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The Inertia Tensor Versus Static Moment and Mass in Perceiving Length and Heaviness of Hand-Wielded Rods

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In 2 experiments, participants haptically estimated length and heaviness of handheld rods while wielding without seeing them. The sets of rods had been constructed such that variation of static moment and the 1st eigenvalue of the inertia tensor (I_1) were separated. Consistent with previous findings, perceived rod length correlated strongly with I_1. However, multiple regressions on current data as well as data from previous studies showed a comparable strong correlation between perceived rod length and static moment plus mass. Contrary to previous findings, perceived heaviness correlated strongly with static moment and only weakly with the eigenvalues of the inertia tensor. These results suggest that the inertia tensor does not provide the sole foundation for a theory of dynamic touch.

In the past decade or so, significant strides have been made in understanding the perception of properties of objects that are held in the hand but not seen. A large body of experimental data has been presented in support of the hypothesis that many instances of haptic perception are governed by a single yet multivalued physical entity, the inertia tensor (**I**). This 3×3 tensor is symmetrical and thus contains six independent numbers. In diagonalized form—that is, when the tensor is expressed with respect to the unique set of axes of symmetrical mass distribution (the so-called principal axes of inertia)—the inertia tensor consists of three elements denoted as the principal moments of inertia or eigenvalues. These principal moments define an object's (invariant) resistance against rotational acceleration around its principal axes of inertia and have been found to be related to perception of an object's length (Solomon & Turvey, 1988; Solomon, Turvey, & Burton, 1989a, 1989b), width and height (Turvey, Burton, Amazeen, Butwill, & Carello, 1998), and heaviness (Amazeen & Turvey, 1996).

In addition, the eigenvectors of the inertia tensor (defining the direction of the principal axes of inertia of the object) have been found to be related to the perception of an object's orientation (Pagano & Turvey, 1998; Turvey, Burton, Pagano, Solomon, & Runeson, 1992). More recently, it has been suggested that perception of grip position is a function of the off-diagonal elements of the inertia tensor (i.e., the products of inertia; Pagano, Kinsella-Shaw, Cassidy, & Turvey, 1994) and that the perception of an object's partial forward length (i.e., the distance from the hand to the forward-directed endpoint of the rod) is a function of both the moments and products of inertia (Carello, Santana, & Burton, 1996; Pagano, Carello, & Turvey, 1996).

In the majority of the studies cited, the invariance of **I** as well as the fact that it provides information about both magnitude and direction, has been emphasized to support the inertia tensor hypothesis. However, when an object is held as still as possible, it appears difficult to perceive its properties through **I** because, by definition, an object's resistance to rotation manifests itself only when the object is rotated. Under static conditions, physical properties such as the static moment (i.e., the first moment of mass distribution; Burton & Turvey, 1990; Carello, Fitzpatrick, Domaniewicz, Chan, & Turvey, 1992) and static torque (i.e., the static moment multiplied by the cosine of the angle of the rod with the horizontal plane) and a combination of weight and static torque (Chan, 1994; Lederman, Ganeshan, & Ellis, 1996) have been suggested to be the main determinants of perceived rod length. In contrast, Carello et al. (1996) argued that, under semistatic conditions, incidental sway or tremor may be sufficient to obtain information about **I** and that prior claims regarding the role of static torque are due to "parasitic" variation of static torque with the products of inertia.

The hypothesized exclusive role of **I** in the perception of object properties has been confirmed in a number of experiments involving active wielding of rods. In cylindrical rods (with or without cylindrical loads attached to them), the first eigenvalue (I_1) is equal to the second eigenvalue (I_2) when they are defined with respect to the endpoint of the rod. Furthermore, the third eigenvalue (I_3) is very small and is generally found to add little to the explained variance in rod wielding experiments. We therefore primarily concentrate on the role of I_1. In the following, further theoretical motives for this choice are presented.

In most experiments, static torque has been ruled out a priori as a candidate physical property on the argument that it is not invariant. For unclear reasons, however, a possible role of the static moment, which is invariant just as is **I,** has not been considered. We submit that the experimental evidence and the arguments that have been given to support the claim for an exclusive role of **I** are

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inconclusive. In the key experiments (e.g., Amazeen & Turvey, 1996; Solomon & Turvey, 1988) the static moment has covaried with I_1, rendering it difficult, if not impossible, to compare the actual role of the static moment with that of I_1. As we show in the present article, this covariation between static moment and I_1 can be prevented by constructing special sets of rods, one with constant I_1 and one with constant static moment.

With respect to the arguments, a general form of the dynamical equation of rotational motion has been put forward in many papers (e.g., Solomon et al., 1989b) as a biomechanical rationale for the exclusive role of **I:**

$$\mathbf{N} = d(\mathbf{I}\boldsymbol{\omega})/dt, \tag{1}$$

where **N** is the sum of torques and $\boldsymbol{\omega}$ is the angular velocity. The left-hand side of the equation has not been explored in studies relating perceptual judgments to **I.** However, splitting the torque into its constituent parts is useful to show the covariation between I_1 and the static moment. Consider a simplified (two-dimensional) wielding task involving a weightless rod containing a point mass *m* attached at a distance *d* to the wrist joint. In this case, splitting the torque into a static torque and a muscular torque and bringing the static torque term to the right-hand side of the equation results in

$$\mathbf{M}_m = I_1\boldsymbol{\alpha} - \mathbf{M}\cos(\theta) = md^2\boldsymbol{\alpha} - dm\mathbf{g}\cos(\theta), \tag{2}$$

where $\mathbf{M}_m$ is the muscular torque, $\boldsymbol{\alpha}$ is the angular acceleration of the wielded object, **g** is the gravitational acceleration, θ is the angle of the rod with the horizontal plane, and *d* is the distance between the point mass *m* and the wrist. The first term on the right-hand side of Equation 2 can be denoted as the inertial term (including the invariant first moment of inertia, $I_1 = md^2$), whereas the second term on the right-hand side is usually called the static torque (including the invariant static moment $\mathbf{M} = dm\mathbf{g}$).

In previous experiments on the role of the inertia tensor in the perception of object properties, I_1 was usually modified by changing either the amount of mass or its spatial distribution. However, it follows from Equation 2 that these modifications of $\mathbf{I_1}$ also cause a change in the static moment, which as mentioned earlier, is an invariant property. Consequently, experimental evidence favoring the role of I_1 in the perception of object properties may have been confounded by accompanying changes in the static moment. Both the static moment and I_1 can be considered mathematical descriptions of mass distribution, because both depend on mass (*m*) as well as on the position (*d*) or distribution of mass. Hence, one may ask whether the difference between the two terms is of a principal nature. With regard to perception, this question should be answered in the affirmative, in that the moment of inertia can be perceived only through angular acceleration, whereas the static moment can be perceived both during motion and during rest. Thus, we are dealing with the question of whether angular acceleration is necessary for the haptic perception of object properties.

To resolve the problem of designing a proper experiment to address this question, it is useful to take a closer look at Equation 2, because it reveals an important difference between the moment of inertia and the static moment. The static moment depends linearly on *d,* whereas the moment of inertia depends on the square of *d.* As a consequence, it is possible to design an experiment in which the effect of one of the two terms is eliminated by holding that one term constant. It follows from Equation 2 that the static moment can be held constant while varying I_1, for instance by doubling *m* and at the same time halving *d.* Similarly, the static moment may be varied while holding I_1 constant by increasing *m* fourfold while halving *d.* To date, as far as we know, such a strategy has not been pursued in experiments on dynamic touch. As a consequence, conclusive evidence on the relative role of static moment and I_1 is lacking.

In reality, of course, masses are not point masses, and the rod and hand are not massless. Still, the same principle can be used: Masses can be placed at specific distances from the wrist to keep either I_1 or the static moment constant. In three dimensions, the same principle holds for the first two eigenvalues defined with respect to the endpoint of the rod. These two moments of inertia (I_1 and I_2), around axes perpendicular to the extent of the rod, are equal when cylindrical rods and masses are used. I_3, the moment of inertia around the length axis of the rod, will depend on the form of the mass but can be held constant within certain boundaries. In the first experiment reported in this article, we focused on distinguishing between I_1 and the static moment, and thus variation in I_3 was minimized. In the second experiment, we extended our investigations by including considerable variation in I_3.

In the present study, we concentrated on the perception of two object properties wherein the inertia tensor and the static moment may have differential effects: perception of length and perception of heaviness. We discuss each property in turn.

In line with the literature, it is hypothesized that perception of the length of a rod that is wielded while being held in the hand at one end is governed by the eigenvalues of the inertia tensor. The rationale behind this hypothesis is that **I,** and, thus, the required muscular torques during wielding vary strongly with length, because in a uniform rod I_1 and I_2 vary with the third power of the rod's length. A second hypothesis is that the inertia tensor is not the only physical entity used for length estimation. The rationale for this hypothesis can be elucidated by exploring the equations defining the length of a solid, uniform cylindrical rod. The following three equations define the length (*L*) of such a rod (see the Appendix):

$$L = m^{-0.5}(3I_1 - 3/2\, I_3)^{0.5}, \tag{3}$$

$$L = \mathbf{g}(3I_1 - 3/2\, I_3)(2\mathbf{M})^{-1}, \tag{4}$$

and

$$L = 2\mathbf{M}(m\mathbf{g})^{-1}, \tag{5}$$

where *m* is mass, **M** is the static moment, and **g** is the gravitational acceleration vector. **M,** I_1, and I_3 are defined with respect to the endpoint of the rod. These three equations are all physically valid descriptions of the length of a uniform rod. Clearly, they all involve two physical entities from the set (**I,** *m*, **M**). This does not necessarily imply that the haptic system uses more than one of these entities. Perhaps participants are able to make assumptions with respect to the second entity. Consequently, errors in estimation of the length of uniform rods will occur when the relation between the two entities varies over rods (e.g., by a change in rod density) and participants do not change their assumption. Conversely, it cannot be excluded a priori that the haptic system uses more than one of the relations in Equations 3–5 to estimate the length of the rod or switches between these relations under different circumstances.

The second perceived object property to be investigated was heaviness. It is hypothesized that perceived heaviness is dominated by the static moment. Support for this hypothesis is found in the subjective observation that the sensation of heaviness does not disappear when an object is held still. From Equation 2, it is evident that without movement, there is no angular acceleration, so the inertia tensor cannot influence the required muscular moment, rendering it impossible to perceive heaviness through **I.** On the other hand, in the absence of static torque due to microgravity, the sensation of heaviness does not disappear (Jones, 1986), although the threshold for mass discrimination increases by a factor of two (Ross & Reschke, 1982). In addition, it is known that mass discrimination improves with movement (Jones, 1986). These observations may hint at a role of the inertia tensor but also a role of inertial mass (i.e., the resistance of an object against linear acceleration).

There is one additional issue that must be addressed at this point. It is often argued that **I** has an advantage over static torque as a candidate entity governing perception in that the moment of inertia is invariant over changes in the angle of a rod with the horizontal plane, whereas static torque is not. Although this is correct, it is also the case that static torque is variable, whereas the static moment is invariant. In the context of testing a theoretical position assuming that (haptic) perception is based on invariants, it would therefore be appropriate to compare the moment of inertia with the static moment rather than static torque. In Equation 2, I_1 (md^2) and the static moment ($dm\mathbf{g}$) are invariant, whereas the inertial term, $md^2\boldsymbol{\alpha}$, depends on the angular acceleration and static torque, $dm\mathbf{g}\cos(\theta)$, depends on the angle. If the static moment were the key entity, wielding a rod could still be useful to determine the maximum of the static torque (i.e., when a rod is held horizontally), because this equals the static moment. It follows from this reasoning that there is no conclusive argument to choose the inertia tensor as the only likely candidate for perception on the ground of its invariance per se.

Experiment 1

The first experiment was conducted to separate effects of I_1 and the static moment in haptically perceiving a rod's length or heaviness. Basically, the experimental setup was comparable to that of Solomon and Turvey (1988). Participants wielded unseen rods with one hand at the wrist joint. Participants were asked to estimate rod extent or heaviness after wielding the rod. Special sets of rods were used in which either the static moment or I_1 was kept constant. It should be noted here that participants were wielding the rods more or less in line with the long axis of the forearm (as in Solomon & Turvey, 1988) and not more or less perpendicular to the long axis of the forearm (as in many other experiments). Consequently, the rotation center of the wrist was quite close to the endpoint of the rod, and we assumed this distance to be zero in calculating the moments of inertia (see the General Discussion section for some further comments and additional analyses with regard to this issue).

Method

Participants. Sixteen healthy right-handed individuals (9 men and 7 women) participated in the experiment after having signed an informed-consent form.

Materials. Two sets of eight rods were used, one set with constant static moment and another set with constant I_1. All rods were made of aluminium and were 0.820 m long and 0.022 m in diameter. The rods weighed 0.148 kg, excluding a lightweight plastic handle that was attached to one end to ensure a constant grip position within and between participants.

Brass cylindrical weights were constructed and attached to the rods in such a way that, within one set of rods, either the radial moment of inertia (I_1 and I_2) or the static moment (both measured with respect to the endpoint of the handle) remained constant. Given the required mass of the brass cylinders, their length and outer diameter were varied in such a way that subsets of rods were created with a constant axial moment of inertia (I_3). The relevant properties of the two sets of rods are described in Table 1.

Procedure. After entering the experimental room, the participant was seated in a chair behind a thin wooden screen. The participant's right arm was positioned on an armrest at the other side of the screen, with the thumb in upward direction and the wrist extending just over the edge of the armrest. Before initiation of the experiment, the height of the armrest was adjusted to the individual seating height. Just proximal to the wrist joint, two vertical supports were attached to the armrest in such a way that rotational movements of the forearm (around its longitudinal axis) were largely prevented. Participants were asked to maintain their forearm on the armrest during rod wielding.

All 16 rods were offered twice in random order. In one of the measurement series, the participants were asked to estimate the length of the rod. In the other series, they were asked to estimate the heaviness of the rod. Eight participants started with length estimation, and the other 8 started with heaviness estimation. The participants were not told that the two series of 16 rods were equal. In addition, the rods were hidden from view for the entire session, and participants were not told that weights had been attached to them. The participants were asked to hold the presented rod firmly in the hand at the handle and to wield it (without lifting the forearm) until they were confident about the rod's length or heaviness.

For length estimation, a vertical planar surface mounted close to the screen could be moved over a rail to and from the participant between 0 and 166 cm, measured from the edge of the right armrest. One experimenter, unaware of the specific rod the participant was wielding, moved the planar surface slowly forward or backward according to the verbal instruction of the participant. This was continued until the participant indicated that, when the planar surface would have extended through the wall, the tip of the rod would just touch the surface when the rod was held horizontally. Then the experimenter, operating the sliding surface, read off the position of the surface with a resolution of 0.5 cm, and this number was defined as the perceived rod length. The participants could not see the length scale and were not informed about the position of the surface.

Before the series of rods were offered for heaviness estimation, the participants were given Rod 12 as a reference. They were told that this rod had an arbitrary heaviness of 10 and that heavier rods were to be given a rating proportionately higher than 10 and lighter rods a rating proportionately lower than 10. After every four rods, the participants were again given the reference rod. The participants were not informed that the reference rod was also one of the experimental rods. No time constraints were imposed on the participants when making their length or heaviness judgments.

Results and Discussion

Perceived rod length. For all participants, regression coefficients with perceived rod length were calculated for I_1, I_3, static moment, and mass over the pooled sets of rods. However, length estimates were too variable to obtain significant correlations for half of the participants. A weak but significant positive correlation with estimated rod length was found for I_1 in the case of 1 participant ($r^2 = .28$). Furthermore, weak but significant correla-

Table 1
Relevant Physical Properties of the Rods Used in Experiments 1 and 2

Rod number	Used in experiment(s)	Mass (kg)	Load distance (m)	I_3 (kg · m²)	Static moment (N · m)	I_1 (and I_2) (kg · m²)
			Constant static moment			
1	1, 2	0.733	0.154	0.00014	1.51	0.048
2	1, 2	0.509	0.248	0.00014	1.51	0.056
3	1	0.409	0.345	0.00014	1.51	0.064
4	1, 2	0.353	0.439	0.00014	1.51	0.073
5	1, 2	0.318	0.529	0.00004	1.51	0.081
6	1	0.293	0.622	0.00004	1.51	0.089
7	1, 2	0.273	0.716	0.00004	1.51	0.097
8	1, 2	0.259	0.809	0.00004	1.51	0.106
			Constant I_1 and I_2			
9	1	2.248	0.159	0.00119	4.00	0.088
10	1	1.788	0.182	0.00118	3.64	0.088
11	1, 2	1.397	0.209	0.00119	3.26	0.088
12	1, 2	1.050	0.246	0.00039	2.86	0.088
13	1, 2	0.761	0.299	0.00039	2.46	0.088
14	1, 2	0.531	0.378	0.00039	2.07	0.088
15	1, 2	0.355	0.515	0.00003	1.68	0.088
16	1, 2	0.231	0.814	0.00003	1.28	0.088
			Constant mass			
17	2	0.498	0.130	0.00046	1.06	0.039
18	2	0.498	0.254	0.00038	1.50	0.056
19	2	0.498	0.378	0.00030	1.94	0.083
20	2	0.498	0.502	0.00022	2.38	0.121
21	2	0.498	0.626	0.00014	2.83	0.170
22	2	0.498	0.750	0.00006	3.27	0.231

Note. I_1, I_2, and I_3 refer to the first, second, and third eigenvalues.

tions (r^2 values ranging from .29 to .77) were found for static moment, mass, and I_3 in the case of 5, 7, and 6 participants, respectively.

Subsequently, perceived rod length was averaged over all participants, and the averaged estimates were plotted against the radial moment of inertia of the rods ($I_1 = I_2$, subsequently indicated as I_1) and against the static moment, the mass, and the axial moment of inertia (I_3) of the rods (Figure 1). As can be seen, the two sets of rods (one with constant I_1 and the other with constant static moment) elicited markedly different response patterns. The averaged data showed that perceived rod length strongly depended on I_1. Indeed, for the eight rods with constant static moment, a highly significant correlation was found between I_1 and perceived rod length ($r^2 = .94$, $p < .001$) as well as between log(I_1) and log(perceived rod length) ($r^2 = .96$, $p < .001$).

Solomon and Turvey (1988) pointed out that if I_1 was indeed an essential parameter in detecting the length of uniform cylindrical rods, the perceived length should be proportional to the third power of I_1. Consequently, a linear regression of log(I_1) versus log(perceived rod length) should yield a slope of about 0.33. The prediction of this coefficient is based on the assumption that I_1 is the only parameter used to estimate rod length. In the rods with a constant static moment, when using I_1 as the only independent variable, a somewhat higher slope of 0.41 ± 0.03 ($p < .001$; $r^2 = .96$) was found in the present experiment.

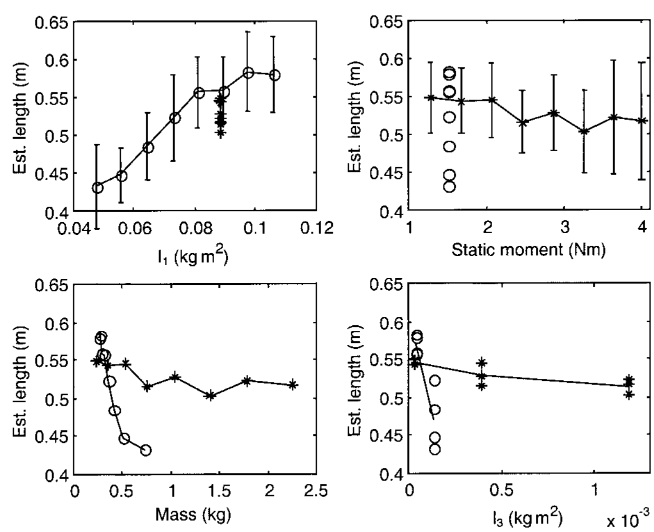

Figure 1. Experiment 1: Estimated (est.) rod lengths for eight rods with constant static moment (open circles) and eight rods with constant I_1 (asterisks), plotted against I_1 (upper left), static moment (upper right), mass (bottom left), and I_3 (bottom right). Error bars indicate one standard error of the mean for the rods with constant static moment in the upper left graph and for the rods with constant I_1 in the upper right graph. I_1 and I_3 refer to the first and third eigenvalues.

A similar approach can be followed for the rods with constant I_1. If the static moment, mass, and I_3 were uniquely responsible for the perception of rod length, the perceived length should be related to the second power of the static moment and to the first power of the mass and I_3, because doubling the length of a uniform rod causes a 2^2 increase in the static moment and a doubling of the mass and I_3 of the rod. Consequently, regression slopes of 0.5, 1, and 1 would be expected in log–log coordinates for the static moment, mass, and I_3, respectively. However, for the rods with constant I_1, only small negative slopes were found: -0.06 ± 0.02 ($p = .014$; $r^2 = .66$) for the static moment, -0.03 ± 0.01 ($p = .015$; $r^2 = .65$) for the mass, and $-.02 \pm .01$ ($p = .018$; $r^2 = .64$) for I_3. If the assumption were to be embraced that estimation of rod length is based on a single parameter, then these results would support the notion that I_1 is uniquely (i.e., not due to covariation with mass or static moment) responsible for perception of rod length. In the rods with constant static moment, mass and I_3 appeared to be related to perceived length in quite a different way. Now the regression coefficients in log–log coordinates were -0.32 ± 0.03 ($p < .001$) for mass and -0.15 ± 0.04 ($p = .005$) for I_3. Again, these negative coefficients do not indicate a unique role in perception of length, because, in that case, they should be positive and close to unity. Furthermore, the coefficients would then be expected to be equal across the two conditions (constant I_1 or constant static moment). Thus, these significant regressions can, at least in part, be attributed to the covariation of mass and I_3 with I_1.

An alternative possibility is that the length of the rod was estimated on the basis of more than one physical entity. We argued in the introduction that, from a mechanical perspective, accurate estimation of the length of a rod on the basis of a single physical entity would require assumptions with respect to one other entity. The possibility that multiple parameters were used in the estimation was tested for the pooled data for both sets of eight rods, averaged over all 16 participants. It should be mentioned in advance that multiple regression analysis might be hampered by covariation of mass and I_3 with static moment and by covariation of sets of two parameters with one or more other parameters. As a result, several models may show up with comparable correlations.

A stepwise multiple regression with I_1, **M,** *m,* and I_3 of the rods as independent variables and perceived length (L_Ψ) as the dependent variables (all in log–log coordinates) resulted in the following significant models: $L_\Psi = 1.279 I_1^{0.356}$ ($r^2 = .81$, $p < .001$, for both coefficients) and $L_\Psi = 1.225 I_1^{0.348} m^{-0.041}$ ($r^2 = .94$, $p < .001$, for all coefficients). When *m* was omitted, the stepwise regression calculated as a second model a very comparable relationship, one involving **M** instead of *m:* $L_\Psi = 1.503 I_1^{0.399} \mathbf{M}^{-0.082}$ ($r^2 = .94$, $p < .001$, for all coefficients). Interestingly, when I_1 was left out of the analysis, the stepwise regression did not produce a significant fit. However, because there was a considerable improvement in r^2 when a second variable was added, the preceding results suggest that the haptic system indeed uses more than one of the parameters (I_1, *m,* **M**). In addition, the coefficients of *m* and **M** have the negative sign predicted by Equations 3 and 4. Nevertheless, the coefficients do not come close to those predicted by Equation 3 or 4, so the haptic system seems to rely predominantly on I_1. In addition, either *m* or **M** might be used.

The inertia tensor hypothesis posits that rod length perception is governed by the eigenvalues of the inertia tensor, implying that the haptic system ignores *m* and **M,** thereby running the risk of making major mistakes in the case of variations in rod radius and rod material density. When both *m* and **M** were left out of the analysis, the multiple regression also resulted in a significant two-parameter model, $L_\Psi = 1.050 I_1^{0.353} I_3^{-0.022}$ ($r^2 = .93$, $p < .001$, for all coefficients) which shows that in the present experiment participants may have completely ignored *m* and **M** and used I_3 instead. On the other hand, the coefficient of I_3 was rather small, and, more important, I_3 itself was very small. Therefore, its effect on muscular torque in the present experiment may have been so minuscule that it may be difficult to extract information about I_3 using the muscular sensory mechanisms. With the use of Equations 3 and 4 for uniform rods, *L* changes 0.1% or less in all 16 rods when I_3 is set to zero. In addition, according to these equations, having information about I_1 and I_3 alone is not sufficient to obtain a reliable estimate of the length of a uniform rod. Hence, we deem it more likely that the last regression model is a consequence of covariation of I_3 with *m* and **M,** but this remains to be proven by additional experimentation.

Perceived heaviness of rods. Both within and between participants, heaviness estimation was more consistent than length estimation. In the pooled sets of rods, significant positive correlations were found in log–log coordinates between static moment and perceived heaviness for all participants ($r^2 = .74 \pm .12$). Possibly as a result of covariation with the static moment, comparable correlations were obtained in simple regression analyses with either mass or I_3 as the independent variable. In contrast, I_1 did not correlate significantly with perceived heaviness in the case of any of the participants.

Averaged over participants, the data indicate that perceived heaviness of the rods varied only minimally with increasing I_1 (Figure 2, top left). In fact, in log–log coordinates, there was even a slightly negative slope (-0.41 ± 0.13, $p = .02$) for the set of

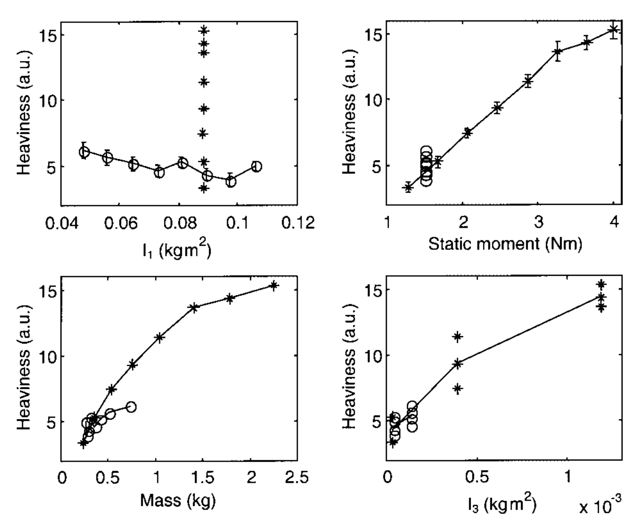

Figure 2. Experiment 1: Perceived heaviness ratings for eight rods with constant static moment (open circles) and eight rods with constant I_1 (asterisks), plotted against I_1 (upper left), static moment (upper right), mass (bottom left), and I_3 (bottom right). Error bars indicate one standard error of the mean for the rods with constant static moment in the upper left graph and for the rods with constant I_1 in the upper right graph. I_1 and I_3 refer to the first and third eigenvalues. a.u. = arbitrary units.

rods with constant static moment. This finding contrasts with those of previous reports (Amazeen, 1997; Amazeen & Turvey, 1996). However, because the increase in I_1 for these eight rods with constant static moment was accompanied by a decrease in rod mass, it is possible that the negative coefficient of I_1 was caused by a decrease in mass. It cannot even be excluded that there was, in fact, a small positive effect of I_1 that was canceled out by a stronger effect of rod mass. Indeed, Amazeen (1997) reported a higher coefficient for mass than for I_1 in a multiple regression analysis in log–log coordinates. It should be noted, however, that Amazeen (1997) did not take the static moment into account. In the set of rods with constant I_1, we found a large and positive effect of static moment on perceived heaviness (slope = 1.35 ± 0.07, $p < .001$; Figure 2, top right).

To obtain a constant I_1, we combined halving of the distance of the load added to the rod with an increase of its mass by a factor of 2^2 when the static moment increased by a factor of 2 (see Table 1). It is therefore not surprising that the regression coefficient of 1.35 for the static moment in log–log coordinates was accompanied by a regression coefficient of roughly half this value (0.66 ± 0.05, $p < .001$) for the mass of the rod.

When both sets of eight rods were pooled and averaged over all 16 participants, significant univariate correlations were found for static moment, mass, and I_3 but not for I_1 (Table 2). This indicates that I_1 does not play an important role in the perception of heaviness. However, because of covariation of static moment, mass, and I_3 in the set of rods with constant I_1, the results of this experiment do not allow us to distinguish among the effects of static moment, mass, and I_3.

Multiple regression analyses showed that only minor improvements in explained variance were obtained when a second independent variable was inserted, from $r^2 = .94$ when only the static moment was used to $r^2 = .97$, $r^2 = .97$, and $r^2 = .96$ when I_1, mass, and I_3 were added, respectively.

In conclusion, the results of the first experiment suggest that, compatible with previous reports, I_1 is a major determinant of perceived rod length. However, contrary to previous reports, I_1 appeared to be unrelated to perception of heaviness.

Experiment 2

The second experiment was aimed at improving differentiation among the effects of mass, static moment, and I_3 by adding a set of rods with a constant mass and with a reversed covariation between static moment and I_3. It was expected that this would allow for distinguishing among I_3, mass, and static moment as a secondary relevant parameter in the perception of rod length and as a primary parameter in the perception of heaviness.

Table 2
Single Correlations Among I_1, Static Moment, Mass, and I_3 and Perceived Length or Perceived Heaviness Averaged Over 16 Participants, in Log Coordinates: Experiment 1

	Perceived length		Perceived heaviness	
Variables	r^2	p	r^2	p
I_1	.812	<.001	.015	.656
Static moment	.005	.801	.944	<.001
Mass	.162	.123	.935	<.001
I_3	.136	.160	.866	<.001

Note. Pooled data from two sets of eight rods were used, one set with constant I_1 and one set with constant static moment. I_1 and I_3 refer to the first and third eigenvalues.

Method

Participants. Twelve individuals (4 men and 8 women) volunteered to participate in the experiment by signing an informed-consent statement. Two of the women were left-handed; the other participants were right-handed. All participants were unaware of the goal of the experiment.

Materials. The setup of the experiment was similar to that of Experiment 1, although now three sets of 6 rods were used, resulting in a total of 18 rods. The two sets of rods from Experiment 1 were used with 2 rods removed from each set. In the set of rods with constant static moment, Rod 3 and Rod 6 were removed. In the set of rods with constant I_1, the 2 heaviest rods were removed (Rod 9 and Rod 10) to reduce possible fatigue effects. A third set was added to allow an examination of the relative contributions of static moment, mass, and I_3 to perceptions of length and heaviness. This set of rods was constructed in the same way as the first two sets, with the mass of the rod held constant. In addition, the covariation of I_3 and static moment was now reversed in comparison with the rod set with constant I_1 (see Table 1).

Small modifications were made to the instrumentation used for length estimates. The vertical supports on the armrest, just proximally to the wrist joint, were set wider so that rotation around the long axis of the forearm was no longer restricted; however, the participants were still instructed to keep their forearm on the armrest. The size of the vertical planar surface was increased to 0.37 × 0.40 m, and a rope and pulley construction (similar to the one introduced by Solomon & Turvey, 1988) was created so that the participants could move the planar surface themselves by rotating a wheel with their left hand. A thin pointer was attached to the plate allowing the experimenter to record length estimates with an accuracy of 1 mm. The participants could not see the length scale and were not informed about the position of the surface.

For the heaviness estimation, Rod 12, the reference rod in Experiment 1, was no longer in the middle range of masses. Therefore, Rod 14 was used as a reference rod with an arbitrary heaviness of 10. After every four rods, participants were again given the reference rod. The participants were not informed that the reference rod was one of the experimental rods.

Procedure. The experimental procedure was comparable to that of Experiment 1, except that three sets of rods were used and all measurements were replicated three times on different days. In each session, two blocks of 18 rods were given to the participants. Within blocks, all rods were offered in randomized order. The order of the blocks was varied systematically between participants and between sessions. The participants were not told that the 18 rods between the two blocks and between the sessions were the same.

Results and Discussion

Order effects. Because each session consisted of one block of length perception measurements and one block of heaviness perception measurements, these measurements could theoretically have affected each other. To test whether this was the case, we conducted an analysis of variance with participant and order of block as independent variables. There was a significant effect of participant on perceived length, $F(11, 624) = 56.10$, $p < .001$, as well as on heaviness, $F(11, 624) = 3.01$, $p = .001$. However, no effects of order were found for length or heaviness perception, $Fs(1, 624) < 0.30$, $ps > .50$, nor were there any interaction effects of participant and order for length perception, $F(11, 624) = 1.33$, $p = .204$, or heaviness perception, $F(11, 624) = 0.43$, $p = .965$.

Length estimation. When the length estimation results for all three sets of rods were plotted against I_1, static moment, mass, and I_3 (Figure 3), the findings of Experiment 1 were essentially replicated. That is, only I_1 showed a consistent and unique relationship with perceived length.

To examine the hypothesis that perceptual judgments of length depended solely on I_1, we calculated univariate correlations in log–log coordinates between perceived length and I_1. These correlations were even higher (see Table 3) than those of Experiment 1 (Table 2), probably as a result of the number of measurements (12 participants and three replications, as opposed to 16 participants and one trial in Experiment 1) and the larger variation in I_1 in the third set of rods. In addition, the slope of the coefficient was 0.334, almost exactly the value of one third predicted by Solomon and Turvey (1988).

Similar to the analyses conducted in the context of Experiment 1, stepwise regressions were performed examining the correlations between perceptual judgments and all relevant independent variables. Averaged over all participants and replications, a log–log stepwise multiple regression analysis resulted in the following significant fits: $L_\Psi = 1.35 I_1^{0.334}$ ($r^2 = .90$, $p < .001$, for both coefficients) and $L_\Psi = 1.28 I_1^{0.328} m^{-0.054}$ ($r^2 = .93$, $p < .002$, for the constant and the I_1 coefficients, and $p = .023$ for the mass coefficient). Unexpectedly, the analysis proceeded with a model in which the coefficient for I_1 became negative (−.037) and nonsignificant ($p = .75$), whereas the static moment entered as a significant contributor. After removal of I_1, the remaining model in the stepwise regression was as follows: $L_\Psi = 0.337 m^{-0.323} \mathbf{M}^{0.534}$ ($r^2 = .96$, $p < .001$).

This raises the possibility that, in fact, L was estimated through a combination of information about mass and static moment and

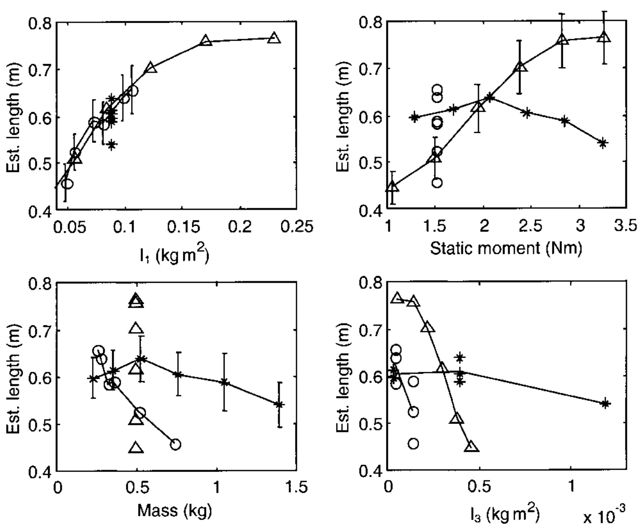

Figure 3. Experiment 2: Estimated (est.) rod lengths for six rods with constant static moment (open circles), six rods with constant I_1 (asterisks), and six rods with constant mass (triangles), plotted against I_1 (upper left), static moment (upper right), mass (bottom left), and I_3 (bottom right). Error bars indicate one standard error of the mean for the rods with constant static moment in the upper left graph, for the rods with constant mass in the upper right graph, and for the rods with constant I_1 in the lower left graph. I_1 and I_3 refer to the first and third eigenvalues.

Table 3
Single Correlations Among I_1, Static Moment, Mass, and I_3 and Perceived Length or Perceived Heaviness Averaged Over 12 Participants, in Log Coordinates: Experiment 2

	Perceived length		Perceived heaviness	
Variables	r^2	p	r^2	p
I_1	.903	<.001	.371	.007
Static moment	.322	.014	.978	<.001
Mass	.069	.291	.512	.001
I_3	.127	.146	.198	<.064

Note. Pooled data from three sets of six rods were used, one set with constant I_1, one set with constant static moment, and one set with constant mass. I_1 and I_3 refer to the first and third eigenvalues.

that I_1 was only a "parasitic" variable. However, the increase in explained variance relative to length estimation based on I_1 was small, so the evidence is inconclusive with respect to the underlying model. In addition, it must be mentioned here that the addition of a set of rods with constant mass in this experiment broke the complete independence of static moment and I_1 in the pooled sets of rods (resulting in $r^2 = .49$ between static moment and I_1). Moreover, a multiple regression analysis of static moment and mass against I_1 yielded an r^2 value of .97. This suggests that these results should be interpreted with care. Nevertheless, multiple regressions for single participants showed a higher correlation for static moment and mass against perceived length than for I_1 against perceived length, and the difference between the two models appeared to be more pronounced (Table 4) in comparison with the averaged data. Nonparametric pairwise tests (Wilcoxon tests) showed that r^2 values for static moment and mass (average $r^2 = .81$) were significantly higher ($p = .006$) than the value for I_1 (average $r^2 = .67$). In addition, r^2 values for static moment and mass against perceived length were also slightly and just significantly ($p = .05$) higher than the multiple regression for I_1 and I_3 (average $r^2 = .75$) against perceived length.

The question arises as to why a combined mass and static moment model did not turn up as an alternative significant model in the stepwise regression in Experiment 1. To answer this question, we conducted a forced mass and static moment regression analysis for the pooled data of Experiment 1, resulting in the following significant fit: $L_\Psi = 0.306 m^{-0.305} \mathbf{M}^{0.537}$ ($r^2 = .90$, $p < .001$).

This model is quite comparable to that produced in Experiment 2. In addition, the r^2 value was higher than that based on I_1 alone (which was .81 in Experiment 1) but smaller than that based on I_1 and I_3 (which was .93 in Experiment 1). The reason that the model did not come up in the stepwise regression for the data of Experiment 1 was that r^2 based on mass or static moment alone is quite low so that the stepwise regression started with a model based on I_1 and did not consider alternative models without I_1. Next, a significant model based on a combination of I_1 and mass produced an r^2 value of .94, so the stepwise regression did not proceed with the model based on mass and static moment, which had a slightly lower r^2 value. It must be noted, though, that in both Experiment 1 and Experiment 2 the coefficients in the combined mass and static moment model were not in accordance with the coefficients

Table 4
Single and Multiple Correlations for All Participants in Experiment 2 Among I_1, Static Moment, Mass, and I_3 and Perceived Length or Perceived Heaviness, in Log Coordinates

	Perceived length			Perceived heaviness			
Participant	I_1	$I_1 + I_3$	Mass + moment	Moment	I_1	$I_1 + I_3$	Mass + I_1 + I_3
1	.706	.769	.768	.838	.374	.589	.784
2	.770	.785	.780	.954	.442	.801	.944
3	.283	.464	.551	.952	.459	.796	.933
4	.594	.664	.865	.828	.319	.725	.855
5	.900	.888	.898	.907	.252	.776	.967
6	.679	.697	.779	.909	.286	.737	.933
7	.786	.790	.837	.946	.315	.800	.969
8	.775	.747	.753	.946	.330	.791	.959
9	.733	.845	.778	.954	.413	.785	.922
10	.676	.773	.823	.908	.408	.799	.901
11	.830	.866	.940	.851	.301	.645	.859
12	.342	.675	.900	.825	.199	.640	.908
M	.673	.747	.806	.902	.342	.740	.911
SD	.178	.109	.096	.050	.076	.072	.052

Note. Values are r^2 values. Pooled data from three sets of six rods were used, one set with constant I_1, one set with constant static moment, and one set with constant mass. I_1 and I_3 refer to the first and third eigenvalues.

based on a model of a uniform rod (Equation 5). The coefficients are therefore more in line with a unique role of I_1.

In conclusion, univariate regressions appeared to support the role of I_1 in perception of rod length. However, in terms of explained variance, multiple regression analyses for individual participants in Experiment 2 provided stronger support for a combined mass and static moment hypothesis than for an inertia tensor hypothesis. In contrast, model coefficients favored an inertia tensor hypothesis rather than a combined mass and static moment hypothesis. Thus, despite the fact that we solved the confounding covariation between I_1 and the static moment in Rod Sets 1 and 2, introduction of multiple regressions showed that neither Experiment 1 nor Experiment 2 produced conclusive evidence on the role of the inertia tensor in length perception during rod wielding.

Heaviness estimation. For the first two sets of rods, the patterns of heaviness estimation were quite comparable to those observed in Experiment 1, although the scales of the graphs (Figure 4) were somewhat different as a result of the lighter reference rod. Comparing the lines of the three sets of rods, an almost constant relationship appeared to exist between heaviness and static moment, whereas I_1, mass, and I_3 exhibited quite variable relations with perceived heaviness. This suggestion was supported by univariate correlations in log–log coordinates (Table 3), showing a high correlation only for the static moment.

A stepwise multiple regression analysis in log–log coordinates of the heaviness (H) data averaged over all participants and replications resulted in $H = 3.29\mathbf{M}^{1.675}$ ($r^2 = .98, p < .001$). The high r^2 value was further increased by adding mass as a second independent variable: $H = 4.17\mathbf{M}^{1.511}m^{0.187}$ ($r^2 = .99, p < .001$).

The graphs as well as the stepwise regressions indicate that **M** is largely and uniquely responsible for the perception of heaviness, at least for rods that are wielded while held at one end. Breaking the covariation between static moment and I_3 (the r^2 value between I_3 and static moment dropped precipitously, from .81 in Experiment 1 to .13 in Experiment 2) also caused the correlation between I_3 and heaviness perception (Experiment 1; see Table 2) to disappear.

Stepwise regressions as well as (forced) single and multiple regressions were also performed for individual participants, all in log–log coordinates (see Table 4). The stepwise regressions produced very robust results in that the static moment surfaced as a first significant parameter for all participants ($r^2 = .90 \pm .050$).

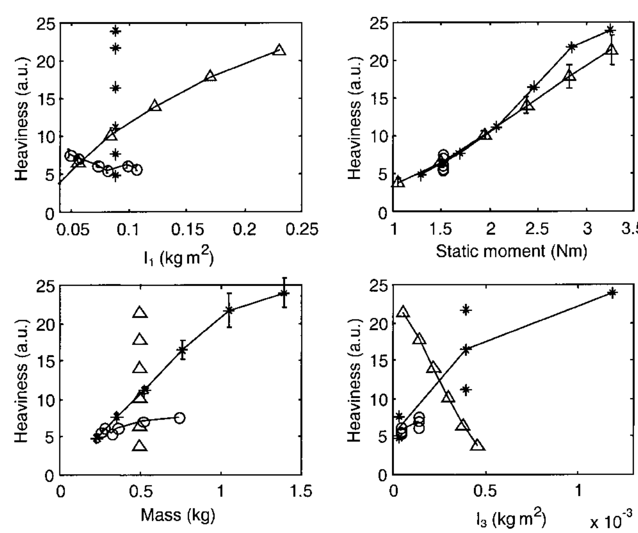

Figure 4. Experiment 2: Perceived heaviness ratings for six rods with constant static moment (open circles), six rods with constant I_1 (asterisks), and six rods with constant mass (triangles), plotted against I_1 (upper left), static moment (upper right), mass (bottom left), and I_3 (bottom right). Error bars indicate one standard error of the mean for the rods with constant static moment in the upper left graph, for the rods with constant mass in the upper right graph, and for the rods with constant I_1 in the lower left graph. I_1 and I_3 refer to the first and third eigenvalues. a.u. = arbitrary units.

Mass was shown to be a second significant variable for 5 of the 12 participants, whereas I_1 and I_3 never revealed themselves as significant parameters. In comparison with the static moment alone, forced single regressions with I_1 resulted in significantly lower correlations ($p < .001$; Wilcoxon paired test), as did forced multiple regressions with I_1 and I_3 ($p = .002$). When, in the latter model, mass was inserted as a third parameter, r^2 values were not significantly different from those for static moment alone ($p = .388$). This result is understandable in that the multiple regression of these three variables with the static moment was also very high ($r^2 = .98$). As stated earlier, interpretation of multiple regressions is hampered by covariation between sets of parameters. In the case of equal correlations, single regressions should be preferred over multiple regressions.

We also tested an alternative three-parameter model recently proposed by Turvey, Shockley, and Carello (1999). This model includes mass, ellipsoid volume, and symmetry. The latter two parameters were calculated from the inertia tensor (for definitions, see Turvey et al., 1999). However, at least in log–log coordinates, r^2 values with perceived heaviness (as well as the correlation with static moment) were exactly equal (i.e., up to three decimals) to the mass, I_1, and I_3 model.

Although the multiple regressions described here do not differ much from those in previous reports (Amazeen, 1997; Amazeen & Turvey, 1996), separation of static moment variation from I_1 variation in Rod Sets 1 and 2 and calculation of single regressions with the static moment in the present study clearly resulted in contrasting conclusions. Contrary to the present results, Amazeen and Turvey (1996) argued that there is evidence that the torque does not play a role in perceptions of length or heaviness. Amazeen and Turvey (1996) concluded that the torque had been varied by varying the speed of wielding and that, because heaviness did not change, the torque could not have played a role in the perception of heaviness. However, Amazeen and Turvey (1996) varied the angular velocity in their Experiment 2, and Equation 2 shows that this indeed results in a change in required muscular torque but not a change in the static moment.

Hence, the invariance of perceived heaviness in the face of changes in angular velocity shown by Amazeen and Turvey (1996, Experiment 2) is not inconsistent with the results of the present study. In Experiments 1, 3, and 5 of Amazeen and Turvey (1996), as well as in Experiment 1 of Amazeen (1997), I_1 and I_2 were manipulated without control for a possibly confounding role of the static moment. The present results suggest that the variations in perceived heaviness that were attributed to variations in I_1 and I_2 are more likely to have resulted from the experimentally induced changes in the static moment. Amazeen and Turvey (1996, Experiments 4 and 6) also reported decreases in perceived heaviness with increasing I_3, as observed in wielding a so-called tensor object with four rods attached to the primary rod in a plane perpendicular to the primary rod. This finding was capitalized on to explain the size–weight illusion. However, the increase in I_3 was always accompanied by a decrease in the static moment (a disc on the central rod was shifted toward the wrist). In light of the present results, this decrease in static moment, rather than the increase in I_3, might be an alternative explanation for their findings.

General Discussion

The present study started from a mechanical analysis revealing that, in addition to the inertia tensor, the static moment and an object's mass may play a role in the perception of object properties such as length and heaviness. This analysis is important in view of recent advances in the understanding of dynamic touch promoting the hypothesis that perception of object properties is governed by the inertia tensor. From a biomechanical point of view, however, this hypothesis may be challenged on the argument that, quantitatively, a larger proportion of the muscular effort needed to manipulate objects in daily life is due to the static moment than to the eigenvalues of the inertia tensor. For the present sets of rods, this can be substantiated by calculating the angular acceleration that would be needed to make, in Equation 2, the inertial term as large as the static torque term (with the rod held in horizontal position).

Stated differently, at what angular acceleration does I_1 cause a muscular torque equal to the muscular torque required for holding the rod in a horizontal position (i.e., compensating for the static moment)? Using Equation 2, this angular acceleration can be calculated by dividing the static moment of each rod by I_1. Using the values of I_1 and static moment from Table 1, it appears that the required angular acceleration ranges from 14.2 rad/s^2 in Rod 22 to 45.2 rad/s^2 in Rod 9. Without any masses attached to the rod, the required angular acceleration would be 17.9 rad/s^2. Although we did not measure angular accelerations, we suspect that these accelerations indicated above are much higher than those that normally occur during rod wielding. Therefore, assuming a major role of muscular tension in dynamic touch (Carello, Fitzpatrick, & Turvey, 1992; Fitzpatrick, Carello, & Turvey, 1994), the signal-to-noise ratio appears to be better for the static moment than for I_1. Extending this argument to the use of muscular sense for detecting resistance against axial rotation of the rod (defined by I_3) shows that this was most improbable, because I_3 was 74 to 3,808 times smaller than I_1 for the present sets of rods. These factors are not a peculiarity of our testing materials but a typical property of rods. For instance, the I_1–I_3 ratio of the rods in Experiment 1 of Stroop, Turvey, Fitzpatrick, and Carello (2000) ranged from 90 to 234. In daily life, when one is dealing with objects other than rods, the ratio of I_1 to I_3 may not be as large as in the present experiment, but I_3 remains, by definition, smaller than I_1.

An issue that is important in this context is whether—and, if so, at what distance—the inertia tensor is translated from the rod endpoint to the wrist joint center, because this considerably affects the size of I_3. In many recent studies, the eigenvalues of the inertia tensor were calculated with respect to the wrist rather than the endpoint of the rod. In our opinion, however, this alternative method of calculation may be disputed because it results in a description of the eigenvalues (principal moments of inertia) with respect to axes that are not aligned with the rod, resulting in a partial transfer of the rod's I_1 to I_3. Moreover, the translation causes inclusion of the rod's static moment in the inertia tensor (i.e., in the product of inertia), which is distributed over the eigenvalues when the tensor is diagonalized. This may lead to overestimation of coefficients of determination in multiple regressions and to confusion with respect to rod properties that are actually related to specific perceptual modalities.

In spite of these concerns, however, we checked to what extent the results of the present experiments would be affected by using the alternative method of calculating the eigenvalues of the inertia tensor (i.e., by translating the inertia tensor and again diagonalizing it). In this recalculation, we used a wrist to rod distance of 3 cm, because the usual 6 cm is appropriate for rods that are held perpendicular to the forearm but not for our experiments, in which the rods were held in line with the longitudinal axis of the forearm. In the averaged data, changes in coefficients of determination were positive for correlations involving I_3, but the effects were always smaller than .008 and can thus be neglected. For individual data, changes were somewhat larger, but the tendencies in our data were unaffected. Therefore, the alternative way of calculating the eigenvalues would not affect our conclusions. It should also be noted that, even without translation to the wrist, our range of I_3 values in Experiment 2 was larger than the range of (translated) I_3 values in Experiment 1 of Stroop et al. (2000).

Chan (1994) hypothesized that perceived rod length is a function of weight and static moment (according to our definition) when a rod is held stationary. Although he reported significant correlations between static moment and perceived rod length for rod holding as well as wielding, he did not perform multiple regressions in which both static moment and mass were entered as independent variables. For holding rods, Lederman et al. (1996) found that a combination of static moment and static torque explained much of the variance in perceived rod length ($r^2 = .96$), just as did a combination of mass and static torque ($r^2 = .92$); again, however, multiple regressions involving mass and static moment were not reported.

Equations 3 to 5 showed that the length of a uniform rod can be defined with three combinations of parameters. More important, these equations suggest that it is physically impossible to reliably estimate the length of a uniform rod on the basis of **I** alone. Of course, this does not prove that participants indeed used more than one physical entity of the set (**I,** *m,* **M**). However, the current results urge for consideration of the possibility that not **I** but a combination of mass and static moment governs the perception of rod length. One might ask why this combination of parameters has not shown up in other experiments involving rod wielding. The answer is simply that this combination was never statistically tested. However, in static holding of rods, Carello, Fitzpatrick, Domaniewicz et al. (1992) made an explicit distinction between static torque and static moment and showed that the static moment was the main determinant of perceived rod extent. In later articles (i.e., Carello et al., 1996; Pagano et al., 1996), these findings, as well as those of Burton and Turvey (1990), were attributed to covariation between the static moment and the products of inertia. In Carello et al. (1996), the combination of static torque and mass was tested once (and not found to be significant) for the perception of partial rod length during holding. On the argument that static torque is not invariant, combinations of mass and static torque or static moment were not tested in subsequent experiments involving rod wielding. The static moment was ignored in the wielding experiments even though, as with **I,** it is invariant.

The alternative explanation of a combination of mass and static moment might also be applicable to other claims with respect to the role of **I,** in, for instance, perception of object orientation, hand position relative to an object, partial object length, object width and height, and limb orientation (Turvey, 1998). To check this, we reanalyzed the data from several recent studies by performing multiple regressions with mass and static moment as independent variables. This resulted in a rather low amount of variance explained by the combination of mass and static moment in one experiment ($r^2 = .62$ in Experiment 2 of Carello et al., 1996). However, in this experiment (concerning rod wielding), as well as in Carello et al.'s Experiment 1 (concerning rod holding), some rods were held at the left side and others at the right side of the center of mass. Introducing the absolute static moment as a third variable into the regression resulted in r^2 values of .81 (Experiment 1) and .86 (Experiment 2).

In a number of other experiments, the amount of variance explained by mass combined with static moment was quite comparable to that by the components of **I:** $r^2 = .91$ for perceived whole rod length and $r^2 = .94$ for perceived partial rod length in Experiment 3 of Carello et al. (1996). For the study of Pagano et al. (1996), we found explained variances of .79 for perceived forward length and .89 for perceived grip position in Experiment 1 and explained variances of .95 for perceived forward length and .87 for perceived grip position in Experiment 2 (all in log–log coordinates). In a recent experiment involving solid objects of varying width and height, Turvey et al. (1998) reported an r^2 value of .95 between I_1 and perceived object height as well between I_3 and perceived object width. Using the data from Turvey et al. (1998), we calculated the following multiple regressions using mass and static moment as independent variables: perceived height $= 0.899 \times \mathbf{M}^{0.364} m^{-0.046}$ ($r^2 = .98$, $p < .001$) and perceived width $= 1.47 \times \mathbf{M}^{-0.317} m^{0.747}$ ($r^2 = .97$, $p < .001$). The analyses just described show that the possibility of mass and static moment governing perceived rod properties is not confined to the present experimental task (wielding rods while holding them at one end and estimating whole length) but also exists for perceived partial rod length and perceived grip position while wielding rods held at other positions, as well as for the perception of width and height of solid objects.

Recent work by Stroop et al. (2000) on static holding of rods provided additional evidence favoring a role of I_1 by showing (in their Experiment 2) that with constant mass and static moment perceived length still varies with I_1. However, it must be added that the actual rod length also varied, so an effect of another, not yet recognized variable cannot be excluded. The same study also provided evidence for a role of the static moment in length perception, although this was not recognized as such. In Experiment 4 of that study, an interaction effect between I_1 and I_3 was reported, resulting in a threefold increase of the coefficient for I_3 in Rod Set 2 relative to Rod Set 1. The mass of the objects was constant within each set, but the static moment was not. In fact, the static moment variation in Rod Set 2 was nearly three times the variation in Rod Set 1. Therefore, the observed interaction effect was probably attributable to variations in the static moment.

Conclusion

The present study shows that it is possible to separately vary static moment and I_1. When analyses of length and heaviness estimation were limited to univariate regressions, the expected main role of I_1 in length perception was corroborated. In addition, evidence was found against a role of I_1 and in favor of a prominent role of the static moment in the perception of heaviness during rod wielding.

Interpretation of multiple regression analyses is hampered by covariation between combinations of independent variables. Nevertheless, the single-variable-based regression of static moment with heaviness estimation appeared to be significantly superior to the two-parameter model (I_1, I_3) proposed by Amazeen and Turvey (1996). The three-parameter models (I_1, I_3, mass) (Amazeen, 1997) and (mass, ellipsoid volume, symmetry) (Turvey et al., 1999) produced exactly equal r^2 values in relation to perceived heaviness. These values were comparable to a statistical model based on static moment alone, but the three-parameter models also correlated highly with the static moment. Given these arguments, we conclude that there is strong support for the hypothesis that the static moment governs the perception of heaviness and that the role of the inertia tensor has been overestimated as a result of confounding covariation. We would like to add here that this conclusion is based on, and thus restricted to, one specific condition, namely wielding rods while holding them at one end. For different situations, for instance when gripping a rod at the center of mass, other variables, especially mass, may also be important.

Length perception appeared to be persistently strongly correlated with I_1 when the covariation between I_1 and the static moment was broken. Another strong argument in favor of I_1 is its coefficient in the statistical model. This coefficient corresponds with previous findings and was predicted theoretically. In this respect, the results of the present study can be interpreted as additional support for previous claims of a prominent and perhaps even exclusive role of the inertia tensor in length perception.

The combined mass and static moment model, which surfaced as an alternative explanation in our study, does not have this appealing logic of coefficients. Accepting the alternative model would imply either that the participants assumed the rods to be uniform but did not weigh the mass and static moment according to the physical model of a uniform rod or that they did not assume the rods to be uniform. Considering these arguments, it appears that the inertia tensor still has the edge relative to the combination of static moment and mass.

At the same time, however, it must be realized that when the haptic system uses two physical entities instead of one to estimate rod length, there is much more freedom to deviate from a physical model, resulting in variations of the actual coefficients. The values of these coefficients may vary depending on the confidence in each of the entities owing to the influence of experimental conditions on the signal-to-noise ratio.

The present study also provides compelling arguments in favor of the alternative model. For example, the combination of mass and static moment appeared to be a significantly better estimator of length perception than I_1 or I_1 combined with I_3. A reanalysis of previous experiments showed that this alternative model also applies to a number of other situations. Another argument for the alternative model is its more favorable signal-to-noise ratio with respect to "muscular sense through muscular effort" relative to I_1.

Given the preceding arguments, we believe that current as well as previous research has not provided sufficient evidence to either accept or reject the hypothesis that the inertia tensor alone governs haptic perception of rod length. Moreover, both the inertia tensor hypothesis and the mass and static moment hypothesis may turn out to be fallible under certain conditions. For instance, when rods of identical mass are held at their center of mass, the only invariant left for detecting rod length is **I.** Conversely, when a rod is held in place vertically by guides and variations are imposed by attaching different weights to the rod at different locations (cf. Lederman et al., 1996), radial angular acceleration is prevented and thus I_1 cannot be used. Furthermore, displacing the weight along the length of the rod does not cause any change in the static torque due to the vertical position of the rod. Consequently, variations in static moment cannot be perceived, so the perception of rod length will have to rely on changes in mass (cf. Lederman et al., 1996). These examples illustrate the need for extracting different physical entities under different circumstances. In so doing, they highlight the need for developing a theory of dynamic touch that allows not only for the possibility that more than one physical entity is used for perceiving rod properties but also for the possibility that different combinations of physical entities are used under different circumstances.

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Appendix

Equations Defining the Length of a Solid, Uniform, Cylindrical Rod

For a solid, cylindrical, homogeneous rod, the first eigenvalue of the inertia tensor (I_1) is defined with respect to the center of mass as follows: $I_1 = \frac{1}{12} m(3R^2 + L^2)$, where m is the mass, R is the radius, and L is the length of the rod. Using the parallel axis theorem, I_1 can be calculated with respect to the endpoint of the rod: $I_1 = \frac{1}{12} m(3R^2 + L^2) + m(\frac{1}{2} L)^2$. This equation can be rewritten to L as a function of I_1:

$$I_1 = \tfrac{1}{12} m(3R^2 + L^2) + m(\tfrac{1}{2} L)^2 \leftrightarrows$$

$$m^{-1}I_1 = \tfrac{1}{4} R^2 + \tfrac{1}{3} L^2 \leftrightarrows$$

$$L = (3m^{-1}I_1 - \tfrac{3}{4} R^2)^{0.5}. \quad \text{(A1)}$$

Furthermore, R can be replaced by the third eigenvalue of the inertia tensor (I_3), because I_3 of a solid homogeneous cylinder is defined as $I_3 = \frac{1}{2} mR^2$, which equals

$$R^2 = 2I_3m^{-1}. \quad \text{(A2)}$$

Inserting Equation A2 into Equation A1 yields $L = (3I_1m^{-1} - \frac{3}{2} I_3m^{-1})^{0.5}$, which can be rewritten as

$$L = m^{-0.5}(3I_1 - \tfrac{3}{2}I_3)^{0.5}. \quad \text{(A3)}$$

Note that the mass has a negative coefficient. Note also that I_3 in our Experiments 1 and 2, as well as in most usual rods, is very small relative to I_1, so its influence on L can be neglected. This simplifies the estimation of L to $L = m^{-0.5}3^{0.5}I_1^{0.5}$.

The static moment of a uniform rod is defined with respect to the endpoint as $\mathbf{M} = \frac{1}{2} Lm\mathbf{g}$, showing that the length of a rod is also defined by

$$L = 2\mathbf{M}(m\mathbf{g})^{-1}. \quad \text{(A4)}$$

Furthermore, Equation A4 can be used to replace the mass by the static moment in Equation A3: $m = 2\mathbf{M}L^{-1}\mathbf{g}^{-1}$. Inserting this into Equation A3 yields

$$L = (2\mathbf{M}L^{-1}\text{g}^{-1})^{-0.5}(3I_1 - \tfrac{3}{2} I_3)^{0.5}$$

$$= L^{0.5}(2\mathbf{Mg}^{-1})^{-0.5}(3I_1 - \tfrac{3}{2} I_3)^{0.5} \leftrightarrows$$

$$L^{0.5} = \mathbf{g}^{0.5}(2\mathbf{M})^{-0.5}(3I_1 - \tfrac{3}{2} I_3)^{0.5} \leftrightarrows$$

$$L = [\mathbf{g}^{0.5}(2\mathbf{M})^{-0.5}(3I_1 - \tfrac{3}{2} I_3)^{0.5}]^2 \leftrightarrows$$

$$L = \mathbf{g}(3I_1 - \tfrac{3}{2}I_3)(2\mathbf{M})^{-1}, \quad \text{(A5)}$$

showing that a combination of **M** and **I** can also be used to define the length of a rod. Equations A3–A5 show that there are three ways to define L, all requiring at least two physical entities from the set (**I,** m, **M**). Stated differently, Equations A3–A5 suggest that the inertia tensor, in itself, does not provide sufficient information to determine the length of a rod. Information about the inertia tensor must be combined with information about either mass or static moment. Suppose that the participants make another assumption concerning the rods, namely that they all have the same density (ρ). Then one might think that there is a way to remove the mass term, because the mass of a uniform rod is defined as $m = \rho\pi R^2L$. Inserting this into Equation A3 yields $L = (\text{d}\pi R^2L)^{-0.5}(3I_1 - \frac{3}{2} I_3)^{0.5}$. However, now we are facing R again. When Equation A2 is used to replace R with I_3, we reintroduce the mass m in the equation. Consequently, even under the assumption of uniform density of rods, knowledge of **I** by itself is not sufficient for reliable estimation of L.

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