The Improbable Differential Attack: Cryptanalysis of Reduced Round CLEFIA

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Outline

A (Very) Short Introduction to Differential Cryptanalysis

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- And others (Higher-order Differential, Boomerang,...)

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Improbable Differential	p	p_0	$p_0 < p$

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Caution

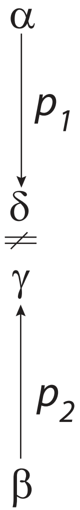
If there are nontrivial differentials from α to β, p_0 becomes bigger than $p \cdot (1 - p')$.

Two Techniques to Obtain Improbable Differentials

Two methods to obtain improbable differentials:

1. Use two differentials that miss in the middle with high probability (almost miss in the middle technique)
2. Expand impossible differentials to improbable diffrentials by adding a differential to the top and/or below the impossible differential (expansion technique)

Almost Miss-in-the-Middle Technique



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Improbable Differentials from Impossible Differentials

$p' = 1$

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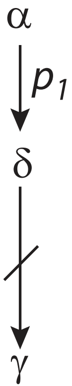
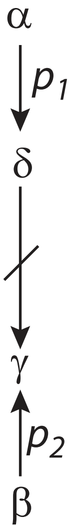


$p' = p_1$

Improbable Differentials from Impossible Differentials



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Pros and Cons of the Expansion Method

Pros:

- Longer differentials
- Attack on more rounds

Cons:

- Data complexity increases (because p_0 increases)
- Time complexity increases (since we use more data)
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Making appropriate changes, these estimates can be used for improbable differential attacks, too.

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Previous attacks where $p_0 < p$

Early examples of improbable differential attack:

- J. Borst, L. Knudsen, V. Rijmen: "Two Attacks on Reduced IDEA"
- L. Knudsen, V. Rijmen: "On the Decorrelated Fast Cipher (DFC) and Its Theory"

CLEFIA

- Developed by Sony in 2007
- *Clef* means *key* in French.
- Block length: 128 bits
- Key lengths: 128, 192, and 256 bits
- Number of rounds: 18, 22, or 26
- Previous best attacks: Impossible differential attacks on 12, 13, 14 rounds for 128, 196, 256-bit key lengths by Tsunoo et al.
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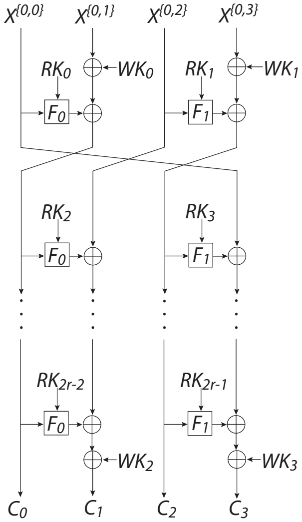
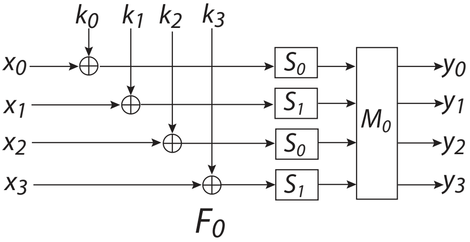
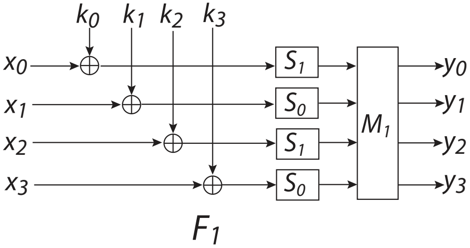
CLEFIA: Encryption Function



CLEFIA: F_0 and F_1 Functions





10-round Improbable Differential

We will use the following two 9-round impossible differentials that are introduced by Tsunoo et al.,

$$[0_{(32)}, 0_{(32)}, 0_{(32)}, [X, 0, 0, 0]_{(32)}] \nrightarrow_{9r} [0_{(32)}, 0_{(32)}, 0_{(32)}, [0, Y, 0, 0]_{(32)}]$$
$$[0_{(32)}, 0_{(32)}, 0_{(32)}, [0, 0, X, 0]_{(32)}] \nrightarrow_{9r} [0_{(32)}, 0_{(32)}, 0_{(32)}, [0, Y, 0, 0]_{(32)}]$$

where $X_{(8)}$ and $Y_{(8)}$ are non-zero differences.

10-round Improbable Differential

We obtain 10-round improbable differentials by adding the following one-round differentials to the top of these 9-round impossible differentials,

$$[[\psi,0,0,0]_{(32)},\zeta_{(32)},0_{(32)},0_{(32)}] \rightarrow_{1r} [0_{(32)},0_{(32)},0_{(32)},[\psi,0,0,0]_{(32)}]$$
$$[[0,0,\psi,0]_{(32)},\zeta'_{(32)},0_{(32)},0_{(32)}] \rightarrow_{1r} [0_{(32)},0_{(32)},0_{(32)},[0,0,\psi,0]_{(32)}]$$

which hold when the output difference of the F_0 function is ζ (resp. ζ') when the input difference is $[\psi,0,0,0]$ (resp. $[0,0,\psi,0]$).

13-round Improbable Differential Attack

We choose ψ and corresponding ζ and ζ' depending on the difference distribution table (DDT) of S_0 in order to increase the probability of the differential. In this way we get $p' \approx 2^{-5.87}$.

We put one additional round on the plaintext side and two additional rounds on the ciphertext side of the 10-round improbable differentials to attack first 13 rounds of CLEFIA that captures RK_1, $RK_{23,1} \oplus WK_{2,1}$, RK_{24}, and RK_{25}.

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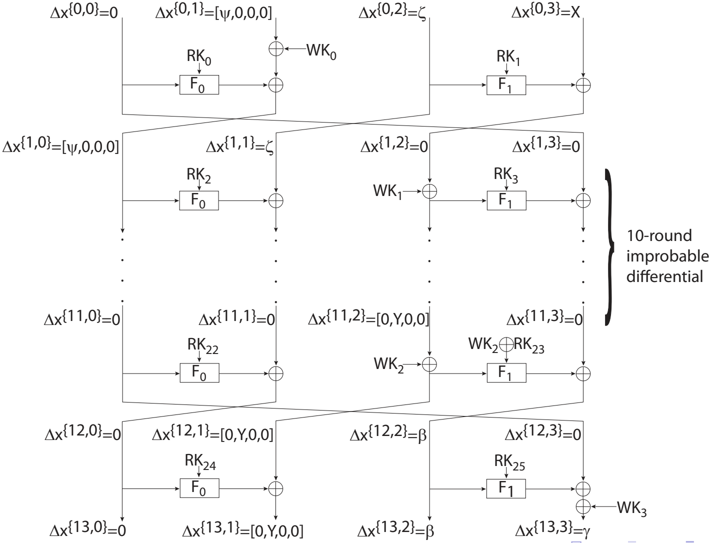

13-round Improbable Differential Attack

Table: Comparison of Tsunoo et al.'s impossible attack with the expanded improbable attack

Rounds	Attack Type	Key Length	Data Complexity	Time Complexity	Memory (blocks)	Success Probability
12	Impossible	128	$2^{118.9}$	2^{119}	2^{73}	-
13	Improbable	128	$2^{126.83}$	$2^{126.83}$	$2^{101.32}$	%99

14 and 15-round Improbable Differential Attacks

By using the similar expansion technique, we can apply improbable differential attack on

- 14-round CLEFIA when the key length is 192 bits
- 15-round CLEFIA when the key length is 256 bits

Conclusion

We provided

1. a new cryptanalytic technique called *improbable differential attack* where a differential holds with less probability when tried with the correct key
2. two techniques to obtain improbable differentials
3. data complexity estimates for improbable differential attacks
4. state of art attacks on the block cipher CLEFIA

Conclusion

THANK YOU FOR YOUR ATTENTION