi^{L_2}(y)G'(\bar{s}_1+\bar{s}_2-y)dy,$$

then we have

$$f'(\bar{s}_2) = (h_2+h_3)\phi^{L_2}(\bar{s}_2)+(1-\alpha_1)\int_{\bar{s}_2}^{\infty}\phi^{L_2}(y)G''(\bar{s}_1+\bar{s}_2-y)dy-(1-\alpha_1)\phi^{L_2}(\bar{s}_2)G'(\bar{s}_1).$$

Since $G'<0$ and $G''>0$, there is $f'(\bar{s}_2)>0$ for $\bar{s}_2\geq 0$. Hence, f is strictly increasing. Note that $f(0)=-h_3+(1-\alpha_1)\int_0^{\infty}\phi^{L_2}(y)G'(\bar{s}_1-y)dy<0$ and $f(\infty)=h_2>0$. Consider the equation $f(\bar{s}_2)=0$. Since $f(0)<0$, $f(\infty)>0$, and $f'>0$, the equation has a unique solution, say, x^*. If $x^*\in(0,\bar{s}_3)$, there will be $\partial\bar{H}_{23}/\partial\bar{s}_2<0$ for $\bar{s}_2<x^*$, $\partial\bar{H}_{23}/\partial\bar{s}_2>0$ for $x^*<\bar{s}_2<\bar{s}_3$, and $\partial\bar{H}_{23}/\partial\bar{s}_2=0$ for $\bar{s}_2\geq\bar{s}_3$. This implies that $\bar{H}_{23}(\bar{s}_1,\bar{s}_2,\bar{s}_3)$ is quasiconvex in $\bar{s}_2$. If $x^*\geq\bar{s}_3$, then there will be $\partial\bar{H}_{23}/\partial\bar{s}_2<0$ for $\bar{s}_2<\bar{s}_3$ and $\partial\bar{H}_{23}/\partial\bar{s}_2=0$ for $\bar{s}_2\geq\bar{s}_3$, which also implies that $\bar{H}_{23}(\bar{s}_1,\bar{s}_2,\bar{s}_3)$ is quasiconvex in $\bar{s}_2$. □

PROOF OF LEMMA 4. From Lemma 3, the necessary first-order condition for the optimal $\bar{s}_2$ is given by (3). As in the proof of Lemma 3, let x^* be the unique solution of the equation $f(\bar{s}_2)=0$, where $f(\bar{s}_2)=-h_3+(h_2+h_3)\Phi^{L_2}(\bar{s}_2)+(1-\alpha_1)\int_{\bar{s}_2}^{\infty}\phi^{L_2}(y)G'(\bar{s}_1+\bar{s}_2-y)dy$. We have shown that if $x^*\in(0,\bar{s}_3)$, there will be $\partial\bar{H}_{23}/\partial\bar{s}_2<0$ for $\bar{s}_2<x^*$, $\partial\bar{H}_{23}/\partial\bar{s}_2>0$ for $x^*<\bar{s}_2<\bar{s}_3$, and $\partial\bar{H}_{23}/\partial\bar{s}_2=0$ for $\bar{s}_2\geq\bar{s}_3$. This implies that x^* is the module supplier's optimal choice of $\bar{s}_2$ when $x^*<\bar{s}_3$. If $x^*\geq\bar{s}_3$, we have $\partial\bar{H}_{23}/\partial\bar{s}_2<0$ for $\bar{s}_2<\bar{s}_3$ and $\partial\bar{H}_{23}/\partial\bar{s}_2=0$ for $\bar{s}_2\geq\bar{s}_3$. This also implies that x^* is the module supplier's optimal choice of $\bar{s}_2$. (Note that if $x^*\geq\bar{s}_3$, then any $\bar{s}_2$ greater than or equal to $\bar{s}_3$ is optimal due to the contingent policy). Hence, we can search for the optimal $\bar{s}_2$ using the equation $f(\bar{s}_2)=0$. The module supplier's optimal choice of $\bar{s}_2$ is independent of $\bar{s}_3$ since $f(\bar{s}_2)$ does not involve $\bar{s}_3$.

It is worthwhile pointing out that although the optimal $\bar{s}_2$ may not be unique due to the contingent policy (e.g., when $x^* \geq \bar{s}_3$), the supplier's optimal cost is unique because the delivery is constrained by $\bar{s}_3$. From the cost perspective, we can view x^* as the unique optimal $\bar{s}_2$. □

PROOF OF LEMMA 5. (1) Taking derivatives of $\bar{H}_1(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ gives

$$\frac{\partial^2 \bar{H}_1}{\partial \bar{s}_1 \partial \bar{s}_2} = (h/p + \alpha_1) \int_0^{\bar{s}_3 - \bar{s}_2} \phi^{l_3}(x)dx \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) G''(\bar{s}_1 + \bar{s}_2 - y)dy.$$

Since $G' > 0$, we have $\partial^2 \bar{H}_1/\partial \bar{s}_1 \partial \bar{s}_2 > 0$. From Lemma 3, we have $\partial^2 \bar{H}_1/\partial \bar{s}_1^2 > 0$. It can be readily shown that $\partial^2 \bar{H}_1/\partial \bar{s}_1 \partial \bar{s}_2 < \partial^2 \bar{H}_1/\partial \bar{s}_1^2$. Hence, we have

$$-1 < \frac{\partial \bar{s}_1(\bar{s}_2, \bar{s}_3)}{\partial \bar{s}_2} = -\frac{\partial^2 \bar{H}_1/\partial \bar{s}_1 \partial \bar{s}_2}{\partial^2 \bar{H}_1/\partial \bar{s}_1^2} < 0.$$

Similarly, we can show $-1 < \frac{\partial \bar{s}_1(\bar{s}_2, \bar{s}_3)}{\partial \bar{s}_3} < 0$ and $-1 < \frac{d\bar{s}_1(\bar{s}_2 + t, \bar{s}_3 + t)}{dt} < 0$.

(2) From Lemma 4, the optimal $\bar{s}_2$ can be determined by using the following equation:

$$f(\bar{s}_2) = -h_3 + (h_2 + h_3)\Phi^{L_2}(\bar{s}_2) + (1 - \alpha_1) \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) G'(\bar{s}_1 + \bar{s}_2 - y)dy = 0.$$

Thus, we have

$$\frac{\partial \bar{s}_2(\bar{s}_1, \bar{s}_3)}{\partial \bar{s}_1} = \frac{\partial \bar{s}_2(\bar{s}_1)}{\partial \bar{s}_1} = -\frac{\partial f/\partial \bar{s}_1}{\partial f/\partial \bar{s}_2},$$

where $\partial f/\partial \bar{s}_1 > 0$, $\partial f/\partial \bar{s}_2 > 0$, and $\partial f/\partial \bar{s}_1 < \partial f/\partial \bar{s}_2$. It follows that $-1 < \frac{\partial \bar{s}_2(\bar{s}_1, \bar{s}_3)}{\partial \bar{s}_1} < 0$.

Taking derivatives of $\bar{H}_{23}(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ gives

$$\frac{\partial^2 \bar{H}_{23}}{\partial \bar{s}_1 \partial \bar{s}_3} = (1 - \alpha_1) \int_{\bar{s}_3 - \bar{s}_2}^{\infty} \phi^{l_3}(x) \int_{\bar{s}_3 - x}^{\infty} \phi^{L_2}(y) G''(\bar{s}_1 + \bar{s}_3 - y - x)dydx.$$

Note that $G' < 0$, $G'' > 0$, and $\partial^2 \bar{H}_{23}/\partial \bar{s}_1 \partial \bar{s}_3 > 0$. Taking the second derivative of $\bar{H}_{23}(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ with respect to $\bar{s}_3$ gives

$$\begin{aligned}
\frac{\partial^2 \bar{H}_{23}}{\partial \bar{s}_3^2} =& h_2 \int_{\bar{s}_3 - \bar{s}_2}^{\infty} \phi^{L_2}(\bar{s}_3 - x)\phi^{l_3}(x)dx + h_3 \int_0^{\bar{s}_2} \phi^{L_2}(x)\phi^{l_3}(\bar{s}_3 - x)dx \\
&+ (1 - \alpha_1) \int_{\bar{s}_3 - \bar{s}_2}^{\infty} \phi^{l_3}(x) \int_{\bar{s}_3 - x}^{\infty} \phi^{L_2}(y) G''(\bar{s}_1 + \bar{s}_3 - y - x)dydx \\
&- (1 - \alpha_1) \int_{\bar{s}_3 - \bar{s}_2}^{\infty} \phi^{l_3}(x)\phi^{L_2}(\bar{s}_3 - x) G'(\bar{s}_1)dx \\
&- \phi^{l_3}(\bar{s}_3 - \bar{s}_2)[-h_3 + (h_2 + h_3)\Phi^{L_2}(\bar{s}_2) + (1 - \alpha_1) \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) G'(\bar{s}_1 + \bar{s}_2 - y)dy].
\end{aligned}$$

When $\bar{s}_2$ is optimally chosen, the last term is equal to zero. Note that $h_2 + h_3 \geq h_3 > 0$, $G' < 0$, and $G'' > 0$. Thus, we have $\partial^2 \bar{H}_{23}/\partial \bar{s}_3^2 > 0$ and $\partial^2 \bar{H}_{23}/\partial \bar{s}_1 \partial \bar{s}_3 < \partial^2 \bar{H}_{23}/\partial \bar{s}_3^2$ since $\bar{s}_2$ is optimally chosen. The result $-1 < \frac{\partial \bar{s}_3(\bar{s}_1, \bar{s}_2(\bar{s}_1))}{\partial \bar{s}_1} < 0$ follows.

(3) Taking derivatives of $\bar{H}_{23}(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ gives

$$\frac{\partial^2 \bar{H}_{23}}{\partial \bar{s}_2 \partial \bar{s}_3} = \phi^{l_3}(\bar{s}_3 - \bar{s}_2)[-h_3 + (h_2 + h_3)\Phi^{L_2}(\bar{s}_2) + (1 - \alpha_1) \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) G'(\bar{s}_1 + \bar{s}_2 - y)dy].$$

From the proof of Lemma 4, we know that $f'(\bar{s}_2) < 0$ for $\bar{s}_2 < \bar{s}_2(\bar{s}_1) = x^*$. This implies that $\partial^2 \bar{H}_{23}/\partial \bar{s}_2 \partial \bar{s}_3 < 0$ and $\partial^2 \bar{H}_{23}/\partial \bar{s}_3^2 > 0$ for $\bar{s}_2 < \bar{s}_2(\bar{s}_1)$ ($\partial^2 \bar{H}_{23}/\partial \bar{s}_3^2$ is given in (2)). The result follows by noticing that $|\partial^2 \bar{H}_{23}/\partial \bar{s}_2 \partial \bar{s}_3| < |\partial^2 \bar{H}_{23}/\partial \bar{s}_3^2|$ for $\bar{s}_2 < \bar{s}_2(\bar{s}_1)$.

(4) Consider $\partial^2 \bar{H}_{23}/\partial \bar{s}_1 \partial \bar{s}_3$ and $\partial^2 \bar{H}_{23}/\partial \bar{s}_3^2$. By the proof of (3) above, there will be $\partial^2 \bar{H}_{23}/\partial \bar{s}_3^2 > 0$ for any $\bar{s}_1 < \hat{s}_1$ if $\bar{s}_2 < \bar{s}_2(\bar{s}_1)$. We have shown that $\partial \bar{s}_2(\bar{s}_1)/\partial \bar{s}_1 < 0$, so $\bar{s}_2 < \bar{s}_2(\hat{s}_1)$ implies $\bar{s}_2 < \bar{s}_2(\bar{s}_1)$. It can be shown that $\partial^2 \bar{H}_{23}/\partial \bar{s}_1 \partial \bar{s}_3 > 0$ and $\partial^2 \bar{H}_{23}/\partial \bar{s}_1 \partial \bar{s}_3 < \partial^2 \bar{H}_{23}/\partial \bar{s}_3^2$ for $\bar{s}_2 < \bar{s}_2(\bar{s}_1)$. The result then follows. □

PROOF OF THEOREM 6. First, we show the existence of a Nash equilibrium in the inventory game. From Lemma 3, $\bar{H}_1(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ is strictly convex in $\bar{s}_1$. Lemma 4 implies that the supplier's action reduces to determining $\bar{s}_3$ only. From Lemma 5, we know that $\partial^2 \bar{H}_{23}/\partial \bar{s}_3^2 > 0$ when $\bar{s}_2$ is optimally chosen. Therefore, there exists a Nash equilibrium in the inventory game by Theorem 1.2 in Fudenberg and Tirole (1991).

Next, we prove the uniqueness of the Nash equilibrium. Since the optimal $\bar{s}_2$ is independent of $\bar{s}_3$, the uniqueness is established if we can show that the best reply mapping $\bar{s}_1(\bar{s}_2, \bar{s}_3)$ and $\bar{s}_3(\bar{s}_1, \bar{s}_2(\bar{s}_1))$ represents a contraction mapping. By Lemma 5, there are $-1 < \frac{\partial \bar{s}_1(\bar{s}_2,\bar{s}_3)}{\partial \bar{s}_3} < 0$ and $-1 < \frac{\partial \bar{s}_3(\bar{s}_1,\bar{s}_2(\bar{s}_1))}{\partial \bar{s}_1} < 0$, and we know that it is a contraction mapping (see Cachon and Netessine 2003). □

PROOF OF THEOREM 7. Imagine the process of searching for $(\bar{s}_1^M, \bar{s}_2^M, \bar{s}_3^M)$, the Nash equilibrium for the LI game in Model M. Suppose we start from $(\bar{s}_1^{(0)}, \bar{s}_2^{(0)}, \bar{s}_3^{(0)}) = (\bar{s}_1^T, \bar{s}_2^T, \bar{s}_3^T)$, the Nash equilibrium for the LI game in Model T. According to Theorem 6, the best reply functions form a contraction mapping and the process will finally converge to $(\bar{s}_1^M, \bar{s}_2^M, \bar{s}_3^M)$ by iteration. In the kth iteration (k = 1,2,…), we specify:

$$(\bar{s}_1^{(k)}, \bar{s}_2^{(k)}, \bar{s}_3^{(k)}) = (\bar{s}_1(\bar{s}_2^{(k-1)}, \bar{s}_3^{(k-1)}), \bar{s}_2(\bar{s}_1^{(k-1)}), \bar{s}_3(\bar{s}_1^{(k-1)}, \bar{s}_2^{(k-1)})).$$

□

Consider k = 1. There is $\bar{s}_1^{(1)} = \bar{s}_1^T$ since the manufacturer's cost function is the same in Model M and Model T. Let $\bar{H}_2^T$ be S2's cost function in Model T and then we have

$$\frac{\partial \bar{H}_{23}}{\partial \bar{s}_2} = \left[\int_0^{\bar{s}_3-\bar{s}_2} \phi^{l_3}(x)dx][-h_3 + (h_2+h_3)\Phi^{L_2}(\bar{s}_2) + (1-\alpha_1)\int_{\bar{s}_2}^{\infty} \phi^{L_2}(y)G'(\bar{s}_1+\bar{s}_2-y)dy\right],$$

$$\frac{\partial \bar{H}_2^T}{\partial \bar{s}_2} = \left[\int_0^{\bar{s}_3-\bar{s}_2} \phi^{l_3}(x)dx][h_2\Phi^{L_2}(\bar{s}_2) + \alpha_2\int_{\bar{s}_2}^{\infty} \phi^{L_2}(y)G'(\bar{s}_1+\bar{s}_2-y)dy\right].$$

It is straightforward to show that $\partial \bar{H}_{23}/\partial \bar{s}_2 < \partial \bar{H}_2^T/\partial \bar{s}_2$ for $\bar{s}_2 \le \bar{s}_3$, which leads to $\bar{s}_2^{(1)} = \bar{s}_2(\bar{s}_1^T) > \bar{s}_2^T$.

Next, we compare $\bar{s}_3^{(1)}$ and $\bar{s}_3^{(0)} = \bar{s}_3^T$. Let $\bar{H}_3^T$ be S3's cost function in Model T. Then, we have $\partial \bar{H}_{23}/\partial \bar{s}_3 = \partial \bar{H}_2^T/\partial \bar{s}_3 + \partial \bar{H}_3^T/\partial \bar{s}_3$. Manipulation shows that

$$\begin{aligned}\frac{\partial \bar{H}_2^T}{\partial \bar{s}_3} =& h_2\int_{\bar{s}_3-\bar{s}_2}^{\infty} \Phi^{L_2}(\bar{s}_3-x)\phi^{l_3}(x)dx \\ &+ \alpha_2\int_{\bar{s}_3-\bar{s}_2}^{\infty} \phi^{l_3}(x)\int_{\bar{s}_3-x}^{\infty}\phi^{L_2}(y)G'(\bar{s}_1 + \bar{s}_3 - y - x)dydx \\ &< h_2\Phi^{L_2}(\bar{s}_2) + \alpha_2\int_{\bar{s}_2}^{\infty}\phi^{L_2}(y)G'(\bar{s}_1+\bar{s}_2-y)dy = 0,\end{aligned}$$

where the inequality is by replacing x by $\bar{s}_3 - \bar{s}_2$ in $\Phi^{L_2}(\bar{s}_3 - x)$ and $G'(\bar{s}_1 + \bar{s}_3 - y - x)$ (note that $G' < 0$ and $G'' > 0$), and the last equality is from S2's first-order condition because $\bar{s}_2 = \bar{s}_2^T$ is optimally chosen in Model T. Therefore, we have $\partial \bar{H}_{23}/\partial \bar{s}_3 < \partial \bar{H}_3^T/\partial \bar{s}_3$ for all $\bar{s}_3$ given $\bar{s}_2 = \bar{s}_2^T$. This implies that $\bar{s}_3^{(1)} = \bar{s}_3(\bar{s}_1^T, \bar{s}_2^T) > \bar{s}_3^T$.

Now consider k = 2. By (1) of Lemma 5, there will be $\bar{s}_1^{(2)} < \bar{s}_1^{(1)}$ since both $\bar{s}_2$ and $\bar{s}_3$ have increased in the first iteration. For the supplier, we have $\bar{s}_2^{(2)} = \bar{s}_2(\bar{s}_1^{(1)}) = \bar{s}_2(\bar{s}_1^{(0)}) = \bar{s}_2^{(1)}$ and $\bar{s}_3^{(2)} = \bar{s}_3(\bar{s}_1^{(1)}, \bar{s}_2^{(1)}) > \bar{s}_3^{(1)}$ by (3) of Lemma 5 (i.e., $\bar{s}_2^T = \bar{s}_2^{(0)} < \bar{s}_2(\bar{s}_1^T) = \bar{s}_2^{(1)}$ and $\bar{s}_1$ remains the same in the first iteration). When k = 3, there will be $\bar{s}_1^{(3)} < \bar{s}_1^{(2)}$ by (1) of Lemma 5, $\bar{s}_2^{(3)} > \bar{s}_2^{(2)}$ by (2) of Lemma 5, and $\bar{s}_3^{(3)} > \bar{s}_3^{(2)}$ by (4) of Lemma 5 (note that in the second iteration, $\bar{s}_1$ has been decreased, $\bar{s}_2$ remains unchanged, and $\bar{s}_3$ has been increased).

The iteration process continues until the unique equilibrium $(\bar{s}_1^M, \bar{s}_2^M, \bar{s}_3^M)$ is reached. As we can see, in each iteration, the manufacturer's base stock level decreases (or at least never increases) and the supplier's base stock levels for both components increase (or at least never decrease). Therefore, there must be $\bar{s}_1^M < \bar{s}_1^T$, $\bar{s}_2^M > \bar{s}_2^T$, and $\bar{s}_3^M > \bar{s}_3^T$.

PROOF OF THEOREM 8. Theorem 7 shows that $\bar{s}_2^M > \bar{s}_2^T$ and $\bar{s}_3^M > \bar{s}_3^T$ hold in equilibrium. Note that the manufacturer's cost is given by

$$\bar{H}_1(\bar{s}_1, \bar{s}_2, \bar{s}_3) = E[G_1(\bar{s}_1 \wedge (\bar{s}_1 + \bar{s}_2 - D^{L_2}) \wedge (\bar{s}_1 + \bar{s}_3 - D^{L_3}))],$$

which is the same in both models. Thus, the proof is complete if we can show that $\bar{H}_1(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ is decreasing in $\bar{s}_2$ and $\bar{s}_3$, given that $\bar{s}_1$ is optimally chosen by the manufacturer. The first-order condition for optimal $\bar{s}_1$ is $\partial \bar{H}_1/\bar{s}_1 = 0$ ($\bar{H}_1$ is given in Equation (2)), which can be decomposed to be

$$\partial \bar{H}_1/\bar{s}_1 = \begin{cases} G_1'(\bar{s}_1) & \text{if } \bar{s}_1 \wedge (\bar{s}_1 + \bar{s}_2 - D^{L_2}) \wedge (\bar{s}_1 + \bar{s}_3 - D^{L_3}) = \bar{s}_1. \\ 0 & \text{otherwise} \end{cases}$$

Hence, there must be $G_1'(\bar{s}_1) < 0$ for the optimal $\bar{s}_1$ (otherwise $\partial \bar{H}_1/\bar{s}_1 = 0$ cannot hold). Since G_1 is convex, we know that $G_1'(x) < 0$ for $x < \bar{s}_1$. Taking derivative with respect to $\bar{s}_2$ gives

$$\frac{d\bar{H}_1}{d\bar{s}_2} = \frac{\partial \bar{H}_1}{\partial \bar{s}_2} + \frac{\partial \bar{H}_1}{\partial \bar{s}_1}\frac{\partial \bar{s}_1(\bar{s}_2, \bar{s}_3)}{\partial \bar{s}_2} = \int_0^{\bar{s}_3-\bar{s}_2} \phi^{l_3}(x)\int_{\bar{s}_2}^{\infty}\phi^{L_2}(y)G_1'(\bar{s}_1+\bar{s}_2-y)dydx < 0,$$

where the inequality follows from $\bar{s}_1 + \bar{s}_2 - y < \bar{s}_1$ in the integrand. Therefore, we have shown that $\bar{H}_1(\bar{s}_1, \bar{s}_2, \bar{s}_3)$ is decreasing in $\bar{s}_2$. The result for $\bar{s}_3$ can be similarly shown. □

References

Benko, C., W. McFarlan. 2003. Metamorphosis in the auto industry. *Strategy & Leadership* **31**(4): 4–8.

Bernstein, F., G. A. DeCroix. 2004. Decentralized pricing and capacity decisions in a multi-tier system with modular assembly. *Manage. Sci.* **50**(9): 1293–1308.

Bernstein, F., G. A. DeCroix. 2006. Inventory policies in a decentralized assembly system. *Oper. Res.* **54**(2): 324–336.

Bernstein, F., G. A. DeCroix, Y. Wang 2007. Incentives and commonality in a decentralized multi-product assembly system. *Oper. Res.* **55**(4): 630–646.

Bernstein, F., G. A. DeCroix, Y. Wang. 2011. The impact of demand aggregation through delayed component allocation in an assemble-to-order system. *Manage. Sci.* **57**(6): 1154–1171.

Bollapragada, R., U. S. Rao, J. Zhang. 2004. Managing inventory and supply performance in assembly systems with random supply capacity and demand. *Manage. Sci.* **50**(12): 1729–1743.

Cachon, G. P. 2001. Stock wars: Inventory competition in a two echelon supply chain with multiple retailers. *Oper. Res.* **49**(5): 658–674.

Cachon, G. P., S. Netessine. 2003. Game theory in supply chain analysis. D. Simchi-Levi, S. D. Wu and Z.-J. Shen, eds. *Supply Chain Analysis in the eBusiness Era*. Kluwer Academic Publishers, Boston, MA, 13–66.

Cachon, G. P., P. H. Zipkin. 1999. Competitive and cooperative inventory policies in a two-stage supply chain. *Manage. Sci.* **45**(7): 936–953.

Caldentey, R., L. Wein. 2003. Analysis of a decentralized production-inventory system. *MSOM* **5**(1): 1–7.

Chappell, L. 2001. Nissan's solution: Modules. *Automotive News*. March, 5, Vol. 75, Issue 5919.

Chen, F., Y.-S. Zheng. 1994. Lower bounds for multiechelon stochastic inventory systems. *Manage. Sci.* **40**(11): 1426–1443.

Clark, A. J., H. Scarf. 1960. Optimal policies for a multiechelon inventory problem. *Manage. Sci.* **6**(4): 475–490.

DeCroix, G. A. 2013. Inventory management for an assembly system subject to supply disruptions. *Manage. Sci.* **59**(9): 2079–2092.

Fang, X., K. C. So, Y. Wang. 2008. Component procurement strategies in decentralized assemble-to-order systems with time-dependent pricing. *Manage. Sci.* **54**(12): 1997–2011.

Federgruen, A., P. H. Zipkin. 1984. Computational issues in an infinite horizon, multiechelon inventory model. *Oper. Res.* **32**: 818–836.

Fudenberg, D., J. Tirole. 1991. *Game Theory*. The MIT Press, Cambridge, MA.

Green, J. 1998. Manufacturing revolution underway in Brazil: Companies build emerging-market templates. *Ward's Auto World* (August 1): 30–32.

Heese, S., J. Swaminathan. 2006. Product line design with component commonality and cost reduction effort. *MSOM* **8**(2): 206–219.

Hsu, V. N., C. Y. Lee, K. C. So. 2006. Optimal component stocking policy for assembly-to-order systems with leadtime-dependent component and product pricing. *Manage. Sci.* **52**(3): 337–351.

Jiang, L., Y. Wang. 2010. Supplier competition in decentralized assembly systems with price-sensitive and uncertain demand. *MSOM* **12**(1): 93–101.

Michaels, D., L. Lunsford. 2005. Streamlined plane making Boeing, Airbus look to car companies' methods to speed up jetliner production. *Wall Street J.*, Eastern edition, April 1.

McClintick, M. 2003. Considerations in Converting from Surface Mount Technology to Die Assemblies. Available at http://webevents.broadcast.com/nsc/webcast062603/transcript.html (accessed date September 1, 2010).

Parker, R., R. Kapuscinski. 2011. Managing a non-cooperative supply chain with limited capacity. *Oper. Res.* **59**(4): 866–881.

Pearson, D., D. Michaels. 2012. Airbus wants suppliers to consolidate. *Wall Street J.*, Nov. 8.

Roundy, R. 1985. 98%-effective integer ratio lot-sizing for one-warehouse multi-retailer systems. *Manage. Sci.* **31**(11): 1416–1430.

Rosling, K. 1989. Optimal inventory policies for assembly systems under random demand. *Oper. Res.* **37**(4): 565–579.

Schmidt, C. P., S. Nahmias. 1985. Optimal policy for a two-stage assembly system under random demand. *Oper. Res.* **33**(5): 1130–1145.

Shang, K. 2008. Note: A simple heuristic for serial inventory systems with fixed order costs. *Oper. Res.* **56**(4): 1039–1043.

Shang, K., J. S. Song. 2007. Supply chains with economies of scale: Single-stage heuristic and approximations. *Oper. Res.* **55**(5): 843–853.

Sheridan, J. H. 1997. Bonds of trust. *Ind. Week*, March 17. pp. 52–62.

Song, J. S., P. H. Zipkin. 2003. Supply chain operations: Assemble-to-order systems. S. Graves, T. de Kok, eds. *Handbooks in Operations Research and Management Science*. Elsevier, Amsterdam. **11**: 561–596.

Swaminathan, J. M., S. R. Tayur. 1998. Managing broader product lines through delayed differentiation using vanilla boxes. *Manage. Sci.* **44**(12): 161–172.

Van Mieghem, J. 2004. Commonality strategies: Value drivers and equivalence with flexible capacity and inventory substitution. *Manage. Sci.* **50**(3): 419–424.

Wang, Y. 2006. Joint pricing-production decisions in supply chains of complementary products with uncertain demand. *Oper. Res.* **54**(6): 1110–1127.

Wang, Y., Y. Gerchak. 2003. Capacity games in assembly systems with uncertain demand. *MSOM* **5**(3): 252–267.

Zhang, F. 2006. Competition, cooperation, and information sharing in a two-echelon assembly system. *MSOM* **8**(3): 273–291.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1. Graphical Illustration of the Results in Tables 1, 2, 4 and 5.