# THE IMPORTANCE OF ACCRETION TORQUES IN PULSING X-RAY SOURCES 

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SUMMARY


#### Abstract

We compare the rates of change of pulse period, of a binary X-ray source, due to accretion torques and due to orbital motion. We show that these rates of change can be comparable, in particular for the sources with pulse periods of a few minutes. For this reason orbital parameters derived for pulsing transient sources may be subject to gross errors. Conversely, once the orbital motion is known, conclusions can be drawn about the structure of the neutron star itself.


## I. INTRODUCTION

A neutron star, which has rotation period $P$, and a strong surface magnetic field ( $\sim \mathrm{I}^{12} \mathrm{G}$ ) and which is a member of a close binary system is likely to be under the influence of a variety of torques. Pulsar emission, which is in general important only if $P \lesssim$ I s, and the 'propeller mechanism' (Fabian 1975; Illarionov \& Sunyaev 1975) tend to increase $P$. Accretion of angular momentum accounts for the gradual decreases in the pulse periods of Her X-r and Cen X-3 (Pringle $\&$ Rees 1972). There is also the possibility of a centrally driven outflow disc around the neutron star (Fabian, Pringle \& Rees 1976). For a given mass flow rate, the changes in $P$ depend on the moment of inertia of the object being spun up or down. The coupling time between the crust/charged particle system and the interior neutron superfluid of a neutron star increases with the rotation period (Baym et al. 1969) and also increases as the neutron star cools (Feibelman 1971). Greenstein (1975) has used these effects to suggest that the crust essentially decouples from the core when $P$ is a few seconds. Subsequent torques applied to the star via the magnetic field, only affect the rotation rate of the stellar crust. The moment of inertia of the crust can be much less than the moment of inertia of the whole star (see Table I). One effect of this is to shorten the estimated spin-up and spin-down time scales (e.g. Fabian 1975) by as much as two orders of magnitude.

The observed rate of change of pulse period in systems such as Her X-I and Cen $\mathrm{X}-3$ is much less than the rate of change associated with orbital motion. In Section 2 we consider under what circumstances the rates of change may be comparable. We discuss the consequences of this possibility in Section 3 .

## 2. COMPARISON OF RATES OF PERIOD CHANGE

Consider a neutron star with mass $M$, radius $R$, and rotational period $P$. Let the accretion rate on to the star be $F$ and define an accretion time scale by $\tau_{a}=M F^{-1}$. Let the moment of inertia of that part of the star which is being spun
up by the accreting material be $I=k^{2} M R^{2}$, where $k R$ is the radius of gyration. Let the specific angular momentum of the accreted material be $h=\left(G M R_{h}\right)^{1 / 2}$. We assume that the angular momenta of the star and of the accreted material are parallel. If the neutron star is magnetized-a reasonable assumption for a pulsing source-and if an accretion disc forms around the star, then $R_{h}$ is roughly equal to the Alfvén radius, $R_{M}$. If no accretion disc forms $R_{h}$ can be much less than $R_{M}$ and, even, less than $R$. In any case, $R_{h}$ is unlikely to exceed the corotation radius $R_{\Omega}=\left(P^{2} G M / 4 \pi^{2}\right)^{1 / 3}$. Then the rate of change of $P$ due to accretion alone, $\dot{P}_{a}$, may be written

$$
\begin{equation*}
\dot{P}_{a}=\frac{P^{2}\left(G M R_{h}\right)^{1 / 2}}{\tau_{a} 2 \pi k^{2} R^{2}} \tag{I}
\end{equation*}
$$

If the star is in a binary system of total mass $\mu M$, semi-major axis $a$ and orbital period $P_{0}$, we may rewrite ( I ) as

$$
\dot{P}_{a}=\frac{P^{2}}{\tau_{a} P_{0}}\left(\frac{R_{h}}{R}\right)^{1 / 2}\left(\frac{a}{R}\right)^{3 / 2} \frac{\mathrm{I}}{k^{2} \mu^{1 / 2}} .
$$

Let us now consider the apparent rate of change of the pulse period $P$ due to orbital motion. Let the radial velocity of the pulsing star at time $t$ be $V_{r}(t)$. Then the apparent rate of change of $P$ due to orbital motion $\dot{P}_{m}$ is given by

$$
\begin{equation*}
\dot{P}_{m} / P=-\dot{V}_{r} / c \tag{2}
\end{equation*}
$$

where $c$ is the velocity of light. Let the angle of inclination of the orbit be $i$, the longitude of periastron be $\omega$ and the eccentricity of the orbit be $e$. We find that

$$
\begin{equation*}
\dot{P}_{m} \lesssim \frac{2 \pi P}{P_{0}} \cdot \frac{K}{c} \frac{F(e, \omega)}{(\mathrm{I}-e)^{3 / 2}} \tag{3}
\end{equation*}
$$

where $K={ }_{2} P_{0}{ }^{-1} a \sin i(\mathrm{I}-e)^{-1 / 2}$ is the semi-amplitude of the radial velocity curve and

$$
\begin{equation*}
F(e, \omega)=\left\{\sin (v+\omega)(I+e \cos v)^{2}\right\} \tag{4}
\end{equation*}
$$

where $v$ satisfies

$$
\begin{equation*}
2 e \sin v \tan (v+\omega)=\mathrm{I}+e \cos v . \tag{5}
\end{equation*}
$$

$F(e, \omega)$ is in general of order unity, and tends to one as $e$ tends to zero.
We may now compare the rates of change of the pulse period due to the two processes. Let

$$
\begin{equation*}
\lambda=\frac{\dot{P}_{a}}{\dot{P}_{m}} \geqslant \frac{P}{\tau_{a}} \cdot \frac{c}{K} \cdot\left(\frac{R_{h}}{R}\right)^{1 / 2} \cdot\left(\frac{a(\mathrm{I}-e)}{R}\right)^{3 / 2} \cdot \frac{\mathrm{I}}{k^{2} \mu^{1 / 2} F(e, \omega)} . \tag{6}
\end{equation*}
$$

Note that $a(\mathrm{I}-e)$ is the periastron distance. Of course for a given binary system, our estimate of $\dot{P}_{m}$ (equation (3)) is in general a generous overestimate since $\dot{P}_{m}$ decreases with decreasing $\sin i$, and $\dot{P}_{m}$ is zero at some points of the orbit. Thus if, for a given system, $\lambda$ is of order unity for $\sin i=\mathrm{I}$, the accretion contribution to the period change cannot be neglected at any point of the orbit.

## 3. DISCUSSION

Consider the $8 \cdot 95$-day binary system $3 \mathrm{U} 0900-40$. Taking $\tau_{a}=10^{9} \mathrm{yr}$, $\mu=16, R=10^{6} \mathrm{~cm}$ and the parameters given by Rappaport \& McClintock

Table I

| Authors |  | $M / M_{\circ}$ | $k_{\text {cr }}{ }^{2}$ | $k_{*}{ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| Pines \& Shaham (1972) |  | $0 \cdot 30$ | -0.050 | -. 18 |
|  |  | $0 \cdot 46$ | 0.030 | 0.21 |
|  |  | $0 \cdot 80$ | 0.009 | - $\cdot 30$ |
|  |  | I.08 | 0.007 | - $\cdot 37$ |
|  |  | I 41 | - | 0.47 |
| Pandharipande et al. (preprint) | TI | $0 \cdot 29$ | $0 \cdot 133$ | $0 \cdot 13$ |
|  |  | $0 \cdot 73$ | 0.230 | $0 \cdot 30$ |
|  |  | I. 08 | 0.202 | 0.28 |
|  |  | I 85 | - 105 | $0 \cdot 35$ |
|  |  | I•93 | 0.056 | 0.35 |
|  | TII | 1 77 | $0 \cdot 114$ | $0 \cdot 34$ |
|  |  | $1 \cdot 90$ | 0.095 | 0.35 |
|  |  | 1-97 | 0.077 | $0 \cdot 35$ |
|  | TI2 | I. 85 | 0.090 | 0.35 |
|  |  | I.90 | 0.072 | $0 \cdot 35$ |
|  |  | I•92 | 0.048 | $0 \cdot 35$ |

(1975) we find

$$
\lambda \geqslant \frac{\mathrm{I} \cdot 2 \times 10^{-2}}{k^{2}}\left(\frac{R_{h}}{R}\right)^{1 / 2}
$$

In Table I we show the values of $k^{2}$ for the crust alone and for the star as a whole from the stellar models of Baym, Bethe \& Pethick (1971) (see Table I of Pines \& Shaham 1972) and Table 4 of Pandharipande et al. (preprint). We see therefore that it is possible for period changes associated with the accretion process substantially to perturb, or even swamp, the velocity curve. Since $\lambda \propto P$, this effect is of particular relevance to sources with periods of a few minutes (Ives, Sanford \& Bell-Burnell 1975; Rosenberg et al. 1975; Mayer 1975; Rappaport \& McClintock 1975; White et al. 1975a, b).

This conclusion has two major consequences. First, any conclusions drawn about the binary orbit of pulsing transient sources such as Airi8-6i and A0535+ 26, for which observations may extend over only a fraction of the true orbital period, may be subject to gross errors. Secondly, observations of the pulse period in sources like $3 \mathrm{U} 0900-40$ over several orbital periods should allow the orbital effects to be disentangled from the (probably highly variable) accretion effects. From this it should be possible to draw conclusions about the structure of the neutron star itself.

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