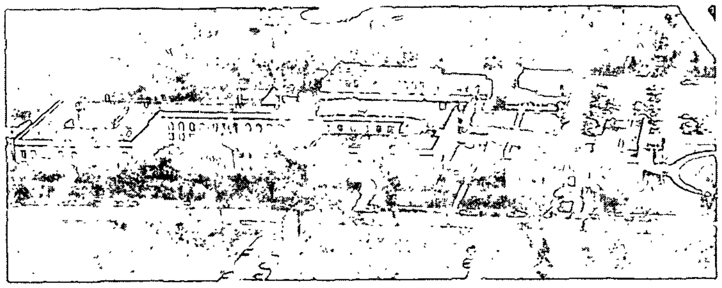

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THE INDENTATION OF A NONPLANAR PUNCH ON AN ORTHOTROPIC ELASTIC HALF SPACE

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INTRODUCTION

The purpose of this paper is to present a simple model for investigatint the indentation of an orthotropic elastic medium. Such an idealized problem is representative of the processes of the printing, calendering, corrugating, scoring and cutting which occur in the paper and paperboard industry.

Simple formulas are derived for the distribution of pressure under a punch of symmetrical profile and for the total load which must be axially applied to achieve this punch. Examples of four typical shapes of punch commonly seen in actual operation are given to illustrate the application of the derived formula.

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The Indentation of a Nonplanar Punch on an Orthotropic Elastic Half Space

ABSTRACT

A method due to Ting [1,2] is employed to investigate the indentation of an orthotropic elastic half space in which, instead of solving a pair of dual integral equations, the inversion of a single integral equation is required. Simple formulas are derived for the distribution of pressure under a punch of symmetrical profile and for the total load which must be axially applied to achieve this punch. The application of these formulas is illustrated for some shapes of punch. It is hoped that the derived formula will provide the basis for further investigations into the detailed mechanics of the printing, calendering, corrugating, scoring, and cutting processes.

Introduction

The problem of an indented half plane seems to have been considered first by Sadowsky [3]. Using the methods of potential theory, under the assumption that the shearing forces vanish on the contact surface, Sadowsky derived a closed form solution of the contact pressure for a flat-ended rigid indenter pressed normally into an isotropic half plane. The same problem with the presence of friction on the contact surface was treated independently with different mathematical techniques by Abramov [4], Muskhelishvili [5], and Ôkubo [6].

The contact stresses between an arbitrary profile and an isotropic half plane were considered by several investigators. Using the theory of Fourier transforms, Sneddon [7] obtained the solution to the punch of a wedge for an isotropic half plane but made incorrect use of Busbridge's solution to the dual integral equations which does not cover the type of the dual integral equations he considered. A simple general solution of the integral equation for the punch of an arbitrary profile was given by Schubert [8], who used Hamel's solution [9] to a singular integral equation with the finite Hilbert transformation, which is the type of equations he investigated.

The contact problems for an orthotropic body with and without friction on the contact surface can be found in Galin's book [10]. The solution for the contact stress between a flat-ended rigid block and an orthotropic half plane was derived by Conway [11], who extended Schubert's solution for an isotropic half plane to an orthotropic half plane.

The present work is concerned with the contact stresses between a rigid indenter of a symmetrical profile and an orthotropic elastic half space. A simple general solution of the integral equation derived in the form of Fourier transforms of a pressure function which is determined in terms of a displacement function, is obtained by following Ting's treatment of the axisymmetrical visco-elastic problem.

Stresses and Displacements for Half Space

Consider an orthotropic half space whose bounding surface corresponds with the y-axis as shown in Fig. 1, and assume that the system of Cartesian coordinates are the principal axes of orthotropy. The rigid indenter of a symmetrical profile is pressed at the origin into an orthotropic half space and it is assumed that the contact on the surface is frictionless. The problem of interest is to determine the stresses σ_{ij} and the displacement u_i in an orthotropic half space and particularly the contact stresses under the rigid indenter.

[Fig. 1 here]

The equations and formulas which determine the stresses and displacements subject to the boundary conditions are the following:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + X_i = 0, \tag{1}$$

$$\varepsilon_{ij} = S_{ijkl}\,\sigma_{kl}, \tag{2}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{3}$$

where X_i are body forces, S_{ijkl} are elastic compliances. In the absence of body forces and introducing the Airy stress function ϕ for the orthotropic plane stress problems the required equations become:

$$\left(\frac{\partial^2}{\partial x^2} + \alpha_1^2 \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2}{\partial x^2} + \alpha_2^2 \frac{\partial^2}{\partial y^2}\right)\phi = 0 \tag{4}$$

$$\varepsilon_x = S_{11}\,\sigma_x + S_{12}\,\sigma_y \tag{5a}$$

$$\varepsilon_y = S_{12}\,\sigma_x + S_{22}\,\sigma_y \tag{5b}$$

$$\gamma_{xy} = S_{66}\,\tau_{xy} \tag{5c}$$

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2},\ \sigma_y = \frac{\partial^2\phi}{\partial x^2},\ \tau_{xy} = -\frac{\partial^2\phi}{\partial x \partial y} \tag{6}$$

$$\alpha_1^2\alpha_2^2 = \frac{S_{11}}{S_{22}},\ \alpha_1^2 + \alpha_2^2 = \frac{S_{66} + 2\,S_{12}}{S_{22}} \tag{7}$$

The above equations also hold in the case of an orthotropic (plain strain) problem if the elastic constants C_{ij}, β_1, β_2 everywhere replace the elastic constants S_{ij}, α_1, α_2 by the following formulas (see Reference [12]):

$$C_{11} = S_{11} - \frac{S_{13}^2}{S_{33}},\ C_{12} = S_{12} - \frac{S_{13}S_{23}}{S_{33}},\ C_{22} = S_{22} - \frac{S_{23}^2}{S_{33}} \tag{8}$$

$$\beta_1^2\beta_2^2 = \frac{C_{11}}{C_{22}},\ \beta_1^2 + \beta_2^2 = \frac{C_{66} + 2\,C_{12}}{C_{22}} \tag{9}$$

$$\sigma_z = -\frac{1}{S_{33}}\,S_{13}\,\sigma_x + S_{23}\,\sigma_y) \tag{10}$$

The boundary conditions for the present problem are

$$\sigma_x = \tau_{xy} = 0 \qquad \text{on } x = 0,\ y > a\ , \tag{11a}$$

$$u(x,y) = D - f(y),\ \tau_{xy} = 0 \qquad \text{on } x = 0,\ y < a\ , \tag{11b}$$

where D is the total depth of penetration, $f(y)$ is the profile of the base of the indenter before contact and is defined so that $f(y) = 0$, a is the width of the contact surface which will be determined from the continuity condition that $\sigma_{\underline{x}} = 0$ at $\underline{x} = 0$, $\underline{y} = \underline{a}$.

The prescribed boundary conditions are of mixed kind. The formulation of mixed boundary value problems often leads to the solution of a pair of dual integral equations or a system of triple integral equations which is frequently difficult to obtain. Alternatively, if the given displacement condition in the case under consideration is replaced by the stress condition by assuming that the contact stresses are known, the transformed boundary conditions will then be uniform which will render the problem tractable. This ingenious idea is due to Ting [1,2], who introduced this technique to make the Laplace transform technique applicable to the more general viscoelastic problems of axisymmetry. The original boundary conditions (10) will therefore be converted as follows:

$$\tau_{\underline{xy}} = 0 \qquad \text{on} \quad \underline{x} = 0, \tag{12a}$$

$$\sigma_{\underline{x}} = -\underline{p}(\underline{y}) \qquad \text{on} \quad \underline{x} = 0,\ \underline{y} < \underline{a}, \tag{12b}$$

where $\underline{p}(\underline{y})$ is assumed known.

Using the theory of Fourier transforms [7], one can show that, in the case of an orthotropic plane stress problem, the stresses and displacements which satisfy the transformed boundary conditions (12) will take the following form:

$$\sigma_{\underline{x}} = -\frac{2}{\pi}\int_0^\infty \frac{\bar{\underline{p}}(\lambda)}{1-\alpha}\left(e^{-\lambda\alpha_1\underline{x}} - \alpha e^{-\lambda\alpha_2\underline{x}}\right)\cos\lambda\underline{y}\,\underline{d}\lambda \tag{13a}$$

$$\sigma_{\underline{y}} = +\frac{2}{\pi}\int_0^\infty \frac{\bar{\underline{p}}(\lambda)}{1-\alpha}\left(\alpha_1{}^2\, e^{-\lambda\alpha_1\underline{x}} - \alpha\alpha_2{}^2\, e^{-\lambda\alpha_2\underline{x}}\right)\cos\lambda\underline{y}\,\underline{d}\lambda \tag{13b}$$

$$\tau_{\underline{xy}} = -\frac{2}{\pi}\int_0^\infty \frac{\bar{\underline{p}}(\lambda)}{1-\alpha}\,\alpha_1\left(e^{-\lambda\alpha_1\underline{x}} - e^{-\lambda\alpha_2\underline{x}}\right)\sin\lambda\underline{y}\,\underline{d}\lambda \tag{13c}$$

$$\underline{u} = -\frac{2}{\pi} \int_0^\infty \frac{\bar{\underline{p}}(\lambda)}{1-\alpha} \left[\underline{S}_{22}\left(\alpha_1{}^3\ e^{-\lambda\alpha_1\underline{x}} - \alpha\alpha_2{}^3\ e^{-\lambda\alpha_2\underline{x}}\right) - \alpha_1(\underline{S}_{11} + \underline{S}_{66}) \right.$$

$$\left. x \left(e^{-\lambda\alpha_1\underline{x}} - e^{-\lambda\alpha_2\underline{x}}\right)\right] \cos \lambda\underline{y}\ \frac{d\lambda}{\lambda} \tag{14a}$$

$$\underline{v} = +\frac{2}{\pi} \int_0^\infty \frac{\bar{\underline{p}}(\lambda)}{1-\alpha}\ \underline{S}_{22}\left(\alpha_1{}^2\ e^{-\lambda\alpha_1\underline{x}} - \alpha\alpha_2{}^2\ e^{-\lambda\alpha_2\underline{x}}\right) - \underline{S}_{12}\left(e^{-\lambda\alpha_1\underline{x}} - \right.$$

$$\left. \alpha e^{-\lambda\alpha_2\underline{x}}\right) \sin \lambda\underline{y}\ \frac{d\lambda}{\lambda} \tag{14b}$$

where $\alpha = \alpha_1/\alpha_2$ and $\bar{\underline{p}}(\lambda)$ is the Fourier transform of the pressure $\underline{p}(\underline{y})$ which is defined by

$$\bar{\underline{p}}(\lambda) = \int_0^{\underline{a}} \underline{p}(\underline{y}) \cos \lambda\underline{y}\ \underline{dy} \tag{15}$$

The mathematical problem is therefore reduced to the determination of the assumed pressure $\underline{p}(\underline{y})$ which must satisfy the replaced displacement condition (11b). Taking $\underline{x} = 0$ in equation (14a), one obtains

$$\underline{u}(o, \underline{y}) = -\frac{2\ \underline{S}_{22}(\alpha_1{}^3 - \alpha\alpha_2{}^3)}{\pi(1-\alpha)} \int_0^\infty \underline{p}(\lambda) \cos \lambda\underline{y}\ \frac{d\lambda}{\lambda}, \tag{16}$$

where $\underline{u}(o, \underline{y}) = \underline{D} - f(\underline{y})$. Substituting equation (15) into equation (16) yields

$$F(\underline{y}) = \int_0^\infty \int_0^{\underline{a}} \underline{p}(\underline{m}) \cos \lambda\underline{m} \cos \lambda\underline{y}\ \lambda^{-1}\ \underline{dmd}\lambda, \tag{17}$$

where

$$F(\underline{y}) = -\frac{\pi(1-\alpha)\ (\underline{u}(o, \underline{y})}{2\ \underline{S}_{22}\ (\alpha_1{}^3 - \alpha\alpha_2{}^3)} \tag{18}$$

Equation (17) is an integral equation for $\underline{p}(\underline{y})$ in terms of displacement $F(\underline{y})$. The total force $\underline{P}$ axially applied to the indenter is

$$\underline{P} = 2 \int_0^{\underline{a}} \underline{p}(\underline{y})\ \underline{dy} \tag{19}$$

It will be found convenient for the next sections to define another constant $\underline{B}$ by

$$\underline{B} = -\frac{\pi(1-\alpha)}{2\,\underline{S}_{22}\,(\alpha_1{}^3 - \alpha\alpha_2{}^3)} = \frac{\pi}{2}\,\frac{1}{\underline{S}_{22}\,\alpha_1\alpha_2\,(\alpha_1 + \alpha_2)} \tag{20a}$$

In the case of an orthotropic plane strain problem $\underline{B}$ is defined by

$$\underline{B} = -\frac{\pi(1-\beta)}{2\,\underline{S}_{22}\,(\beta_1{}^3 - \beta\beta_2{}^3)} = \frac{\pi}{2}\,\frac{1}{\underline{S}_{22}\,\beta_1\beta_2\,(\beta_1 + \beta_2)}\,, \tag{20b}$$

where $\beta = \beta_2/\beta_1$. Identifying the elastic constants $\alpha = \alpha_1 = 1$, $\underline{S}_{11} = \underline{S}_{22} = 1/\underline{E}$, $\underline{S}_{12} = 2(1 + \nu)/\underline{E}$, and making use of L'Hospital rule to evaluate the indeterminate form, one can find, in the case of an isotropic plane stress problem,

$$\underline{B} = \frac{\pi}{4}\,\underline{E} \tag{20c}$$

and, in the case of an isotropic plane strain problem,

$$\underline{B} = \frac{\pi\underline{E}}{4(1 - \nu^2} \tag{20d}$$

Once the pressure $\underline{p}(\underline{y})$ is determined, the problem is completely solved.

Solution of the Integral Equation

If the original boundary conditions (11) are used, one has to solve the following dual integral equations [7]:

$$\int_0^\infty \lambda^{-1}\,\underline{A}(\lambda)\cos\lambda\underline{y}d\lambda = F(\underline{y}) \qquad 0 < \underline{y} < \underline{a}, \tag{21a}$$

$$\int_0^\infty \underline{A}(\lambda)\cos\lambda\underline{y}d\lambda = 0 \qquad \underline{y} > \underline{a}, \tag{21b}$$

where $\underline{A}(\lambda) = \underline{p}(\lambda)/\lambda^2(1-\alpha)$. The problem of solving a pair of dual integral equations is, in general, difficult. However, the inversion of the alternative integral equation (17) is suggested from one of the above dual integral equations by observing that

$$\int_0^\infty J_0(a\lambda)\cos\lambda y d\lambda = 0 \qquad \text{for } y > a, \tag{22a}$$

$$J_0(ay) = \frac{2}{\pi}\int_0^a \frac{\cos uy\, du}{\sqrt{a^2 - u^2}} \tag{22b}$$

Thus, one can proceed as follows:

Multiplying equation (17) by $(y^2 - m^2)^{-1/2}$ and integrating with respect to m from 0 to y one finds, on making use of equation (22b),

$$\int_0^y \frac{F(m)\,dm}{\sqrt{y^2 - m^2}} = \frac{\pi}{2}\int_0^a p(\lambda)\int_0^\infty J_0(yn)\frac{\cos n\lambda}{n}\,dnd\lambda \tag{23a}$$

Differentiating equation (23a) with respect to y yields

$$\frac{\partial}{\partial y}\int_0^y \frac{F(m)\,dm}{\sqrt{y^2 - m^2}} = -\frac{\pi}{2}\int_0^a p(\lambda)\int_0^\infty J_1(yn)\cos n\lambda\,dnd\lambda$$

But

$$\int_0^\infty J_1(yn)\cos n\lambda\,dn = \frac{1}{y}\left[1 - \frac{\lambda}{\sqrt{\lambda^2 - y^2}}\right],$$

$$2\int_0^a p(\lambda)\,d\lambda = P$$

Hence,

$$\frac{2}{\pi}y\frac{\partial}{\partial y}\int_0^y \frac{F(m)\,dm}{\sqrt{y^2 - m^2}} = -\frac{P}{2} + \int_y^a \frac{p(m)\,mdm}{\sqrt{m^2 - y^2}} \tag{23b}$$

Taking the well-known formula

$$\int_m^n \frac{rdr}{\sqrt{(n^2 - r^2)(r^2 - m^2)}} = \frac{\pi}{2},$$

and multiplying equation (23b) by a similar factor $\lambda/\sqrt{\lambda^2 - y^2}$ and integrating from y to a, one gets, on changing the order of integration,

$$\frac{2}{\pi}\int_{y}^{a}\frac{\lambda d\lambda}{\sqrt{\lambda^2 - y^2}}\lambda \cdot \frac{\partial}{\partial\lambda}\int_{0}^{\lambda}\frac{F(m)\,dm}{\sqrt{\lambda^2 - m^2}}$$

$$= -\frac{P}{2}\int_{y}^{a}\frac{\lambda d\lambda}{\sqrt{\lambda^2 - y^2}} + \int_{y}^{a}\frac{\lambda d\lambda}{\sqrt{\lambda^2 - y^2}}\int_{\lambda}^{a}\frac{p(m)\,mdm}{\sqrt{m^2 - \lambda^2}}$$

$$= -\frac{P}{2}\sqrt{a^2 - y^2} + \int_{y}^{a} p(m)\,mdm\int_{y}^{m}\frac{\lambda d\lambda}{\sqrt{(m^2 - \lambda^2)(\lambda^2 - y^2)}}$$

$$= -\frac{P}{2}\sqrt{a^2 - y^2} + \frac{\pi}{2}\int_{y}^{a} p(m)\,mdm \qquad (23c)$$

Differentiating equation (23c) with respect to y gives

$$p(y) = \frac{P}{\pi\sqrt{a^2 - y^2}} - \frac{4}{\pi^2}\frac{1}{y}\frac{\partial}{\partial y}\int_{y}^{a}\frac{\lambda d\lambda}{\sqrt{\lambda^2 - y^2}}\lambda\frac{\partial}{\partial\lambda}\int_{0}^{\lambda}\frac{F(m)\,dm}{\sqrt{\lambda^2 - m^2}} \qquad (24)$$

Equation (24) is the solution of equation (17). One can proceed to simplify equation (24) by imposing an additional physical condition as follows:

Let

$$G(\lambda) = \lambda\frac{\partial}{\partial\lambda}\int_{0}^{\lambda}\frac{F(m)\,dm}{\sqrt{\lambda^2 - m^2}}$$

Integrating by part and then differentiating gives

$$G(\lambda) = \int_{0}^{\lambda}\frac{mdm}{\sqrt{\lambda^2 - m^2}}\frac{dF(m)}{dm} \qquad (25a)$$

Substituting equation (25a) into equation (24) and proceeding in the same way as above the following is obtained:

$$p(y) = \frac{1}{\pi\sqrt{a^2 - y^2}}\left[P + \frac{4G(a)}{\pi}\right] - \frac{4}{\pi^2}\int_{y}^{a}\frac{d\lambda}{\sqrt{\lambda^2 - y^2}}\frac{dG(\lambda)}{d\lambda} \qquad (25b)$$

Imposing the continuity condition that $p(y) = 0$ at $y = a$, one obtains from equation (25),

$$P = -\frac{4}{\pi} G(a) \tag{25c}$$

Substituting $F(m) = B[D-f(m)]$, $dF(m)/dm = -B[df(m)/dm]$ into equations (25) yields

$$G(\lambda) = -B\int_0^{\lambda} \frac{m\,dm}{\sqrt{\lambda^2 - m^2}} \frac{df(m)}{dm} \tag{26a}$$

$$p(y) = -\frac{4}{\pi^2}\int_y^a \frac{d\lambda}{\sqrt{\lambda^2 - y^2}} \frac{dG(\lambda)}{d\lambda} \tag{26b}$$

$$P = -\frac{4}{\pi} G(a) = -\frac{4B}{\pi}\int_0^a \frac{m\,dm}{\sqrt{a^2 - m^2}} \frac{df(m)}{dm} \tag{26c}$$

Equations (26) are the formulas for the pressure under the indenter and the total load exerted on the indenter. Some special cases of the application of these formulas are illustrated in the next section.

Examples

Results for four particular shapes are given in terms of B. One can get the pressure distribution $p(y)$ and the total force P by substituting the expression B given in equations (20).

1. Flat-ended block: Since the profile of the punch is not smooth at $y = a$, equation (24) must be used to determine the pressure distribution. Thus, for $f(y) = 0$,

$$p(y) = \frac{P}{\pi\sqrt{y^2 - a^2}}$$

which shows that the pressure is independent of elastic constants.

2. Wedge: $f(y) = y \cot\theta$, where θ = semiangle of wedge.

It follows from equations (26) that

$$p(y) = \frac{4B \cot \theta}{\pi^2} \cosh^{-1}\left(\frac{a}{y}\right)$$

$$P = \frac{4Ba \cot \theta}{\pi}$$

3. Circle (classical circular indenter): $f(y) = y^2/2R$.

Substituting expression $f(y)$ into equations (26) gives

$$p(y) = \frac{2B}{\pi R} \sqrt{a^2 - r^2}$$

$$P = \frac{a^2 B}{R}$$

4. Profile with n-th order polynomial: $f(y) = \sum_{n=0}^{n} c_n y^n$.

Integrating equations (26) yields

$$p(y) = \frac{2B}{\pi^{3/2}} \sum_{n=0}^{n} n^2 c_n \frac{\Gamma\left(\frac{n}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{n}{2} + 1\right)} \int_y^a \frac{\lambda^{(n-1)} d\lambda}{\sqrt{\lambda^2 - y^2}}$$

$$P = \frac{2B}{\sqrt{\pi}} \sum_{n=0}^{n} n c_n a^n \frac{\Gamma\left(\frac{n}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{n}{2} + 1\right)}$$

Conclusion

A method due to Ting is employed to obtain the contact stresses between the rigid indenter and the orthotropic half space. The contact stresses for the isotropic half space can be obtained by identifying the elastic constants and taking the limit $\alpha_1 \to \alpha_2 \to 1$. The components of stresses and displacements for the isotropic half plane derivable from the passage of limit of the orthotropic half plane can also be obtained by following Lamb's treatment [13] of Boussinesq's problem. It has been shown that the pressure distribution under a flat-ended block is independent of elastic constants of the elastic half space.

It is surprising to note that the total depth of penetration D in the plane contact problems cannot be obtained from the stress continuity that $\sigma_x = 0$, at $x = 0$, $y = a$, which, in contrast, will determine the total depth of penetration D in contact problems of three-dimensions. Hence, the value D must be determined from equation (17) by taking the limit $y \to 0$. However, the depth of penetration D, given by the integral equation (17), has a singularity at the origin in view of the trigonometric kernel $\lambda^{-1} \cos \lambda y$ as $\lambda \to 0$. Thus, the actual value of D is an arbitrary one which must be given as the prescribed value, a peculiar behavior quite unexpected.

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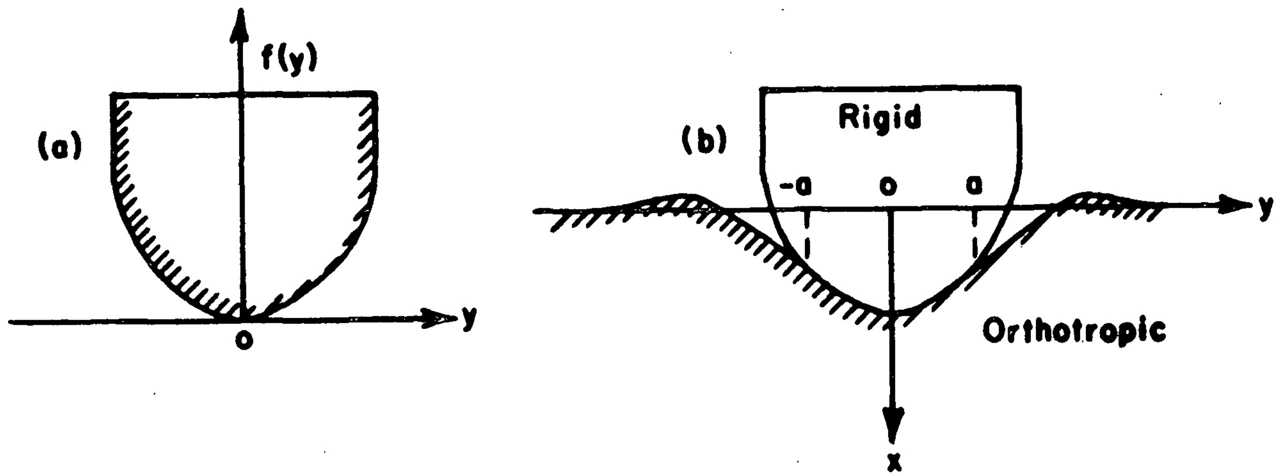

Fig. 1. The rigid profile indented into an orthotropic half space