

## The importance of hydraulic groundwater theory in catchment hydrology: The legacy of Wilfried Brutsaert and Jean-Yves Parlange

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[1] Based on a literature overview, this paper summarizes the impact and legacy of the contributions of Wilfried Brutsaert and Jean-Yves Parlange (Cornell University) with respect to the current state-of-the-art understanding in hydraulic groundwater theory. Forming the basis of many applications in catchment hydrology, ranging from drought flow analysis to surface water-groundwater interactions, hydraulic groundwater theory simplifies the description of water flow in unconfined riparian and perched aquifers through assumptions attributed to Dupuit and Forchheimer. Boussinesq (1877) derived a general equation to study flow dynamics of unconfined aquifers in uniformly sloping hillslopes, resulting in a remarkably accurate and applicable family of results, though often challenging to solve due to its nonlinear form. Under certain conditions, the Boussinesq equation can be solved analytically allowing compact representation of soil and geomorphological controls on unconfined aquifer storage and release dynamics. The Boussinesq equation has been extended to account for flow divergence/convergence as well as for nonuniform bedrock slope (concave/convex). The extended Boussinesq equation has been favorably compared to numerical solutions of the three-dimensional Richards equation, confirming its validity under certain geometric conditions. Analytical solutions of the linearized original and extended Boussinesq equations led to the formulation of similarity indices for baseflow recession analysis, including scaling rules, to predict the moments of baseflow response. Validation of theoretical recession parameters on real-world streamflow data is complicated due to limited measurement accuracy, changing boundary conditions, and the strong coupling between the saturated aquifer with the overlying unsaturated zone. However, recent advances are shown to have mitigated several of these issues. The extended Boussinesq equation has been successfully applied to represent baseflow dynamics in catchment-scale hydrological models, and it is currently considered to represent lateral redistribution of groundwater in land surface schemes applied in global circulation models. From the review, it is clear that Wilfried Brutsaert and Jean-Yves Parlange stimulated a body of research that has led to several fundamental discoveries and practical applications with important contributions in hydrological modeling.

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## 1. Introduction

[2] Quantifying catchment-scale hydrological processes, states, and fluxes remains difficult due to the scale of the chosen control volume and the intrinsic heterogeneities of soil, vegetation, and topographic parameters that define these processes. A bottom-up approach (i.e., building large-scale predictions explicitly from direct measurement of parameters reflecting basic principles of underlying small-scale processes) that attempts to characterize all relevant heterogeneities seems to be infeasible with current technology [McDonnell *et al.*, 2007], although important progress has been made using geophysical and remote sensing methods (e.g., ground penetrating radar and electrical resistance tomography to scan subsurface soil properties at hillslope scales, visible and near IR remote sensing to map vegetation properties). We also lack solid theory that links small-scale variability of soil and vegetation properties to large-scale fluxes at the land surface and the subsurface [Troch *et al.*, 2008]. Therefore, analysis of catchment data, based on appropriate hydrological theory, is a reasonable approach to progress the understanding of catchment-scale processes.

[3] This approach is at the heart of the research executed by Wilfried Brutsaert and Jean-Yves Parlange. While they have addressed many different aspects of hydrology, here we focus on their groundbreaking contributions regarding the rainfall-runoff response at the catchment scale. In a time when distributed hydrological models became popular and seemingly the way out of the intractable problem of identifying catchment response in light of landscape heterogeneity, they emphasized the scientific method: formulate the problem in ways that preserve the main physical properties but reduce the dimensionality, find exact or approximate solutions to the resulting flow and transport equations to better understand the dominant modes of response, test these simplified solutions with real catchment data, develop methods to derive catchment-scale parameter values from these data sets, and inspire colleagues to explain observed inconsistencies between theory and observations.

[4] Their combined work on hydraulic groundwater theory is a case study in the power of the scientific method that they embraced. Hydraulic groundwater theory is simple enough to allow derivation of exact or approximate analytical solutions to specific flow situations defined by the initial and boundary conditions, yet it preserves the main physics that drives the flow (i.e., advection and diffusion driven flow and transport). Its simplicity is reflected in the amount of parameters required to fully characterize the dynamic response of a hillslope or a catchment. This limited amount of system parameters is easily informed from the available data sets of catchment response (hydrographs and rainfall measurements). Agreement between theory and observations, quantified by validating predictions against measurements, can be interpreted as closure at the catchment scale. However, there are several alternative explanations that lead to equally valid predictions, opening the debate of what really controls catchment response. This fact does not diminish the scientific method of Brutsaert and Parlange, it strengthens it. It leads to the formulation of novel hypotheses, the discovery of methods that allow testing between these alternative hypotheses, and it can inspire new field measurements that collect data required to accept or reject

these hypotheses. In short, it is the scientific method that guarantees advancement in our understanding rather than parameter estimation in absence of insight, as so often encountered in the literature.

[5] This review paper describes the importance of hydraulic groundwater theory in catchment hydrology, and the role Brutsaert and Parlange have played. Because of this focus, we selected contributions that have played pivotal roles in developing catchment-scale theory of hydrologic response, and have left out many other important contributions employing hydraulic groundwater theory. Section 2 summarizes the main concepts leading to hydraulic groundwater theory of riparian and perched aquifers that drain into the catchment's channel network. Section 3 reviews several exact and approximate analytical solutions to the main equations of hydraulic groundwater theory, many of which were proposed by Brutsaert, Parlange, their students, and colleagues. Section 4 focuses on how hydraulic groundwater theory can be used to interpret recession flow at catchment scales. Section 5 extends the baseflow analysis methods proposed by Brutsaert and Parlange to quantify other hydrological fluxes, such as precipitation and evaporation, as well as develop parsimonious rainfall-runoff models. Section 6 reviews several extensions recently made to hydraulic groundwater theory that attempt to relax some of the important assumptions inherent in traditional hydraulic groundwater theory. In section 7, we present studies that have formulated hydrologic similarity theory based on the pioneering work of Brutsaert and Parlange. Finally, in Section 8, we review recent contributions to understanding hydrologic response to climate change based on hydraulic groundwater theory.

## 2. Hydraulic Groundwater Theory

### 2.1. The Dupuit-Forchheimer Assumptions

[6] Hydraulic groundwater theory of unconfined flow in a horizontal or sloping aquifer is founded on the Dupuit-Forchheimer assumptions. In 1863, Dupuit postulated that given the aspect ratio prevalent in groundwater systems that accurate descriptions of flow could be obtained by ignoring the impact of vertical fluxes, and instead compute groundwater flux assuming that water moves largely horizontally in proportion to the saturated aquifer thickness and the slope of the free surface of the aquifer [Dupuit, 1863]. Thus, the Dupuit-Forchheimer formulation neglects any vertical gradient in hydraulic heads (equivalent to assuming no energy losses in the vertical [Kirkham, 1956]), i.e., all vertical profiles can be considered hydrostatic, resulting in a groundwater flow pattern that is parallel to the underlying bedrock. At larger scales (e.g., that of a hillslope), where the horizontal dimensions are considerably larger than the thickness of the aquifer, this assumption does not introduce significant errors in prediction. However, near the boundaries of the aquifer where vertical components in groundwater flow may be expected, the simplified groundwater equations will introduce a predicted groundwater table shape which deviates significantly from the actual one, which is an inescapable consequence of taking into account the Dupuit-Forchheimer assumptions [Childs, 1971]. Strikingly, however, in many contexts, the slightly shorter flow paths of this approach almost exactly balance the slightly

lower hydraulic gradients, and thus often give rise to flux predictions more accurate than the predictions of watertable position [Brutsaert, 2005, p. 388]. The validity (and limits of applicability) of the hydraulic assumption, i.e., the Dupuit-Forchheimer assumptions, has been dealt with experimentally by Ibrahim and Brutsaert [1965] and numerically by Verma and Brutsaert [1970, 1971].

## 2.2. The Boussinesq Equation

### 2.2.1. Horizontal Aquifer

[7] When dealing with analytical solutions to the Boussinesq equation, the majority of the researchers have considered one-dimensional (1D) flow along a hillslope of constant width. Adopting the Dupuit-Forchheimer assumptions allows the hydraulic head along a vertical section (i.e., perpendicular to the underlying impermeable basis) of the saturated aquifer thickness to be equated with the elevation of the groundwater table (setting the elevation head datum at the underlying impermeable layer). Vertically integrating Darcy's law over the saturated thickness  $h$  yields an expression for the flow per unit width  $q$ :

$$q = -kh \frac{\partial h}{\partial x} \quad (1)$$

[8] When this is combined with the continuity equation

$$f \frac{\partial h}{\partial t} = -\frac{\partial h}{\partial x} dx + N dx, \quad (2)$$

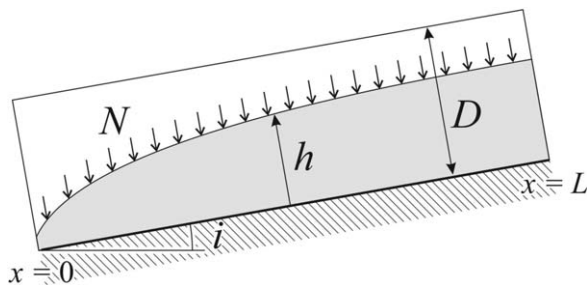
[9] One obtains the Boussinesq equation for a horizontal aquifer, considering a spatially constant recharge rate  $N$  [ $\text{m s}^{-1}$ ]:

$$\frac{\partial h}{\partial t} = -\frac{k}{f} \frac{\partial}{\partial x} \left( h \frac{\partial h}{\partial x} \right) + \frac{N}{f} \quad (3)$$

where  $q$  [ $\text{m}^2 \text{s}^{-1}$ ] is the flow rate in the  $x$ -direction per unit width of the aquifer,  $k$  [ $\text{m s}^{-1}$ ] is the spatially constant saturated hydraulic conductivity,  $f$  [-] is the spatially constant drainable porosity,  $t$  [s] is time, and  $h$  [m] is the hydraulic head.

### 2.2.2. Sloping Aquifer

[10] When a sloping aquifer is considered, it is advised to consider the  $(x, h)$  coordinate system as displayed in Figure 1.



**Figure 1.** Definition sketch of the cross-section of a hillslope resting on an impermeable layer of slope  $i$ . For easy display, the sketch is distorted. In reality, the length  $L$  of the hillslope is much larger than the total depth  $D$  of the phreatic aquifer.

Only in such formulation, the Dupuit-Forchheimer assumption of having a streamlines parallel to the underlying bedrock can hold [Bear, 1972]. In this coordinate system, the Darcy equation then becomes [Boussinesq, 1877; Childs, 1971]:

$$q = -kh \left( \frac{\partial h}{\partial x} \cos i + \sin i \right) \quad (4)$$

where  $i$  is the slope angle of the underlying impermeable layer. Combining this equation with the continuity equation, one obtains the Boussinesq equation for a sloping aquifer given a spatially constant recharge rate  $N$  to the groundwater table:

$$\frac{\partial h}{\partial t} = \frac{k}{f} \left[ \cos i \frac{\partial}{\partial x} \left( h \frac{\partial h}{\partial x} \right) + \sin i \frac{\partial h}{\partial x} \right] + \frac{N}{f} \quad (5)$$

## 3. Exact and Approximate Solutions of the Boussinesq Equation

### 3.1. Exact Solutions

[11] Boussinesq himself provided the first exact solution to the equation for unsteady flow bearing his name using the technique of separation of variables [Boussinesq, 1877]. Boussinesq considered the problem of an initially saturated horizontal aquifer of unit width draining, under sudden drawdown, to a channel. The initial conditions and boundary conditions are as follows:

$$\begin{aligned} h &= 0 & x &= 0 & t &\geq 0 \\ \frac{\partial h}{\partial x} &= 0 & x &= L & t &\geq 0 \\ h &= D & x &= L & t &= 0 \end{aligned} \quad (6)$$

[12] The solution is referred to as being late-time, or long-time, because the curvilinear water table that arises from these boundary conditions will only take such a shape after drainage has proceeded for some time.

[13] A half-century later, Polubarinova-Kochina [1962] considered a similarly initially saturated, horizontal aquifer subject to sudden drawdown, but assumed a semi-infinite aquifer ( $L \rightarrow \infty$ ). Thus, the initial and boundary conditions are formulated as follows:

$$\begin{aligned} h &= 0 & x &= 0 & t &\geq 0 \\ h &= D & x &> 0 & t &= 0 \end{aligned} \quad (7)$$

[14] Polubarinova-Kochina [1962] used the Boltzmann's transformation to reduce the Boussinesq equation to an ordinary differential equation. The solution of Polubarinova-Kochina [1962] is valid only so long as the flow can proceed as if the aquifer were infinitely wide, therefore is referred to as an early, or short-term solution.

[15] Subsequently, the Boussinesq equation (3) has been solved exactly for only a limited number of boundary and/or initial conditions, all in the absence of rainfall recharge. Parlange et al. [2000] found a solution for initially parabolic water tables in a horizontal and finite aquifer. For what is, as far as we know, the only known exact solution for a sloping aquifer, Daly and Porporato [2004] described the evolution of a groundwater mound in an aquifer that is

infinite in both the upslope and downslope direction. *Rupp and Selker* [2005] generalized the Boussinesq equation to include a hydraulic conductivity that increased as a power function of height above bedrock, and solved the generalized equation under the same conditions given by equation (6).

### 3.2. Approximate Solutions

[16] Approximate analytical solutions have been found to the Boussinesq equation for a wider variety of initial and boundary conditions. By far most of these consider the horizontal case. Here, we cite some examples to which Jean-Yves Parlange contributed. *Hogarth and Parlange* [1999], following *Polubarinova-Kochina* [1962], recast the Boussinesq equation as a Blasius equation and solved it for the same conditions given by equation (7), but to a much higher degree of accuracy than *Polubarinova-Kochina*'s original solution. *Hogarth et al.* [1999] and *Lockington et al.* [2000] considered flow into an initially dry aquifer from a channel with a water level that varies in time. *Parlange et al.* [2001] found a single solution to the initially saturated aquifer that unified the early-time and late-time solutions of *Polubarinova-Kochina* [1962] and *Boussinesq* [1904], respectively. Solutions for a sloping aquifer are much rarer: *Rupp and Selker* [2006b] found a late-time solution for discharge only (not water table height) for an initially saturated finite aquifer under sudden drawdown and hydraulic conductivity that increased as a power function of height above bedrock.

[17] A rich set of approximate solutions has been found by means of first linearizing the Boussinesq equation. The assumption is that  $h$  varies in  $x$  relatively little compared to the value of  $h$  itself (since  $h \ll L$ ), so that the first  $h$  on the right-hand side of equation (5) can be made a constant equal to  $pD$ , yielding:

$$\frac{\partial h}{\partial t} = \frac{kpD \cos i}{f} \frac{\partial^2 h}{\partial x^2} + \frac{k \sin i}{f} \frac{\partial h}{\partial x} + \frac{N}{f} \quad (8)$$

[18] In general,  $0 < p \leq 1$ . This linearization parameter is best determined by treating it as an additional calibration parameter [*Brutsaert*, 1994].

[19] Note that there are other ways to linearize equation (5) (see, e.g., review in *Rupp and Selker* [2006b]). One of these is to make the equation linear in  $h^2$ , but equation (8) is the most common [*Brutsaert*, 2005, Chapter 10.4]. *Brutsaert and Ibrahim* [1966] contrasted solutions using both linearizations ( $h$  and  $h^2$ ) and compared the solutions to experimental results.

[20] The first solution to equation (8) can be traced back to *Boussinesq* [1877] for the same conditions given by equation (6). The solution can be given as the summation of an infinite series, from which *Boussinesq* [1877] retained but the first term, whereas, according to *Brutsaert* [2005], the full series was presented by *Dumm* [1954] and *Kraijenhoff van de Leur* [1958], as is also shown in *Verhoest et al.* [2002].

[21] An exhaustive accounting of all the literature concerning solutions to equation (8) for a horizontal aquifer is beyond the scope of this review. Examples of J.-Y. Parlange's contributions include flow into an initially dry aquifer with a time-varying boundary condition at the channel/aquifer interface

[*Hogarth et al.*, 1997] and tidal-aquifer interaction with a moving shoreline [*Li et al.*, 2000].

[22] In the less tractable case of the sloping aquifer, *Brutsaert* [1994] made a breakthrough contribution. He used a Laplace transform to solve equation (8) for the classical conditions given by equation (6), arriving at an infinite summation series for aquifer discharge. He further noted that arbitrary inputs (as rainfall recharge or evaporation) could be considered as well through convolution. *Brutsaert*'s solution was expanded to include constant recharge [*Verhoest and Troch*, 2000], drainage at both the downslope and upslope boundaries [*Verhoest et al.*, 2002], and temporally varying recharge [*Pauwels et al.*, 2002, 2003]. Extensions have been made to the linearized Boussinesq equation to consider a nonuniform shape in the planar profile with sloping bedrock [e.g., *Troch et al.*, 2003], for which some analytical solutions have been found [*Troch et al.*, 2004]. We expand on this topic in section 6.

## 4. Streamflow Recession: Theoretical Analysis of Riparian Aquifer Properties

[23] In this section, we focus on methods developed based on hydraulic groundwater theory to investigate catchment storage dynamics from baseflow analysis under the assumption that streamflow is the main catchment-scale flux that results from storage release/increase. In section 5, we will extend this review by including contributions that look at all water balance fluxes at the catchment scale.

### 4.1. Basic Methodology

[24] The overall objective of this type of analysis is to use the characteristics of the baseflow recession to estimate lumped catchment hydraulic parameters, more specifically  $D$ ,  $k$ , and  $f$ . Employing solutions to the linearized Boussinesq equation, one could use the decreasing limbs of observed hydrographs and fit a theoretically derived expression for the recession to these data. However, this implies that the exact instance of the onset of the recession is known, since all baseflow recession models explicitly depend on this variable. For this reason, *Brutsaert and Nieber* [1977, henceforth referred to as B&N] argued that a solution to this problem might be the examination of relationships between the baseflow fluxes and their first time derivatives. This eliminates the variable time from the analysis. B&N then showed that three solutions to the Boussinesq equation can be recast to have the following exact power-law relationship between the outflow and its first time derivative:

$$\frac{dQ_b}{dt} = -aQ_b^b \quad (9)$$

[25]  $Q_b$  stands for the baseflow ( $\text{m}^3 \text{s}^{-1}$ ), and  $a$  and  $b$  are parameters depending on the amount of water stored in the aquifer.

[26] A first solution, valid for small values of time, uses a fully saturated aquifer as initial condition. As the aquifer starts to drain, the geometry of the aquifer at the most distant location from the outflow point (i.e., the hillcrest) does not yet influence the outflow rate. Under these conditions, based on an analytical solution under the assumption of an infinitely long horizontal aquifer [*Polubarinova-Kochina*,

1962] (see section 3), the parameters  $a$  and  $b$  in equation (9) can be written as:

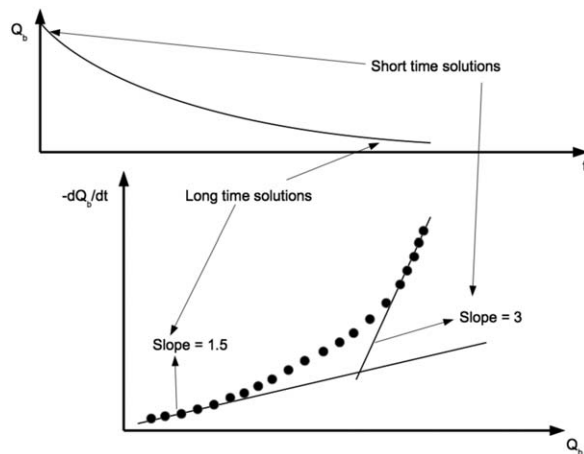
$$\begin{cases} a_1 = \frac{1.133}{kfD^3W_0^2} \\ b_1 = 3 \end{cases} \quad (10)$$

[27]  $W_0$  stands for the total length of streams in the watershed, and the subscripts of  $a$  and  $b$  indicate this first solution. A second solution, valid for late times in a horizontal aquifer, uses an inverse incomplete Beta function as initial condition. The parameters in equation (9) can now be written as [Polubarinova-Kochina, 1962]:

$$\begin{cases} a_2 = \frac{4.804W_0\sqrt{k}}{f\sqrt{A^3}} \\ b_2 = \frac{3}{2} \end{cases} \quad (11)$$

[28]  $A$  is the catchment drainage area.  $W_0$  and  $A$  can be determined through an analysis of topographic data.

[29] If one has a rough estimate of the drainable porosity, for example through pumping tests, equations (10) and (11) form a system of two equations with two unknowns ( $k$  and  $D$ ). In order to solve this system of equations, a time series of discharge needs to be available. The decreasing limbs of the hydrographs are retained for further analysis. For these data, the absolute values of the first time derivative of the discharges are plotted as a function of the discharges themselves, in a double-logarithmic plot. If the data in the plots would consist of only baseflow data for short and large values of time, the intercepts of two regressions through the data set, with slope 3 and 1.5, would be estimates of the values of  $a_1$  and  $a_2$ , respectively (Figure 2). However, the discharge data measured during recessions will generally consist of both baseflow and some surface runoff data. Furthermore, by taking into account discharge data only, one is unable to determine whether the short or large time approximations are valid or not. Under the assumption that baseflow corresponds to lower absolute values of the first time derivative than surface runoff (baseflow being the



**Figure 2.** Illustration of baseflow recession in time (upper panel) and the same data plotted on a Brutsaert-Nieber plot.

slow component of discharge), B&N suggested to use the lower envelopes of the data set. In other words, the intercepts of the lower envelopes with slope 3 and 1.5 correspond to the values of  $a_1$  and  $a_2$ . Troch *et al.* [1993] showed that the order of magnitude of the estimates of  $k$  and  $D$  obtained in this manner did not alter strongly with the exact location of these lower envelopes.

[30] This methodology to estimate catchment-averaged aquifer hydraulic parameters has been used in numerous studies since. However, since its development, a number of improvements have been suggested.

#### 4.2. Modifications to the Original Algorithm

[31] A first improvement was suggested by Huyck *et al.* [2005]. In their study, which is based on the linearized Boussinesq equation, two extra aquifer properties have been taken into account, of which the first is the aquifer shape. More specifically, instead of assuming the width of the aquifer to be constant, an exponentially varying width is assumed:

$$W(x) = W_0e^{-\beta x} \quad (12)$$

[32]  $W_0$  is total length of streams in the watershed and  $\beta$  is a shape parameter or the convergence rate. A second aquifer property that has been taken into account is the aquifer slope angle,  $i$ . Taking into account these properties, Huyck *et al.* [2005] derived the following expression for short values of time:

$$\begin{cases} a_1 = \frac{\pi}{8kfD^3W_0^2p\cos i} \\ b_1 = 3 \end{cases} \quad (13)$$

[33] The similarity between equations (10) and (13) is apparent, as in both equations the parameters  $k$ ,  $f$ ,  $D$ , and  $W_0$  appear to the same power, and  $b_1$  is in both cases equal to 3. For late times, the following relationship is valid:

$$\begin{cases} a_2 = K\left(a^2 + \frac{z_1^2}{L^2}\right) - U\beta \\ b_2 = 1 \end{cases} \quad (14)$$

$z_1$  is the first solution of the following equation:

$$\tan z = -\frac{z}{L(\beta - a)} \quad (15)$$

[34]  $a$ ,  $K$ , and  $U$  can be calculated as:

$$\begin{cases} K = \frac{kpD \cos i}{f} \\ U = \frac{k \sin i}{f} \\ a = -\frac{U - K\beta}{2K} \end{cases} \quad (16)$$

[35] For large values of time, a slope of 1 in the discharge time derivative versus discharge plots is obtained, which can be explained by the use of the linearized version of the Boussinesq equation.

[36] A second improvement was developed by Rupp and Selker [2006a]. In their study, they demonstrated that care

must be taken in the analysis of the  $-dQ_b/dt$  versus  $Q_b$  plots, and that one must account for the resolution of the underlying data. More specifically, it was shown that the time step used in the calculation of the first time derivatives should not always be taken as constant for a correct estimation of the aquifer properties.

[37] As pointed out in section 4.1, the original algorithm uses a system of two equations (early-time and late-time transient discharge) to solve for the two unknown aquifer parameters. In cases where the early-time regime ( $b_1 = 3$ ) is ambiguous or not identifiable from discharge observations, a third modification to the algorithm is to use the water table recession to provide the second equation. For example, a similar envelope fitting method can be applied using late-time water table observations where the second equation takes the form  $dh/dt = -ah^b$  [Rupp and Selker, 2005] or  $Q = ah^b$  [Rupp et al., 2009], where  $a$  and  $b$  are again parameterized from solutions of the Boussinesq equation.

#### 4.3. Extensions and Criticism

[38] Most of the applications of the Brutsaert and Nieber approach have been applied to natural systems. However, man-made drainage systems can provide several useful advantages. First, the idealized geometry of a well-defined effective depth and width of the aquifer system are rigorously valid, and known a priori. Further, well-defined water table depths can be obtained from known positions along the flow paths. This context provides exceptional opportunities to employ and validate the methodology. Rupp et al. [2004] focused on this application, and the approach led to development of a simple and powerful method to obtain field-scale values of hydraulic conductivity and drainable porosity based only on measurements of outflow as a function of time, as well as on knowledge of the spacing and depth of the drainage system. Given the wide-spread use of drainage systems in agriculture, the method should be broadly applicable.

[39] Akin to the example provided by Brutsaert and Nieber [1977], Rupp et al. [2004] recognized the difficulty in identifying an effective “time zero” at which a drainage process could be thought to have started, and moreover, of identifying a transition between early-time and late-time during natural precipitation events. Following the B&N strategy, Rupp et al. [2004] used ratios of successive measurements to eliminate the need for a specific start time. They presented four approaches to estimate hydraulic conductivity, all of which used solutions to the nonlinear version of the Boussinesq equation, two of which applied to outflow measurements, while the other two make use of measurements of groundwater heads in the field, either at the center point between tiles, or at a well-defined distance from the tile. Two of the solutions employed the integral of flow rather than time derivatives (i.e., they avoided the computation of  $dQ/dt$ ), thus avoiding many of the pitfalls pointed out in Rupp et al. [2006a].

[40] Rupp et al. [2004] compared the estimated field-scale hydraulic conductivities with results from 40 soil cores to find that the field values reproduced the median value obtained from cores, but were eight times lower than the mean value from the cores, suggesting that, in their fields, up-scaled processes respected Darcian flow through

the bulk soil rather than preferential flow. While the method presented by Rupp et al. [2004] was proposed in the context of drained fields, the solutions obtained share the basic assumptions that B&N and the articles that followed employed, hence they should be expected to be applicable in a much broader range of settings.

[41] Although a baseflow recession analysis is widely recognized as a valuable tool to characterize aquifers, fundamental criticism has also emerged. This mainly originates from the observation that the theoretically derived slopes are hardly ever precisely obtained when discharge data are analyzed. One possible explanation is the assumption of homogenous parameters in the derivation of the relationships between  $dQ_b/dt$  and  $Q_b$ . The impact of vertical parameter heterogeneity was studied by Rupp and Selker [2005, 2006b], while the impact of horizontal aquifer heterogeneity was studied by Harman and Sivapalan [2009b]. Chapman [1999] suggested that the deviations arose from the planform convergence of streamlines in hillslope hollows (an effect captured by Troch et al. [2003]’s extension of the Boussinesq equation). A second possible explanation is the implicit assumption that the behavior of a collection of hillslopes is well approximated by the behavior of a single hillslope, an assumption challenged by Harman et al. [2009]. All these studies, which are further explained in the next sections, demonstrate that parameter heterogeneity explains deviations from the theoretically derived slopes.

[42] Van de Giesen et al. [1994, 2005] offered an alternative explanation of the fact that in many catchment applications the theoretical values of  $b$  in (9), derived from exact solutions to Boussinesq’s equation, are rarely observed, and the fact that in many instances  $b$  tends to 1 for large time (which can only be explained using Boussinesq’s equation after linearization). They used analytical solutions to Laplace equation with linearized surface boundary conditions [Van de Giesen et al., 1994] to show that  $b \rightarrow 1$  for  $t \rightarrow \infty$  and that  $b \rightarrow \infty$  for  $t = 0$  after sudden drawdown (a situation similar to the one assumed by Polubarinova-Kochina [1962]). They further showed that the transition of values of  $b$  depends on the time of application of recharge rates to the aquifer. The work of Van de Giesen et al. [1994, 2005] should be further explored to shed light on the applicability and limitations of the Dupuit-Forchheimer assumptions in catchment hydrology.

#### 4.4. Impact of Horizontal Heterogeneity

[43] The assumption of uniform aquifer parameters along the slope ignores the potential influence of spatial variations in hydraulic conductivity to groundwater flow through the hillslope. Szilagyi et al. [1998] examined the impact of spatial heterogeneity in conductivity fields on recession curves using a 2D numerical model of the Boussinesq equation where slope effects were negligible. They found the B&N method for estimating effective catchment-scale parameters was robust for the degree of heterogeneity they imposed in their synthetic catchments.

[44] Harman and Sivapalan [2009b] further examined the impact of spatial heterogeneity for a sloping aquifer. Their results showed that heterogeneity tended to increase the rising limb of the aquifer response to recharge, delay the approach to steady-state discharge, and extend the

recession limb (relative to an aquifer with a uniform hydraulic conductivity equal to the log-mean of the heterogeneous field). This led to a higher exponent for the recession curve, mimicking the early time behavior even for late time. The effects were also found to depend on the relative importance of the diffusive and advective terms, as captured by the Hillslope number (see section 7). For low-gradient aquifers, lateral variations in hydraulic conductivity induced compensating variations in the water table, driving flow away from low conductivity areas, and toward high conductivity areas. As a result, the effect of the spatial variability on the recession behavior was mitigated. For steeper hillslopes, where the topographic gradient dominates, flow accumulated behind low-conductivity road-blocks, creating localized stores of water that were subsequently released slowly from the hillslope. This produced recession curves with high exponents at long times.

[45] The effect of these types of heterogeneities have been partially captured in reformulations of the Boussinesq equation using various stochastic extensions. *Cayar and Kavvas* [2008] developed a stochastic form of the Boussinesq equation for horizontal aquifers that is applicable to a low level of variance. *Harman et al.* [2010] developed a time-subordinated version of the kinematic-wave approximation of the Boussinesq equation. By comparison with numerical solutions of the full Boussinesq equation, they showed that this approximation produced analytical solutions for the recession curve from a highly heterogeneous sloping aquifer when advective forces dominate the flow.

#### 4.5. Impact of Vertical Heterogeneity

[46] *El-Kadi and Brutsaert* [1985] studied the effects of spatial variability on outflows from a 2D (vertical slice) aquifer, and found that only under a narrow set of circumstances could a set of effective “homogeneous equivalent” parameters effectively reproduce the recession behavior over a range of flows. They used a random field of soil properties. Field evidence of decreasing saturated hydraulic conductivity with depth in some hillslopes [e.g., *Brooks et al.*, 2004] prompted *Rupp and Selker* [2005, 2006b] to generalize the Boussinesq equation by allowing the vertically averaged  $k$  to increase with saturated thickness  $h$  above the impermeable bedrock:

$$k(h) = \frac{k_D}{(m+1)} \left(\frac{h}{D}\right)^m \quad (17)$$

where  $k_D/(m+1)$  is the vertically averaged saturated hydraulic conductivity when the water table is at the surface. An approximate solution to this generalized equation for a horizontal aquifer valid for early-time (given the initial and boundary conditions in section 3.1) yields equation (9) with parameters:

$$\begin{aligned} a_1 &\approx \frac{(1-\mu)(m+1)(m+2)}{(1-2\mu)2k_D f D^3 W_0^2} \\ b_1 &\approx 3 \end{aligned} \quad (18)$$

where  $\mu$  is given by:

$$\mu = \frac{4 - 3\gamma - \sqrt{\gamma^2 - 2\gamma - 4}}{4 - 2\gamma} \quad (19)$$

and

$$\gamma = 2(m+2)B(m+2,2) \quad (20)$$

with  $B(m+2,2)$ , the beta function evaluated at  $m+2$  and 2 [*Rupp and Selker*, 2005]. Again, note the similarities between equations (10), (13), and (17): in all equations, the parameters  $k$ ,  $f$ ,  $D$ , and  $W_0$  appear to the same power, and  $b_1$  is in all cases equal to 3.

[47] For late-time, an exact solution (see the initial and boundary conditions in section 3.1) also yields equation (9), though with parameters:

$$\begin{aligned} a_2 &= \frac{(m+2)}{f} \left[ \frac{k_D W_0^2 B((m+2)/(m+3), 1/2)}{2^m (m+1)(m+3) D^m A^{m+3}} \right]^{1/(m+2)} \\ b_2 &= \frac{2m+3}{m+2} \end{aligned} \quad (21)$$

where  $B((m+2)/(m+3), 1/2)$  is the beta function evaluated at  $(m+2)/(m+3)$  and  $1/2$  [*Rupp and Selker*, 2005]. The range of values taken by  $b_2$  varies from  $3/2$  to 2, as  $m$  varies from 0 to  $\infty$ .

[48] Lacking an analytical solution to their generalized Boussinesq equation for sloping aquifers, *Rupp and Selker* [2006b] ran a suite of numerical simulations with the classic boundary and initial conditions given in section 3.1. At late-time, the recession discharge converged to the form given by equation (9) regardless of the initial conditions and the parameters could be approximated as follows:

$$\begin{aligned} a_2 &= \frac{(m+1)^2}{mfA} \left[ \frac{2k_D W_0 \sin i}{(m+1)D^m} \right]^{1/(m+1)} \\ b_2 &= \frac{2m+1}{m+1} \end{aligned} \quad (22)$$

[49] For sloping aquifers, as a consequence, the range of values taken by  $b_2$  varies from 1 to 2, as  $m$  varies from 0 to  $\infty$ .

[50] These results demonstrate how decreasing conductivity from surface to bedrock expands the range of theoretical values of  $b_2$  and provides one plausible explanation for the fact that we observe a range of values in the field and not only the classic late-time values of  $3/2$  and 1 predicted by the traditional Boussinesq theory under the assumption of a homogeneous aquifer.

#### 4.6. Impact of Between-Hillslope Variability

[51] Implicit in the method of *Brutsaert and Nieber* [1977] is the assumption that the entire watershed can be replaced by an effective hillslope whose parameters can be identified from the recession curve. *Clark et al.* [2009] observed that recessions at the Panola Mountain Research Watershed varied as a function of watershed scale, with exponents around 1 at the hillslope scale and exponents approaching 3 in the 41 ha watershed. They proposed that this behavior could be explained by the superposition of several sources of water with exponents of 1 but with different time constants, ranging from fast (in the steep headwaters) to slow in the floodplain riparian aquifer.

[52] *Harman et al.* [2009] derived this effect using superstatistics. They parameterized the watershed as a collection

of parallel linear reservoirs whose time constants are given by a probability distribution, such as a gamma distribution. They then derived the unit hydrograph for the watershed and the watershed response to various idealized inputs. The results demonstrated that the exponent of the B&N recession slope curves could be related to the coefficient of variation of the distribution describing the individual linear reservoir time constants. This result also demonstrated that recession curves ought to vary depending on the time-history of recharge, especially in areas with high spatial variability between hillslopes. Note that the above formulations can produce the full range of recession curves predicted from the original theory. This suggests that, when applied to heterogeneous watersheds, the exponents of the B&N recession slope curves may reflect the combined effect of the geomorphological and geological structure of the watershed, rather than the hydraulics of a single effective watershed.

#### 4.7. Analysis of Baseflow Hydrograph Rising Limbs

[53] Another approach to estimate aquifer parameters through the examination of baseflow records was recently developed by *Pauwels and Troch* [2010]. In this study, the argument was raised that full saturation of an aquifer prior to recession occurs very rarely. This implies that the use of the short-time relationship described in equation (10) can lead to errors in the obtained parameter values. However, based on an analytical solution to the linearized Boussinesq equation, and starting from an initially empty aquifer, *Pauwels and Troch* [2010] derived the following recession relationship:

$$\frac{dQ_b}{dt} \approx 8W_0^2 N^2 \frac{K}{\pi} Q_b^{-1} \quad (23)$$

[54]  $N$  is the recharge rate and  $K$  is a diffusion coefficient defined according to equation (15). Equation (23) can be used, in combination with equation (13), to estimate hydraulic conductivity values of the lower parts of an aquifer. First, the points need to be selected for which equation (23) is valid. In other words, the aquifer needs to be sufficiently dry. It can easily be proven [*Pauwels and Troch*, 2010] that for these points the following relationship is true:

$$\frac{Q_b(t)}{\frac{dQ_b(t)}{dt}} \approx 2\Delta t \quad (24)$$

[55]  $\Delta t$  is the time step of the measurements [s]. A linear regression through a plot with  $Q_b N^{-2}$  as abscissa and  $dQ_b/dt$  as ordinate, of all the points for which equation (23) is valid, should then result in a slope of  $-1$ . The intercept of this regression is denoted  $I_2$  ( $m^4 s^{-1}$ ). Next, in the  $-dQ_b/dt$  versus  $Q_b$  plot, the lowest values of  $Q_b$  are retained. The intercept of a linear regression with slope 1 through these data is denoted as  $I_1$  ( $s^{-1}$ ). The following system of equations is then solved in order to find the two unknowns  $k$  and  $pD$

$$\begin{cases} I_1 = \frac{kpD \cos i}{f} \left[ \left( \frac{\beta pD \cos i - \sin i}{2pD \cos i} \right)^2 + \frac{z_n^2}{L^2} \right] + \frac{\beta k \sin i}{f} \\ I_2 = \frac{8W_0^2 kpD \cos i}{f\pi} \end{cases} \quad (25)$$

[56] Through a number of synthetic studies, *Pauwels and Troch* [2010] demonstrated the accuracy of this methodology, and the potential of the use of this equation for the estimation of aquifer lower layer hydraulic conductivity values.

## 5. Streamflow Recession: Water Balance Applications at the Catchment Scale

[57] A lasting contribution of the work of Wilfried Brutsaert and Jean-Yves Parlange is their contributions to a suite of tools for making inferences about the properties of watersheds and their internal hydrologic dynamics directly from the streamflow record. Given that streamflow data are often the best (or only) observed component of the catchment water balance, these tools are very useful. Methods have been developed to make inferences about the evapotranspiration, recharge, and storage dynamics from the streamflow recession directly. Furthermore, the functional form and parameters of the relationship between streamflow and its time derivative can be investigated, aiding in model identification and prediction, and providing a fundamental metric of catchment behavior that can be compared to theory, to other watersheds, and tracked over time. The streamflow recession analysis techniques of *Brutsaert and Nieber* [1977] and the hydraulic groundwater theory are central components of these techniques.

### 5.1. Streamflow Recession and Catchment Water Balance

[58] Observations of diurnal and seasonal variations in groundwater level and streamflow prompted early efforts to estimate daily evapotranspiration by inference from variations in other components of the water balance [*White*, 1932]. Although multiple factors contribute to changes in catchment storage  $S$ , in general the water balance reads:

$$\frac{dS}{dt} = R - E - Q \quad (26)$$

where  $R$  is the total amount of rainfall per unit of time [ $m^3 s^{-1}$ ] and  $E$  corresponds to the total amount of evapotranspiration in the catchment per unit of time [ $m^3 s^{-1}$ ]. Through consideration of the water balance, *Tschinkel* [1963] suggested a method for estimating evapotranspiration during periods when  $R = 0$  by comparing the diurnal deviations in streamflows from the “potential” rate, which (implicitly) depends only on the increment of storage and can be estimated from nocturnal peak streamflows after periods of low potential evapotranspiration. *Daniel* [1976] later extended this to estimate evapotranspiration from the analysis of recession curves over longer periods by assuming a nonlinear functional relationship (derived from hydraulic groundwater arguments) between discharge and riparian groundwater storage. *Brutsaert* [1982] suggested a generalization of this approach by adopting a general nonlinear storage-discharge relation of the form  $Q = f(S) = KS^n$  and the use of equation (9) to determine the “potential” rate of streamflow recession that would occur in the absence of evapotranspiration. If the recession curve can be characterized by  $aQ^b$  when evapotranspiration is negligible, equation (9) can be expressed for other times as:



$$-\frac{dQ}{dt} = aQ^b \left(1 + \frac{E}{Q}\right) \quad (27)$$

allowing *Brutsaert* [1982] to suggest that evapotranspiration can be estimated from the daily record of streamflow recession as:

$$E = -\left(\frac{Q^{1-b}}{a} \frac{dQ}{dt} + Q\right) \quad (28)$$

[59] *Zecharias and Brutsaert* [1988] used a quasi-steady-state solution to the linearized Boussinesq equation [equation (8)] that results in  $b = 1$  to estimate evapotranspiration from groundwater in 19 watersheds in the Allegheny Mountains, and found that it was a minor part of the overall groundwater balance. *Szilagyi et al.* [2007] used a detailed 2D aquifer simulation to validate the method. *Wittenberg* [1994, 1999], *Wittenberg and Rhode* [1997], and *Wittenberg and Sivapalan* [1999] developed an alternative technique for simultaneously identifying nonlinear storage-discharge relations of the form  $Q = f(S) = KS^n$  during seasons when evapotranspiration was low. *Wittenberg and Sivapalan* [1999] used these to estimate not only evapotranspiration but also the full catchment groundwater balance (recharge and the time series of relative storage) directly from the record of streamflow recessions.

[60] Recently, *Kirchner* [2009] proposed a generalization of this approach by defining the “discharge sensitivity function”  $g(Q)$  as:

$$g(Q) = dQ/dS \quad (29)$$

[61] The advantage of this approach over the previous ones is that the functional form of the storage-discharge relationship is not assumed a priori. This allows for a wider class of relationships between storage and discharge to be obtained depending on the observed functional form of  $g(Q)$ . *Kirchner* [2009] notes when the *Brutsaert and Nieber* [1977] analysis suggests that equation (9) is a good fit to the dynamics, the discharge sensitivity function is given by:

$$g(Q) = \frac{dQ}{dS} = \frac{\frac{dQ}{dt}}{-Q} = aQ^{(b-1)} \quad (30)$$

[62] The storage-discharge relationship can be obtained by integrating equation (29). *Kirchner* [2009] distinguished between three different classes of solutions: in case  $b < 2$ ,  $f(S)$  is a generalized form of the *Coutagne* [1948] nonlinear function which includes  $S_0$  the residual storage for which  $Q = 0$ :

$$Q = Q_{ref} \left[ \frac{S - S_0}{\kappa} \right]^{1/(2-b)} \quad (31)$$

where  $Q_{ref}$  is an arbitrary reference discharge and  $\kappa = f(a, b, Q_{ref})$  is a scaling constant. In case  $b = 2$ ,  $f(S)$  becomes an exponential function:

$$Q = Q_{ref} e^{a(S-S_0)} \quad (32)$$

such that  $S_0$  is the value of  $S$  for which  $Q = Q_{ref}$ ; in case  $b > 2$ ,  $S_0$  becomes an asymptotical upper storage limit [see also *Rupp and Woods*, 2008]. In contrast to the use of seasonal variations in potential evapotranspiration adopted by *Wittenberg* [1999] to identify  $f(S)$ , *Kirchner* [2009] applied this method to hourly records of streamflow from catchments in Wales using only night-time recession flows from periods when potential evapotranspiration was low. The use of hourly streamflow records and focus on the marginal (rather than absolute) relationship between storage and discharge makes the *Kirchner* [2009] approach reminiscent of the methods developed by *Tschinkel* [1963]. The use of night-time recessions to estimate the “potential” curve is problematic when the night-time storage recovery in riparian aquifers is large, in particular during later stages of recession when streamflow losses become small in comparison to daytime ET. Analyzing the composite daily streamflow recession across different discharge conditions in the Rietholzbach, *Teuling et al.* [2010] noted an increasing time lag as well as a strong diurnal cycle (including discharge increase during morning recession) at low discharge values. An integration of the *Kirchner* [2009] approach with those previously developed by *Tschinkel* [1963], *Brutsaert* [1982], and others may help to overcome these challenges.

## 5.2. Comparison With the Boussinesq Model

[63] Given that the above approaches yield insights into the catchment-scale storage-discharge relations derived from data, it is interesting to compare their results to the theoretical derivations based on the Boussinesq equation as applied to hillslopes. Different applications of the Boussinesq or other equations result in a range of values for  $b$ , shown in Table 1, below. For comparison, alternative model formulations are also shown.

[64] The *Brutsaert-Nieber* [1977] (or similar) recession data analysis has been applied to many catchments. *Wittenberg* [1999], using a power law  $S(Q)$  relationship applied to ~100 German streams, found (equivalently)  $b$  ranging from 1 to 2, with a peak in the range 1.5–1.7. *Lyon and Troch* [2007] found  $b = 1.5$ –1.6 for hillslope strips in the steep (~40°) New Zealand Maimai catchment and  $b = 1.29$  for the moderately steep (~10°) Troy (Idaho, USA) hillslope. *Kirchner* [2009] analyzing the Severn and Wye headwaters at Plynlimon, Wales, UK, found  $b = 2.17$  and  $b = 2.10$ , respectively.

[65] *Teuling et al.* [2010] applied the discharge sensitivity method to the Swiss Rietholzbach catchment. Here, even more so than in Plynlimon, the  $\log g(Q)$  function was found to depend nonlinearly on  $\log Q$ . Instead of using a single quadratic expression, as *Kirchner* [2009] used, they applied a piecewise linear fit (in log-log space). It was found that low flows were characterized by  $b = 2.6$ , intermediate flows by  $b = 2$ , and high flows by  $b = 1.69$ . These results were interpreted in terms of storage-dependent changes in dominant processes. Under dry conditions, the discharge sensitivity is low, but increases when the catchment storage increases. *Birkel et al.* [2011] applied the method to two Scottish catchments. Plots of  $\log g(Q)$  versus  $\log Q$  showed little if any nonlinearities. Equivalent values of  $b$  were high, ranging from 2.38 to 2.52.

[66] In all these examples, it is clear that the majority of steep catchments show  $b > 1.5$ , which is not consistent with

**Table 1.**  $b$ -Values for Different Aquifer Representations (Late-Time Behavior in Case of Boussinesq Equation)

Model	$b$ Value
Kinematic wave model; steep hillslope; homogeneous aquifer	$b = 0$ (P. W. Bogaart, D. E. Rupp, J. S. Selker, and Y. van der Velde, Late-time drainage from a sloping Boussinesq aquifer, under review with <i>Water Resources Research</i> , 2013)
Boussinesq equation; steep hillslope; homogeneous aquifer	$b = 0$ (P. W. Bogaart et al., manuscript in review, 2013)
Linear reservoir	$b = 1$ [Brutsaert, 2005]
Boussinesq equation; horizontal aquifer; homogeneous aquifer	$b = 1.5$ [Brutsaert and Nieber, 1977]
Kinematic wave model; steep hillslope; exp. decreasing conductivity	$b = 2$ [Troch et al., 1993b]
Boussinesq equation; horizontal aquifer; power-law $k$ -profile	$b = 1.5$ – $2$ [Rupp and Selker, 2006b]
Kirchner [2009] equation (17)	$b > 2$ [Kirchner, 2009]

any of the models listed in Table 1 that assume a homogeneous conductivity profile. Exceptions are Rupp and Selker's [2005] power-law profile, and the exponential model (31) that is equivalent with the formulation used in TOPMODEL [Beven and Kirkby, 1979]. Several explanations for high  $b$  values related to the heterogeneity of the landscape are given in section 4, and generalizations of the Boussinesq equation that could account for high values of  $b$  are discussed in section 6 of this paper.

[67] Inappropriate application of the Brutsaert and Nieber [1977] technique may also explain some of the high observed values of  $b$ . Chapman [2003] suggested that the assumption that recharge goes to zero soon after rainfall events may not be realistic in many cases, and the form of the streamflow recession may owe much to extended drainage from the unsaturated zone. Evapotranspiration can have a significant effect on the shape of the recession, an effect exploited by Brutsaert [1982] and others to estimate the water balance as described in section 5.1. Szilagyi et al. [2007] showed that  $ET$  affects both the apparent slope  $b$  and intercept  $a$ . Shaw and Riha [2012] analyzed individual recession events within the  $-dQ/dt$  "data-cloud," and found that  $b$  values deviated considerably from those obtained by taking the lower envelope, but also showed considerable seasonal variation. This is shown in Figure 3, which shows the seasonal trend in the intercept parameter  $a$  for several watersheds in New York State, USA. This effect could be explained by the seasonal cycle of  $ET$ . This suggests that Brutsaert and Nieber [1977] recession curve analysis can be improved by methods that account for evapotranspiration, such as those of Brutsaert [1982], Wittenberg and Sivapalan [1999], and Kirchner [2009].

[68] However, even the Kirchner approach cannot fully exclude all confounding effects, in particular during later stages of recession when streamflow losses become small in comparison to daytime evapotranspiration. Analyzing the composite daily streamflow recession across different discharge conditions in the Rietholzbach, Teuling et al. [2010] noted an increasing time lag as well as a strong diurnal cycle (including discharge increase during morning recession) at low discharge values. Both observations constitute serious complications to current approaches trying to link theoretical values of  $b$  to those derived from late recession analysis in natural catchments.

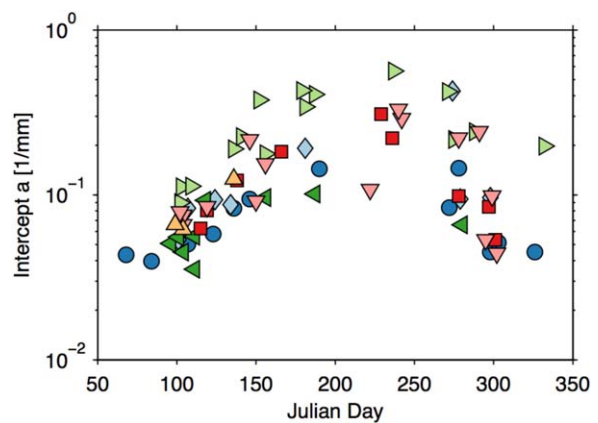
### 5.3. Storage

[69] A useful application of both Wittenberg's [1999] approach to the nonlinear storage-discharge function  $f(S)$

and the Kirchner [2009] discharge sensitivity function  $g(Q)$  is the estimation of the catchment dynamic storage (or "active" storage as Wittenberg [1999] named it), defined as the difference in storage between dry and wet periods, and found by integrating the reciprocal of the hydrological sensitivity [Kirchner, 2009]:

$$dS = \int \frac{1}{g(Q)} dQ \quad (33)$$

[70] Wittenberg [1999] found that estimates of storage variability obtained using an equivalent form of this method compared well with water table observations, but noticed a nonlinear discrepancy that suggested aquifer drainable porosity varied with depth. Application of equation (32) to Plynlimon yielded a dynamic storage of 62 mm (Wye) to 98 mm (Severn), which is approximately 3–5% of net annual precipitation. Application to the Swiss Rietholzbach [Teuling et al., 2010] yields a dynamic storage of  $\sim 175$ – $200$  mm which corresponds to  $\sim 20\%$  of net annual precipitation. Birkel et al. [2011] applied both the discharge sensitivity method and a tracer-constrained conceptual rainfall-runoff model to assess dynamic storage for two



**Figure 3.** Relationship between time of year and the intercept of regression lines (with slope  $b=2$  in log-log space) fit to individual recession event  $dQ/dt$  versus  $Q$  data for several catchments in New York State, U.S. As indicated in the figure, the intercept of recession curves tends to be shifted upward in the summer and early fall months (Julian Day 150–270) due to the impact of evapotranspiration losses. Data taken from Shaw and Riha [2012].

nested catchments in the Scottish Highlands. They found that the tracer-constrained estimates (approx. 10–15% of  $P-E$ ) exceeded the estimates based on  $g(Q)$  (approx. 2–5%). However, an analysis based on oxygen isotope mixing indicates a total storage of about 100% of annual  $P-E$ . This result, which is corroborated by alternative, independent approaches, suggests that the chemical response time of catchments cannot easily be estimated from the hydrological response time.

#### 5.4. Future Directions

[71] The use of a general nonlinear storage discharge relationship [Brutsaert, 1982; Wittenberg, 1994] or discharge sensitivity approach [Kirchner, 2009] can be seen as a generalization of the Brutsaert and Nieber [1977] recession analysis technique. These approaches allow the empirical functional properties of watersheds and the dynamics of the groundwater balance to be inferred directly from analysis of streamflow. However, when applied to steep catchments, the empirical properties of the nonlinear stores are difficult to reconcile with the Boussinesq equation. The deviations may be due to the effects of unconstrained recharge or evapotranspiration dynamics, or they may arise from the structural properties of natural watersheds that deviate from the assumptions of the Boussinesq equation. The assumption of vertically homogeneous hydraulic conductivity within a relatively thin soil layer may be inappropriate in many natural soils. In addition, tracer-based estimates of the total catchment storage involved in mixing indicates that passive storage exceeds active storage by a factor 5 or more. Again, this result is difficult to reconcile with the assumptions underlying the Boussinesq model. An alternative set of assumptions, such as a decreasing saturated hydraulic conductivity with depth, might be more appropriate for these cases. Examples include the TOPMODEL assumption of exponentially decreasing conductivity, or the power-law conductivity model used in the Boussinesq framework by Rupp and Selker [2006b]. Both the recession analysis ( $b=2$  for TOPMODEL,  $b=1$  to 3 for Boussinesq) and the storage volume (large to infinite for TOPMODEL, small, and finite for Boussinesq) point in that direction. Also, Kirchner's [2009] interpretation of  $S_0$  in case of  $b > 2$  in terms of a maximum (instead of residual) storage is in line with the TOPMODEL use of storage deficit as a state variable, rather than storage itself.

[72] A possible solution to reconcile the evidence for high  $b$  values found in steep catchments, and the success of the Boussinesq equation applied to more gentle terrain is discussed in Brutsaert [2005, p. 431], where it is suggested that in steeper catchments the hydrological response in the early stages of recession is dominated by the steep hillslopes, but during later stages of recession, when the catchment dries and saturated areas shrink, the hydrologic response is therefore dominated by riparian areas. These areas are not only characterized by gentle slopes, but also consist of sediment of fluvial origin, such that the vertical variation of hydraulic conductivity is either layered or relatively homogeneous and indicate that the assumptions underlying the Boussinesq equation might be valid here. This suggests that steep catchments with pronounced valley bottoms might experience a shift from TOPMODEL-like behavior under wet conditions (when the hillslopes dominate) toward more Boussinesq-like

conditions under dry conditions (when the valley bottom aquifers dominate).

[73] Future extensions to recession analysis should therefore focus on the unraveling of at least three classes of unknowns. One is the relative contribution of drainage versus recharge from the unsaturated zone and evapotranspiration in the water balance, while a second is the relative contribution of hillslope and valley bottom domains, and their associated hydraulic architecture, under various levels of wetness. Moreover, a third unknown, which has only recently received much attention, is the relative roles that geomorphology and a dynamic river network play in determining the shape of the recession curve [Biswal and Marani, 2010; Mutzner et al., 2013].

## 6. Extensions to the Boussinesq Equation

[74] The Boussinesq equation and its analytical solutions, as presented in previous sections, have been instrumental in creating a better understanding of subsurface flow and storage phenomena along hillslopes. Traditionally, the work has been aimed at increasing our understanding by seeking analytical solutions, which are typically only to be obtained under strict geometric conditions (e.g., one-dimensional, straight slopes). Real hillslopes, however, have more complex, three-dimensional (3D) shapes. The papers reviewed in this section aim to extend the applicability of Boussinesq's original work, and the work that followed from it, to complex hillslope geometries, leaky aquifers, and the unsaturated zone.

### 6.1. The Hillslope-Storage Kinematic Wave Model

[75] The papers of Fan and Bras [1998] and Troch et al. [2002] are amongst the first aimed at relaxing the strict conditions regarding hillslope geometry that are traditionally required when applying a Boussinesq model. Both papers make use of a simplified form of Boussinesq's equation, a kinematic wave approach that assumes the second-order diffusive term in the Boussinesq equation is negligible, and introduce a conceptualization whereby the three-dimensional soil mantle overlying the hillslope bedrock is collapsed into a one-dimensional storage profile. Hereafter, this model will be referred to as the hillslope-storage kinematic-wave (hsKW) model. The subsurface mass balance equation for both Fan and Bras [1998] and Troch et al. [2002] reads:

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = NW \quad (34)$$

where  $W$  is the hillslope width function that in this case is not necessarily modeled according to equation (12). The storage function is defined as:

$$S = Whf \quad (35)$$

[76] As already mentioned, it is further assumed that the flow rate can be described using a kinematic form of Darcy's equation:

$$Q = -k \frac{S}{f} \frac{\partial \xi}{\partial x} \quad (36)$$

where  $\xi$  is the bedrock height (m).

[77] In *Fan and Bras* [1998], these equations are applied to a set of three realistic hillslopes of which the geometric properties (i.e., hillslope width and bedrock topography) are measured, while the work of *Troch et al.* [2002] applies this approach to a set of hillslopes ranging from convergent to divergent in both plan and bedrock shape, yielding nine characteristic combinations. In both papers, analytical solutions are presented using the method of characteristics. *Fan and Bras* [1998] use a second-order polynomial to describe the bedrock topography, while *Troch et al.* [2002] use a more general power function, and perform a dimensional analysis that allows for scaling of results to differently shaped hillslopes. Both papers illustrate clearly the major influence that hillslope shape and steepness exert on runoff and storage dynamics.

## 6.2. The Hillslope-Storage Boussinesq Model

[78] The assumption of a negligibly small second-order diffusion term in the hsKW model is thought to hold only for particular flow conditions where the water table gradients are small in comparison to bedrock gradients (e.g., for steep hillslopes). In the two-part paper by *Troch et al.* [2003] and *Paniconi et al.* [2003], the full Boussinesq equation is cast in a hillslope-storage formulation. The resulting hillslope-storage Boussinesq (hsB) model can thus account for complex hillslope plan shapes and diffusive processes.

[79] *Troch et al.* [2003] provide a comprehensive description of the hsB model, along with several linearized and simplified versions. These variants, as well as the hsKW model, are compared against the full hsB model for the same set of characteristic hillslopes described in *Troch et al.* [2002], for two average slope gradients, 5 and 30%, and for both a free-drainage scenario and a recharge scenario. The hillslope-storage Boussinesq equation is derived by combining the mass-balance formulation in equation (33) with a Boussinesq-type flow equation:

$$Q = -\frac{kS}{f} \left[ \cos i \frac{\partial}{\partial x} \left( \frac{S}{fw} \right) + \sin i \right] \quad (37)$$

to obtain

$$f \frac{\partial S}{\partial t} = \frac{k \cos i}{f} \frac{\partial}{\partial x} \left[ \frac{S}{w} \left( \frac{\partial S}{\partial x} - \frac{S}{w} \frac{\partial w}{\partial x} \right) \right] + k \sin i \frac{\partial S}{\partial x} + fNw \quad (38)$$

[80] The full hsB model is solved by means of numerical integration. In *Paniconi et al.* [2003], the hsB model is benchmarked against a fully 3D numerical Richards equation model.

[81] In a follow-up paper by *Hilberts et al.* [2004], the condition of noncurved bedrock (i.e., a constant bedrock slope) that was inherent in the hsB model presented in *Troch et al.* [2003] is relaxed, and a series of benchmark tests are conducted on the same hillslopes used in *Troch et al.* [2003] and *Paniconi et al.* [2003]. Comparisons are made between this extended hsB model, the fully 3D numerical Richards equation model, and the hsKW model. In addition, a dimensional analysis is performed, and the results of the study are presented using dimensionless variables. The conclusions are that the hsB model and the fully

3D numerical Richards equation model compare very favorably in terms of both simulated runoff and storage dynamics along the hillslope. Similar to what was found in *Troch et al.* [2003], it is again found that hsKW performance improves as hillslopes become steeper and more divergent.

[82] *Troch et al.* [2004] derived an analytical solution for a linearized version of the hsB equation under the specific condition of a hillslope width function of exponential form. This variant of the hsB model is then compared against a numerically integrated hsB model, and the results show an almost exact match, thus confirming the validity of the numerical results obtained in the earlier papers. Moreover, *Troch et al.* [2004] extend the analysis by expressing the Peclet number, often used in groundwater transport studies, as a function of hillslope characteristics. This aspect is further discussed in section 7.

## 6.3. Modeling Unsaturated Zone Effects in a Boussinesq Aquifer

[83] Traditionally, model parameters such as the drainable porosity  $f$  are considered constant when applying the Boussinesq equation, and generally a value equal to the saturated soil moisture content minus the residual soil moisture content,  $(\theta_s - \theta_r)$ , is attributed to  $f$ . In a benchmark paper by *Parlange and Brutsaert* [1987], the capillarity effect on groundwater systems is modeled by assuming a deep profile for which  $\theta = \theta_r$  holds at the land surface and instantaneous equilibrium in the unsaturated zone. An analytical expression that accounts for capillarity effects is thus derived and can be added to the Boussinesq equation. *Barry et al.* [1996] extended the equations derived by *Parlange and Brutsaert* [1987] to include higher-order capillarity effects. *Hilberts et al.* [2005] on the other hand intervene directly on the drainable porosity parameter, treating it as a function of the water table height  $h$  and the soil depth  $D$ . Assuming an unsaturated zone soil moisture profile that is in equilibrium with the water table, and using an alternative parameterization of the van Genuchten relationships to describe the unsaturated zone characteristics that is amenable to analytical integration, the authors derive the following expression for drainable porosity:

$$f(h) = (\theta_s - \theta_r) \left\{ 1 - \left( 1 + \left[ \alpha \frac{h-D}{\cos i} \right]^n \right)^{-\frac{(n+1)}{n}} \right\} \quad (39)$$

where  $\alpha$  and  $n$  are van Genuchten parameters that describe the retention behavior of the unsaturated zone. The hsB model is then rederived using equation (38) for what is now a state variable-dependent drainable porosity parameter. Drainage simulations with this revised hsB model are then compared against the original hsB model (equation (37)) as well as against experimental discharge and water table observations from a laboratory hillslope set up for two plan shapes (divergent and convergent) and for three gradients (5, 10, and 15%). It is found that both the new and original hsB models match the observed discharge rates well, but that the revised model with a more general representation of  $f$  allows a much more accurate description of water table dynamics.

[84] In follow-up work to *Hilberts et al.* [2005], *Hilberts et al.* [2007] couple a one-dimensional Richards equation

model to the hsB model with a variable drainable porosity. This allows the recharge rate  $N$  to be modeled as a function of unsaturated zone storage dynamics, so that a more accurate representation of the rainfall to aquifer recharge transformation process is possible. Results for an experiment consisting of constant rainfall followed by pure drainage are compared against both the original hsB model and the fully 3D numerical Richards equation model. The three-way comparison indicates a much improved performance of the coupled hsB model over the original version. The results also highlight some interesting features of potentially rapidly rising water tables as a result of a soil moisture pulse propagating through the unsaturated zone and encountering the capillary fringe above the water table.

#### 6.4. Incorporating Bedrock Leakage and Other Processes

[85] In addition to neglecting the unsaturated zone, another basic assumption of classical Boussinesq and hsB models is that the unconfined aquifer overlies an impermeable bedrock. In upslope areas where water tables are deeper, allowing leakage or percolation through the bottom of the unconfined aquifer is essential for capturing the process of groundwater recharge to confined aquifers. In downslope regions where water tables are shallower and typically intersect a lake or stream, the leakage process can occur in the reverse direction, with flow from a deep aquifer traversing the unconfined aquifer to discharge into the stream. Representing this contribution to the stream hydrograph is critical for baseflow estimation.

[86] *Koussis et al.* [1998] introduced a leakage term into a linearized version of Boussinesq's equation, approximating the exchange flux across the aquitard as a Darcy expression involving the aquitard properties and the head difference across this unit. *Broda et al.* [2011] use an approach similar to the comparative studies conducted in the hsB papers previously cited (i.e., simulating with a 3D numerical Richards equation model idealized hillslopes of uniform, convergent, and divergent shape at different slope angles), but at the same time they extend the vertical horizon to include an unconfined and a confined aquifer separated by an aquitard of specified thickness. Their results show that leakage can be downward or upward (in the reverse direction) at different points along the hillslope and at different times during a drainage or recharge event, and that these dynamics are strongly influenced by hillslope geometry (for instance, convergent hillslopes contain the largest portions of upward directed leakage), boundary conditions, and aquifer hydraulic properties.

#### 6.5. Extending the Hillslope-Storage Boussinesq Model to Catchment-Scale Applications

[87] Having added complex geometry, unsaturated zone effects, bedrock leakage, and other factors and processes to the classical Boussinesq model as described in the preceding sections, a natural next step is to try to extend the applicability of the hsB model to catchment and river basin scales, under the guiding principle that hillslopes can be considered fundamental flow units or building blocks in watershed hydrology. The basic idea is to couple an hsB representation for shallow subsurface flow to models that will handle deeper groundwater flow and overland and

channel flow routing. The numerous challenges in doing so include ensuring that the process submodels (hsB, groundwater, surface routing) are compatible and that the exchanges between each component (flows between hillslope units as well as flows between submodels) can be reasonably well represented or estimated.

[88] *Matonse and Kroll* [2009] focus their study on a small steep headwater catchment and on low flow estimation, so deep aquifer and overland flow processes are not considered. They find that a higher level of partitioning (discretization of the catchment into more hillslope units), by enhancing the ability of the model to represent hydrogeologic heterogeneity, improves the model performance. *Broda et al.* [2012] couple the hsB model to an equally parsimonious and computationally efficient analytic element model of deep regional groundwater flow. Iterative updating of the hydraulic head levels in the shallow subsurface and deep aquifers is used to determine the exchange flux (leakage across a hypothetical aquitard) between the hillslope and regional groundwater units. In comparisons against a benchmark 3D Richards equation model, good matches for head, discharge, and interaquifer exchange are generally obtained, with the poorest performance noted for very steep and convergent hillslopes.

[89] *Carrillo et al.* [2011] used the hillslope-storage Boussinesq model as the core of a semidistributed watershed model. They combined a root zone water and energy balance model with the hsB equation to account for unsaturated-saturated zone interactions on runoff generation and hydrological partitioning. Vegetation effects are accounted for to influence interception and transpiration, while deep aquifer dynamics are modeled as in *Kirchner* [2009]. *Carrillo et al.* [2011] applied their model to 12 catchments across a climate gradient in the USA, and concluded that it can be used to produce models that can capture hydrological dynamics at different temporal scales, from decades to daily.

### 7. Similarity Analysis of Baseflow Recession

[90] Geometric similarity in relation to Boussinesq was already discussed nearly 50 years ago in the experimental analysis by *Ibrahim and Brutsaert* [1965]. Based on analytical solutions to the governing dynamic equations, it is also possible to develop similarity indices capable of discerning how similar or dissimilar landscapes are with respect to their hydrological response [*Wagner et al.*, 2007].

#### 7.1. The Hillslope Number

[91] *Brutsaert* [1994] derived an analytical solution to a linearized Boussinesq equation to study the hillslope subsurface flow unit response, corresponding to the free drainage of an unconfined aquifer (hereafter the characteristic response function or CRF). The motivation for his work was to provide a direct link between the underlying physical mechanisms of hillslope subsurface flow and the general linear theory of catchment hydrology [*Dooge*, 1973]. The analytical approach provides a powerful framework to analyze the influence of the different characteristics (hydraulic and geometric) of hillslopes on the shape of its hydrological response [*Berne et al.*, 2005]. *Brutsaert's* [1994] work suggests that, under given boundary

conditions, the variables in the equations describing the governing dynamics for subsurface flow in shallow, sloping aquifers can be scaled producing a dimensionless number for straight (nonconverging) hillslopes called the groundwater hillslope number ( $Hi$ ).  $Hi$  is defined as:

$$Hi = \frac{L \tan i}{\eta_0} \quad (40)$$

with  $\eta_0$  is the average water table height within the hillslope. This dimensionless parameter represents the relative magnitude of the slope term (that is the effect of gravity) with respect to the diffusion term [Brutsaert, 2005].

### 7.2. The Péclet Number

[92] Continuing along this line of exploration and starting from the Laplace domain solution derived by Troch *et al.* [2004] to the linearized hsB equation, an additional dimensionless number for advective-diffusive subsurface flow dynamics along complex hillslopes can be derived. This hillslope Péclet ( $Pe$ ) number as a similarity parameter extends the Brutsaert  $Hi$  number by explicitly accounting for exponentially converging/diverging hillslopes in the landscape. This extension is possible through the development of low-dimensional hillslope storage dynamics models capable of handling three-dimensional hillslope structures in a parsimonious way [e.g., Troch *et al.*, 2003]. Applied to flow response of a hillslope of length  $L$ , the  $Pe$  number becomes:

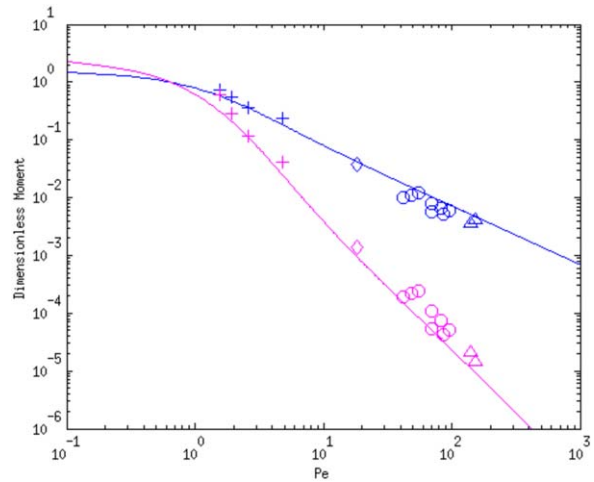
$$Pe = \frac{UL}{2K} \quad (41)$$

where  $U$  is a characteristic advective velocity equal to  $U - K\beta$  in equation (15) and  $K$  is the complex-hillslope diffusion coefficient defined according to equation (15). Troch *et al.* [2004] showed that the  $Pe$  number can be expressed in terms of only quantifiable geomorphic properties:

$$Pe = \frac{L \tan i}{2pD} - \frac{\beta L}{2} \quad (42)$$

where  $pD$  is the effective aquifer depth and  $\beta$  is the rate of (exponential) plan shape convergence (or divergence) of the hillslope [equation (12)].

[93] Berne *et al.* [2005] used moment generating functions to analytically relate the hillslope  $Pe$  number to the moments of the CRF of subsurface flow. In addition, they showed the  $Pe$  number as an efficient similarity parameter to describe the hillslope subsurface flow response and used laboratory data from a scaled hillslope model to validate the derived relationships (Figure 4). The  $Pe$  number has also been successfully applied to “real-world” data (i.e., the hydrological response determined from hillslope-scale field experiments) at the hillslope-scale by Lyon and Troch [2007] and at the small catchment scale by Lyon and Troch [2010]. For these later similarity applications, effective parameters to capture the subsurface heterogeneity at hillslope scales were required and assessed based on recession flow analysis [i.e., Brutsaert and Nieber, 1977], which has been demonstrated throughout the literature in various climatologic and geomorphologic settings [Lyon *et al.*,



**Figure 4.** Theoretical scaling relationships between the Péclet number and the dimensionless moments of the characteristic response function (first order in blue, second order in magenta). Symbols represent observations from artificial hillslopes (+); real-world hillslopes located in Maimai, New Zealand (circles), and Idaho, USA (diamonds); and small catchments in Arizona, USA (triangles).

2008; Troch *et al.*, 1993a; Zecharias and Brutsaert, 1988]. Lyon *et al.* [2010] have shown further utility of similarity approaches like those considered via the  $Hi$  and  $Pe$  numbers through simple empirical estimation of climatic impacts on catchment-scale water travel times brought about through alterations of landscape storage and structure.

### 7.3. The Combined Hydraulic-Geomorphology-Climate-Boundary Conditions Number

[94] The linearization approach and the derived analytical solutions depend on the identification of an effective value of the aquifer thickness  $pD$ . As the previous work had shown, the average storage thickness  $\eta_0$  determines whether the advective or diffusive terms of the Boussinesq equation play the dominant role in the system behavior. However, that storage is itself an outcome of the interplay between the magnitude and variability of recharge (and leakage through the bedrock), the boundary condition at the toe of the hillslope, and the hydrologic response itself. This point was made by Beven [1982], when he showed that the kinematic approximation to the steady-state Boussinesq equation was adequate for low values of the  $\lambda$ -index proposed by Henderson and Wooding [1964]. This index is given by:

$$\lambda = \frac{4N \cos i}{k \sin 2i} \quad (43)$$

[95] The dependence of the effective thickness  $pD$  on the history of recharge excludes the application of these Boussinesq-based similarity parameters to sequences of events [Stagnitti *et al.*, 2004]. Rupp and Selker [2006a, 2006b] have also commented on potential shortcomings with the use of such a linearization.

[96] Harman and Sivapalan [2009a] relaxed the assumptions of the linearized solutions in order to investigate these

controls in the full Boussinesq equation. To simplify the analysis, they cast the equations in a dimensionless form (similar to that of *Henderson and Wooding* [1964] and *Koussis* [1992]) using the characteristic length and time scales  $l^* = Ltan i$  and  $t^* = Lf/(k \sin i)$ . The timescale can be obtained from the advective time scale of *Berne et al.* [2005] for straight hillslopes, and is the characteristic response time scale of a kinematic wave moving through the hillslope.

[97] Asymptotic analysis of the dimensionless equations showed that a range of storage-discharge relations and recession curves could be obtained from the resulting equations, depending largely on the average value of the dimensionless storage thickness  $\bar{\eta} = \eta_0/(Ltan i)$ , which is the dynamic equivalent of the reciprocal of the Hillslope number. For instance, scaling arguments were used to show how, when  $\bar{\eta} \gg 1$  (or equivalently  $Hi \ll 1$ ), the assumption of a fixed, vanishingly small flow depth at the hillslope toe produced storage-discharge relations (equation (29)) with an exponent  $b$  of 2, but the assumption of a fixed gradient boundary condition ( $\frac{dh}{dx}|_{x=0} = 0$ ) produced an exponent of  $b = 1$ .

[98] *Harman and Sivapalan* [2009a] also showed how the concept of “regimes” presented in *Robinson and Sivapalan* [1997] could be applied to understand the role of recharge variability on the behavior, under the simplifying assumption of rectangular recharge pulses. They defined the “regime storage” as:

$$\eta_R = \frac{\lambda}{4} \frac{1 - e^{-t_r/t^*}}{1 - e^{-(t_r+t_b)/t^*}} \quad (44)$$

where  $t_r$  is the duration of a recharge pulse, and  $t_b$  is the time between recharge pulses. This storage can then be used to determine the appropriate value of the storage thickness controlling the response under different boundary conditions, hillslope configurations, and regimes of recharge variability.

## 8. Relevance of Hydraulic Groundwater Theory in Understanding Climate Impacts on Hydrologic Response

[99] Boussinesq relations and the methods pioneered by Brutsaert and Parlange provide powerful tools in the context of climate-induced and human-induced changes in groundwater storage dynamics. Consider the example of permafrost thawing in northern landscapes. Permafrost change and thawing has been identified as a key proxy for changes in climate. Direct observations of permafrost change, however, are difficult to perform at scales larger than the local (borehole) scale, creating the need for indirect detection methods of permafrost change and its effects on larger scales. Recent work by *Lyon et al.* [2009] outlined a theoretical connection between Brutsaert-Nieber recession flow analysis and changes in permafrost at the catchment scale, assuming storage changes, as reflected in the recession parameters and drainage time scale, are due to thawing. This differs from previous recession analysis work in northern systems [*Carey and Woo*, 2001; *Yamazaki et al.*, 2006] in that *Lyon et al.* [2009] related long-term changes in recession characteristics to storage-discharge dynamics via Boussinesq relationships.

[100] This application of B&N analysis to assess permafrost thaw has been successfully applied across scales and geological settings in Scandinavia [*Lyon et al.*, 2009; *Sjöberg et al.*, 2012] and the Yukon river basin [*Lyon and Destouni*, 2010]. *Sjöberg et al.* [2012] further demonstrated that the approach is consistent with results from a physically based three-phase flow groundwater model [i.e., *Frampton et al.*, 2011] that directly solves for coupled fluid flow and heat transport in partially saturated porous media. Brutsaert recently extended upon this approach with application to Eurasia [*Brutsaert and Hiyama*, 2012].

[101] Such applications of Boussinesq to assess long-term changes are not limited to climate-impacted cold regions as was demonstrated by a recent analysis from an arid-region application in *Brutsaert* [2012]. There Brutsaert demonstrates via assessment of groundwater storage changes from low-flow dynamics that for several large North American deserts (namely, the hot Chihuahua, Sonora, and Mojave deserts and the cold Great Basin desert), there is no discernible shift in groundwater storage or corresponding increases in aridity in these systems. From this, it can be concluded that deterioration of groundwater conditions in these systems [e.g., *Tillman et al.*, 2008] are likely attributable to human activities rather than purely to climatic shifts.

[102] The Boussinesq theory is the common denominator between this and the aforementioned permafrost work. *Brutsaert* [2008] illuminates on this connection for investigating changes in aquifer storage. Put simply, as the areal average underground water storage in a natural catchment is directly manifested by the low flow or baseflow features, it follows that baseflow data can therefore provide a quantitative estimate of the basin-scale groundwater storage changes that have taken place over a period of record. As such, any systematic changes in storage-discharge dynamics (assessed, for example, through the B&N analysis) are conditionally brought about due to changes in either storage (as is the case in the desert example) or in the aquifer itself (as is the case in the permafrost examples). This approach has been used in assessing long-term aquifer changes in the US [*Brutsaert*, 2008, 2010] and Japan [*Sugita and Brutsaert*, 2009]. Taken together with remote sensing methods for assessing groundwater storage change (e.g., GRACE), there is good potential for continued application of the Boussinesq theory (with connection to the methods pioneered by Brutsaert and Parlange) by future generations of hydrologists.

## 9. Discussion and Outlook

[103] Hydraulic groundwater theory originated with French engineers and scientists of the 19th century, prominently Dupuit and Boussinesq, and was further developed into a coherent theoretical framework for catchment-scale investigations of hydrological processes by Brutsaert and Parlange. Many researchers have used this framework to develop methods to study baseflow dynamics, rainfall-runoff processes and even the impact of climate change on catchment-scale processes. Several other researchers have questioned the validity of hydraulic groundwater theory and have argued that other properties of riparian aquifers are responsible for the observed dynamics in streamflow. Such criticism is welcome as it invites us to think deeply

about the meaning of what we observe and to develop alternative hypotheses that can form the basis of a novel theoretical framework of catchment-scale response to climate forcing. The aim of this review article is to provide background information to hydrologists new to this body of work and to encourage them to make contributions to this important field of catchment science. So far, streamflow observations are the main data used to explore the validity of hydraulic groundwater theory, but since the way water flows through catchments affects many other processes, such as geochemical weathering and ecosystem dynamics, it is hoped that future investigations will take advantage of emerging data sets, generated by national programs in the United States such as the Critical Zone Observatories, to interrogate the applicability of the theoretical framework based on hydraulic groundwater theory and its alternatives.

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