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The inflationary bispectrum with curved field-space

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Abstract. We compute the covariant three-point function near horizon-crossing for a system of slowly-rolling scalar fields during an inflationary epoch, allowing for an arbitrary field-space metric. We show explicitly how to compute its subsequent evolution using a covariantized version of the separate universe or ‘ δN ’ expansion, which must be augmented by terms measuring curvature of the field-space manifold, and give the nonlinear gauge transformation to the comoving curvature perturbation. Nonlinearities induced by the field-space curvature terms are a new and potentially significant source of non-Gaussianity. We show how inflationary models with non-minimal coupling to the spacetime Ricci scalar can be accommodated within this framework. This yields a simple toolkit allowing the bispectrum to be computed in models with non-negligible field-space curvature.

Keywords: Inflation, non-Gaussianity, non-canonical, bispectrum

1 Introduction

Considerable effort has recently been invested in the study of multiple-field models of inflation. There are three principal motivations: First, unless curvature couplings flatten their potentials at high energy density, the Standard Model has not produced scalar fields which can successfully inflate. This has stimulated the search for realizations of inflation in theories beyond the Standard Model. Second, single-field inflationary models often require super-Planckian field excursions. Unfortunately, well-rehearsed arguments suggest that we should not expect the scalar potential to be stable against renormalization-group running over such large distances in field space. Multiple-field models may evade this problem by allowing sub-Planckian excursions. Finally, interactions between several fields can give rise to observable non-Gaussianity. This may make some multiple-field models sufficiently predictive that they can be falsified by observation.

It has been known for some time that, under slow-roll conditions, multiple-field models with canonical kinetic terms generate unobservable three- and four-point functions at horizon-crossing [1–4]. Observable effects can arise only later, from nonlinear processes operating on superhorizon scales. A reasonably clear picture has emerged in which these nonlinear effects can be understood as the deformation of a Gaussian probability distribution by the phase space flow associated with the theory [5–8].

Noncanonical kinetic terms offer new possibilities. In some theories the Lagrangian becomes an arbitrary function of the kinetic energy $2X = -\partial_\mu\varphi\partial^\mu\varphi$. Where this yields a reduced sound speed for perturbations there can be a significant enhancement of the three- and four-point functions [9–15]. But this is not the only type of noncanonical Lagrangian. In many examples descending from our present ideas about physics at very high energies, including string theory and supergravity, the kinetic energy must be written $2X = -G_{IJ}\partial_\mu\varphi^I\partial^\mu\varphi^J$, where G_{IJ} is an arbitrary, symmetric function of the fields φ^I . The simplest example is the nonlinear σ -model of Gell-Mann & Lévy, originally introduced to describe spin-0 mesons.

The matrix G_{IJ} can be thought of as a metric on the space of scalar fields and will generically exhibit nonzero curvature. Are there interesting enhancements of non-Gaussian effects in these models? Estimates of the three-point function have been made by a number of authors [14–18], but as yet no complete formalism exists which allows the bispectrum to be followed from horizon-crossing up to the time of observation. Moreover, Gong & Tanaka recently pointed out that the most widely-used formulation of nonlinear perturbation theory is not covariant [19]. They introduced a covariant description, to be discussed in §2.1, and constructed the action for fluctuations up to third-order in the scenario of Langlois et al. [14]. A similar argument was later made by Saffin [18]. Covariance is a convenience rather than a physical principle, so its absence does not invalidate earlier results. Nevertheless, it is a considerable convenience: there are subtleties associated with time evolution of the two-point function on curved field-space which are most clearly expressed in covariant language. These take the form of a contribution to the effective mass-matrix from the Riemann curvature tensor. In this paper we show that a similar phenomenon occurs for the three-point function.

In flat field-space, time evolution of superhorizon modes may be taken into account using the ‘separate universe’ method [20–23]. This enables the time dependence of each fluctuation to be determined from the relative behaviour of separated spatial regions following slightly displaced phase-space trajectories. It can be effected using a Taylor expansion to compare

two solutions of the slow-roll equation. But in curved field-space we must be cautious when comparing the relative motion of neighbouring trajectories. In the analogous case of general relativity one would use the equation of geodesic deviation, or ‘Jacobi equation’. In flat field-space this can be integrated to reproduce the Taylor expansion [8]. When promoted to curved field-space the Jacobi approach is automatically covariant and accounts naturally for time-dependent effects generated by the Riemann curvature, including its known contribution to the effective mass-matrix. Beyond linear order there are new contributions which influence the three-point function. These must appear in any correct formulation but are expressed most clearly and economically in terms of the field-space curvature.

At present, the covariant approach cannot be used to generate observable predictions beyond the power spectrum. To do so would require a determination of the initial value of the three-point function near horizon-crossing, together with a prescription to evolve it into the primordial curvature perturbation. It is only the curvature perturbation which can be connected with observable quantities. Neither of these pre-requisites has yet been provided.

In this paper we compute the initial value of the covariant three-point function at horizon exit and use the Jacobi approach to determine its time evolution. (Partial expressions for the noncovariant three-point function were given by Langlois et al. [14] and Renaux-Petel et al. [15].) As a concrete example we give the analysis for the σ -model Lagrangian $\mathcal{L} = X + V$, although our methods generalize to more complex cases. We show that this initial value can be smoothly connected to the subsequent Jacobi evolution. In particular, the evolutionary effects described above which depend on the Riemann tensor can be matched to new infrared divergences in the three-point function. We demonstrate this matching explicitly to subleading order in both time-dependent perturbation theory and the slow-roll expansion. A further benefit of the Jacobi approach is that time evolution can be computed very simply using ordinary differential equations.

In §2 we specialize the results of Gong & Tanaka to the σ -model Lagrangian and obtain the action for fluctuations to third-order. In §3 we compute the corresponding two- and three-point functions near horizon-crossing in the spatially flat gauge. The two-point function has been known since the work of Sasaki & Stewart [24], but the computation of the covariant three-point function is new. In §4 we use the Jacobi method discussed above to compute the evolution of these correlation functions after horizon-crossing.

In §5 we show that our results can be applied to models in which the scalar fields are coupled non-minimally to gravity by making a suitable conformal transformation [25]. Such couplings arise naturally in the low-energy limit of higher-dimensional theories including supergravity, string theory and Kaluza–Klein models [26–28], or as counterterms in curved spacetime [29, 30]. Finally, we conclude in §6.

Notation. Throughout, we work in units where $c = \hbar = 1$ and express the gravitational coupling in units of the reduced Planck mass, $M_{\text{pl}}^{-2} \equiv 8\pi G$. Upper-case Latin indices I, J, K, \dots , label the species of scalar fields, and Greek letters label spacetime indices. We use a modified index convention for bilocal tensors (‘bitensors’), to be described in §3.3. The covariant derivative compatible with the field-space metric G^{IJ} is ∇_I . For any tensor $\mathcal{F}...$ we write $\nabla_I \mathcal{F}... = \mathcal{F}...;I$. Our sign convention for the curvature tensor is defined by the Ricci identity, $[\nabla_I, \nabla_J]V_K = R_{IJKL}V^L$. Finally, it is useful to define covariant versions of the derivatives with respect to coordinate time t , conformal time $\eta = \int dt/a$ and e-folds

$N = \int H dt$ as

$$\mathcal{D}_t = \frac{d\phi^I}{dt} \nabla_I, \quad \mathcal{D}_\eta = \frac{d\phi^I}{d\eta} \nabla_I, \quad \mathcal{D}_N = \frac{d\phi^I}{dN} \nabla_I. \quad (1.1)$$

2 The action and its perturbations

Consider an inflationary epoch driven by \mathcal{N} scalar fields φ^I (with $I = 1, 2, \dots, \mathcal{N}$), minimally coupled to gravity and self-interacting through a potential $V(\varphi)$,

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [M_{\text{pl}}^2 R - G_{IJ} g^{\mu\nu} \partial_\mu \varphi^I \partial_\nu \varphi^J - 2V]. \quad (2.1)$$

As explained in §1, the field space is to be understood as a manifold with metric $G_{IJ}(\varphi)$. This metric is used to raise and lower tangent-space indices, and distinguishes the model from canonical scenarios where $G_{IJ} = \delta_{IJ}$.

During inflation the φ^I take approximately homogeneous but time-dependent values which we label $\phi^I(t)$. Small inhomogeneities around this homogeneous background are controlled by a perturbative expansion of (2.1). When quantized they will seed the primordial inflationary fluctuation.

2.1 Covariant perturbations

In what follows we perturbatively expand the action for these fluctuations to third-order. On curved field-space it is very helpful to arrange this expansion so that covariance is manifest. Each inhomogeneity is a coordinate displacement $\delta\varphi^I = \varphi^I(\mathbf{x}, t) - \phi^I(t)$, but for finite length this displacement does not lie in the tangent space at ϕ^I and therefore does not have a tensorial transformation law. To obtain a covariant description we must find an alternative characterization which associates each displacement with a tangent-space vector. This construction was given by Gong & Tanaka [19], whose method we briefly describe.

We assume the field-space metric to be smoothly differentiable in the neighbourhood of the background trajectory. Within a normal neighbourhood, the points $\varphi^I(\mathbf{x}, t)$ and $\phi^I(t)$ are linked by a unique geodesic, labelled by a parameter λ . We adjust the normalization so that $\lambda = 0$ corresponds to the unperturbed coordinate ϕ^I and $\lambda = 1$ corresponds to the perturbed coordinate $\phi^I + \delta\varphi^I$. The coordinate displacement $\delta\varphi^I$ may be expressed as a formal Taylor series along this geodesic

$$\delta\varphi^I \equiv \left. \frac{d\varphi^I}{d\lambda} \right|_{\lambda=0} + \frac{1}{2!} \left. \frac{d^2\varphi^I}{d\lambda^2} \right|_{\lambda=0} + \frac{1}{3!} \left. \frac{d^3\varphi^I}{d\lambda^3} \right|_{\lambda=0} + \dots \quad (2.2)$$

Eq. (2.2) is independent of the normalization of λ , so our particular choice was merely a convenience. The geodesic satisfies

$$\mathcal{D}_\lambda^2 \varphi^I = \frac{d^2\varphi^I}{d\lambda^2} + \Gamma_{JK}^I \frac{d\varphi^J}{d\lambda} \frac{d\varphi^K}{d\lambda} = 0, \quad (2.3)$$

where $\mathcal{D}_\lambda \equiv Q^I \nabla_I$ and $Q^I \equiv d\varphi^I/d\lambda|_{\lambda=0}$. Using (2.3), the expansion (2.2) can be reorganized as a power series in Q^I , yielding

$$\delta\varphi^I = Q^I - \frac{1}{2!} \Gamma_{JK}^I Q^J Q^K + \frac{1}{3!} (\Gamma_{LM}^I \Gamma_{JK}^M - \Gamma_{JK,L}^I) Q^J Q^K Q^L + \dots, \quad (2.4)$$

where the coefficients $\Gamma_{JK}^I, \Gamma_{JK,L}^I, \dots$, are evaluated at $\lambda = 0$. In flat field-space all terms but the first vanish and therefore $\delta\varphi^I = Q^I$. It was to achieve this correspondence that we adopted our normalization for λ . Although (2.4) is not itself covariant, it can be used to exchange an expansion in $\delta\varphi^I$ for an expansion in Q^I . Since Q^I *does* lie in the tangent space at ϕ^I , the perturbative expansion of any tensorial object will be manifestly covariant if expressed in powers of Q^I . We label tangent-space indices at the perturbed position $\phi^I + \delta\varphi^I$ by primed indices I', J' and tangent-space indices at the original position ϕ^I by unprimed indices. For any tensor $\mathcal{F}_{I\dots J}$ we obtain

$$\mathcal{F}_{I'\dots J'} = G_{I'I} \dots G_{J'J} \left(\mathcal{F}_{I\dots J}|_{\lambda=0} + \mathcal{D}_\lambda \mathcal{F}_{I\dots J}|_{\lambda=0} + \frac{1}{2!} \mathcal{D}_\lambda^2 \mathcal{F}_{I\dots J}|_{\lambda=0} + \dots \right), \quad (2.5)$$

where $G_{I'I}$ is the parallel propagator, which expresses parallel transport along the geodesic connecting ϕ^I with $\phi^I + \delta\varphi^I$. For details, see Poisson, Pound & Vega [31].

2.2 Gauge choice and infrared-safe observables

Inhomogeneities in φ^I couple to gravity and therefore induce fluctuations in the metric. The description of this mixing is simplified using ADM variables [32], in terms of which the metric can be written

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (2.6)$$

where N is the lapse function and N^i is the shift vector. Spatial indices are raised and lowered using the 3-metric h_{ij} , which has determinant h . In these variables, the action can be written

$$S = \frac{1}{2} \int d^4x \sqrt{h} \left\{ M_{\text{pl}}^2 \left[NR^{(3)} + \frac{1}{N} (E_{ij}E^{ij} - E^2) \right] + \frac{1}{N} \pi^I \pi_I - N \partial_i \varphi^I \partial^i \varphi_I - 2NV \right\}, \quad (2.7)$$

where $R^{(3)}$ is the Ricci scalar built from the 3-metric and E_{ij} is proportional to the extrinsic curvature of slices of constant t ,

$$2E_{ij} = \dot{h}_{ij} - N_{i|j} - N_{j|i}. \quad (2.8)$$

We have defined E to be the trace E^i_i , and a vertical bar denotes the covariant derivative compatible with h_{ij} . Finally, $\pi^I = \dot{\varphi}^I - N^j \varphi^I_{|j}$ and an overdot denotes a derivative with respect to t .

The lapse and shift appear in (2.7) without time derivatives. Therefore they are not propagating modes, but yield constraint equations. These determine N and N^i as functions of the physical degrees of freedom, and also generate gauge symmetries associated with translations in time and space. After imposing the constraints and removing redundant gauge modes we are left with \mathcal{N} scalar degrees of freedom plus two polarizations of the graviton which transform as a spin-2 mode, but we are free to choose how the scalar modes are divided between the φ^I and h_{ij} by making gauge transformations.

Infrared-safe observables. When selecting a gauge we must balance competing demands. First, consider a single-field model where $\mathcal{N} = 1$. A common gauge choice is to arrange slices of constant t to coincide with slices of constant φ , leaving a scalar metric mode which we label ζ . This scalar mode is a perturbation to the volume element, $\zeta = (1/6)\delta \ln \det h$. The

disadvantage of this choice is that the calculation is long [33]. Multiple partial integrations are required to bring the action to a suitable form, simultaneously generating boundary terms which must be retained [34–36].

The advantage is a precise type of technical simplicity. With only a single field it is a classical theorem that ζ is time independent [37, 38]. The same is true in the quantum theory, at least at tree-level, for the correlation functions of ζ [39].¹ If present, time dependence would manifest itself as a divergence in the vertex integrals appearing in each n -point function [43–45]. Each integral sums the amplitude for an interaction to occur at a specific time, and a divergence means only that interactions continue arbitrarily far into the future. At tree-level the great merit of ζ -gauge for single-field models is that the n -point functions are free of divergences: each vertex integral receives significant contributions only from interactions which occur near horizon-crossing. Physically, predictions for ζ decouple from the infrared dynamics of the theory.

For this reason we describe ζ as ‘infrared safe’.² In practice this means that subleading terms in the action map to subleading terms in each correlation function. This makes it simple to impose approximation schemes, such as the slow-roll expansion.

The enormous convenience of infrared safety means that most single-field calculations have made use of this gauge. An alternative is to fix slices of constant t to carry a flat metric, leaving a perturbation in the scalar field value $\varphi(\mathbf{x}, t)$. In view of the discussion in §2.1 we denote the covariant representation of this perturbation Q^I . Calculation of the lowest-order action for Q^I is simpler than in the ζ -gauge [1]. But unlike ζ the field perturbation is not infrared safe because the integrals which define its correlation functions receive contributions from all times, not just those near horizon crossing [43, 47–50]. Therefore they are sensitive to the infrared dynamics of the theory. As a result, subleading terms in the action can be enhanced by divergences and special action is required to deal with them. In a single-field model, Maldacena argued that they can be accounted for by a gauge transformation to ζ just after horizon crossing [33].

Multiple-field models. In a multiple-field model the situation is more complex. Because ζ is evolving, the divergences can no longer be accounted for using only a gauge transformation on a fixed time-slice. The solution proposed in Ref. [1] was to evaluate each n -point function in the spatially flat gauge a few e-folds after horizon crossing. This choice prevents enhanced subleading terms from spoiling the lowest-order slow-roll prediction, and therefore we can take advantage of the computational simplicity of this gauge. The price we pay is that some means must be found to express the correlation functions at a subsequent time in terms of their values near horizon crossing. However this is done, the result must equal what would have been obtained if we had been able to evaluate the original divergent integrals.

¹A similar statement can be made at one-loop level, ignoring internal graviton lines [40, 41]. Even if present, loop-generated time dependences are typically strongly suppressed [42]. Although they might be important to describe the evolution of correlations over very large time and distance scales, it seems probable they would be negligible for the description of a phase of observable slow-roll inflation.

²The terminology is borrowed from gauge theories, where an infrared-safe observable dominated by a hard subprocess occurring near energy E does not depend on the details of other processes (such as hadronization and confinement) occurring at energies much less than E . A similar discussion was given by Weinberg [46], who focused on the conditions under which (in our language) observables might be infrared-safe. (However, there is no reason of principle for any physical observable of interest to be infrared-safe.)

Therefore, in a perturbative expansion, it must reproduce the same divergences. For canonical fields on flat field-space, the nonlinear separate universe or ‘ δN ’ formalism introduced by Lyth & Rodríguez can be used for this purpose [23].

On curved field-space we expect new divergences involving the field-space Riemann curvature [19], and we must be aware that these will modify the time-evolution of the correlation functions. We will argue in §4 that these can be understood as an analogue of the geodesic-deviation effect for freely-falling observers in curved spacetime, and show how they can be incorporated in a new version of the ‘ δN ’ formula.

The conclusion of this discussion is that we are still entitled to choose the spatially flat gauge in order to simplify the calculation. However, we must take care to study the effect of divergent terms. Although we will carry the QFT calculation of the three-point function only as far as horizon-crossing, where the divergences are harmless, it is important to check carefully that whatever technique we employ to account for time dependence correctly reproduces all these divergent pieces.

3 Two- and three-point functions of the fluctuations

Flat gauge calculation. With this in mind, we specialize to the spatially flat gauge where $h_{ij} = a^2 \delta_{ij}$. To expand (2.1) to third-order in Q^I we need only solve the constraints for the first-order components of N and N^i [11, 33]. The shift vector can be decomposed into irrotational and solenoidal parts, yielding $N^i \equiv \partial^i \vartheta + \beta^i$ where $\partial_i \beta^i = 0$. The first-order solenoidal component β_1^i appears uncoupled to the other perturbations in the second-order action $S_{(2)}$, yielding the constraint equation $\beta_1^i = 0$. The remaining metric perturbations can be expanded in powers of Q ,

$$\begin{aligned} N &= 1 + \alpha_1 + \alpha_2 + \dots, \\ \vartheta &= \vartheta_1 + \vartheta_2 + \dots. \end{aligned} \tag{3.1}$$

In what follows we determine these metric fields and use them to obtain the two- and three-point functions for the Q^I .

3.1 Linear order

At linear order we find

$$S_{(1)} = \int d^4x \left\{ a^3 \left[3M_{\text{pl}}^2 H^2 - \frac{1}{2} \dot{\phi}_I \dot{\phi}^I - V \right] \alpha_1 - \left[\mathcal{D}_t(a^3 \dot{\phi}_I) + a^3 V_{,I} \right] Q^I \right\}, \tag{3.2}$$

where we have integrated by parts and removed total derivatives. The background field equations follow after varying this action with respect to α_1 and Q^I ,

$$3M_{\text{pl}}^2 H^2 = \frac{1}{2} \dot{\phi}_I \dot{\phi}^I + V, \tag{3.3}$$

$$\mathcal{D}_t \dot{\phi}_I + 3H \dot{\phi}_I = -V_{,I}. \tag{3.4}$$

The slow-roll regime in curved field-space was discussed by Sasaki & Stewart [24] and later Nakamura & Stewart [49]. Inflation occurs when $\epsilon \equiv -\dot{H}/H^2 < 1$. For a successful phenomenology we also require that inflation is sufficiently prolonged. Therefore ϵ should not

change significantly in a Hubble time. To satisfy these requirements we must choose

$$\epsilon = \frac{1}{2M_{\text{pl}}^2} \frac{\dot{\phi}^I \dot{\phi}_I}{H^2} \ll 1 \quad \text{and} \quad \eta \equiv \frac{1}{H} \frac{d \ln \epsilon}{dt} = \frac{2}{H} \frac{\dot{\phi}^I \mathcal{D}_t \dot{\phi}_I}{\dot{\phi}^J \dot{\phi}_J} + 2\epsilon \ll 1. \quad (3.5)$$

The condition $\eta \ll 1$ requires the tangential component of the acceleration vector $H^{-1} \mathcal{D}_t \dot{\phi}^I$ (measured in Hubble units) to be much smaller than the tangent vector to the trajectory.

One can verify that the slow-roll equation

$$3H\dot{\phi}_I + V_{,I} \approx 0 \quad (3.6)$$

gives a self-consistent realization of these conditions if the potential is sufficiently flat. Detailed conditions are given in Refs. [24, 49]. However, global flatness of the potential is not necessary. This possibility has received recent attention [51–54]. In this paper we do not assume any particular properties of the background theory, except that it realizes an era of inflation with $\epsilon \ll 1$ and in which H varies smoothly during horizon exit. Such scenarios are not the only cases of interest, but a dedicated analysis is required where the background evolution exhibits a feature [34, 55].

When we compute the two- and three-point functions we will do so only for field-space directions which are light during horizon exit. Therefore our results will not apply to any heavy directions generated by a steep potential orthogonal to the inflationary trajectory. (However, when estimating the magnitude of terms in the second- and third-order actions, we quote powers of $\dot{\phi}^I/H$ to emphasize that individual components of this vector are not necessarily of order $\epsilon^{1/2}$.) This is sufficient to estimate the statistics of the primordial curvature perturbation in an approximation where the fluctuations in massive directions at horizon exit are neglected. In simple models this is acceptable because large masses rapidly drive any fluctuations to extinction. In more complex models it has been suggested that modest corrections can occur where the phase-space flow drives power from massive modes into the curvature perturbation before decay [56–58]. To capture these effects would require an extension of the formalism of §§3.2–3.3 used to compute initial conditions, although we expect that the subsequent transfer of power would be correctly described by the method of §4.

3.2 Second order

Expanding the action to second order, performing multiple partial integrations and removing total derivatives, we find

$$\begin{aligned} S_{(2)} = \frac{1}{2} \int d^4x a^3 \left\{ \alpha_1 \left[-6M_{\text{pl}}^2 H^2 \alpha_1 + \dot{\phi}_I \dot{\phi}^I \alpha_1 - 2\dot{\phi}_I \mathcal{D}_t Q^I - 2V_{,I} Q^I \right] \right. \\ \left. - \frac{2}{a^2} \partial^2 \vartheta_1 \left[2M_{\text{pl}}^2 H \alpha_1 - \dot{\phi}_I Q^I \right] \right. \\ \left. + R_{KIJL} \dot{\phi}^K \dot{\phi}^L Q^I Q^J + \mathcal{D}_t Q_I \mathcal{D}_t Q^I - h^{ij} \partial_i Q_I \partial_j Q^I - V_{,IJ} Q^I Q^J \right\}. \end{aligned} \quad (3.7)$$

The momentum and energy constraints can be obtained by varying the action with respect to α_1 and ϑ_1 . We find

$$2M_{\text{pl}}^2 H \alpha_1 = \dot{\phi}_I Q^I, \quad (3.8)$$

$$-2M_{\text{pl}}^2 \frac{H}{a^2} \partial^2 \vartheta_1 = 6M_{\text{pl}}^2 H^2 \alpha_1 - \alpha_1 \dot{\phi}_I \dot{\phi}^I + \dot{\phi}_I \mathcal{D}_t Q^I + V_{,I} Q^I. \quad (3.9)$$

In writing Eqs. (3.7) and (3.9) we have used the spatial Laplacian $\partial^2 \equiv \partial_i \partial_i$, with the convention that spatial indices which are both written in the covariant position are summed using the flat Euclidean metric δ^{ij} . Employing Eqs. (3.8) and (3.9) we can eliminate the metric perturbations in $S_{(2)}$ to obtain

$$S_{(2)} = \frac{1}{2} \int d^4x a^3 \left\{ \mathcal{D}_t Q_I \mathcal{D}_t Q^I - h^{ij} \partial_i Q_I \partial_j Q^I - \mathcal{M}_{IJ} Q^I Q^J \right\}, \quad (3.10)$$

where the symmetric mass matrix \mathcal{M}_{IJ} satisfies

$$\mathcal{M}_{IJ} = V_{,IJ} - R_{LIJM} \dot{\phi}^L \dot{\phi}^M - \frac{1}{M_{\text{pl}}^2 a^3} \mathcal{D}_t \left(\frac{a^3}{H} \dot{\phi}_I \dot{\phi}_J \right). \quad (3.11)$$

This is identical to the canonical case except for the covariant derivatives and the term involving the Riemann tensor, which was first obtained by Sasaki & Stewart [24]. (See also Nakamura & Stewart [49] and Gong & Stewart [59, 60].) We will find similar terms in the third-order action (3.14) below. Their meaning is not immediately clear because they imply that promotion to curved field-space requires more than ‘minimal coupling’ to curvature. In the mass matrix, the Riemann term changes the effective mass of modes orthogonal to $\dot{\phi}$ but also alters the coupling of these modes to each other. We discuss these effects in more detail in §4.

Power spectrum at horizon-crossing. To compute the power spectrum and all higher n -point functions we must use the ‘in-in’ formulation of quantum field theory, which entails doubling all field degrees of freedom. For details, we refer to the literature [4, 46, 61, 62]. In curved field-space an extra complication is caused by the necessity to give each n -point function the correct tensorial transformation properties. As we describe in §3.3 below, this is enforced by transport of the tangent-space basis along the inflationary trajectory.

The power spectrum at lowest order in \mathcal{M}_{IJ} was calculated by Sasaki & Stewart [24] and can be obtained from (A.10) or (A.11). Taking the equal-time limit in either equation, it follows that the two-point function evaluated a little later than horizon-exit satisfies

$$\langle Q^I(\mathbf{k}_1) Q^J(\mathbf{k}_2) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \frac{H^2}{2k^3} G^{IJ}. \quad (3.12)$$

This estimate becomes valid when the decaying power-law terms in (A.10) and (A.11) have become negligible [63]. However, because Q^I is not infrared safe there are growing terms at subleading order [48, 49]. Therefore (3.12) soon becomes untrustworthy, and its validity extends only for a very narrow range of e-folds. This is the problem of enhanced subleading terms discussed in §2.2. In Eq. (4.16) we use the separate universe method to give a more accurate expression which remains valid until late times.

3.3 Third order

The third order action is

$$\begin{aligned}
S_{(3)} = \frac{1}{2} \int d^4x a^3 \left\{ 6M_{\text{pl}}^2 H^2 \alpha_1^3 + 4M_{\text{pl}}^2 \frac{H}{a^2} \alpha_1^2 \partial^2 \vartheta_1 - \frac{M_{\text{pl}}^2 \alpha_1}{a^4} \left(\partial_i \partial_j \vartheta_1 \partial_i \partial_j \vartheta_1 - \partial^2 \vartheta_1 \partial^2 \vartheta_1 \right) \right. \\
- \dot{\phi}_I \dot{\phi}^I \alpha_1^3 + 2\alpha_1^2 \dot{\phi}_I \mathcal{D}_t Q^I + \frac{2}{a^2} \alpha_1 \dot{\phi}_I \partial_i \vartheta_1 \partial_i Q^I - \alpha_1 R^{L(IJ)M} \dot{\phi}_L \dot{\phi}_M Q_I Q_J \\
- \alpha_1 \left(\mathcal{D}_t Q_I \mathcal{D}_t Q^I + \frac{1}{a^2} \partial_i Q_I \partial_i Q^I \right) - \frac{2}{a^2} \partial_i \vartheta_1 \mathcal{D}_t Q_I \partial_i Q^I + \frac{4}{3} R_{I(JK)L} \dot{\phi}^L \mathcal{D}_t Q^I Q^J Q^K \\
\left. + \frac{1}{3} R_{(I|LM|J;K)} \dot{\phi}^L \dot{\phi}^M Q^I Q^J Q^K - \frac{1}{3} V_{;(IJK)} Q^I Q^J Q^K - V_{;(IJ)} \alpha_1 Q^I Q^J \right\}. \tag{3.13}
\end{aligned}$$

We have indicated the symmetric index combinations picked out by each product of the Q^I .³ The lapse and shift can be eliminated using (3.8)–(3.9). The resulting expression is exact and does not invoke an expansion in powers of slow-roll. Nevertheless, the argument of §2.2 implies that we need only compute lowest-order contributions provided we evaluate the three-point function near horizon-crossing. In this region infrared divergences cannot enhance subleading terms. Focusing only on the low-order contributions and rewriting in terms of conformal time, we find [19]

$$\begin{aligned}
S_{(3)} \supseteq \int d^3x d\eta \left\{ - \frac{a^2}{4M_{\text{pl}}^2 H} \dot{\phi}^I Q_I \mathcal{D}_\eta Q^J \mathcal{D}_\eta Q_J - \frac{a^2}{4M_{\text{pl}}^2 H} \dot{\phi}^I Q_I \partial_i Q^J \partial_i Q_J \right. \\
+ \frac{a^2}{2M_{\text{pl}}^2 H} \dot{\phi}^I \partial_i \partial^{-2} \mathcal{D}_\eta Q_I \partial_i Q_J \mathcal{D}_\eta Q^J \\
\left. + \frac{2a^3}{3} R_{I(JK)L} \dot{\phi}^L \mathcal{D}_\eta Q^I Q^J Q^K + \frac{a^4}{6} R_{(I|LM|J;K)} \dot{\phi}^L \dot{\phi}^M Q^I Q^J Q^K \right\}. \tag{3.14}
\end{aligned}$$

The first two lines of (3.14) are a covariantization of the action obtained in flat field-space [1]. We will sometimes describe them as the ‘canonical’ terms. Subleading corrections begin at order $\mathcal{O}(\dot{\phi}/H)^3$ and are negligible unless enhanced by divergences. The third line includes terms containing the Riemann tensor, analogous to the Riemann term in the mass matrix (3.11). These were first obtained by Gong & Tanaka [19] and are a new feature associated with the curvature of field space. Unlike the lowest-order ‘canonical’ terms they produce divergences. To track the influence of these as clearly as possible we have retained Riemann terms up to $\mathcal{O}(\dot{\phi}/H)^2$.

Although operators involving the curvature at $\mathcal{O}(\dot{\phi}/H)^2$ are subleading, we can reliably compute contributions to the three-point function at this level because Eq. (3.11) shows that next-order corrections from the propagator are themselves $\mathcal{O}(\dot{\phi}/H)^2$, and therefore enter the three-point function only at $\mathcal{O}(\dot{\phi}/H)^3$. The same is true for corrections from the scale factor and Hubble rate. After expanding around the horizon-crossing time for a fiducial scale k_* , the explicit $\mathcal{O}(\dot{\phi}/H)^2$ term in (3.14) is accompanied by one further contribution at the same order from the time-dependence of $R_{I(JK)L}$. [See Eqs. (A.15) and (A.22).] We will retain both these terms when estimating the three-point function. The advantage of doing so is

³Our symmetrization conventions are $2A_{(IJ)} = A_{IJ} + A_{JI}$ and $6A_{(IJK)} = A_{IJK} + \{5 \text{ perms}\}$. Bars delimit indices excluded from symmetrization.

that we can perform a more stringent test of our matching to the superhorizon evolution in §4.

Three-point function. In Appendix A we calculate the contribution to the three-point function from each of these operators. As explained in §3.1, our computation applies only to light field-space directions for which \mathcal{M}_{IJ} can be neglected.

To express the result it will be necessary to perform parallel transport along the inflationary trajectory. This can be accomplished using the parallel propagator,⁴

$$\Pi^I{}_i = \text{P exp} \left(- \int_{\sigma}^{\tau} d\eta \Gamma^{I' M' N'} \frac{d\phi^{M'}}{d\eta} \right) G^{n_i}, \quad (3.15)$$

where the integral is computed along the inflationary trajectory between conformal times σ and τ . The symbol ‘P’ denotes path ordering along this trajectory. (More details and sample calculations are given in Appendix A.) We refer to $\Pi^I{}_i$ as the *trajectory propagator*. In writing (3.15) we have adopted a notation in which the index I is associated with a basis for the tangent space at time τ , whereas the index i is associated with an independent basis for the tangent space at σ . Primed indices label the tangent space at a time corresponding to the integration variable η . The rightmost N' index lies at time σ and contracts with G^{n_i} . Therefore $\Pi^I{}_i$ is a *bitensor*: it is an object with mixed indices [31], transforming like a contravariant vector at τ and a covariant vector at σ . In what follows we set σ to be the horizon-crossing time for the reference scale k_* .

When computing n -point functions we typically measure time in e-folds of expansion, evaluating each n -point function at $N = -\ln |k_*\tau|$ e-folds since the fiducial scale k_* passed outside the horizon. With these conventions we find

$$\langle Q^I(\mathbf{k}_1) Q^J(\mathbf{k}_2) Q^K(\mathbf{k}_3) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{H_*^4}{4 \prod_i k_i^3} \Pi^I{}_i \Pi^J{}_j \Pi^K{}_k A^{ijk}(N). \quad (3.16)$$

The bitensor $A^{ijk}(N)$ transforms as a scalar at time N and a rank-three tensor at the time of horizon-crossing for k_* . However, the three-point function $\langle Q^I Q^J Q^K \rangle$ transforms as a rank-three tensor at N . To match these transformation properties, the propagators $\Pi^I{}_i$, $\Pi^J{}_j$ and $\Pi^K{}_k$ are necessary. In Appendix A we explain how they arise in terms of Feynman diagrams. We set $k_t = k_1 + k_2 + k_3$ to be the total scalar momentum and define $\kappa^2 = \sum_{i < j} k_i k_j$. Under

⁴In (2.5), to match the notation of Ref. [31], we denoted the parallel propagator evaluated along a geodesic as $G_{I'}{}^I$. We reserve the symbol Π to indicate parallel propagation along an inflationary trajectory.

the same conditions which were required for validity of (3.12), we find

$$\begin{aligned}
A^{ijk}(N) = & \frac{1}{M_{\text{pl}}^2} \frac{\dot{\phi}_*^i}{H_*} G_*^{jk} \left(-2 \frac{k_2^2 k_3^2}{k_t} + \frac{k_1}{2} \mathbf{k}_2 \cdot \mathbf{k}_3 \right) \\
& + \frac{4}{3} R_*^{i(jk)m} \frac{\dot{\phi}_m^*}{H_*} \left[k_1^3 \left(\gamma_E - N + \ln \frac{k_t}{k_*} \right) - k_1^2 k_t + \frac{k_1^2 k_2 k_3}{k_t} \right] \\
& + \frac{1}{3} R_*^{i(mn|j;k)} \frac{\dot{\phi}_m^*}{H_*} \frac{\dot{\phi}_n^*}{H_*} \left[k_1^3 \left(N - \ln \frac{k_t}{k_*} - \gamma_E - \frac{1}{3} \right) + \frac{4}{9} k_t^3 - k_t \kappa^2 \right] \\
& - \frac{4}{3} R_*^{i(jk)m;n} \frac{\dot{\phi}_m^*}{H_*} \frac{\dot{\phi}_n^*}{H_*} \left[\frac{k_1^3}{2} \left(N^2 - \gamma_E^2 + \frac{\pi^2}{12} - \left[2\gamma_E + \ln \frac{k_t}{k_*} \right] \ln \frac{k_t}{k_*} \right) \right. \\
& \quad \left. + k_1^2 k_t \left(\ln \frac{k_t}{k_*} + \gamma_E - 1 \right) - \frac{k_1^2 k_2 k_3}{k_t} \left(\gamma_E + \ln \frac{k_t}{k_*} \right) \right] \\
& + \text{cyclic},
\end{aligned} \tag{3.17}$$

where $\gamma_E \approx 0.577$ is the Euler–Mascheroni constant. The first line of (3.17) covariantizes Eq. (69) of Ref. [1], and subsequent lines arise from the Riemann terms in (3.14). The sum over cyclic permutations includes the two permutations generated by simultaneous exchange of $\{i, \mathbf{k}_1\}$ with either $\{j, \mathbf{k}_2\}$ or $\{k, \mathbf{k}_3\}$.

4 Evolution after horizon exit

Terms in (3.17) involving N are divergent in the late-time limit $\tau \rightarrow 0$ and are responsible for spoiling infrared safety, as described in §2.2. They generate time evolution after horizon exit [43–45] and rapidly invalidate the expressions derived in §3. In this section we show that the divergent Riemann curvature terms in (3.17) have a geometrical origin and explain how their contribution can be taken into account using a covariant version of the ‘separate universe’ method [8, 23]. The terms diverging like a single power of N are

$$A_{1\text{-log}}^{ijk} = \frac{1}{3} N k_1^3 \left(R^{i(mn|j;k)} \frac{\dot{\phi}_m}{H} \frac{\dot{\phi}_n}{H} - 4 R^{i(jk)m} \frac{\dot{\phi}_m}{H} \right)_* + \text{cyclic}. \tag{4.1}$$

Eq. (3.17) also contains an explicit double logarithm (a term proportional to N^2)

$$A_{2\text{-log}}^{ijk} = -\frac{4}{6} N^2 k_1^3 \left(R^{i(jk)m;n} \frac{\dot{\phi}_m}{H} \frac{\dot{\phi}_n}{H} \right)_* + \text{cyclic}. \tag{4.2}$$

In addition, it implicitly contains terms of all powers in N from higher-order corrections we have not evaluated.

Separate universe approach. Evolutionary effects on superhorizon scales can be understood using causality and classicality. After smoothing over small-scale structure, widely separated regions locally evolve like an unperturbed or ‘separate’ Friedmann universe [20–22]. Lyth & Rodríguez [23] extended this method to n -point functions for $n \geq 3$. In their formulation, the principal tool was a Taylor expansion of the background solutions in small deviations from a chosen initial condition.

In Ref. [8], the separate universe approach was applied without directly invoking this Taylor expansion. The inflationary trajectories in phase space are interpreted as the integral curves of a flow, and can converge or disperse. In Ref. [8] it was shown that the Taylor coefficients used by Lyth & Rodríguez can be understood as (derivatives of) the Jacobi fields which describe this dispersion. The growth and decay of fluctuations, and the processes by which power is transferred between modes, can be understood as the local dilation, shear and twist of a narrow bundle of trajectories.

4.1 Jacobi equation for separate universes

The Jacobi method provides a simple way to implement the separate universe approach in curved field-space. Consider two separate universes, with slightly displaced initial conditions, which correspond to neighbouring trajectories on phase space. The displacement between these trajectories can be described covariantly using a tangent-space vector Q^I (the ‘connecting vector’) as described in §2.1.

Deviation equation. Each universe evolves according to the field equation

$$\frac{1}{3}\mathcal{D}_N^2\varphi^I + \left(1 - \frac{\epsilon}{3}\right)\mathcal{D}_N\varphi^I = u^I, \quad (4.3)$$

where $u_I = -V_{,I}/3H^2$. Under the slow-roll approximation the acceleration term $\mathcal{D}_N^2\varphi^I$ is negligible along each trajectory. In flat field-space this means that the change in acceleration term between neighbouring trajectories also contributes at higher-order in slow-roll. On curved field-space this is no longer true because derivatives do not commute. Therefore we must retain the acceleration term when studying how trajectories disperse.

The evolution of Q^I can be determined by making a Taylor expansion of (4.3) along a geodesic connecting the adjacent trajectories, as in §2.1. To describe evolution of the two- and three-point functions we require this expansion up to second-order. Dropping the explicit $O(\epsilon)$ term, which can contribute only at higher order in the slow-roll expansion, and discarding a common factor of the parallel propagator we find

$$\left(\mathcal{D}_\lambda + \frac{1}{2}\mathcal{D}_\lambda^2\right)\left(\frac{1}{3}\mathcal{D}_N^2\varphi^I + \mathcal{D}_N\varphi^I\right) = u^I{}_{;J}Q^J + \frac{1}{2}u^I{}_{;JK}Q^JQ^K. \quad (4.4)$$

Using the Ricci identity to commute \mathcal{D}_λ with \mathcal{D}_N and employing the Bianchi identities to symmetrize resulting curvature terms, we conclude that Q^I evolves according to the Jacobi or ‘deviation’ equation

$$\mathcal{D}_N Q^I = w^I{}_J Q^J + \frac{1}{2}w^I{}_{(JK)}Q^JQ^K + \dots, \quad (4.5)$$

where the coefficients $w^I{}_J$ and $w^I{}_{(JK)}$ satisfy

$$w_{IJ} = u_{(I;J)} + \frac{1}{3}R_{L(IJ)M}\frac{\dot{\phi}^L}{H}\frac{\dot{\phi}^M}{H}, \quad (4.6)$$

$$w_{I(JK)} = u_{(I;JK)} + \frac{1}{3}\left(R_{(I|LM|J;K)}\frac{\dot{\phi}^L}{H}\frac{\dot{\phi}^M}{H} - 4R_{I(JK)L}\frac{\dot{\phi}^L}{H}\right). \quad (4.7)$$

To obtain (4.6)–(4.7) we have imposed the slow-roll approximation to determine $\mathcal{D}_N^2 Q^I$ in terms of $\mathcal{D}_N Q^I$. As usual, the background trajectory is denoted $\phi^I(t)$. All curvature quantities and derivatives of u^I are evaluated on this trajectory and therefore powers of slow-roll can be counted in the usual way. Because we have used the slow-roll approximation, Eqs. (4.6)–(4.7) are trustworthy only to lowest slow-roll order in the derivatives of u^I , and to $\mathcal{O}(\dot{\phi}/H)^2$ in terms multiplying the Riemann tensor and its derivatives. This accuracy is sufficient to make a comparison with the divergent terms retained in §3.3.

Although both terms in (4.6) are automatically symmetric under exchange of IJ we have indicated this explicitly. However, w_{IJK} is symmetric only on JK . This is different to the case of flat field-space, where terms involving the Riemann tensor are absent and each coefficient on the right-hand side of the Jacobi equation is always a symmetric combination of partial derivatives. When writing $w^I_{(JK)}$ we add redundant brackets to emphasize this symmetry.

Time-evolution operators. The Jacobi equation (4.5) is a first-order differential equation, and therefore its solution can be expanded in powers of the initial conditions Q_*^m ,⁵

$$Q^I = T^I_m Q_*^m + \frac{1}{2} T^I_{(mn)} Q_*^m Q_*^n + \dots \quad (4.8)$$

To write (4.8) we have used the index convention introduced in §3.3. The fluctuation Q^I is evaluated at some late time N , and its index I transforms as a contravariant vector in the tangent space at this time. Conversely, Q_*^m and its index m transform as a contravariant vector at an earlier time N_* . Like the trajectory propagator (3.15), the coefficients T^I_m and $T^I_{(mn)}$ are bitensors. In particular, T^I_m transforms like a contravariant vector on I and a covariant vector on m , whereas $T^I_{(mn)}$ transforms like a contravariant vector in the tangent space at N and a covariant rank-two tensor in the tangent space at N_* . The initial conditions require $T^I_m = \delta_m^I$ and $T^I_{(mn)} = 0$ when $N = N_*$.

Eq. (4.8) is a solution to the Jacobi equation (4.5) provided the T coefficients satisfy

$$\mathcal{D}_N T^I_m = w^I_J T^J_m, \quad (4.9)$$

$$\mathcal{D}_N T^I_{(mn)} = w^I_J T^J_{(mn)} + w^I_{(JK)} T^J_m T^K_n. \quad (4.10)$$

(Recall that N in the derivative \mathcal{D}_N is not a field-space index, but the number of e-folds.) Eq. (4.8) can be summarized as the Taylor expansion of Q^I in terms of its value at some earlier time N_* , and defines a ‘separate universe’ approximation for curved field-space. We describe the coefficients $T^I_{(m\dots n)}$ collectively as ‘time-evolution operators’. They are covariant analogues of the coefficients $\partial\phi^I/\partial\phi_*^m$ (and its higher derivatives) which occur when applying the separate-universe method in flat field-space [65, 66]. We could obtain them by solving φ^I and using (2.5) to compute its derivatives with respect to the initial conditions,⁶ but in practice it is much easier to integrate (4.9)–(4.10) directly.

Divergences. We now show that (4.5)–(4.7) reproduce the divergences of (3.17). The argument is similar to that of Zaldarriaga [43]. Solving Eqs. (4.9)–(4.10) perturbatively

⁵The quantities T^I_m and T^I_{mn} were written Γ^I_m and Γ^I_{mn} in Refs. [8, 64]. In this paper we reserve Γ to mean the Levi-Civita connection compatible with the field-space metric G^{IJ} .

⁶To reproduce all information in (4.6)–(4.7) it would be necessary to retain the connection term from $\mathcal{D}_t^2\varphi^I$.

yields a power series in N . The lowest-order terms are

$$T^I_m = \Pi^I_m + \Pi^I_i \left[w^i_m \right]_* N + \frac{1}{2} \Pi^I_i \left[w^i_k w^k_m + \mathcal{D}_N w^i_m \right]_* N^2 + \dots, \quad (4.11)$$

$$T^I_{(mn)} = \Pi^I_i \left[w^i_{(mn)} \right]_* N + \frac{1}{2} \Pi^I_i \left[\mathcal{D}_N w^i_{(mn)} + w^i_k w^k_{(mn)} + w^i_{(mk)} w^k_n + w^i_{(nk)} w^k_m \right]_* N^2 + \dots, \quad (4.12)$$

where $N = -\ln |k_* \tau|$ as above.

Eq. (4.11) shows that the time-evolution operator T^I_m can be understood as a modification of the trajectory propagator to include the effect of time-dependence along the inflationary trajectory in addition to parallel transport. This follows because the trajectory propagator Π^I_m satisfies (4.9) with $w^I_J = 0$.

At linear order in N , the two- and three-point functions following from (4.11)–(4.12) are

$$\langle Q^I(\mathbf{k}_1) Q^J(\mathbf{k}_2) \rangle \supseteq (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \frac{N H_*^2}{k_*^3} \Pi^I_i \Pi^J_j w_*^{ij}, \quad (4.13)$$

$$\langle Q^I(\mathbf{k}_1) Q^J(\mathbf{k}_2) Q^K(\mathbf{k}_3) \rangle \supseteq (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{N H_*^4}{4 \prod_i k_i^3} \Pi^I_i \Pi^J_j \Pi^K_k w_*^{i(jk)} k_1^3 + \text{cyclic}. \quad (4.14)$$

The symmetry properties of w^{ij} and $w^{i(jk)}$ ensure that these expressions are symmetric under simultaneous permutations of the indices I, J, K and their associated momenta $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$. At quadratic order in N and lowest order in slow-roll there is a contribution only from the $\mathcal{D}_N w^i_{(mn)}$ term in (4.12). This gives

$$\begin{aligned} \langle Q^I(\mathbf{k}_1) Q^J(\mathbf{k}_2) Q^K(\mathbf{k}_3) \rangle &\supseteq (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{N^2 H_*^4}{4 \prod_i k_i^3} \\ &\times \Pi^I_i \Pi^J_j \Pi^K_k \left(-\frac{4}{6} R^{i(jk)m;n} \frac{\dot{\phi}_m \dot{\phi}_n}{H H} \right)_* k_1^3 + \text{cyclic}. \end{aligned} \quad (4.15)$$

It can be checked that (4.13) reproduces the divergence in the two-point function (including the term involving the Riemann tensor) found by Nakamura & Stewart [49].⁷ Comparing (4.14) and (4.7), it can also be checked that the terms in $w^{i(jk)}$ involving the Riemann tensor reproduce the divergence in (4.1). It was to enable a nontrivial check of this matching that we elected to keep divergences up to $\mathcal{O}(\dot{\phi}/H)^2$ in the Riemann-tensor terms of (3.14). Finally, comparing (4.15) and (4.2) it can be checked that the lowest-order double-logarithmic divergence is also correctly reproduced. At the accuracy of our present calculation it is not possible to check whether the divergences proportional to $u_{(i;jk)}$ also agree. These are a covariantized version of the divergences which appear in flat field-space. Since the same is true for the ‘canonical’ operators in the Lagrangian (3.13) we should expect agreement.

In principle, higher-order terms in N and $\dot{\phi}/H$ could be retained in the perturbative expansions (4.11)–(4.12), which would enable a check of matching at all orders. Although such a check would be interesting, the matching at single- and double-logarithm order provides no reason to believe it would fail. We will not attempt it in this paper.

⁷In Refs. [49, 59, 60] the factors of Π^I_i were omitted.

4.2 Transport equations

The time-evolution operators enable us to determine each n -point function after horizon exit. Translating the formulae of Lyth & Rodríguez, we obtain

$$\langle Q^I(\mathbf{k}_1)Q^J(\mathbf{k}_2) \rangle = T^I{}_m T^J{}_n \langle Q^m(\mathbf{k}_1)Q^n(\mathbf{k}_2) \rangle_* \quad (4.16)$$

and

$$\begin{aligned} \langle Q^I(\mathbf{k}_1)Q^J(\mathbf{k}_2)Q^K(\mathbf{k}_3) \rangle &= T^I{}_l T^J{}_m T^K{}_n \langle Q^l(\mathbf{k}_1)Q^m(\mathbf{k}_2)Q^n(\mathbf{k}_3) \rangle_* \\ &+ T^I{}_{(lm)} T^K{}_r T^J{}_s \int \frac{d^3q}{(2\pi)^3} \langle Q^l(\mathbf{k}_1 - \mathbf{q})Q^r(\mathbf{k}_2) \rangle_* \langle Q^m(\mathbf{q})Q^s(\mathbf{k}_3) \rangle_* + \text{cyclic}, \end{aligned} \quad (4.17)$$

where ‘cyclic’ denotes the two permutations of the second line in (4.17) obtained by exchanging $\{I, \mathbf{k}_1\}$ with $\{J, \mathbf{k}_2\}$ or $\{K, \mathbf{k}_3\}$. When there is no time evolution and $w^I{}_J = 0$, Eq. (4.17) reproduces (3.16). In combination, Eqs. (3.17), (4.6)–(4.7), (4.9)–(4.10) and (4.16)–(4.17) constitute the principal results of this paper. For comparison with observation it only remains to make a gauge transformation from Q^I to the curvature perturbation ζ , for which we supply the relevant formulae in §4.3.

Up to this point we have worked in a holonomic frame derived from the field-space coordinates, but other possibilities exist. Since an n -point function of the Q^m evaluated at N_* transforms as a rank- n tensor in the tangent-space at time N_* , Eqs. (4.16)–(4.17) are manifestly covariant. As a result, we are free to select a basis for the tangent space independently at the early and late times N_* and N .

Equations for two- and three-point functions. The approach given above is simple and emphasizes its similarity with familiar δN methods, but it is also possible to write evolution equations for the n -point functions directly. In Ref. [8] these were described as *transport equations*.

We write the two-point function as

$$\langle Q^I(\mathbf{k}_1)Q^J(\mathbf{k}_2) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \frac{\Sigma^{IJ}}{2k^3}, \quad (4.18)$$

where Σ^{IJ} is symmetric. The amplitude of the local mode of the three-point function can be parametrized

$$\langle Q^I(\mathbf{k}_1)Q^J(\mathbf{k}_2)Q^K(\mathbf{k}_3) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \left[\frac{\alpha^{I(JK)}}{k_2^3 k_3^3} + \frac{\alpha^{J(IK)}}{k_1^3 k_3^3} + \frac{\alpha^{K(IJ)}}{k_1^3 k_2^3} \right]. \quad (4.19)$$

Direct differentiation followed by use of (4.5) yields the equations

$$\mathcal{D}_N \Sigma^{IJ} = w^I{}_L \Sigma^{LJ} + w^J{}_L \Sigma^{LI} + \dots, \quad (4.20)$$

$$\mathcal{D}_N \alpha^{I(JK)} = w^I{}_L \alpha^{L(JK)} + w^J{}_L \alpha^{I(LK)} + w^K{}_L \alpha^{I(JL)} + w^I{}_{(LM)} \Sigma^{LJ} \Sigma^{MK} + \dots, \quad (4.21)$$

where the omitted terms involve higher-order correlation functions and are negligible in typical inflationary theories. Following the method described in Ref. [8] it can be verified that (4.20)–(4.21) reproduce Eqs. (4.16)–(4.17).

4.2.1 Interpretation of Riemann terms

It is now possible to understand the significance of the interactions in (3.17) mediated by the Riemann curvature, and the infrared divergences to which they give rise. They correspond directly to those terms in the separate-universe Jacobi equation (4.5) which measure deviation between nearby trajectories. The derivation of §4.1 makes clear that this effect is entirely analogous to geodesic deviation between freely-falling observers in curved spacetime.

These new sources of time dependence arise mathematically from tidal effects in field space. Their physical meaning can be understood as follows. An initial perturbation Q_*^m generically represents a mix of adiabatic and isocurvature fluctuations. The isocurvature fluctuations differentiate between ‘separate universes’, and correspond to a choice of inflationary trajectory measured from the fiducial trajectory at $Q^I = 0$. As these trajectories flow over field-space the proper distance between them will vary depending on the metric G_{IJ} . Each trajectory evolves independently in the absence of gradient terms which couple spatially separated regions. Therefore the isocurvature component of Q^I represents a displacement to a fixed trajectory and must respond to this variation.

These effects generate new sources of dilation and shear which contribute to the redistribution of power between adjacent trajectories. In addition, the *nonlinear* terms in $w^I_{(JK)}$ generate new sources of non-Gaussianity. The precise form of these nonlinear terms is unfamiliar only because they are irrelevant in familiar applications of geodesic deviation—such as the focusing and defocusing of a bundle of light rays, where the connecting vector can be taken to be infinitesimal.

The Riemann tensor is antisymmetric on its first and second pairs of indices. Since $\dot{\phi}^I$ is proportional to the tangent to the curve, we conclude that the Riemann contribution to w^I_J is zero when either index is aligned with the adiabatic direction. Gong & Tanaka emphasized that this leads only to new couplings between isocurvature modes [19]. Eq. (4.20) shows that these couplings influence how the isocurvature modes share power between themselves, but do not cause power to flow between the isocurvature and adiabatic directions. Such a flow must be mediated by the potential through $u_{(I;J)}$.

In the special case where the trajectory is an exact geodesic its tangent vector is parallel-transported proportional to itself. In this case, the adiabatic mode decouples completely and no power flows to or from it.

4.3 Gauge transformation to the curvature perturbation

For comparison with microwave background experiments or galaxy surveys we must compute the n -point functions of the primordial curvature perturbation, ζ . This is a measure of the local excess expansion between a spatially flat hypersurface and a uniform density hypersurface with which it coincides on average. In curved field-space the computation can be performed economically using the method of Ref. [64]. We expand N as a function of the density ρ . Taking $\Delta\rho$ to be the displacement from a point of fixed density ρ_c to an arbitrary initial location, we find

$$\Delta N = \frac{dN}{d\rho}\Delta\rho + \frac{1}{2}\frac{d^2N}{d\rho^2}(\Delta\rho)^2 + \dots \quad (4.22)$$

To determine the variation of (4.22) under a change in the initial location we expand along a geodesic, as in §2.1, along which both $\Delta\rho$ and the differential coefficients will vary. The

variation of $\Delta\rho$ satisfies

$$\delta(\Delta\rho) = -V_{;I}Q^I - \frac{1}{2}V_{;IJ}Q^IQ^J + \dots \quad (4.23)$$

Therefore, up to second order, we can express ζ as

$$\zeta = \delta(\Delta N) = N_I Q^I + \frac{1}{2}N_{IJ}Q^IQ^J + \dots \quad (4.24)$$

The coefficients N_I and N_{IJ} satisfy

$$N_I = -\frac{dN}{d\rho}V_{;I}, \quad (4.25)$$

$$N_{IJ} = -\frac{dN}{d\rho}V_{;IJ} + \frac{d^2N}{d\rho^2}V_{;I}V_{;J} + \frac{1}{M_{\text{pl}}^2}(A_{I;J} + A_{J;I}), \quad (4.26)$$

where

$$A_I = \frac{V_{;I}}{V_{;J}V_{;J}} - \frac{2V}{(V_{;J}V_{;J})^2}V^{;K}V_{;IK}, \quad (4.27)$$

$$\frac{dN}{d\rho} = -\frac{1}{M_{\text{pl}}^2}\frac{V}{V_{;I}V_{;I}}, \quad (4.28)$$

$$\frac{d^2N}{d\rho^2} = -\frac{1}{M_{\text{pl}}^2}\frac{1}{V_{;I}V_{;I}} + \frac{2}{M_{\text{pl}}^2}\frac{V}{(V_{;I}V_{;I})^3}V^{;J}V^{;K}V_{;JK}. \quad (4.29)$$

δN coefficients. Eqs. (4.25)–(4.26) are defined at a single point in field space; they are not bilocal in the sense of the coefficients T^I_m and $T^I_{(mn)}$. We can obtain analogues of these bilocal coefficients using (4.8) to relate the Q^I to their values at horizon-crossing. This yields

$$\zeta(N) = N_m Q_*^m + \frac{1}{2}N_{(mn)}Q_*^m Q_*^n + \dots, \quad (4.30)$$

where N_m and $N_{(mn)}$ transform as scalars in the tangent space at N , and (respectively) rank-one and rank-two tensors in the tangent space at N_* . We describe them as ‘ δN coefficients’. They satisfy

$$N_m = N_I T^I_m, \quad (4.31)$$

$$N_{(mn)} = N_I T^I_{(mn)} + N_I N_J T^I_m T^J_n. \quad (4.32)$$

It follows that the two- and three-point functions of ζ are determined by

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2) \rangle = N_m N_n \langle Q^m(\mathbf{k}_1)Q^n(\mathbf{k}_2) \rangle_*, \quad (4.33)$$

and

$$\begin{aligned} \langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle &= N_l N_m N_n \langle Q^l(\mathbf{k}_1)Q^m(\mathbf{k}_2)Q^n(\mathbf{k}_3) \rangle_* \\ &+ N_{(lm)}N_r N_s \int \frac{d^3q}{(2\pi)^3} \langle Q^l(\mathbf{k}_1 - \mathbf{q})Q^r(\mathbf{k}_2) \rangle_* \langle Q^m(\mathbf{q})Q^s(\mathbf{k}_3) \rangle_* + \text{cyclic}, \end{aligned} \quad (4.34)$$

where ‘cyclic’ indicates the usual combination of permutations, as in Eq. (4.17).

Eq. (4.32) implies that $N_{(mn)}$ is the covariant derivative of N_m with respect to a change in the initial conditions. Therefore (4.30) agrees with the covariant δN expansion discussed by Saffin [18]. A similar expansion has already been used by Peterson & Tegmark [17].

Observable quantities. The statistical properties of ζ are typically expressed in terms of its spectrum and bispectrum

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle \equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) P_\zeta(k_1), \quad (4.35)$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3). \quad (4.36)$$

Constraints on the spectrum are expressed in terms of a dimensionless quantity \mathcal{P}_ζ

$$\mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2} P_\zeta(k) = N_m N_n G^{mn}(k) \frac{H(k)^2}{4\pi^2}, \quad (4.37)$$

where the argument k denotes evaluation at the horizon-crossing time for k .

The bispectrum contains considerably more information. Its terms can be divided into two types: first, those which come from interference effects involving decaying modes near horizon exit; and second, those arising from interactions between only growing modes far outside the horizon. The first type can have arbitrary dependence on the k -modes k_1, k_2, k_3 which appear in the three-point correlation function. The second type appear only in the ‘local’ combination $(k_1^3 k_2^3)^{-1}$ or its permutations. With canonical kinetic terms, applying global slow-roll conditions to the potential and assuming that only light fields contribute to ζ , Lyth & Zaballa [67] showed that the result of Ref. [1] implies only the ‘local’ bispectrum shape can be observable.

With a nontrivial field-space metric, interactions involving the Riemann tensor may change this conclusion. Inspection of (3.17) shows that these interactions contribute terms of both types. Depending on the field-space curvature it is possible that ‘nonlocal’ contributions in (3.17) could be enhanced, but to determine whether this happens would require an extension of the analysis in Refs. [67, 68]. We leave this interesting question for future work. On the other hand these terms certainly modify the evolution of the amplitude of each local shape, as discussed in §§4.1–4.2.

Where the Riemann curvature is sufficiently small to make nonlocal contributions negligible, Eq. (4.34) yields an analogue of the familiar ‘ δN ’ formula for the amplitude of the local bispectrum,

$$f_{\text{NL}}^{\text{local}} \approx \frac{5}{6} \frac{N_m N_n N^{(mn)}}{(N_r N_r)^2}, \quad (4.38)$$

where N_m and $N_{(mn)}$ were defined in (4.31)–(4.32). In this case, Eq. (3.17) can be interpreted to mean (as in the case of canonical kinetic terms) that the bispectrum generated at horizon exit is negligible: subsequent time evolution is necessary to generate an observable non-Gaussian signal.

4.3.1 Backwards formalism

To track the evolution of mixed two- and three-point functions for the complete set of fluctuations, including isocurvature modes, it is necessary to solve for all components of T^I_m and $T^I_{(mn)}$. In an \mathcal{N} -field model, the linear coefficient T^I_m has \mathcal{N}^2 independent components and the quadratic coefficient $T^I_{(mn)}$ has $\mathcal{N}^2(\mathcal{N} + 1)/2$ independent components. But for some purposes we may require only the two- and three-point functions for ζ given by (4.33)–(4.34). If so, we can reduce the computational complexity by tracking only the \mathcal{N} components of N_n and the $\mathcal{N}(\mathcal{N} + 1)/2$ components of $N_{(mn)}$.

An evolution equation for N_n was given by Yokoyama, Suyama & Tanaka [69, 70]. Assuming the local shape dominates, only the combination $N_m N_{(mn)} N_n$ is required. Yokoyama, Suyama & Tanaka supplied an integral expression from which this could be computed. In Ref. [8] this was extended to an explicit evolution equation for $N_{(mn)}$. Equivalent expressions were later given by Mazumdar & Wang [71].

Eq. (4.9) expresses the evolution of T^I_m with N . It can be verified that its evolution with N_* satisfies an analogous equation

$$\mathcal{D}_{N_*} T^I_m = -T^I_n w^n_m. \quad (4.39)$$

Here, the covariant derivative applies to tangent-space indices at N_* and therefore operates only on m . The index I labels a tangent-space basis at N and is inert. Using (4.39) together with (4.31)–(4.32) we conclude

$$\mathcal{D}_{N_*} N_n = -N_m w^m_n, \quad (4.40)$$

$$\mathcal{D}_{N_*} N_{(mn)} = -N_{(rn)} w^r_m - N_{(mr)} w^r_n - N_r w^r_{(mn)}. \quad (4.41)$$

Like Eqs. (4.9)–(4.10) these are covariantized versions of the evolution equations in flat field-space, using the correct coefficients w^m_n and $w^m_{(rs)}$ which appear in the Jacobi equation.

Eqs. (4.40)–(4.41) should be solved by fixing N to be the late time of interest. During inflation the initial conditions would correspond to (4.25)–(4.26). One should then integrate backwards until N_* corresponds to the horizon-crossing time for the fiducial scale k_* .

5 Non-minimally coupled models

In a multiple-field model with potential V and non-minimal coupling to the Ricci scalar, the action in the Jordan frame can be written as

$$S = \int d^4x \sqrt{-\hat{g}} \left[\frac{1}{2} M_{\text{pl}}^2 f(\varphi) \hat{R} + \hat{X} - \hat{V}(\varphi) \right], \quad (5.1)$$

where $f(\varphi)$ is positive definite but otherwise arbitrary. We distinguish Jordan frame quantities with a circumflex. The kinetic energy is \hat{X} , assumed to be an arbitrary second-order combination of the field derivatives in the form

$$\hat{X} = -\frac{1}{2} \hat{G}_{IJ} \hat{g}^{\mu\nu} \partial_\mu \varphi^I \partial_\nu \varphi^J, \quad (5.2)$$

where $\hat{G}_{IJ}(\varphi)$ is the Jordan frame field-space metric. It has been shown that this action can be rewritten in the Einstein frame after a conformal transformation [25]

$$g_{\mu\nu} = f \hat{g}_{\mu\nu}. \quad (5.3)$$

The Einstein frame action is

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\text{pl}}^2 R + X - V \right], \quad (5.4)$$

where $V = \hat{V}/f^2$ is the potential and

$$X = -\frac{1}{2} G_{IJ} g^{\mu\nu} \partial_\mu \varphi^I \partial_\nu \varphi^J, \quad G_{IJ} = \frac{1}{f} \hat{G}_{IJ} + \frac{3}{2} M_{\text{pl}}^2 \frac{f_{,I} f_{,J}}{f^2}. \quad (5.5)$$

Cosmological observables themselves are not altered by this procedure [72–75]. However, one must be careful to restrict attention to clearly defined physical quantities; in particular, the curvature perturbation ζ is not itself an observable [76].

The Einstein-frame action (5.4) is of the same form as Eq. (2.1). Therefore the results of §§3–4 are also applicable to models with non-minimal coupling.

6 Conclusions

In this paper we have computed the covariant three-point function near horizon-crossing for a collection of slowly-rolling scalar fields with a nontrivial field-space metric. After making a conformal transformation, this framework is sufficiently general to include scenarios where one or more fields are nonminimally coupled to the Ricci scalar. The subsequent superhorizon evolution can be expressed using a version of the ‘separate universe’ approach.

We concentrate on the broad class of models described by a σ -model Lagrangian $\mathcal{L} = X + V$, where $2X = -G_{IJ}\partial_\mu\varphi^I\partial^\mu\varphi^J$, and obtain expressions for the two- and three-point functions. The presence of a nontrivial field-space metric leads to technical subtleties. First, to obtain a covariant formalism we must be careful to define perturbations as tangent-space vectors Q^I using the method of Gong & Tanaka. When computing correlation functions involving the Q^I , the Feynman diagram expansion introduces explicit factors of the ‘trajectory propagator’ which implements parallel transport along the inflationary trajectory.

Second, new interaction vertices appear which involve explicit factors of the Riemann curvature tensor. Therefore the two- and three-point functions receive modifications of two types. The first follow from promotion of the flat field-space perturbation $\delta\varphi^I$ to Q^I and covariantize the result for $G^{IJ} = \delta^{IJ}$. As in general relativity, covariantization is achieved by exchanging partial derivatives for covariant derivatives and contracting all indices with the field-space metric. The second involve explicit factors of the field-space Riemann curvature. These modify both quantum interference effects operating near horizon exit, and the interactions between growing modes far outside the horizon.

In §4 we developed a covariant version of the ‘separate universe’ formalism to account for these superhorizon-scale interactions. The covariant Jacobi equation (4.5) automatically incorporates curvature contributions which influence the evolution of the two- and three-point function. We have shown that this correctly reproduces the time-dependent growing modes near horizon-crossing generated by the apparatus of quantum field theory. In particular, it matches the two lowest-order divergences at single-logarithm order and the leading divergence at double-logarithm order.

The Jacobi approach leads to covariant ‘time evolution operators’ T^I_m and $T^I_{(mn)}$ which can be obtained straightforwardly by direct integration of (4.9)–(4.10). Together with the covariant gauge transformations derived in §4.3 these yield covariant ‘ δN coefficients’ N_m and $N_{(mn)}$ which define the ‘separate universe’ expansion of the curvature perturbation ζ in Eq. (4.30). This provides a clear and economical framework enabling perturbations to be evolved in a slow-roll inflationary model with nontrivial field-space metric.

We always retain the option to abandon manifest covariance and work with the coordinate variation $\delta\varphi^I$. The traditional separate-universe expansion for $\delta\varphi^I$ is unchanged by the presence of a nontrivial field-space metric, and our predictions for the autocorrelation

functions of ζ cannot vary because ζ is a field-space scalar. The advantage of the covariant formulation is one of convenience and practicality.

The phenomenology of the new Riemann-tensor terms may be interesting. In a canonical scenario, interactions among superhorizon modes are suppressed by three powers of $\dot{\phi}/H$, and are therefore relatively slow. (Note that one should not regard this suppression as an indication of how large the bispectrum can become. Rather, it is an indication of the *timescale* over which it can evolve.) However, Eq. (3.17) shows that curvature-mediated interactions are suppressed by only a single power of $\dot{\phi}/H$. In a model where the field-space curvature is $O(1)$ these could lead to much more rapid evolution. It will be interesting to study these effects in more detail, and we hope to return to this in future work.

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A n -point functions in the in–in formalism

The general theory necessary to compute n -point functions was set out in the papers by Maldacena [33] and Weinberg [46], and has been reviewed elsewhere [4, 61, 62]. In curved field-space this is complicated by the necessity to ensure that all quantities satisfy the correct tensor transformation laws. This means that the structure of the two- and higher n -point functions becomes more elaborate. Special factors, given by line integrals of the connection over field space, are required to ensure their indices reside in the proper tangent space.

In this Appendix we briefly review necessary elements from the general theory, focusing on the changes necessary due to field-space curvature.

A.1 Two-point function

After transforming to conformal time η , defined by $dt = a d\eta$, the in–in generating functional can be written to quadratic order,

$$Z = \int [dQ_+^I dQ_-^I] \exp \left(-\frac{i}{2} \int_{\tau_0}^{\tau} d^3x d\eta a^2 \bar{\mathbf{Q}}^I \begin{pmatrix} \Delta & \\ & -\Delta \end{pmatrix}_{IJ} \mathbf{Q}^J + \delta\text{-fn terms} \right), \quad (\text{A.1})$$

where $\bar{\mathbf{Q}}^I = (Q_+^I, Q_-^I)$ and an overbar denotes matrix transposition. The time τ_0 should be set well before horizon crossing of the modes under discussion, and τ is the time at which we

wish to compute each correlation function. The δ -function terms have support at τ_0 and τ , and enforce boundary conditions, to be described below. Finally, the differential operator Δ satisfies

$$\Delta_{IJ} = G_{IJ} \left(\mathcal{D}_\eta^2 + 2\frac{a'}{a}\mathcal{D}_\eta - \partial_i\partial_i \right) + a^2\mathcal{M}_{IJ}, \quad (\text{A.2})$$

where \mathcal{M}_{IJ} is the mass matrix (3.11).

We write the time-ordered two-point function between contravariant components of Q_+ as D_{++} ,

$$D_{++}^{JK'}(\eta, \mathbf{x}; \sigma, \mathbf{y}) = \langle \mathbb{T} Q_+^J(\eta, \mathbf{x}) Q_+^{K'}(\sigma, \mathbf{y}) \rangle, \quad (\text{A.3})$$

with analogous definitions for D_{-+} , D_{+-} and D_{--} . In this appendix, we use unprimed indices to label the tangent space at time η and primed indices to label the tangent space at time σ . Each two-point function is a field-space bitensor and a spacetime biscalar. The operator \mathbb{T} denotes time ordering, with the convention that all ‘-’ fields are taken later than all ‘+’ fields and that time ordering on the ‘-’ contour is in the reverse sense to the ‘+’ contour.

The rules of Gaussian integration enable us to calculate the $D_{\pm\pm}$. They are obtained by inverting the quadratic structure Δ_{IJ} ,

$$ia^2 \begin{pmatrix} \Delta & \\ & -\Delta \end{pmatrix}_{IJ} \begin{pmatrix} D_{++} & D_{+-} \\ D_{-+} & D_{--} \end{pmatrix}^{JK'} = G_I^{K'} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \delta(\eta - \sigma) \delta(\mathbf{x} - \mathbf{y}). \quad (\text{A.4})$$

Since an expectation value $\langle \mathcal{O} \rangle$ inherits the tensor transformation properties of \mathcal{O} , the associated two-point functions between covariant or mixed components of Q_\pm can be obtained by raising or lowering the J and K' indices. To do so, one should use the metric evaluated at η or σ , respectively. In (A.4) we have suppressed coordinate labels, but the differential operator acts only on η and \mathbf{x} .

Mass matrix. For the remainder of this section, we ignore the mass matrix \mathcal{M}_{IJ} and treat each mode as massless. Small masses can be accommodated perturbatively if desired, but we will not do so in this paper. Note that this does not imply that we ignore all couplings between modes *after* horizon crossing: these are certainly important, because they describe how power is transferred from isocurvature perturbations to the adiabatic mode. These couplings will be retained when we discuss time evolution in §4. We are ignoring them only for a brief period around horizon exit.

Tensor structure. First, consider D_{++} . The \mathbf{x} and \mathbf{y} dependence can be diagonalized by passing to Fourier space,

$$D_{++}^{JK'} = \int \frac{d^3k}{(2\pi)^3} D_{++}^{JK'}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}. \quad (\text{A.5})$$

Neglecting the mass matrix as explained above, the mode function $D_{++}^{JK'}(\mathbf{k})$ satisfies

$$G_{IJ} \left(\mathcal{D}_\eta^2 + 2\frac{a'}{a}\mathcal{D}_\eta + k^2 \right) D_{++}^{JK'} = -\frac{i}{a^2} G_I^{K'} \delta(\eta - \sigma). \quad (\text{A.6})$$

To factorize the tensor structure we introduce a bitensor $\Pi^I_{K'}$ which is required to solve the equation $\mathcal{D}_\eta \Pi^I_{K'} = 0$,

$$\mathcal{D}_\eta \Pi^I_{K'} = \frac{d\Pi^I_{K'}}{d\eta} + \Gamma^I_{MN} \frac{d\phi^M}{d\eta} \Pi^N_{K'} = 0. \quad (\text{A.7})$$

The solution can formally be written as an ordered exponential,

$$\Pi^I_{K'} = \text{P exp} \left(- \int_\sigma^\eta d\tau \Gamma^{I''}_{M''N''} \frac{d\phi^{M''}}{d\tau} \right) G^{N'}_{K'}, \quad (\text{A.8})$$

where the integral is computed along the inflationary trajectory and the symbol ‘P’ denotes path ordering along it. Eq. (A.8) is the *trajectory propagator* introduced in §§3.2–3.3. It is simply the parallel propagator evaluated on the inflationary trajectory. In (A.8) we have chosen boundary conditions so that when $\eta \rightarrow \sigma$ the trajectory propagator satisfies $\Pi^I_{K'} \rightarrow G^I_K$.

The trajectory propagator allows the index structure in (A.6) to be factorized. Taking $D^{JK'}_{++} = \Pi^{JK'} \Delta_{++}$ (where indices on $\Pi^I_{K'}$ are raised and lowered using the usual rules for a bitensor), it follows that the scalar factor Δ_{++} satisfies the same equation as the propagator in flat field-space [1],

$$\left(\mathcal{D}_\eta^2 + 2\frac{a'}{a} \mathcal{D}_\eta + k^2 \right) \Delta_{++} = -\frac{i}{a^2} \delta(\eta - \sigma). \quad (\text{A.9})$$

The same factorization can be made for each Green’s function, so $D_{\pm\pm}^{JK'} = \Pi^{JK'} \Delta_{\pm\pm}$. With vacuum boundary conditions, the δ -function terms at τ_0 in (A.1) require Δ_{++} to be approximately positive frequency there, and Δ_{-+} to be approximately negative frequency. The δ -function terms at τ require $\Delta_{++}(\tau, \sigma) = \Delta_{-+}(\tau, \sigma)$ for all σ . Finally, Δ_{--} and Δ_{+-} are the Hermitian conjugates of Δ_{++} and Δ_{-+} , respectively.

If the initial conditions at τ_0 correspond to the vacuum, then D_{++} should be approximately positive frequency at that time. If τ_0 is well before horizon-exit, then

$$\langle \text{T} Q^I_+(\mathbf{k}_1, \eta) Q^{J'}_+(\mathbf{k}_2, \sigma) \rangle \simeq (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \Pi^{IJ'} \frac{H_*^2}{2k^3} \times \begin{cases} (1 - ik\eta)(1 + ik\sigma) e^{ik(\eta - \sigma)} & \eta < \sigma \\ (1 + ik\eta)(1 - ik\sigma) e^{ik(\sigma - \eta)} & \sigma < \eta \end{cases}, \quad (\text{A.10})$$

where k is the common value of $|\mathbf{k}_1|$ and $|\mathbf{k}_2|$, and this estimate is valid for values of η and σ within a few e-folds of horizon-crossing at $|k\eta| = |k\sigma| = 1$. A subscript ‘*’ denotes evaluation precisely at the horizon-crossing time. In Eq. (A.10) the trajectory propagator plays an essential role in ensuring that the right-hand side has the correct bitensorial transformation law.

Now consider D_{-+} . Imposing the boundary condition that D_{-+} is approximately negative frequency at the initial time, and equals D_{++} at time τ , we find

$$\langle \text{T} Q^I_-(\mathbf{k}_1, \eta) Q^{J'}_+(\mathbf{k}_2, \sigma) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \Pi^{IJ'} \frac{H_*^2}{2k^3} (1 + ik\eta)(1 - ik\sigma) e^{ik(\sigma - \eta)}. \quad (\text{A.11})$$

A.2 Three-point function

Each term in the third-order action (3.14) makes a contribution to the three-point correlation function $\langle Q^I_{\mathbf{k}_1} Q^J_{\mathbf{k}_2} Q^K_{\mathbf{k}_3} \rangle$, or equivalently A^{ijk} . Vertices are constructed from two copies of the

action, one for each of the ‘+’ fields and ‘-’ fields of Eq. (A.1). Vertices for ‘+’ fields appear with a factor of +i; vertices for ‘-’ fields appear with a factor of -i. We apply Wick’s theorem to produce all ways of pairing indices, using G^{++} of (A.10) to pair two ‘+’ indices and its complex conjugate G^{--} to pair two ‘-’ indices. We use G^{-+} in (A.11) or its complex conjugate G^{+-} to pair a mix of ‘+’ and ‘-’ indices. Finally, we integrate over all possible spacetime positions for each vertex.

Trajectory propagator. In curved field-space, this procedure is modified by the appearance of the trajectory propagator in each two-point function. Consider a typical interaction appearing the the third-order action, such as the term

$$S_{(3)} \supseteq \int d^3x d\eta \left\{ - \frac{a^2}{4M_{\text{pl}}^2 H(\eta)} \dot{\phi}^{I'} Q_{I'} \mathcal{D}_\eta Q^{J'} \mathcal{D}_\eta Q_{J'} \right\}. \quad (\text{A.12})$$

To proceed we should expand background quantities around a reference scale k_* , in the direction of the inflationary trajectory. This can be determined by analogy with the Taylor expansion (2.5), which was constructed along a geodesic. To separate the various tangent-space indices which appear, we adopt the following conventions: tangent-space indices at the time of observation, τ , are labelled I, J, K . (In the main text, the time of observation is expressed as an e-folding number N .) Tangent-space indices at the time of horizon-crossing for the reference scale k_* are labelled i, j, k . (In the main text, the time of horizon-crossing is expressed as N_* .) Finally, indices associated with the integration variable η are given primed indices I', J', K' . The background factor $\dot{\phi}/H$ gives

$$\frac{\dot{\phi}^{I'}}{H(\eta)} = \Pi^{I'}{}_i \left(\frac{\dot{\phi}^i}{H} + \mathcal{D}_N \frac{\dot{\phi}^i}{H} N + \dots \right)_*, \quad (\text{A.13})$$

where $N = -\ln|k_*\eta|$ represents the number of e-folds since horizon exit of the reference scale. We should make an analogous expansion for the metric which is used to contract the two copies of $\mathcal{D}_\eta Q^{J'}$,

$$G^{J'K'} = \Pi^{J'}{}_j \Pi^{K'}{}_k G_*^{jk}. \quad (\text{A.14})$$

In this case, higher-order terms in the Taylor expansion vanish because the metric is covariantly constant. A final example is the Riemann tensor, whose series expansion will be required to compute the higher-order contribution (A.22),

$$R^{I'(J'K')L'} \frac{\dot{\phi}^{L'}}{H} = \Pi^{I'}{}_i \Pi^{J'}{}_j \Pi^{K'}{}_k \left(R^{i(jk)l} \frac{\dot{\phi}^l}{H} + \mathcal{D}_N R^{i(jk)l} \frac{\dot{\phi}^l}{H} N + \dots \right)_*. \quad (\text{A.15})$$

One can see that the three factors of the trajectory propagator in Eq. (A.15) will be common to all terms in the three-point function. These propagators carry dependence on the integration variable η as well as the reference time N_* . However, each two-point function which connects an external field with a field at the vertex will introduce a propagator factor $\Pi^I{}_{I'}$, which is a function of η and N . Therefore all η dependence cancels, leaving a propagator that relates the two tangent spaces at N and N_* . This may be factored out of the vertex integral. The remaining details of the calculation correspond to those in flat field-space.

Results. Three of the contributing terms are covariantized versions of those present in flat field-space:

- $-\frac{a^2}{4M_{\text{pl}}^2 H} \dot{\phi}^I Q_I \mathcal{D}_\eta Q^J \mathcal{D}_\eta Q_J$

$$A^{ijk} \supseteq -\frac{1}{2M_{\text{pl}}^2} \frac{\dot{\phi}_*^i}{H_*} G_*^{jk} k_2^2 k_3^2 \left(\frac{1}{k_t} + \frac{k_1}{k_t^2} \right) + \text{cyclic}, \quad (\text{A.16})$$

- $-\frac{a^2}{4M_{\text{pl}}^2 H} \dot{\phi}^I Q_I \partial_i Q^J \partial_i Q_J$

$$A^{ijk} \supseteq \frac{1}{2M_{\text{pl}}^2} \frac{\dot{\phi}_*^i}{H_*} G_*^{jk} (\mathbf{k}_2 \cdot \mathbf{k}_3) \left(k_t - \frac{\kappa^2}{k_t} - \frac{k_1 k_2 k_3}{k_t^2} \right) + \text{cyclic}, \quad (\text{A.17})$$

- $\frac{a^2}{2M_{\text{pl}}^2 H} \dot{\phi}^I \partial_i \partial^{-2} \mathcal{D}_\eta Q_I \partial_i Q^J \mathcal{D}_\eta Q_J$

$$A^{ijk} \supseteq \frac{1}{2M_{\text{pl}}^2} \frac{\dot{\phi}_*^i}{H_*} G_*^{jk} \left[(\mathbf{k}_1 \cdot \mathbf{k}_2) k_3^2 \left(\frac{1}{k_t} + \frac{k_2}{k_t^2} \right) + (\mathbf{k}_1 \cdot \mathbf{k}_3) k_2^2 \left(\frac{1}{k_t} + \frac{k_3}{k_t^2} \right) \right] + \text{cyclic}. \quad (\text{A.18})$$

These expressions are valid under the same conditions as the power spectrum (3.12): we must wait sufficiently many e-folds that decaying power-law terms have become negligible, but Eqs. (A.16)–(A.18) are unreliable when $N = -\ln |k_* \tau| \gg 1$. These three contributions can be combined to give

$$A^{ijk} \supseteq \frac{1}{M_{\text{pl}}^2} \frac{\dot{\phi}_*^i}{H_*} G_*^{jk} \left[-2 \frac{k_2^2 k_3^2}{k_t} + \frac{1}{2} k_1 (\mathbf{k}_2 \cdot \mathbf{k}_3) \right] + \text{cyclic}. \quad (\text{A.19})$$

We now consider the remaining two terms involving the Riemann tensor:

- $\frac{2a^3}{3} R^{I(JK)L} \dot{\phi}_L \mathcal{D}_\eta Q_I Q_J Q_K$

$$A^{ijk} \supseteq \frac{4}{3} R_*^{i(jk)m} \frac{\dot{\phi}_m^*}{H_*} \left[k_1^3 \left(\gamma_E - N + \ln \frac{k_t}{k_*} \right) - k_t k_1^2 + \frac{k_1^2 k_2 k_3}{k_t} \right] + \text{cyclic}, \quad (\text{A.20})$$

- $\frac{a^4}{6} R^{(I|LM|J;K)} \dot{\phi}_L \dot{\phi}_M Q_I Q_J Q_K$

$$A^{ijk} \supseteq \frac{1}{3} R_*^{i|mn|j;k} \frac{\dot{\phi}_m^*}{H_*} \frac{\dot{\phi}_n^*}{H_*} \left[k_1^3 \left(N - \ln \frac{k_t}{k_*} - \gamma_E - \frac{1}{3} \right) + \frac{4}{9} k_t^3 - k_t \kappa^2 \right] + \text{cyclic}. \quad (\text{A.21})$$

Finally we consider the next-order correction to the $R^{I(JK)L} \dot{\phi}_L$ term:

- $\frac{2a^3}{3H} R^{I(JK)L;M} \dot{\phi}_L \dot{\phi}_M N \mathcal{D}_\eta Q_I Q_J Q_K$

$$A^{ijk} \supseteq \frac{2}{3} R_*^{i(jk)m;n} \frac{\dot{\phi}_m^*}{H_*} \frac{\dot{\phi}_n^*}{H_*} \left[-k_1^3 N^2 + k_1^3 \left(\gamma_E^2 - \frac{\pi^2}{12} + \ln \frac{k_t}{k_*} \left(2\gamma_E + \ln \frac{k_t}{k_*} \right) \right) \right. \\ \left. - 2k_t k_1^2 \left(\ln \frac{k_t}{k_*} + \gamma_E - 1 \right) + 2 \frac{k_1^2 k_2 k_3}{k_t} \left(\ln \frac{k_t}{k_*} + \gamma_E \right) \right] + \text{cyclic}. \quad (\text{A.22})$$

The presence of terms proportional to positive powers of N , which spoil the validity of these formulae when $N \gg 1$, is explicit in Eqs. (A.20)–(A.22).

References

- [1] D. Seery and J. E. Lidsey, *Primordial non-Gaussianities from multiple-field inflation*, *JCAP* **0509** (2005) 011, [[arXiv:astro-ph/0506056](#)], [[doi:10.1088/1475-7516/2005/09/011](#)].
- [2] D. Seery, J. E. Lidsey, and M. S. Sloth, *The inflationary trispectrum*, *JCAP* **0701** (2007) 027, [[arXiv:astro-ph/0610210](#)], [[doi:10.1088/1475-7516/2007/01/027](#)].
- [3] D. Seery, M. S. Sloth, and F. Vernizzi, *Inflationary trispectrum from graviton exchange*, *JCAP* **0903** (2009) 018, [[arXiv:0811.3934](#)], [[doi:10.1088/1475-7516/2009/03/018](#)].
- [4] X. Chen, *Primordial Non-Gaussianities from Inflation Models*, *Adv.Astron.* **2010** (2010) 638979, [[arXiv:1002.1416](#)].
- [5] D. J. Mulryne, D. Seery, and D. Wesley, *Moment transport equations for non-Gaussianity*, *JCAP* **1001** (2010) 024, [[arXiv:0909.2256](#)], [[doi:10.1088/1475-7516/2010/01/024](#)].
- [6] D. J. Mulryne, D. Seery, and D. Wesley, *Moment transport equations for the primordial curvature perturbation*, *JCAP* **1104** (2011) 030, [[arXiv:1008.3159](#)], [[doi:10.1088/1475-7516/2011/04/030](#)].
- [7] J. Elliston, D. J. Mulryne, D. Seery, and R. Tavakol, *Evolution of f_{NL} to the adiabatic limit*, *JCAP* **1111** (2011) 005, [[arXiv:1106.2153](#)], [[doi:10.1088/1475-7516/2011/11/005](#)].
- [8] D. Seery, D. J. Mulryne, J. Frazer, and R. H. Ribeiro, *Inflationary perturbation theory is geometrical optics in phase space*, [arXiv:1203.2635](#).
- [9] M. Alishahiha, E. Silverstein, and D. Tong, *DBI in the sky*, *Phys.Rev.* **D70** (2004) 123505, [[arXiv:hep-th/0404084](#)], [[doi:10.1103/PhysRevD.70.123505](#)].
- [10] D. Seery and J. E. Lidsey, *Primordial non-Gaussianities in single field inflation*, *JCAP* **0506** (2005) 003, [[arXiv:astro-ph/0503692](#)], [[doi:10.1088/1475-7516/2005/06/003](#)].
- [11] X. Chen, M.-x. Huang, S. Kachru, and G. Shiu, *Observational signatures and non-Gaussianities of general single field inflation*, *JCAP* **0701** (2007) 002, [[arXiv:hep-th/0605045](#)], [[doi:10.1088/1475-7516/2007/01/002](#)].
- [12] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, *The Effective Field Theory of Inflation*, *JHEP* **0803** (2008) 014, [[arXiv:0709.0293](#)], [[doi:10.1088/1126-6708/2008/03/014](#)].
- [13] F. Arroja, S. Mizuno, and K. Koyama, *Non-gaussianity from the bispectrum in general multiple field inflation*, *JCAP* **0808** (2008) 015, [[arXiv:0806.0619](#)], [[doi:10.1088/1475-7516/2008/08/015](#)].
- [14] D. Langlois, S. Renaux-Petel, D. A. Steer, and T. Tanaka, *Primordial perturbations and non-Gaussianities in DBI and general multi-field inflation*, *Phys.Rev.* **D78** (2008) 063523, [[arXiv:0806.0336](#)], [[doi:10.1103/PhysRevD.78.063523](#)].
- [15] S. Renaux-Petel, S. Mizuno, and K. Koyama, *Primordial fluctuations and non-Gaussianities from multifield DBI Galileon inflation*, *JCAP* **1111** (2011) 042, [[arXiv:1108.0305](#)], [[doi:10.1088/1475-7516/2011/11/042](#)].
- [16] C.-Y. Sun and D.-H. Zhang, *Non-gaussianity of general multiple-field inflationary models*, *Int.J.Mod.Phys.* **D15** (2006) 1259–1266, [[arXiv:astro-ph/0510709](#)], [[doi:10.1142/S0218271806008887](#)].
- [17] C. M. Peterson and M. Tegmark, *Testing Multi-Field Inflation: A Geometric Approach*, [arXiv:1111.0927](#).
- [18] P. M. Saffin, *The covariance of multi-field perturbations, pseudo-susy and f_{NL}* , [arXiv:1203.0397](#).

- [19] J.-O. Gong and T. Tanaka, *A covariant approach to general field space metric in multi-field inflation*, *JCAP* **1103** (2011) 015, [[arXiv:1101.4809](#)], [[doi:10.1088/1475-7516/2011/03/015](#), [10.1088/1475-7516/2012/02/E01](#)].
- [20] A. A. Starobinsky, *Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations*, *Phys.Lett.* **B117** (1982) 175–178, [[doi:10.1016/0370-2693\(82\)90541-X](#)].
- [21] D. Lyth, *Large Scale Energy Density Perturbations and Inflation*, *Phys.Rev.* **D31** (1985) 1792–1798, [[doi:10.1103/PhysRevD.31.1792](#)].
- [22] A. A. Starobinsky, *Multicomponent de Sitter (Inflationary) Stages and the Generation of Perturbations*, *JETP Lett.* **42** (1985) 152–155.
- [23] D. H. Lyth and Y. Rodríguez, *The Inflationary prediction for primordial non-Gaussianity*, *Phys.Rev.Lett.* **95** (2005) 121302, [[arXiv:astro-ph/0504045](#)], [[doi:10.1103/PhysRevLett.95.121302](#)].
- [24] M. Sasaki and E. D. Stewart, *A General analytic formula for the spectral index of the density perturbations produced during inflation*, *Prog. Theor. Phys.* **95** (1996) 71–78, [[arXiv:astro-ph/9507001](#)], [[doi:10.1143/PTP.95.71](#)].
- [25] D. I. Kaiser, *Conformal Transformations with Multiple Scalar Fields*, *Phys.Rev.* **D81** (2010) 084044, [[arXiv:1003.1159](#)], [[doi:10.1103/PhysRevD.81.084044](#)].
- [26] M. Rainer and A. Zhuk, *Tensor-multi-scalar theories from multidimensional cosmology*, *Phys.Rev.* **D54** (1996) 6186–6192, [[arXiv:gr-qc/9608020](#)], [[doi:10.1103/PhysRevD.54.6186](#)].
- [27] U. Gunther and A. Zhuk, *Gravitational excitons from extra dimensions*, *Phys.Rev.* **D56** (1997) 6391–6402, [[arXiv:gr-qc/9706050](#)], [[doi:10.1103/PhysRevD.56.6391](#)].
- [28] T. Appelquist, A. Chodos, and P. Freund, *Modern Kaluza-Klein theories*. Frontiers in physics. Addison-Wesley, 1987.
- [29] N. Birrell and P. Davies, *Quantum fields in curved space*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 1982.
- [30] I. Buchbinder, S. Odintsov, and I. Shapiro, *Effective action in quantum gravity*. Taylor & Francis, 1992.
- [31] E. Poisson, A. Pound, and I. Vega, *The motion of point particles in curved spacetime*, *Living Reviews in Relativity* **14** (2011), no. 7.
- [32] R. L. Arnowitt, S. Deser, and C. W. Misner, *The Dynamics of general relativity*, [arXiv:gr-qc/0405109](#). Gravitation: an introduction to current research, Louis Witten ed. (Wiley 1962), chapter 7, pp 227-265.
- [33] J. M. Maldacena, *Non-Gaussian features of primordial fluctuations in single field inflationary models*, *JHEP* **0305** (2003) 013, [[arXiv:astro-ph/0210603](#)].
- [34] P. Adshead, W. Hu, C. Dvorkin, and H. V. Peiris, *Fast Computation of Bispectrum Features with Generalized Slow Roll*, *Phys.Rev.* **D84** (2011) 043519, [[arXiv:1102.3435](#)], [[doi:10.1103/PhysRevD.84.043519](#)].
- [35] F. Arroja and T. Tanaka, *A note on the role of the boundary terms for the non-Gaussianity in general k-inflation*, *JCAP* **1105** (2011) 005, [[arXiv:1103.1102](#)], [[doi:10.1088/1475-7516/2011/05/005](#)].
- [36] C. Burrage, R. H. Ribeiro, and D. Seery, *Large slow-roll corrections to the bispectrum of noncanonical inflation*, *JCAP* **1107** (2011) 032, [[arXiv:1103.4126](#)], [[doi:10.1088/1475-7516/2011/07/032](#)].
- [37] D. H. Lyth, K. A. Malik, and M. Sasaki, *A General proof of the conservation of the curvature perturbation*, *JCAP* **0505** (2005) 004, [[arXiv:astro-ph/0411220](#)],

- [doi:10.1088/1475-7516/2005/05/004].
- [38] G. I. Rigopoulos and E. P. S. Shellard, *The Separate Universe Approach and the Evolution of Nonlinear Superhorizon Cosmological Perturbations*, *Phys. Rev.* **D68** (2003) 123518, [arXiv:astro-ph/0306620], [doi:10.1103/PhysRevD.68.123518].
- [39] S. Weinberg, *A Tree Theorem for Inflation*, *Phys.Rev.* **D78** (2008) 063534, [arXiv:0805.3781], [doi:10.1103/PhysRevD.78.063534].
- [40] L. Senatore and M. Zaldarriaga, *On Loops in Inflation*, *JHEP* **1012** (2010) 008, [arXiv:0912.2734], [doi:10.1007/JHEP12(2010)008].
- [41] G. L. Pimentel, L. Senatore, and M. Zaldarriaga, *On Loops in Inflation III: Time Independence of zeta in Single Clock Inflation*, *JHEP* **1207** (2012) 166, [arXiv:1203.6651], [doi:10.1007/JHEP07(2012)166].
- [42] C. Burgess, L. Leblond, R. Holman, and S. Shandera, *Super-Hubble de Sitter Fluctuations and the Dynamical RG*, *JCAP* **1003** (2010) 033, [arXiv:0912.1608], [doi:10.1088/1475-7516/2010/03/033].
- [43] M. Zaldarriaga, *Non-Gaussianities in models with a varying inflaton decay rate*, *Phys.Rev.* **D69** (2004) 043508, [arXiv:astro-ph/0306006], [doi:10.1103/PhysRevD.69.043508].
- [44] D. Seery, *One-loop corrections to the curvature perturbation from inflation*, *JCAP* **0802** (2008) 006, [arXiv:0707.3378], [doi:10.1088/1475-7516/2008/02/006].
- [45] D. Seery, *Infrared effects in inflationary correlation functions*, *Class.Quant.Grav.* **27** (2010) 124005, [arXiv:1005.1649], [doi:10.1088/0264-9381/27/12/124005].
- [46] S. Weinberg, *Quantum contributions to cosmological correlations*, *Phys.Rev.* **D72** (2005) 043514, [arXiv:hep-th/0506236], [doi:10.1103/PhysRevD.72.043514].
- [47] T. Falk, R. Rangarajan, and M. Srednicki, *The Angular dependence of the three point correlation function of the cosmic microwave background radiation as predicted by inflationary cosmologies*, *Astrophys.J.* **403** (1993) L1, [arXiv:astro-ph/9208001].
- [48] E. D. Stewart and D. H. Lyth, *A More accurate analytic calculation of the spectrum of cosmological perturbations produced during inflation*, *Phys.Lett.* **B302** (1993) 171–175, [arXiv:gr-qc/9302019], [doi:10.1016/0370-2693(93)90379-V].
- [49] T. T. Nakamura and E. D. Stewart, *The Spectrum of cosmological perturbations produced by a multicomponent inflaton to second order in the slow roll approximation*, *Phys.Lett.* **B381** (1996) 413–419, [arXiv:astro-ph/9604103], [doi:10.1016/0370-2693(96)00594-1].
- [50] D. Seery, K. A. Malik, and D. H. Lyth, *Non-gaussianity of inflationary field perturbations from the field equation*, *JCAP* **0803** (2008) 014, [arXiv:0802.0588], [doi:10.1088/1475-7516/2008/03/014].
- [51] A. J. Tolley and M. Wyman, *The Gelaton Scenario: Equilateral non-Gaussianity from multi-field dynamics*, *Phys.Rev.* **D81** (2010) 043502, [arXiv:0910.1853], [doi:10.1103/PhysRevD.81.043502].
- [52] A. Achúcarro, J.-O. Gong, S. Hardeman, G. A. Palma, and S. P. Patil, *Mass hierarchies and non-decoupling in multi-scalar field dynamics*, *Phys.Rev.* **D84** (2011) 043502, [arXiv:1005.3848], [doi:10.1103/PhysRevD.84.043502].
- [53] A. Achúcarro, J.-O. Gong, S. Hardeman, G. A. Palma, and S. P. Patil, *Features of heavy physics in the CMB power spectrum*, *JCAP* **1101** (2011) 030, [arXiv:1010.3693], [doi:10.1088/1475-7516/2011/01/030].
- [54] A. Avgoustidis, S. Cremonini, A.-C. Davis, R. H. Ribeiro, K. Turzynski, *et al.*, *Decoupling Survives Inflation: A Critical Look at Effective Field Theory Violations During Inflation*, *JCAP* **1206** (2012) 025, [arXiv:1203.0016], [doi:10.1088/1475-7516/2012/06/025].

- [55] P. Adshead, C. Dvorkin, W. Hu, and E. A. Lim, *Non-Gaussianity from Step Features in the Inflationary Potential*, *Phys.Rev.* **D85** (2012) 023531, [[arXiv:1110.3050](#)], [[doi:10.1103/PhysRevD.85.023531](#)].
- [56] X. Chen and Y. Wang, *Quasi-Single Field Inflation and Non-Gaussianities*, *JCAP* **1004** (2010) 027, [[arXiv:0911.3380](#)], [[doi:10.1088/1475-7516/2010/04/027](#)].
- [57] X. Chen and Y. Wang, *Quasi-Single Field Inflation with Large Mass*, [arXiv:1205.0160](#).
- [58] L. McAllister, S. Renaux-Petel, and G. Xu, *A Statistical Approach to Multifield Inflation: Many-field Perturbations Beyond Slow Roll*, [arXiv:1207.0317](#).
- [59] J.-O. Gong and E. D. Stewart, *The Density perturbation power spectrum to second order corrections in the slow roll expansion*, *Phys.Lett.* **B510** (2001) 1–9, [[arXiv:astro-ph/0101225](#)].
- [60] J.-O. Gong and E. D. Stewart, *The Power spectrum for a multicomponent inflaton to second order corrections in the slow roll expansion*, *Phys.Lett.* **B538** (2002) 213–222, [[arXiv:astro-ph/0202098](#)], [[doi:10.1016/S0370-2693\(02\)02004-X](#)].
- [61] D. Seery, *One-loop corrections to a scalar field during inflation*, *JCAP* **0711** (2007) 025, [[arXiv:0707.3377](#)], [[doi:10.1088/1475-7516/2007/11/025](#)].
- [62] K. Koyama, *Non-Gaussianity of quantum fields during inflation*, *Class.Quant.Grav.* **27** (2010) 124001, [[arXiv:1002.0600](#)], [[doi:10.1088/0264-9381/27/12/124001](#)].
- [63] E. Nalson, A. J. Christopherson, I. Huston, and K. A. Malik, *Quantifying the behaviour of curvature perturbations during inflation*, [arXiv:1111.6940](#).
- [64] G. J. Anderson, D. J. Mulryne, and D. Seery, *Transport equations for the inflationary trispectrum*, [arXiv:1205.0024](#).
- [65] C. DeWitt-Morette, *The Semiclassical Expansion*, *Annals Phys.* **97** (1976) 367–399, [[doi:10.1016/0003-4916\(76\)90041-5](#)].
- [66] J. Lewandowski, E. Newman, and C. Rovelli, *Variations of the parallel propagator and holonomy operator and the Gauss law constraint*, *J.Math.Phys.* **34** (1993) 4646–4654, [[doi:10.1063/1.530362](#)].
- [67] D. H. Lyth and I. Zaballa, *A Bound concerning primordial non-Gaussianity*, *JCAP* **0510** (2005) 005, [[arXiv:astro-ph/0507608](#)], [[doi:10.1088/1475-7516/2005/10/005](#)].
- [68] F. Vernizzi and D. Wands, *Non-gaussianities in two-field inflation*, *JCAP* **0605** (2006) 019, [[arXiv:astro-ph/0603799](#)], [[doi:10.1088/1475-7516/2006/05/019](#)].
- [69] S. Yokoyama, T. Suyama, and T. Tanaka, *Primordial Non-Gaussianity in Multi-Scalar Slow-Roll Inflation*, *JCAP* **0707** (2007) 013, [[arXiv:0705.3178](#)], [[doi:10.1088/1475-7516/2007/07/013](#)].
- [70] S. Yokoyama, T. Suyama, and T. Tanaka, *Primordial Non-Gaussianity in Multi-Scalar Inflation*, *Phys.Rev.* **D77** (2008) 083511, [[arXiv:0711.2920](#)], [[doi:10.1103/PhysRevD.77.083511](#)].
- [71] A. Mazumdar and L. Wang, *Separable and non-separable multi-field inflation and large non-Gaussianity*, [arXiv:1203.3558](#).
- [72] D. I. Kaiser, *Frame independent calculation of spectral indices from inflation*, *Phys. Lett. B* (1995) [[arXiv:astro-ph/9507048](#)].
- [73] E. E. Flanagan, *The Conformal frame freedom in theories of gravitation*, *Class.Quant.Grav.* **21** (2004) 3817, [[arXiv:gr-qc/0403063](#)], [[doi:10.1088/0264-9381/21/15/N02](#)].
- [74] T. Chiba and M. Yamaguchi, *Extended Slow-Roll Conditions and Rapid-Roll Conditions*, *JCAP* **0810** (2008) 021, [[arXiv:0807.4965](#)], [[doi:10.1088/1475-7516/2008/10/021](#)].
- [75] J.-O. Gong, J.-c. Hwang, W.-I. Park, M. Sasaki, and Y.-S. Song, *Conformal invariance of curvature perturbation*, *JCAP* **1109** (2011) 023, [[arXiv:1107.1840](#)],

[[doi:10.1088/1475-7516/2011/09/023](https://doi.org/10.1088/1475-7516/2011/09/023)].

- [76] J. White, M. Minamitsuji, and M. Sasaki, *Curvature perturbation in multi-field inflation with non-minimal coupling*, *JCAP* **1207** (2012) 039, [[arXiv:1205.0656](https://arxiv.org/abs/1205.0656)], [[doi:10.1088/1475-7516/2012/07/039](https://doi.org/10.1088/1475-7516/2012/07/039)].