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The influence of earthquakes on the Chandler wobble during 1977–1983

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Summary. A direct calculation is made of the effect on the Chandler wobble of 1287 earthquakes that occurred during 1977–1983. The hypocentral parameters (location and origin time) and the moment tensor representation of the best point source for each earthquake as determined by the 'centroid-moment tensor' technique were used to calculate the change in the Chandler wobble's excitation function by assuming this change is due solely to the static deformation field generated by that earthquake. The resulting theoretical earthquake excitation function is compared with the 'observed' excitation function that is obtained by deconvolving a Chandler wobble time series derived from *LAGEOS* polar motion data. Since only 7 years of data are available for analysis it is not possible to resolve the Chandler band and determine whether or not the theoretical earthquake excitation function derived here is coherent and in phase with the 'observed' excitation function in that band. However, since the power spectrum of the earthquake excitation function is about 56 dB less than that of the 'observed' excitation function at frequencies near the Chandler frequency, it is concluded that earthquakes, via their static deformation field, have had a negligible influence on the Chandler wobble during 1977–1983. However, fault creep or any type of aseismic slip that occurs on a time-scale much less than the period of the Chandler wobble could have an important (and still un-modelled) effect on the Chandler wobble.

Key words: Chandler wobble, earthquakes, excitation

1 Introduction

The Chandler wobble is the Earth's realization of the 'free nutation' predicted by Euler in 1765. Euler deduced that any non-spherical rigid body rotating about some axis that is not a principal moment of inertia axis of that body should wobble as it rotates. This motion of the

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Earth was not detected, however, until 1891 when S. C. Chandler, a merchant by trade but an astronomer by avocation, discovered that the motion of the rotation pole actually consists of two harmonic components – one at an annual frequency and the other corresponding to Euler's free wobble. The annual term is a forced motion of the rotation pole and is believed to be caused by meteorological events (e.g. Wahr 1983; Wilson & Haubrich 1976). The other term, now known as the Chandler wobble in honour of its discoverer, should quickly (on a geological time-scale) dampen out – its *e*-folding amplitude decay time has been estimated to be about 40 yr (Wilson & Haubrich 1976). However, the Chandler wobble has been under systematic observation for more than 80 years and during part of that time its amplitude is actually observed to increase, indicating that some mechanism, or mechanisms, are acting to keep the Chandler wobble excited.

One of the earliest proposed excitation mechanisms was earthquakes. In 1876 Kelvin speculated about the effect on the Earth's rotation pole of sudden changes in the Earth's inertia tensor. Cecchini (1928) suggested that a connection exists between changes in the Chandler wobble's amplitude and the increase in seismic activity that took place at the beginning of the twentieth century. More recently, apparent correlations have been found between the amplitude of the Chandler wobble, the annual number of large earthquakes and the annual seismic energy release of great earthquakes (e.g. Kanamori 1977; Anderson 1974; Myerson 1970; Press & Briggs 1975).

Munk & MacDonald (1960, pp. 163–164) calculated the effect of earthquakes on the Chandler wobble by assuming a simple block fault model for the earthquake displacement field. They found the change induced in the excitation pole to be about 0.00001 arcsec and consequently concluded that earthquakes play a negligible role in exciting the Chandler wobble. However, Press (1965) showed that displacements due to a dislocation source in a homogeneous elastic half-space are not insignificant even at teleseismic distances from the source. Mansinha & Smylie (1967) realized that this could have important consequences relating to the question of earthquake excitation of the Chandler wobble, although a homogeneous half-space model of the Earth is inadequate to draw any firm conclusions about the effect of this non-negligible teleseismic displacement field on the Chandler wobble. Dahlen (1971, 1973) extended elastic dislocation theory to deal with a point shear dislocation in a spherically symmetric, self-gravitating earth model containing a fluid core and thereby computed the change in the Earth's inertia tensor due to such an earthquake source model. He then classified each of 1201 large (magnitude 7.0 or greater) earthquakes that occurred during 1904–1964 into one of three possible types based upon its location. Using estimates of the moment of each of these earthquakes obtained from an empirical moment–magnitude relationship and the average shift in the Chandler wobble's excitation pole expected for each earthquake type he concluded that earthquakes could account for no more than 10 per cent of the Chandler wobble's power. O'Connell & Dziewonski (1976) computed the change in the Earth's inertia tensor due to an earthquake by representing the static displacement field generated by the earthquake as a superposition of normal modes. An empirical moment–magnitude relationship was used to estimate the moments of 234 large ($M_S \geq 7.8$) earthquakes that occurred during 1901–1970 and the earthquakes were again assigned to one of three types based upon their location. By computing a synthetic Chandler wobble time series due to the effect of these 234 earthquakes and comparing it to observations they concluded that earthquakes can contribute about 25 per cent to the Chandler wobble's power. However, this result has been criticized by Kanamori (1976) and Wilson & Haubrich (1977). Kanamori (1976) observes that the magnitudes used by O'Connell & Dziewonski (1976) in obtaining the earthquakes' moments are biased too large by an average amount of 0.3 leading to an overestimate of the seismic moments and hence an overestimate of the effect of these earthquakes on the Chandler wobble (although for a conflicting opinion see Mansinha, Smylie & Chapman 1979). Wilson & Haubrich (1977) agree with O'Connell & Dziewonski (1976) that earthquakes could account for about 25 per cent of the power of the Chandler wobble (assuming the moment values are accurate) but by computing the phase spectrum and the coherence between the theoretical earthquake excitation function and an excitation function of the Chandler wobble that is derived from observations they find no evidence for a supporting correlation between the synthetic earthquake-derived Chandler wobble of O'Connell & Dziewonski (1976) and the observations.

In this study a direct calculation of the effect on the Chandler wobble of 1287 earthquakes that occurred during 1977–1983 is made by resorting to neither an empirical moment–magnitude relationship nor statistical arguments. This is made possible by the recent availability of a set of moment tensors representing the source mechanisms of 1287 earthquakes that occurred during 1977–1983. In Section 2 this moment tensor data set is described along with the theory used to calculate the expected change in the Chandler wobble's excitation function. Section 3 describes the outcome of this calculation and compares the resulting theoretical earthquake excitation function to an 'observed' excitation function that is derived from observations of polar motion. Section 4 discusses the results of this comparison and Section 5 contains a summary and conclusions.

2 Method and data

As seen in a frame of reference attached to the rotating Earth the motion of the rotation pole associated with the Chandler wobble is that of an inward decaying, counter-clockwise (as viewed looking down on the North Pole) spiral having a period T of about 434 day and an amplitude decay time constant $\tau = TQ/\pi$ of about 40 yr (Wilson & Haubrich 1976). The equation governing this motion of the rotation pole has been derived, e.g. by Munk & MacDonald (1960) and is written here as:

$$\psi(t) = m(t) + \frac{i}{\sigma_0}\dot{m}(t) \tag{2.1}$$

where the dot denotes time differentiation, σ_0 is the complex-valued (thereby including damping) frequency of the Chandler wobble:

$$\sigma_0 = \frac{2\pi}{T}\left(1 + \frac{i}{2Q}\right) \tag{2.2}$$

and $m(t)$ is the location of the rotation pole given as a (small) angular offset from some reference axis:

$$m = \frac{1}{\Omega}(\omega_x + i\omega_y) \tag{2.3}$$

where Ω is the mean angular rotation rate of the Earth and ω_x and ω_y are the x- and y-components, respectively, of the Earth's instantaneous angular velocity vector $\boldsymbol{\omega}$.

$\psi(t)$, known as the excitation function of the Chandler wobble, is the term in equation (2.1) responsible for keeping the Chandler wobble excited. If ψ were to be held constant in time then the solution to equation (2.1) would represent an inward decaying spiral centred on the constant position ψ. However, geophysical events such as earthquakes or meteorological disturbances cause $\psi(t)$ to vary with time causing the rotation pole to revolve about a varying centre keeping the Chandler wobble excited. Any single geophysical event influences $\psi(t)$ via (Wahr 1982):

$$\psi(t) = \frac{1.61}{\Omega^2(C-A)} (\Omega^2 \Delta c - i\Omega\Delta\dot{c} + \Omega h - i\dot{h} + L) \tag{2.4}$$

where the dot denotes time differentiation, C and A are the greatest and least, respectively, principal moments of inertia of the Earth, $L = L_x + iL_y$ represents any external torques acting on the system, $\Delta c = \Delta c_{xz} + i\,\Delta c_{yz}$ represents changes in the Earth's products of inertia induced by the excitation event and $h = h_x + ih_y$ represents any relative angular momentum induced by the event. The factor 1.61 is due primarily to the core not participating in the Chandler wobble and by assuming that the solid earth and oceans respond linearly to changes in the angular velocity vector (see, e.g. Wahr 1982; Smith & Dahlen 1981).

Earthquakes affect the Chandler wobble by rearranging the mass of the Earth which in turn changes the Earth's inertia tensor. This redistribution of mass is accomplished during a time interval much shorter than the period of the Chandler wobble and it is therefore assumed here that the change in the inertia tensor occurs instantaneously:

$$\Delta c(t) = \Delta c H(t - t_j) \tag{2.5}$$

where t_j is the time of the earthquake. This assumption has several consequences. Dahlen (1971) has pointed out that assuming the inertia tensor changes instantaneously greatly simplifies the calculation of the amount the inertia tensor changes because of the earthquake. It is only necessary to calculate the static deformation field induced by the earthquake while the dynamics of the problem can be ignored. Chao (1984) has explicitly shown that the h-terms in equation (2.4) are much smaller than the c-terms for a sudden seismic event. It is also easily shown that the $\Omega\Delta\dot{c}$ term in equation (2.4) is much smaller than the $\Omega^2\Delta c$ term when $\Delta c(t)$ is given by equation (2.5). Finally, assuming that the inertia tensor changes instantaneously implies that the earthquake itself occurs instantaneously. Since earthquake rupture times are much less than the period of the Chandler wobble this assumption is probably adequate here. Therefore, in this investigation earthquakes will be assumed to influence the Chandler wobble through its excitation function by:

$$\psi(t) = \frac{1.61}{C-A} \Delta c. \tag{2.6}$$

The change in the products of inertia Δc will be computed from the static deformation field generated by the earthquake.

Many studies have recently computed the change in the Earth's inertia tensor due to the static displacement field generated by a given earthquake. Dahlen (1971, 1973), Israel, Ben-Menahem & Singh (1973) and Mansinha *et al.* (1979) (correcting a mistake in Smylie & Mansinha 1971) have all, independently, solved this problem directly. O'Connell & Dziewonski (1976) represented the static displacement field as a superposition of normal modes. Smith (1977) used normal mode excitation theory to compute the change in the Chandler wobble's excitation function using an elliptical rotating earth model. The results of all the above calculations for the 1960 Chilean and the 1964 Alaskan earthquakes are in basic agreement with each other (see Lambeck 1980, pp. 220–238 for a review of this subject). In view of this current agreement the results of Dahlen (1973) were used in this study since they were in a convenient form. The inertia tensor C is defined by:

$$\mathsf{C} = \int_v \rho_0(r)\,[(\mathbf{r}\cdot\mathbf{r})\mathsf{I} - \mathbf{rr}]\;dV \tag{2.7}$$

where $\rho_0(r)$ is the density of the medium at the point $\mathbf{r}$ in a mass of volume V. I is the second-order identity tensor. Therefore the change $\Delta\mathsf{C}$ in the inertia tensor due to a change $\rho_1(\mathbf{r})$ in the density field is simply given by:

$$\Delta\mathsf{C} = \int_v \rho_1(\mathbf{r})\,[(\mathbf{r}\cdot\mathbf{r})\mathsf{I} - \mathbf{rr}]\;dV \tag{2.8}$$

where the volume integral now extends over the deformed body. The density change ρ_1 is related to the static displacement field $\mathbf{u}(\mathbf{r})$ by the linearized continuity equation (conservation of mass):

$$\rho_1 = -\boldsymbol{\nabla}\cdot(\rho_0\mathbf{u}). \tag{2.9}$$

The static displacement field $\mathbf{u}(\mathbf{r})$ is obtained by solving the elastic-gravitational equations of motion (e.g. Dahlen 1973). Let $\mathbf{f}$ be the body force equivalent to the moment tensor M associated with some earthquake source:

$$\mathbf{f}(\mathbf{r}) = -\mathsf{M}\cdot\boldsymbol{\nabla}\,\delta(\mathbf{r}-\mathbf{r}_0) \tag{2.10}$$

where $\mathbf{r}_0$ is the location of the earthquake. It is assumed here that the earthquake source can be adequately represented by a point source. The static displacement field $\mathbf{u}(\mathbf{r})$ is then obtained by solving:

$$\begin{aligned} &-\rho_0\boldsymbol{\nabla}\phi_1 - \rho_1\boldsymbol{\nabla}\phi_0 - \boldsymbol{\nabla}[\mathbf{u}\cdot(\rho_0\boldsymbol{\nabla}\phi_0)] + \boldsymbol{\nabla}\cdot\mathsf{E} + \mathbf{f} = 0 \\ &\boldsymbol{\nabla}^2\phi_1 = 4\pi G\rho_1 \end{aligned} \tag{2.11}$$

where ϕ_1 is the perturbation to the initial gravitational potential ϕ_0 due to the density perturbation ρ_1 (which is in turn given by equation 2.9). E is the stress tensor associated with the displacement field $\mathbf{u}(\mathbf{r})$:

$$\mathsf{E} = \lambda(\boldsymbol{\nabla}\cdot\mathbf{u})\mathsf{I} + \mu[\boldsymbol{\nabla}\mathbf{u} + (\boldsymbol{\nabla}\mathbf{u})^{\mathrm{T}}] \tag{2.12}$$

where λ and μ are the elastic Lamé parameters. The solution to this system of equations obtained by Dahlen (1973) for the change in the products of inertia is written here in terms of the six independent components of the moment tensor M:

$$\begin{aligned} \Delta c_{xz} = {} & \tilde{\Gamma}_1(h)\,[\tfrac{1}{2}(M_{\theta\theta} - M_{\phi\phi})\sin 2\theta\cos\phi - 2M_{\theta\phi}\sin\theta\sin\phi] \\ & - \tilde{\Gamma}_2(h)\,M_{rr}\sin 2\theta\cos\phi \\ & + \tilde{\Gamma}_3(h)\,[-M_{r\theta}\cos 2\theta\cos\phi + M_{r\phi}\cos\theta\sin\phi] \end{aligned} \tag{2.13}$$

$$\begin{aligned} \Delta c_{yz} = {} & \tilde{\Gamma}_1(h)\,[\tfrac{1}{2}(M_{\theta\theta} - M_{\phi\phi})\sin 2\theta\sin\phi + 2M_{\theta\phi}\sin\theta\cos\phi] \\ & - \tilde{\Gamma}_2(h)\,M_{rr}\sin 2\theta\sin\phi \\ & - \tilde{\Gamma}_3(h)\,[M_{r\theta}\cos 2\theta\sin\phi + M_{r\phi}\cos\theta\cos\phi] \end{aligned}$$

where θ is the colatitude, ϕ is the east longitude and h is the depth of the earthquake source. The $\tilde{\Gamma}_i(h)$ functions depend upon the particular earth model used to solve the system of equations (2.11) and are shown in Fig. 1. Formulae (2.13) will be used here to compute the

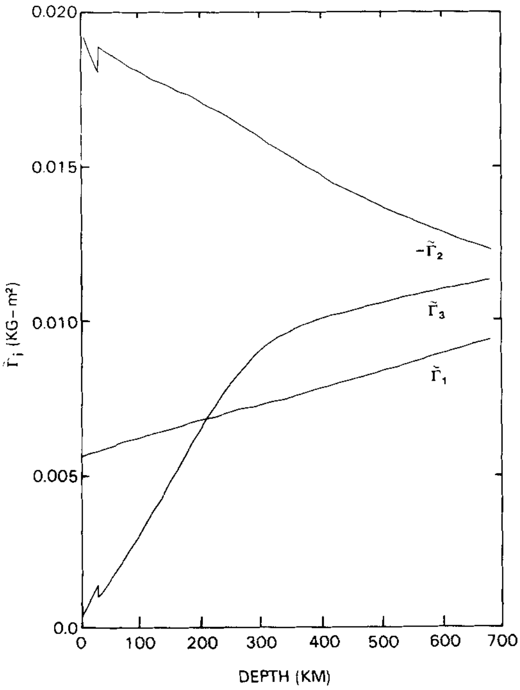

Figure 1. The values of $\tilde{\Gamma}_i(h)$ plotted here were obtained by hand digitizing an enlargement of fig. 2 of Dahlen (1973). A spline interpolate was then used to obtain the $\tilde{\Gamma}_i$ values at any desired depth h. Note that the negative of $\tilde{\Gamma}_2(h)$ is plotted here. The low amplitude variations seen in these curves were introduced by the hand-digitization procedure.

instantaneous change in the products of inertia due to an earthquake given its location and the moment tensor representation of its source.

The set of moment tensors and hypocentral parameters used in this investigation were computed by the 'centroid-moment tensor' (CMT) solution technique described in Dziewonski, Chou & Woodhouse (1981) and Dziewonski & Woodhouse (1983). Briefly, this solution technique attemps to find the best point source that fits the observed seismograms of some earthquake. The hypocentral coordinates, the origin time and the six independent components of the moment tensor are obtained by iteratively comparing the observed long-period body (and, for large events, mantle) waveforms to synthetic seismograms. The synthetic long-period waveforms are obtained by summing 5000 (for a body wave cutoff period of 45 s) normal modes computed using the PREM earth model (Dziewonski & Anderson 1981). Since the $M_{r\theta}$ and $M_{r\phi}$ elements of the moment tensor are not well-resolved for shallow events the focal depth of the best point source is constrained to be no less than 10 km. In addition, since the observed waveforms are dominated by shear energy the isotropic part of the moment tensor is not well-resolved and therefore the trace of the moment tensor is constrained to be zero: $M_{rr} + M_{\theta\theta} + M_{\phi\phi} = 0$. However, the moment tensor is not constrained to represent a double couple source mechanism.

The determination of these moment tensors is made possible by the high quality digital data obtained from the Global Digital Seismic Network (GDSN) and the IDA (International Deployment of Accelerometers) network (Agnew *et al.* 1976). The wide dynamic range of these instruments allows for moment tensor inversion of events having scalar moments M_0 ranging from as low as 10^{24} dyne cm ($m_b \approx 5.5$) to as large as 10^{30} dyne cm ($M_s \approx 8.5$). In fact, the CMT method is currently being used to obtain the best point source model routinely for all events having $M_0 \geq 10^{24}$ dyne cm (Dziewonski & Woodhouse 1983; Dziewonski *et al.* 1983c; Dziewonski, Friedman & Woodhouse 1983b; Dziewonski, Franzen & Woodhouse 1983a, 1984a, b; Giardini 1984). The data set used in this study consists of these routinely determined moment tensors and hypocentral parameters for earthquakes having magnitudes $m_b \geq 5.5$ that occurred during 1981–1983 (202 during 1981, 308 during 1982, 428 during 1983). During 1977–1980 deep ($h > 100$ km) earthquakes having magnitude $m_b \geq 5.5$ and shallow ($h \leq 100$ km) earthquakes of magnitude $m_b \geq 6.5$ were also included, making a total of 1287 events for which moment tensors and hypocentral parameters are available. The 1287 solutions for the moment tensor, hypocentral location and origin time of earthquakes that occurred during 1977–1983 are used here to calculate directly the effect of these earthquakes on the excitation of the Chandler wobble.

3 Results

The change in the products of inertia Δc_{xz} and Δc_{yz} due to the static deformation field generated by any earthquake is computed by equations (2.13). The $\tilde{\Gamma}_i(h)$ values were obtained by hand-digitizing an enlarged version of fig. 2 of Dahlen (1973). A cubic spline function was then fitted to the digitized values in order to automate the process of obtaining the $\tilde{\Gamma}_i$ values at any depth h (Fig. 1). The change in the Chandler wobble's excitation function caused by the corresponding change in the products of inertia is computed by equation (2.6) where $C-A$ was taken to be 2.61×10^{35} kg m^2. Note that the computed earthquake excitation function does not depend upon any assumed value for the Q of the Chandler wobble. A value for the Q of the Chandler wobble needs to be specified only when the excitation function is finally convolved with the Earth's impulse response in order to generate the Chandler wobble time series.

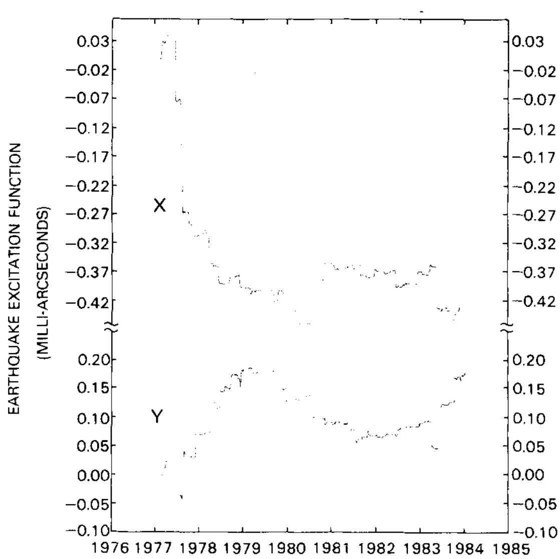

Figure 2. The predicted cumulative effect of 1287 earthquakes on the x- and y-components of the Chandler wobble's excitation function during 1977–1983. Between earthquakes the excitation function is assumed to remain unchanged. Note that the ordinate has units of milli-arcseconds.

Table 1. The origin time, location and size of the 10 earthquakes that have affected the Chandler wobble the most during 1977–1983. The origin time and the hypocentral parameters are those given by the 'centroid-moment tensor' solution. The scalar moment M_0 is the average of the absolute values of the two dominant eigenvalues of the moment tensor. The static total moment M_T^0 is that obtained by Silver & Jordan (1983) and is shown for comparison. The body-wave m_b and surface-wave M_S magnitudes are those determined by the USGS. The calculated amplitude and phase of the shift in the position of the Chandler wobble's excitation pole due to these earthquakes is also shown.

No	origin time		latitude	longitude	depth	M_0	M_T^0	m_b	M_S	amplitude	phase
	mm/dd/yy	hh:mm:ss.s	°N	°E	km	10^{27} dyne-cm				arc-sec	°E
1	08/19/77	06:08:93.1	-11.14	118.23	23.3	35.9	24.	7.0	7.9	0.000212	157
2	06/22/77	12:08:87.3	-22.52	-175.64	59.1	14.3	23.	6.8		0.000129	-164
3	05/26/83	03:00:18.3	40.44	138.87	12.6	4.55		6.8	7.7	0.000101	135
4	07/17/80	19:42:63.1	-12.44	165.94	34.0	4.84		5.8	7.9	0.000049	-31
5	10/04/83	18:52:37.8	-26.01	-70.56	38.7	3.38		6.4	7.3	0.000045	110
6	11/23/77	09:26:48.4	-31.22	-67.69	20.8	1.86		6.3	7.4	0.000044	114
7	06/12/78	08:14:45.5	38.10	142.14	43.4	2.04	2.1	6.8	7.7	0.000037	132
8	03/18/83	09:05:66.2	-4.86	153.34	69.9	4.63		6.4	7.8	0.000037	-81
9	12/06/78	14:02:35.5	44.74	145.82	181.0	6.40	3.6	6.7		0.000036	125
10	12/12/79	07:59:67.0	2.32	-78.81	19.7	16.9	25.	6.4	7.7	0.000036	-140

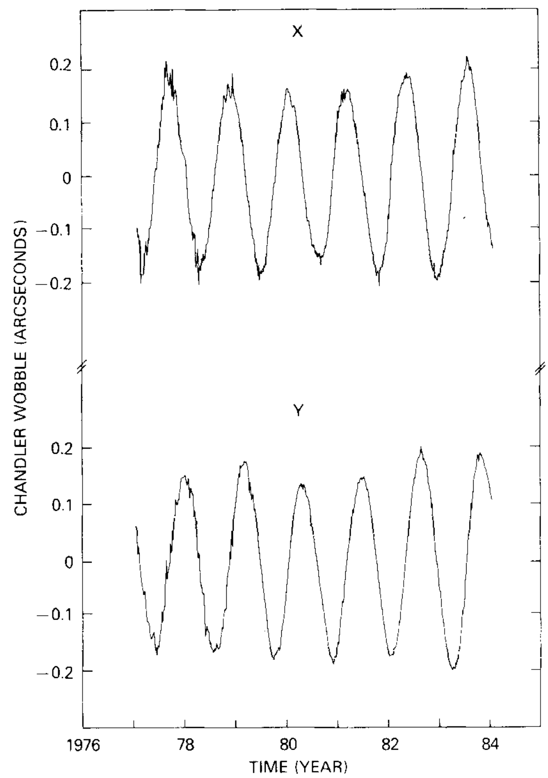

Figure 3. The x- and y-components of the Chandler wobble obtained by removing a mean, a linear trend and the annual wobble from the *LAGEOS* polar motion observations.

The result for the 1287 earthquakes for which moment tensors and hypocentral parameters are available is shown in Fig. 2 (before the time of the first earthquake the excitation pole was assumed to be at rest at the origin of the coordinate system). Note that the scale on this plot is in milli-arcseconds. The peak-to-peak amplitude variation seen in this figure is some 0.0002 arcsec (not including the two large earthquakes that occurred in 1977). Dahlen (1973) obtained shifts of amplitude 0.01 and 0.007 arcsec for the 1960 Chilean and the 1964 Alaskan earthquakes, respectively. Thus, the cumulative effect on the Chandler wobble's excitation function of 1287 earthquakes that occurred during 1977–1983 is more than an order of magnitude smaller than the individual effect of the Chilean or Alaskan earthquakes. Table 1 lists the earthquakes that have produced the largest change (computed here) in the Chandler wobble's excitation function. The largest shift (of amplitude 0.000212 arcsec) was due to the largest earthquake that occurred during 1977–1983: the great Sumba, Indonesia earthquake of 1977 August 19.

Shown in Fig. 3 is the Chandler wobble time series obtained by removing a mean, a trend and an annual term from the observations of the polar motion obtained by laser ranging to the *LAGEOS* satellite during 1977–1983 (Gross & Chao 1985). It is clearly evident that during this time period the amplitude of the Chandler wobble has changed. This implies that some excitation mechanism has acted during this time period in order to bring about this change.

Fig. 4 shows the excitation function that is required to explain the amplitude and phase changes that have taken place in the observed Chandler wobble during 1977–1983. The excitation function shown here was obtained by Gross & Chao (1985) by deconvolving a

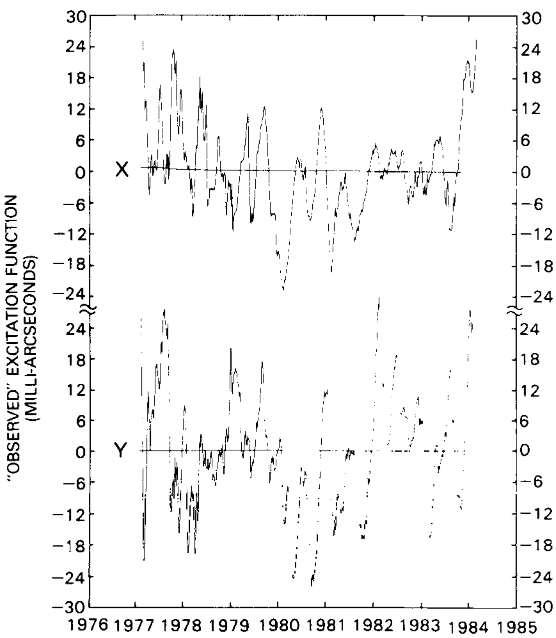

Figure 4. The x- and y-components of the Chandler wobble's excitation function obtained by deconvolving a Chandler wobble time series derived from *LAGEOS* polar motion data (Gross & Chao 1985). The nearly horizontal traces represent, to scale, the theoretical earthquake excitation function reproduced from Fig. 2.

Chandler wobble time series obtained from *LAGEOS* polar motion data. Their deconvolution method contained a parameter used, in practice, to reduce the level of noise in the output of the deconvolution filter and the result shown in Fig. 4 corresponds to setting this damping parameter αN to 0.5. This is the value that they considered achieved a reasonable tradeoff between the competing requirements of producing a well-resolved yet error-free result. They showed that this excitation function is able to reproduce the input observed Chandler wobble time series apart from the noise contained therein. As can be seen the peak-to-peak amplitude variations of this 'observed' excitation function are about 0.03 arcsec or some 150 times larger than the variations seen in the theoretical earthquake excitation function (Fig. 2). For comparison, this theoretical earthquake excitation function is superimposed on the plot of the 'observed' excitation function and it is clearly seen that on this scale the earthquake excitation function is essentially constant.

A consequence of equation (2.1) is that the only spectral components of the excitation function that exert any influence on the Chandler wobble are those near the Chandler frequency (see, e.g. Gross & Chao 1985). In the frequency domain equation (2.1) becomes:

$$M(\omega) = G(\omega)\,\Psi(\omega) + N(\omega) \tag{3.1}$$

where $M(\omega)$ and $\Psi(\omega)$ are the Fourier transforms of $m(t)$ and $\psi(t)$, respectively, $N(\omega)$ represents the frequency content of the noise contained in the observations of $m(t)$ and $G(\omega)$ is the Earth's transfer function defined by:

$$G(\omega) = \frac{\sigma_0}{\sigma_0 - \omega}. \tag{3.2}$$

As can be seen $G(\omega)$ has the functional form of a spectral peak of width (half-power, full bandwidth) ω_0/Q. The excitation function $\Psi(\omega)$ is passed through the Earth's transfer function $G(\omega)$ in order to generate the Chandler wobble $M(\omega)$. Since $G(\omega)$ is highly peaked at the Chandler frequency ω_0 only those frequency components of $\Psi(\omega)$ near ω_0 will have an appreciable effect on the Chandler wobble. Therefore in order to isolate the Chandler band the comparison between the 'observed' and theoretical excitation functions should in

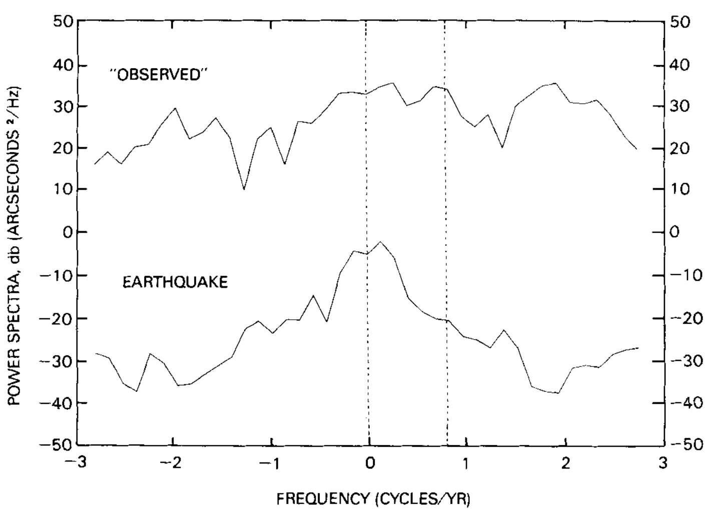

Figure 5. The power spectra of the theoretical earthquake excitation function and the Chandler wobble's 'observed' excitation function between −3 and +3 cpy. The dashed lines indicate frequencies at 0.0 cpy and at the Chandler frequency (assuming a period of 434.1 day, Wilson & Haubrich 1976).

general be done in the frequency domain. A complete comparison in the frequency domain requires the computation of not only the power spectra but also the phase spectrum and the coherence between the two excitation functions in the Chandler band. Fig. 5 shows the power spectra between -3 and $+3$ cycle yr^{-1} (cpy) of the 'observed' and theoretical excitation functions. The available frequency resolution is readily apparent in these plots of the power spectra. With only 7 years of data the interval between points in the frequency domain is 0.14 cpy which is larger than the width of the Chandler band (usually taken to be of width 0.1 cpy between 0.8 and 0.9 cpy, e.g. Wilson & Haubrich 1976; Wahr 1983). A Hann window (a three-point frequency domain smoother) was used in computing these power spectra which further degrades the frequency resolution. This lack of frequency resolution will seriously restrict the comparison of the two excitation functions in the frequency domain. However, keeping in mind the limitations imposed by the lack of frequency resolution, it is seen that near the Chandler frequency the power of the theoretical earthquake excitation function is about 56 dB less than the power needed to maintain the observed Chandler wobble.

The coherence R_{12}^2 between two time series is essentially a measure of their correlation as a function of frequency and is estimated here by:

$$R_{12}^2(j) = \frac{\left\| \sum_{k=j-p}^{j+p} X_1^*(k)\, X_2(k) \right\|^2}{\sum_{k=j-p}^{j+p} \left\| X_1(k) \right\|^2 \sum_{k=j-p}^{j+p} \left\| X_2(k) \right\|^2} \tag{3.3}$$

where the * denotes complex conjugation, $X_1(k)$ is the value at the kth frequency of the Fourier transform of the first complex-valued time series and $X_2(k)$ is the corresponding quantity for the second complex-valued time series. It is seen that the coherence estimate $R_{12}^2(j)$ can take on values between 0 and 1 and is a measure of the average coherence in a band of width $(2p + 1)/N\Delta$ where N is the number of data points in the time series and Δ is the spacing between the data points in the time domain. Note that if $p = 0$ the coherence is identically 1 at all frequencies. Smoothing (non-zero values of p) is necessary in order to avoid this degeneracy of the sample coherence estimates. By letting $p = 1$ (thereby averaging over three elemental frequency intervals) the average coherence in a band of width 0.43 cpy is obtained in the present case. This is more than 4 times the width of the Chandler band (assuming it is of width 0.1 cpy) making it impossible to resolve the coherence between the 'observed' and synthetic earthquake excitation functions in the Chandler band. However, for the sake of completeness Fig. 6 (top) shows the coherence of these two excitation functions between the frequencies -3 and $+3$ cpy that is obtained by setting $p = 1$. The horizontal dashed line is the 95 per cent confidence limit for the coherence. Values less than this limit are not significantly different (at the 95 per cent confidence level) from zero. This confidence limit is obtained by the formula (Brillinger 1975, p. 317):

$$\sigma_\alpha^2 = 1 - (1 - \alpha)^{2/\nu - 2} \tag{3.4}$$

where σ_α^2 is the 100α per cent confidence limit and ν is the number of degrees of freedom of the spectral window (in the present case this was taken to be twice the number of elemental frequency intervals being averaged over, or $\nu = 6$). As can be seen in Fig. 6 the coherence near the Chandler frequency is small but as explained above this result is of questionable usefulness. It makes no sense to assign to the Chandler band a value for the coherence that has been averaged over a band of width 0.43 cpy when the Chandler band itself has a width of at most 0.1 cpy. Therefore the coherence result shown in Fig. 6 is of limited usefulness in deciding whether or not the 'observed' and synthetic earthquake excitation functions are correlated at frequencies near the Chandler frequency.

The phase spectrum is a measure of the amount the frequency components of one time series lags (or leads) the frequency components of another time series and is estimated by:

$$\phi_{12}(j) = \tan^{-1} \frac{\mathrm{Im}[X_1(j)/X_2(j)]}{\mathrm{Re}[X_1(j)/X_2(j)]} \tag{3.5}$$

where j denotes the jth frequency value, Re[ ] denotes the real part of the bracketed complex-valued quantity and Im[ ] denotes the imaginary part of the bracketed complex-valued quantity. Fig. 6 (bottom) shows the phase spectrum of the 'observed' and theoretical earthquake excitation functions between -3 and $+3$ cpy. The frequency resolution problem in computing the phase spectrum is not quite as severe as that encountered in computing the coherence since the phase spectrum can be obtained in bands of width $1/N\Delta$, or 0.14 cpy. As seen in Fig. 6 the value of the phase spectrum near the Chandler frequency is about -40° (indicating that the earthquake excitation function leads the 'observed' excitation function by about 40°) but due to the limited amount of frequency resolution and the large amount of fluctuation evident in the values of the phase spectrum it is difficult to judge the importance of this result.

In summary, then, with only 7 years of data available for analysis it is not possible to resolve the Chandler band and determine whether or not the theoretical earthquake

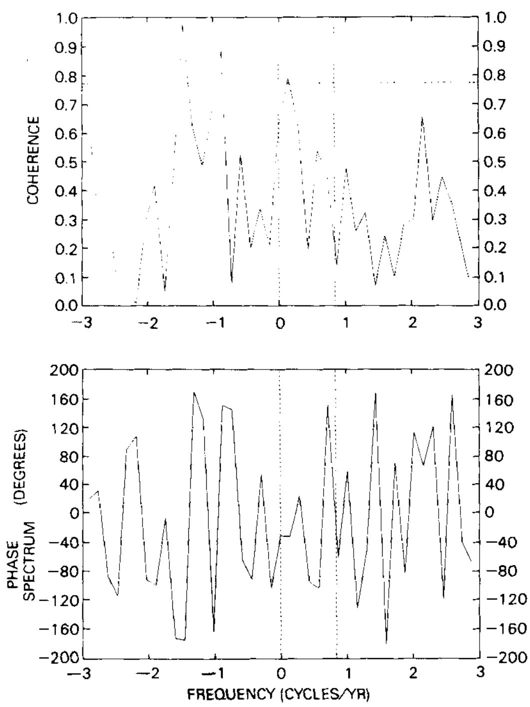

Figure 6. The coherence (top) and phase spectrum (bottom) of the theoretical earthquake and 'observed' excitation functions between -3 and $+3$ cpy. The vertical dashed lines indicate frequencies of 0.0 cpy and the Chandler frequency. The horizontal dashed line (at a coherence value of 0.776) represents the 95 per cent confidence limit for the coherence estimates. Positive values for the phase (in degrees) implies that the earthquake excitation function lags the 'observed' excitation function by that amount.

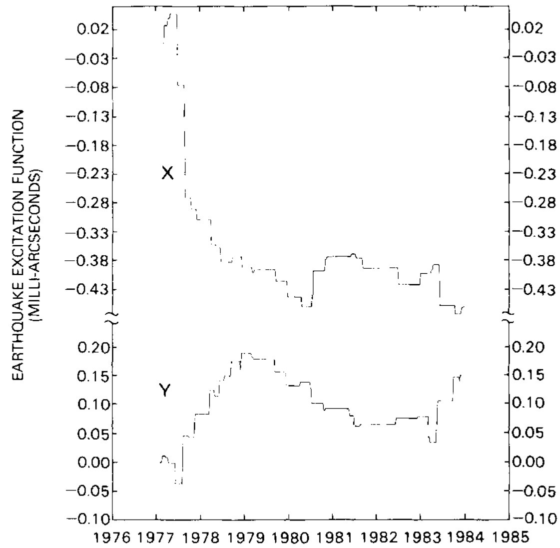

Figure 7. The predicted cumulative effect on the x- and y-components of the Chandler wobble's excitation function of 37 earthquakes having moments $M_0 \geqslant 10^{27}$ dyne cm that occurred during 1977–1983.

excitation function computed here is coherent and in phase (in the Chandler band) with the Chandler wobble's excitation function that is obtained from observations. However, since the synthetic earthquake excitation function clearly lacks the power to maintain the Chandler wobble during 1977–1983 I conclude that earthquakes, via the static deformation field generated by them, have had a negligible influence on the Chandler wobble during 1977–1983.

4 Discussion

The conclusion that earthquakes have had a negligible influence on the excitation of the Chandler wobble during 1977–1983 depends upon the completeness of the earthquake data set used, the accuracy of the moment tensors to reflect the actual source mechanisms of the earthquakes and the adequacy of the static deformation field generated by the earthquakes to reflect the actual mass redistribution that takes place within the Earth. In this study a direct calculation of the influence on the Chandler wobble of 1287 earthquakes that occurred during 1977–1983 was performed. All earthquakes having body wave magnitude $m_b \gtrsim 6.5$ that occurred during this period were included. Deep earthquakes that occurred during 1977–1980 and earthquakes occurring at all depths during 1981–1983 with magnitudes in the range $5.5 \lesssim m_b \lesssim 6.5$ were also included. Neglecting all earthquakes of magnitude less than 5.5 as well as the shallow earthquakes of magnitude less than 6.5 that occurred during 1977–1980 does not materially affect the conclusions drawn from this study. The amplitude of the change in the position of the excitation pole due to some earthquake is linearly proportional to the scalar moment M_0 (the 'size' of the moment tensor) of the earthquake. By averaging over the effects due to the earthquake location and fault plane orientation on the change induced in the products of inertia due to an earthquake (Dahlen 1973, table 3) it is found that, on average and neglecting differences due to earthquake type, an earthquake will change the position of the excitation pole by roughly an amount:

$$<\|\psi\|> \approx 10^{-32} M_0 \tag{4.1}$$

where M_0 is given in dyne cm and the average shift amplitude $<\|\psi\|>$ is given in arc seconds. For an earthquake of magnitude 5.5 ($M_0 \approx 10^{24}$ dyne cm) this amounts to 10^{-8} arcsec. For an earthquake of magnitude 6.5 ($M_0 \approx 10^{27}$ dyne cm) it amounts to 10^{-5} arcsec. Thus on average a magnitude 5.5 earthquake changes the location of the excitation pole roughly three orders of magnitude less than will a magnitude 6.5 earthquake. This is simply a reflection of the well-known argument that only the largest magnitude earthquakes will be able to influence the Chandler wobble to an observable level.

Although it is true that there are many more magnitude 5.5 earthquakes occurring each year than magnitude 6.5 earthquakes, the phase of the shift in the position of the excitation pole due to these earthquakes will be (from a statistical viewpoint) randomly determined. Earthquake excitation of the Chandler wobble behaves like red noise meaning that the path of the excitation pole due to the cumulative effect of the earthquakes will be a two-dimensional random walk. Thus even though there are many more smaller magnitude earthquakes than larger magnitude earthquakes the cumulative effect of the numerous smaller earthquakes is much less than the effect of the larger earthquakes on the position of the excitation pole. This is illustrated in Fig. 7 where the cumulative change in the excitation function due to the 37 earthquakes that occurred during 1977–1983 having moments $M_0 \geqslant 10^{27}$ dyne cm ($m_b \geqslant 6.5$) is plotted. As can be seen by comparing this result to the excitation function resulting from the entire set of 1287 earthquakes (Fig. 2) the numerous smaller events have not materially affected the path of the excitation pole. The position of the excitation pole at any given time t_0 is due primarily to the largest earthquakes that occurred previous to t_0. The numerous smaller earthquakes that occur only add finer-scale detail to the excitation pole path.

Since the moment tensors used in this study are obtained by comparing the observed waveforms generated by the earthquakes to synthetically generated waveforms any influence, such as lateral heterogeneity, that affects the observed waveform but is not included in calculating the synthetic waveform will cause the derived moment tensors to be in error. By comparing the moment tensor solutions obtained by the CMT technique to those obtained by other investigators using different techniques the reliability and accuracy of the CMT solutions can be gauged. Giardini (1984) has compared the CMT solutions for some larger events to the moment tensors obtained by other investigators and concludes that the results show good agreement between the different methods used. He also compared the focal plane mechanisms obtained from the CMT solutions with precisely determined focal mechanisms published by others and again concludes that the CMT technique generates reliable solutions. Dziewonski *et al.* (1981) found the results of the CMT technique for 14 earthquakes to be highly consistent with those obtained by other investigators using different methods and data sources. Dziewonski *et al.* (1983c) gives further evidence that the solutions obtained by the CMT method are accurate and reliable. Ward (1983) computed the moment tensors of 11 plate-bending earthquakes using an algorithm especially suited for shallow earthquakes and compared his results to those obtained by the CMT technique. He finds the tension and compression axes usually agree within 15° in azimuth and 5° in dip. The scalar moments he obtained averaged only 18 per cent larger than those obtained via the CMT solutions. Again, the CMT solutions are found to be reliable and accurate. Table 1 gives the static (zero-frequency) moment obtained by Silver & Jordan (1983) for five of the 10 events found here to cause the greatest displacements of the excitation pole. The CMT method yields a solution corresponding to the source spectrum at zero frequency if the assumed source duration time is accurate (Giardini 1984). Thus the scalar moment M_0 obtained from the CMT solution (defined as the average of the absolute values of the two dominant eigenvalues of the moment tensor) and the static total moment M_T^0 can be directly compared with each other. As can be seen they are within a factor of 2 of each other. If we take this factor of 2 as an estimate of the accuracy of the CMT solution for large events then we conclude that any errors in these solutions cannot overcome the discrepancy noted between the 'observed' and theoretical earthquake excitation functions. Even if all of the CMT solutions had moments underestimated by a factor of 2 the power spectrum of the earthquake excitation function would only be increased by 6 dB – not enough to overcome the 56 dB discrepancy between the 'observed' and earthquake excitation functions.

Of greater concern is the assumption that the static displacement field generated by the earthquake accurately reflects all of the mass redistribution that occurs. It has been suggested that large aseismic mass movements can occur near the time of an earthquake (Kanamori & Cipar 1974). Spence (1986) has concluded that the Sumba, Indonesian event of 1977 August 19, was due to slab-pull forces and was a decoupling event. He suggests that the subducted slab moved about 8 m in its downdip direction after the main shock. Gross & Chao (1985) speculate that this motion of the subducted slab is responsible for a step-like change in the 'observed' Chandler wobble's excitation function (Fig. 4) that occurred during the 20 days following the main Sumba shock. Aseismic mass movement of this kind is not accounted for in the calculation of the static displacement field. Any type of aseismic slip or fault creep that occurs on a time-scale much less than the period of the Chandler wobble is capable of exciting the Chandler wobble. It is currently an open question as to the degree that these aseismic motions influence the Chandler wobble. In this study it is only found that the static deformation field (representing only one type of mass redistribution associated with earthquakes) cannot be responsible for maintaining the Chandler wobble during 1977–1983.

At first sight it might appear surprising that earthquakes have had such a small influence upon the Chandler wobble during 1977–1983. Dahlen (1973) concluded that earthquakes can contribute no more than about 10 per cent (for a Chandler wobble Q of 100) to the power of the Chandler wobble by studying 1201 earthquakes of magnitude 7.0 or larger that occurred during 1904–1964. The present study does not invalidate this conclusion. The reason that earthquakes have played such a minor role in the Chandler wobble's excitation during 1977–1983 is that no really large earthquakes have occurred during this time period. The largest (in terms of moment) earthquake to have occurred was the Indonesian event of 1977 August 19 having a magnitude $m_b = 7.0$. This was the only earthquake of body-wave magnitude 7.0 or larger to have occurred during 1977–1983. As argued above, only the largest magnitude earthquakes will be able to influence the Chandler wobble to an observable level. When an earthquake as large as the 1960 Chilean or the 1964 Alaskan event next occurs it should produce a change (of amplitude about 0.01 arcsec) in the Chandler wobble's excitation function that should be able to be detected in the excitation functions determined from the polar motion time series obtained by the modern laser-ranging or VLBI techniques.

5 Conclusions

A direct calculation of the effect on the Chandler wobble of 1287 earthquakes that occurred during 1977–1983 was made in this study. Unfortunately, with only 7 years of data available it is not possible to resolve the Chandler band and determine if the synthetic earthquake excitation function is coherent and in phase with the 'observed' excitation function in this band. However, since the power spectrum of this theoretically derived earthquake excitation function is 56 dB less (at frequencies near the Chandler frequency) than the power spectrum of the excitation function derived from observations of the Chandler wobble I conclude that earthquakes (via the static deformation field directly generated by them) have played a negligible role in exciting the Chandler wobble during 1977–1983. Earthquakes cannot be the mechanism responsible for causing the 'observed' excitation function to continuously vary during 1977–1983 (see Fig. 4). The reason that earthquakes have played such a minor role in exciting the Chandler wobble during 1977–1983 is because there have occurred no really large earthquakes during this time period. A shift of amplitude 0.01 arcsec in the position of the excitation pole should be readily detectable in 'observed' excitation functions derived from polar motion time series determined by the modern laser-ranging and VLBI techniques (see Fig. 4). Since an earthquake as energetic as the 1960 Chilean or 1964 Alaskan event should cause the excitation pole to shift by this amount, the effect on the Chandler wobble of the next such earthquake should be observable.

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