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The Influence of Formation Material Properties on the Response of Water Levels in Wells to Earth Tides and Atmospheric Loading

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The water level in an open well can change in response to deformation of the surrounding material, either because of applied strains (tidal or tectonic) or surface loading by atmospheric pressure changes. Under conditions of no vertical fluid flow and negligible well bore storage (static-confined conditions), the sensitivities to these effects depend on the elastic properties and porosity which characterize the surrounding medium. For a poroelastic medium, high sensitivity to applied areal strains occurs for low porosity, while high sensitivity to atmospheric loading occurs for high porosity; both increase with decreasing compressibility of the solid matrix. These material properties also influence vertical fluid flow induced by areally extensive deformation and can be used to define two types of hydraulic diffusivity which govern pressure diffusion, one for applied strain and one for surface loading. The hydraulic diffusivity which governs pressure diffusion in response to surface loading is slightly smaller than that which governs fluid flow in response to applied strain. Given the static-confined response of a water well to atmospheric loading and Earth tides, the in situ drained matrix compressibility and porosity (and hence the one-dimensional specific storage) can be estimated. Analysis of the static-confined response of five wells to atmospheric loading and Earth tides gives generally reasonable estimates for material properties.

INTRODUCTION

Fluctuations in water level due to atmospheric loading, Earth tides, and seismic events have long been noted in many wells. These fluctuations are of interest to geophysicists and hydrologists for two reasons: they indicate that water wells can be sensitive indicators of crustal strain, and they provide information about the material properties of the rock that the wells tap. When the response of the water level in a well to areally extensive deformation occurs under conditions where neither well bore storage or water table drainage are important, water level changes directly reflect the undrained response of the formation. Following hydrologic convention, we define water level changes under these conditions as the static-confined response. This response will not always be observed in a well. The response to high-frequency deformation may be influenced by well bore storage; the response to low-frequency deformation may be influenced by drainage to the water table. However, if the static-confined response can be found, it is a useful geophysical and hydrologic parameter. The static-confined response generally represents the maximum sensitivity of a well to aseismic strain. It is also representative of the elastic properties and porosity of the formation around the well.

Many workers have given theories for the static response of wells to atmospheric loading and Earth tides under confined conditions. *Jacob* [1940] recognized that the undrained response of rock to atmospheric loading depended on the formation's elastic properties and porosity. *Bredehoeft* [1967] noted that the undrained response to Earth tides was proportional to the formation's response to atmospheric loading. Relations between areally extensive deformation and formation response were also developed by *Robinson and Bell* [1971] and *Rhoads and Robinson* [1979]. *Van der Kamp and Gale* [1983] extended the results of *Jacob* [1940] and *Bredehoeft* [1967] to allow for grain compressibility.

The potential of water wells as strain meters has been discussed in detail [*Bredehoeft*, 1967; *Bodvarsson*, 1970; *Rojstaczer*, 1988*a*], and some attempts have been made to use the response of the water level in a well to known strains for calibration. *Johnson et al.* [1973, 1974] used the response of a well near the San Andreas Fault to atmospheric loading to calibrate its response to creep events. *Sterling and Smets* [1971] quantified the response of a well in Belgium to atmospheric loading and Earth tides and described its behavior as a strain seismograph. *Bower and Heaton* [1978] calibrated a well near Ottawa, Canada, using the local Earth tide and noted that its coseismic response to the great Alaskan earthquake of 1964 could not be explained on the basis of the static strain field produced by the earthquake.

Much work has focused on using the static-confined response of wells to atmospheric loading and Earth tides to determine material properties of the formation around the well. *Bredehoeft* [1967] showed that it was possible to estimate formation compressibility and porosity from the undrained response to atmospheric loading and Earth tides; while his results have been criticized [*Narasimhan et al.*, 1984], investigations by *Van der Kamp and Gale* [1983] and *Hsieh et al.* [1988] reaffirm their correctness. Analyses similar to that of *Bredehoeft* [1967] have been made by *Robinson and Bell* [1971], *Marine* [1975], *Rhoads and Robinson* [1979], and *Hanson* [1980], all in an effort to determine formation elastic properties and/or porosity.

Several assumptions are commonly made in analyzing the response of wells to atmospheric loading and Earth tides.

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One is that the observed response is independent of frequency and reflects the static-confined response of the formation. If fluid flow influences the response, this assumption can lead to a severe underestimation of the undrained sensitivity of the formation to strain [*Rojstaczer*, 1988*a*, *b*; S. Rojstaczer and F. Riley, The influence of vertical fluid flow on the response of the water level in a well to atmospheric loading under unconfined conditions, submitted to *Water Resources Research*, 1988] (hereinafter referred to as (RR, 1988)). A second assumption is that the matrix compressibility is much greater than the solids compressibility; while this assumption is appropriate for the response of unconsolidated materials, it is likely to be inappropriate for most rock [*Van der Kamp and Gale*, 1983]. A third assumption is that uniform surface loading causes only vertical deformation, but this is not generally appropriate [*McGarr*, 1988].

In this paper we remove the second and third assumptions and find the static response of wells (under confined conditions) to Earth tides, tectonic strain, and atmospheric loading in the framework of the *Biot* [1941] theory of poroelasticity. We give a simpler derivation of the results of *Van der Kamp and Gale* [1983] which describe the response of wells to Earth tides. We also amend their result on the response to surface loading by including the influence of horizontal deformation through a simple model which describes the areal strain produced by a uniform load over an elastic half-space [*Love*, 1929; *Farrell*, 1972]. When horizontal deformation due to surface loading is included in the analysis, we find that the elastic parameter which governs vertical pressure diffusion in response to areally extensive strain (the hydraulic diffusivity) differs for atmospheric loading and applied strains. We apply our results to the measured response of five wells to strain, and after correcting for the influence of fluid flow obtain in situ estimates of drained matrix compressibility, porosity, and specific storage.

Theoretical Static-Confined Response: Implications for Well Sensitivity to Applied Strains and Loads

In this section we describe the static-confined response of a well to applied strains and loads; to do so, we make several idealizations:

1. The intake of the well is assumed to penetrate a porous elastic medium with uniform properties. These properties are those specified by the theory of *Biot* [1941] (as reexpressed by *Rice and Cleary* [1976] and *Green and Wang* [1986]), namely the compressibilities of the solid phase β_u, the fluid phase β_f, and the porous matrix when drained of fluid β, together with the Poisson's ratio ν of the matrix, the porosity ϕ, and the permeability κ.

2. The response is confined, meaning that no vertical flow takes place between the fluid around the well intake and the water table above. When considering the effects of atmospheric pressure, we also neglect the force that such pressure changes exert directly on the water table at low frequencies [*Yusa*, 1969; *Weeks*, 1979; *Rojstaczer*, 1988*b*].

3. We also neglect the influence of any flow between the medium and the well bore, so that the height in the well is a proxy for the pore pressure of the fluid. This quasi-static analysis is a good approximation if we assume that the well is open but that the bore is so narrow that little flow is needed to cause height changes in it. In practice the height in the well may differ from the equivalent head of pore fluid because of the time required for fluid to diffuse through the medium and drain into the well; such time-dependent well bore storage effects have been discussed by *Cooper et al.* [1965], *Bredehoeft* [1967], *Hsieh et al.* [1987], and *Rojstaczer* [1988*b*].

The stress-strain relationship for a poroelastic medium may be written as [*Rice and Cleary*, 1976]

$$\varepsilon_{ij} = \frac{\beta}{3}\left(\frac{1+\nu}{1-2\nu}\sigma_{ij} - \frac{\nu}{1-2\nu}\sigma_{kk}\delta_{ij} + \alpha p \delta_{ij}\right) \tag{1}$$

where repeated subscripts imply summation. Here ε_{ij} and σ_{ij} are the macroscopic strain and stress (reckoned positive for extension), p is the pressure of the pore fluid (reckoned positive for compression), and α is an additional constant of the medium, which under many conditions can be taken to be [*Nur and Byerlee*, 1971]

$$\alpha = 1 - \beta_u/\beta \tag{2}$$

Under undrained conditions (no flow takes place) we also have the relation between stress and fluid pressure,

$$p = -B\sigma_{kk}/3 \tag{3}$$

where B, often known as "Skempton's coefficient," is given by

$$B = \frac{(\beta - \beta_u)}{(\beta - \beta_u) + \phi(\beta_f - \beta_u)} \tag{4}$$

As noted by *Rice and Cleary* [1976], this definition for B assumes that the rock matrix is homogeneous and all the pore space is interconnected. We may now use (1) and (3) to get the relationship between p (which is related to the water height in the well) and the variable of interest under different conditions.

Volumetric Strain

For this case we wish to get the relationship between p and the volume strain $\varepsilon_v = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$. The use of (1) and (3) yields the simple form

$$\varepsilon_v = -\bar{\beta}p/B \tag{5}$$

where $\bar{\beta}$ is the undrained compressibility of the formation

$$\bar{\beta} = \beta(1 - B\alpha) \tag{6}$$

The increase of water depth in the well is given by $w = -p/\rho g$, where ρ is the water density and g the acceleration due to gravity. The coefficient of response for water depth given an applied volume strain under static-confined conditions is then

$$\frac{w}{\varepsilon_v} = \frac{B}{\rho g \bar{\beta}} \tag{7}$$

Although this result is trivial to obtain and has been noted by others [e.g., *Roeloffs*, 1988], it is worthwhile to examine its consequences for well response to volumetric strain. Figure 1 shows this quantity, in millimeters of depth per 10^{-9} strain (mm/nε), over a realistic range of ϕ and β. As shown in the figure, water well sensitivity is largely independent of matrix compressibility unless this is very low (a stiff material), and in general the response is greater for low-porosity formations.

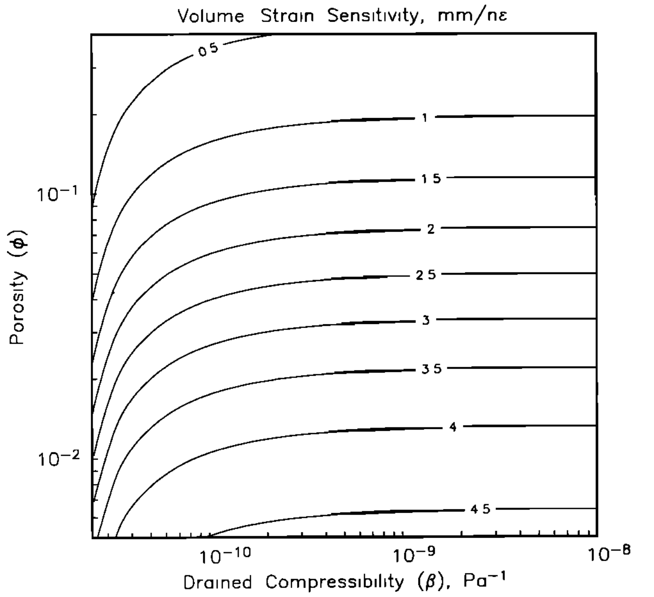

Fig. 1. Sensitivity of a well to volume strain under static-confined conditions (sensitivity is expressed in terms of millimeters change in water level depth per volume nanostrain). Sensitivity is plotted as a function of porosity and drained matrix compressibility. The compressibility of the grains, β_u, is fixed at 2×10^{-11} Pa^{-1}; for the fluid, the compressibility β_f is 4.4×10^{-10} Pa^{-1}.

Areal Strain

Strictly speaking, pore fluids respond to cubic strain. However, for strains imposed by Earth tides or tectonic events the cubic strain is not well known a priori; it is more convenient to examine the response of water levels in wells to areal strain, $\varepsilon_a = \varepsilon_{11} + \varepsilon_{22}$. At the surface of a half-space not subject to surface tractions, we must have $\sigma_{33} = 0$. This free-surface boundary condition is a good approximation for depths much less than the wavelength of the applied strain, which is tens of kilometers or more for tectonic strains and thousands of kilometers for tidal strains. Under these conditions, (1) and (3) yield

$$\varepsilon_a = -\frac{(1-\nu_u)\bar{\beta}p}{(1-2\nu_u)B} \tag{8}$$

where ν_u is the undrained Poisson's ratio of the formation [*Rice and Cleary*, 1976]

$$\nu_u = \frac{3\nu + B(1-2\nu)\alpha}{3 - B(1-2\nu)\alpha} \tag{9}$$

Equation (8) was obtained (in a different form) by *Van der Kamp and Gale* [1983]. From (8) we may define the static-confined, areal strain sensitivity A_s' as

$$A_s' = \frac{w}{\varepsilon_a} = \frac{(1-2\nu_u)B}{\rho g(1-\nu_u)\bar{\beta}} \tag{10}$$

The response of a water well to Earth tides is of particular value because the areal strains produced by Earth tides can be fairly well determined (within a factor of 2) from theoretical calculations [*Beaumont and Berger*, 1975; *Berger and Beaumont*, 1976]. The tidal response thus serves to calibrate the response to other sorts of strain (such as tectonic strain) and, as will be shown below, makes it possible to estimate some of the properties of the medium the well penetrates.

It is useful to compare the response of a well to areal strain with the response to volume strain. Figure 2 plots the contours of A_s' in the same units and over the same range of ϕ and β as in Figure 1. The response is substantially less, largely because the free-surface condition means that ε_{33} is opposite in sign to ε_a, so that the $\varepsilon_v \ll \varepsilon_a$. As before, high sensitivity is favored by low porosity; however, the sensitivity is a strong function of matrix compressibility, with high matrix compressibilities causing low sensitivity. This is because the presence of the pore fluid causes ε_{33} to approach $-\varepsilon_a$ (the undrained Poisson's ratio, ν_u, approaches 0.5 at high matrix compressibilities), so that little volume strain occurs.

Since it might be expected that deep wells would tap relatively stiff rock of low porosity, the results shown in Figure 2 are in accordance with the observations of *Roeloffs* [1988] that Earth tide sensitivity tends to increase with well depth. If we use Earth tide sensitivity as an indicator of sensitivity to tectonic strain, the implication of Figure 2 for the use of water wells as strain meters is clear: independent of any fluid flow considerations, installation of water wells for strain monitoring purposes should be done in stiff, low-porosity formations. A typical formation might have $\beta = 1 \times 10^{-10}$ Pa^{-1} and $\phi = 0.10$, giving an areal strain sensitivity of A_s' of 0.6 mm/nε, somewhat greater than that of the wells examined below.

Surface Loading

For changes in atmospheric pressure, the surface may no longer be regarded as stress free; instead of the vertical stress σ_{33} being 0, we have

$$\sigma_{33} = -p_b \tag{11}$$

where p_b is the change in barometric pressure, positive for compression. It has generally been assumed in the hydrological literature [e.g., *Jacob*, 1940] that application of a

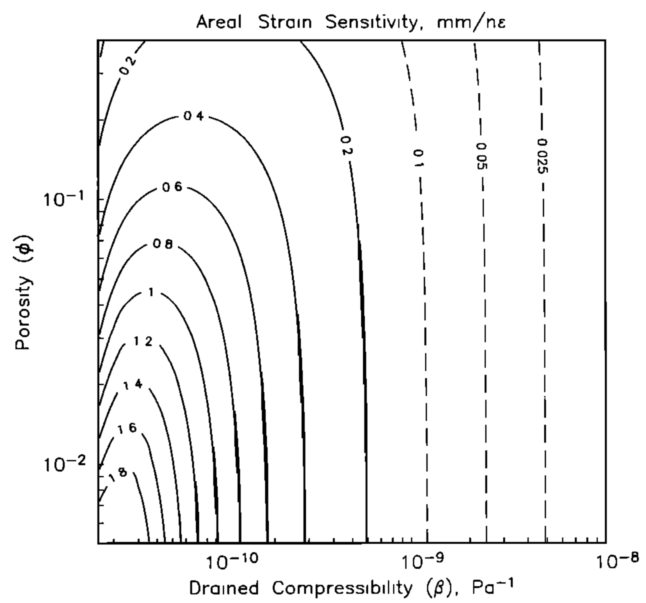

Fig. 2. Sensitivity of a well to areal strain A_s' (no vertical stress applied) under static-confined conditions. Elastic constants as for Figure 1, with Poisson's ratio $\nu = 0.25$. Dashed curves indicate change in contour interval.

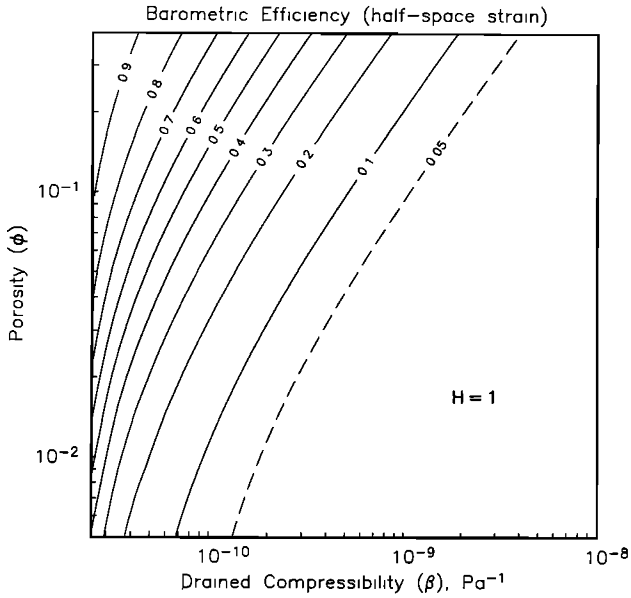

Fig. 3. Static-confined barometric efficiency (change in water depth, in terms of pressure, compared to barometric pressure change) under conditions of $\varepsilon_{33} = \varepsilon_a$ as a function of porosity and matrix compressibility. Dashed curves indicate change in contour interval. Elastic constants are the same as that given in Figure 2.

uniform pressure causes only vertical strain in the medium. However, this is not generally appropriate. A simple model to account for a change in atmospheric load (a uniform load over a homogeneous elastic half-space) indicates that the induced areal strain ε_a equals the vertical strain ε_{33} [*Love*, 1929; *Farrell*, 1972]. Of course, this model is an idealization, and in Appendix A we discuss a slightly more realistic case: loading of a layer over a homogeneous half-space. The results of Appendix A suggest that the ratio of horizontal to vertical strain in the layer, H, is roughly inversely proportional to the ratio $\bar{\beta}_1/\bar{\beta}_2$ where $\bar{\beta}_1$ and $\bar{\beta}_2$ are the undrained compressibilities of the overlying layer and the half-space, respectively. The use of (6) indicates that rocks with typical drained compressibility and porosity ($\beta \approx 1 \times 10^{-10}$ Pa^{-1} and $\phi \approx 0.1$) will have an undrained compressibility $\bar{\beta}$ of the order of 5×10^{-11} Pa^{-1}. Highly compressible and porous sediments ($\beta > 10^{-8}$ Pa^{-1} and $\phi \approx 0.3$) will have an undrained compressibility $\bar{\beta}$ approaching 2×10^{-10} Pa^{-1}. Hence even if observations of pore pressure are made in a well tapping sediment underlain by bedrock, horizontal strains produced by surface loading can be expected to be significant relative to vertical strains and influence the static-confined well response. For greater generality we assume that

$$\varepsilon_a = H\varepsilon_{33} \tag{12}$$

where H varies between 0 (the traditional assumption) and 1 (the half-space case). If we apply this relationship, together with the expression for σ_{33} (equation (11)), to (1) and (3), we find that the loading efficiency of a formation, γ, [*Van der Kamp and Gale*, 1983] or the ratio between aquifer pressure and surface load is

$$\gamma = \frac{p}{p_b} = \frac{B(1 + H)(1 + \nu_u)}{3[1 - (1 - H)\nu_u]} \tag{13}$$

The static-confined response of a well to air pressure changes is usually expressed in terms of the (dimensionless) barometric efficiency E_B':

$$E_B' = w\rho g/p_b \tag{14}$$

Since the well is assumed to be open to the atmosphere, the change in water depth, w will depend on the difference in forces between the pore fluid pressure and the atmospheric pressure, so that

$$w = (p_b - p)/\rho g \tag{15}$$

Combining these equations gives

$$E_B' = 1 - \gamma \tag{16}$$

which for $H = 1$ implies

$$E_B' = 1 - \gamma_1 \tag{17a}$$

$$\gamma_1 = 2B(1 + \nu_u)/3 \tag{17b}$$

somewhat different than the result of *Van der Kamp and Gale* [1983], who assumed $H = 0$. Figures 3 and 4 show the quasi-static barometric efficiency of a well under confined conditions as a function of compressibility and porosity, for $H = 1$ and 0. The responses for $H = 1$ and $H = 0$ are qualitatively very similar. Including the effects of horizontal deformation causes the contours to essentially shift to lower compressibilities and higher porosities. The response to changes in atmospheric loading, like the response to areal strain, depends strongly on matrix compressibility, with low compressibility favoring high sensitivity; unlike the response to volume or areal strain, a large water level response to atmospheric loading is favored by high porosity. The positive correlation between porosity and static-confined barometric efficiency can be explained by the relatively small pore volume strains that take place in high porosity materials subject to deformation (all elastic properties taken to be

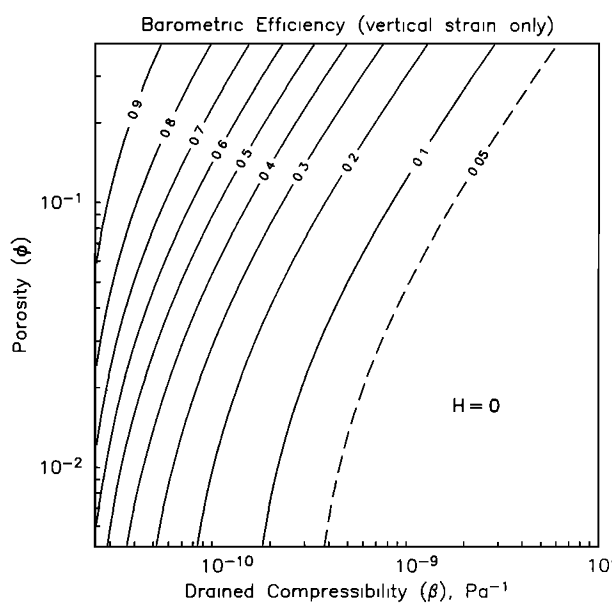

Fig. 4. Static-confined barometric efficiency under conditions of $\varepsilon_a = 0$ as a function of porosity and matrix compressibility. Dashed curves indicate change in contour interval. Elastic constants are the same as that given in Figure 2.

TABLE 1. Description of Wells

Well Identification	Location (in California)	Open Interval, m	Rock Type
GD	Parkfield	18–88	granodiorite
TF	Parkfield	152–177	marine sediments
JC	Parkfield	147–153	diatomaceous sandstone and siltstone
SC2	Mammoth Lakes	66–70	fractured basalt
LKT	Mammoth Lakes	152–296	largely rhyolite

equal). For formations with drained matrix compressibility exceeding about 3×10^{-10} Pa^{-1}, the barometric efficiency is so low that the response to pressure changes may be difficult to detect; since Earth tide sensitivities are also low for this case, it will probably be difficult to then determine the properties of the formation. This problem is addressed in further detail elsewhere [*Hsieh et al.*, 1988].

DETERMINATION OF MATERIAL PROPERTIES FROM THE STATIC-CONFINED RESPONSE OF A WELL

As has been noted by others [*Bredehoeft*, 1967; *Hsieh et al.*, 1988], it is in theory possible to estimate the elastic properties and porosity of a formation if the static-confined response of a well to atmospheric loading and Earth tides can be found. In the analysis of *Bredehoeft* [1967], the solids compressibility β_u was assumed to be zero, and atmospheric loading was assumed to induce only vertical strain. The matrix compressibility and porosity of the formation could then be found from the static-confined response given the Poisson's ratio. Because we have included more material parameters, more assumptions must be made to perform the same analysis using the formulas given here.

From (17) (i.e., we assume that $H = 1$) we find that

$$B = \frac{3\gamma_1}{2(1 + \nu) + \alpha(1 - 2\nu)\gamma_1} \tag{18}$$

and from (10) that

$$\beta = \frac{-3B(1 - 2\nu)}{\rho g A_s'[2\alpha B(1 - 2\nu) - 3(1 - \nu)]} \tag{19}$$

Given A_s' (in meters per unit strain) and γ_1, if we assume values for ν, β_u, and an initial guess at β, we may use (2) to find α and (18) to find B; then (19) will give us an improved value for β, which we may use again in (2) and (18). A few iterations between these equations will converge to stable values for B and β; once values of β and B have been found, the porosity may be determined from

$$\phi = \frac{(\beta - \beta_u)(1 - B)}{B(\beta_f - \beta_u)} \tag{20}$$

We used this procedure to estimate the matrix compressibility and porosity of formations tapped by the five wells described in Table 1. All five have been monitored for purposes of detecting tectonic strain. Three of the wells (TF, GD, and JC) are located near Parkfield, California; the other two (SC2 and LKT) are near Mammoth Lakes, California. Table 2 gives the observed areal strain sensitivities for the O_1 and M_2 tides, along with the inferred static-confined, areal strain sensitivities (based on the observed M_2 and O_1 responses for each well), and barometric efficiencies. We found the observed values through a cross-spectral analysis of the water level against the atmospheric pressure and theoretical strain tide [*Rojstaczer*, 1988*a*]. The theoretical tide included the body tide only, with no allowance for ocean loading or for topographic or geologic distortions. The static-confined responses were inferred from the observed ones through a procedure which adjusted for any effects of drainage to the water table; Appendix B gives the details.

Figure 5 shows the observed barometric efficiencies of the five wells as a function of frequency. As is discussed in detail elsewhere [*Rojstaczer*, 1988*a*, *b*; RR, 1988], these barometric efficiencies can be a strong function of frequency because of drainage to the water table; this is seen at three wells (GD, TF, and JC). Water table drainage also influences the observed areal strain sensitivities (determined for the tides); this partly explains the observed difference between the sensitivities observed for O_1 and M_2, with the O_1 sensitivity being lower because water table drainage can cause more attenuation at lower frequencies. The additional difference between the sensitivities is likely due to deviation from the body tide–induced strain at Parkfield [*Roeloffs et al.*, 1989]. Ignoring water table drainage for these three walls and assuming that the response is in fact confined would lead to biased estimates of material properties. At two of the Parkfield wells (TF and JC) the difference between the observed and corrected sensitivities is slight, but at well GD the difference is considerable.

Table 3 shows the material properties of the formations estimated from the barometric efficiency and corrected M_2 areal strain sensitivity in Table 2. In order to make estimates of matrix compressibility and porosity, we need to assume values for the solids compressibility (β_u, taken to be 2×10^{-11} Pa^{-1}), the Poisson's ratio (ν, taken to be 0.25), and the compressibility of the pore fluid (β_f, taken to be 4.4×10^{-10} Pa^{-1}). There are no independent measurements of β, but the estimates made from the well responses are reasonable compared with laboratory measurements of the compressibility of rock [*Haas*, 1981]. The porosity estimates are also usually within the realm of expected values [*Wolff*, 1981], except at SC2 and perhaps JC; in the latter case, a reduction in the assumed value of β_u will make the value of ϕ more reasonable. Table 3 also includes estimates of the specific storages, S_s and S_a, defined in the next section.

It is worth examining briefly possible sources of error in these estimates. The largest sources of error are probably the use of the body tide to represent the actual tidal strain and the assumption that the drained Poisson's ratio is 0.25. The results of *Beaumont and Berger* [1975] and *Berger and Beaumont* [1976] suggest that actual strain tides can some-

TABLE 2. Atmospheric and Tidal Responses of the Wells

Well Identification	Inferred E_B'	Observed Tidal Sensitivities, mm/nε		Inferred A_s', mm/nε	
		M_2	O_1	M_2	O_1
GD	0.10	0.30	0.24	0.38	0.34
TF	0.37	0.34	0.29	0.33	0.30
JC	0.67	0.28	0.22	0.27	0.21
SC2	0.74	0.13	0.07	0.13	0.07
LKT	0.48	0.34	0.31	0.34	0.31

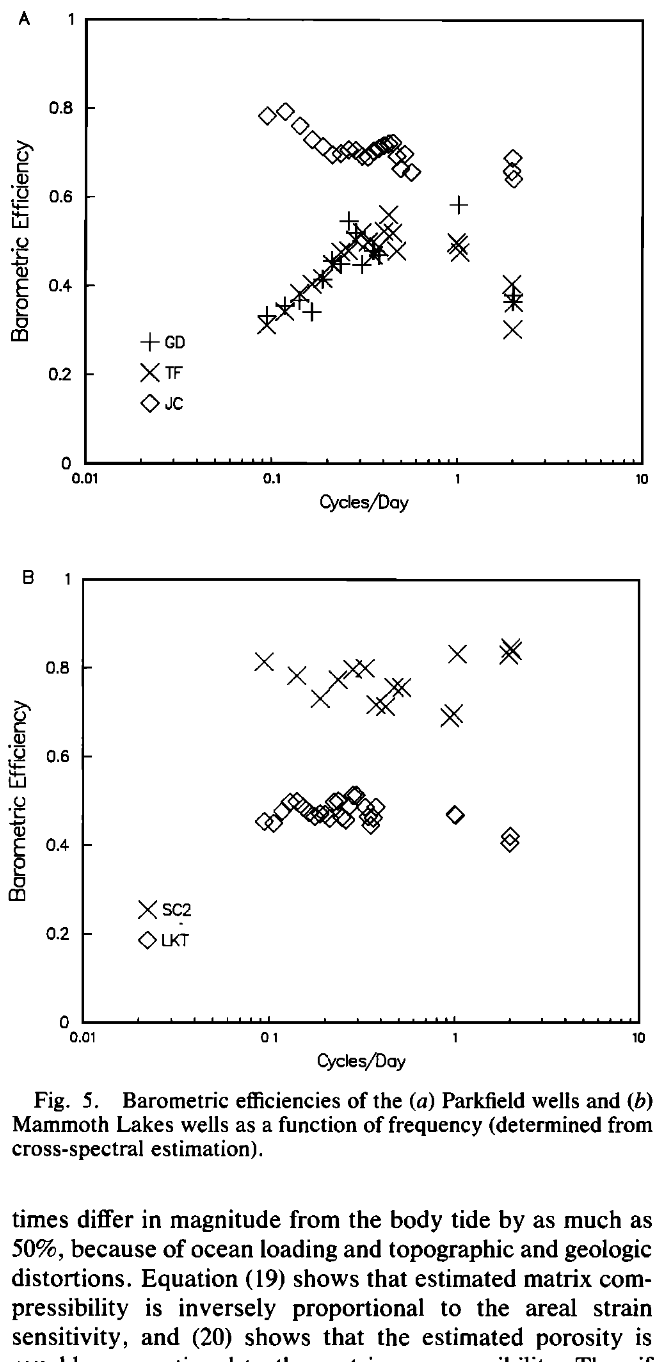

Fig. 5. Barometric efficiencies of the (*a*) Parkfield wells and (*b*) Mammoth Lakes wells as a function of frequency (determined from cross-spectral estimation).

times differ in magnitude from the body tide by as much as 50%, because of ocean loading and topographic and geologic distortions. Equation (19) shows that estimated matrix compressibility is inversely proportional to the areal strain sensitivity, and (20) shows that the estimated porosity is roughly proportional to the matrix compressibility. Thus if the theoretical tidal strain were to be twice the actual tidal strain, the estimates of both parameters will be about half what they should be. Better estimates of the true strain tides would reduce the error in estimating material properties. If the drained Poisson's ratio is not 0.25, the estimate of material properties will also be in error. Numerical experiments on the values in Table 2 show that varying the drained Poisson's ratio over the range 0.15–0.35 causes the inferred compressibilities and porosities to vary by a factor of about 2.5, with higher values of both being associated with lower Poisson's ratio. Obviously, the material properties estimated for the wells in Table 2, while generally reasonable, should be regarded as approximate values only.

TABLE 3. Material Properties and Specific Storages Estimated From Analysis

Well Identification	Matrix Compressibility β, Pa^{-1} $\times$ 10^{-10}	Porosity ϕ	Specific Storage, cm^{-1} $\times$ 10^{-8} S_s	S_a
GD	2.7	0.05	1.6	2.3
TF	1.9	0.19	1.8	2.2
JC	1.1	0.37	2.1	2.3
SC2	1.8	0.88	4.5	4.9
LKT	1.5	0.22	1.7	2.0

MATERIAL PROPERTIES GOVERNING VERTICAL PRESSURE DIFFUSION IN RESPONSE TO DEFORMATION

The preceding sections show that the static response of the water level in a well to deformation under confined conditions depends on the matrix and solids compressibility, the Poisson's ratio, and the porosity of the formation. In this section we examine how these material properties affect pressure diffusion. As noted above, the static-confined response of the well is a reflection of the undrained response of the formation, but in practice, fluid flow will influence water well response. If we assume that formations are of large extent laterally and the frequency of the deformation is low enough that well bore storage effects are negligible, then the influence of horizontal fluid flow can be neglected. Vertical fluid flow may occur due to water table drainage [*Rojstaczer*, 1988*b*; RR, 1988] or vertical variations in formation elastic properties [*Bower and Heaton*, 1978; *Gieske and de Vries*, 1985]. If we assume homogeneity, the response of pore pressure to changes in mean rock stress can be derived from the results of *Biot* [1941] and *Nur and Byerlee* [1971] and can be written in the form [*Rice and Cleary*, 1976]

$$\kappa \frac{\partial^2 p}{\partial z^2} = \frac{3\beta(\nu_u - \nu)}{B^2(1-2\nu)(1+\nu_u)} \frac{\partial}{\partial t}\left(p + B\frac{\sigma_{ii}}{3}\right) \tag{21}$$

Utilizing (10) and (21), pore pressure response to areal strains such as that produced by Earth tides and tectonic strain can be written as

$$c\frac{\partial^2 p}{\partial z^2} = \frac{\partial p}{\partial t} + \rho g A_s' \frac{\partial \varepsilon_a}{\partial t} \tag{22}$$

where c is a hydraulic diffusivity which is identical to that which governs diffusion of fluid mass in a poroelastic material [*Rice and Cleary*, 1976],

$$c = \kappa \frac{B^2(1-\nu)(1-2\nu)(1+\nu_u)^2}{3\beta(1+\nu)(1-\nu_u)(\nu_u-\nu)} \tag{23}$$

Utilizing (13) and (21), pore pressure response to areally extensive atmospheric loading is governed by

$$c\Upsilon \frac{\partial^2 p}{\partial z^2} = \frac{\partial p}{\partial t} - (1 - E_B')\frac{\partial p_b}{\partial t} \tag{24}$$

where Υ is a term which accounts for the influence of one-dimensional pore pressure diffusion on horizontal deformation,

$$\Upsilon = \frac{1-\nu}{1-\nu(1-H)} + \frac{H[3\nu + \alpha B(1-2\nu)](1-\nu)}{[3(1-\nu) - 2\alpha B(1-2\nu)][1-\nu(1-H)]} \tag{25}$$

Equations (22)–(24) indicate that if the material properties of the formation are known and the applied loads or strains can be measured or estimated a priori, it is relatively straightforward to solve for time-dependent pressure diffusion driven by atmospheric loading and Earth tides. The solution to some near-surface pressure diffusion problems which involve a periodic imposed deformation are given elsewhere [*Rojstaczer*, 1988*a*, *b*; RR, 1988].

Because $c\Upsilon$ is smaller than or nearly equal to c, pressure diffusion driven by atmospheric loading can be dampened (all material properties being equal) relative to pressure diffusion driven by Earth tides. Examination of (25) indicates that the term Υ will always be fairly close to unity, even if the areal strain produced by atmospheric loading is significant ($H = 1$). For highly compressible rock with low porosity and Poisson's ratio, Υ can be expected to be no lower than 0.5. In stiff, highly porous rock with high Poisson's ratio, Υ approaches 1. Hence the relative dampening of pressure diffusion driven by atmospheric loading can be expected to be small (no greater than a factor of 2).

The hydraulic diffusivity c is identical to the term k/S_s developed by *Van der Kamp and Gale* [1983], where k is the hydraulic conductivity and S_s is their one-dimensional specific storage coefficient,

$$S_s = \rho g\{\alpha\beta[1 - (2\alpha(1 - 2\nu)/3(1 - \nu))] + \phi(\beta_f - \beta_u)\} \tag{26}$$

As noted by *Van der Kamp and Gale* [1983], the one-dimensional specific storage coefficient is the poroelastic property of the formation generally assumed by hydrologists to relate changes in fluid mass to changes in pore pressure. The equivalence of c and k/S_s and the use of S_s as a poroelastic coefficient is discussed in detail elsewhere (D. Green and H. Wang, Specific storage as a poroelastic coefficient, submitted to *Water Resources Research*, 1988).

Following hydrologic convention, we can define the identity $c\Upsilon = k/S_a$ where S_a is a specific storage under conditions of surface loading. Assuming $H = 1$ yields

$$S_a = S_s/\Upsilon = \rho g\{\alpha\beta[1 - (\alpha(1 - 2\nu)/3)] + \phi(\beta_f - \beta_u)\} \tag{27}$$

This specific storage which relates changes in fluid mass to changes in pore pressure under conditions of atmospheric loading differs from that given by *Van der Kamp and Gale* [1983] because they assumed that atmospheric loading induces only vertical deformation (which makes $\Upsilon = 1$). The specific storages S_s and S_a determined for the five wells of this study are shown in Table 3. The one-dimensional specific storages S_s are 0.7–0.9 times the loading storages S_a, indicating that Υ is significantly less than unity, and that for the formations examined, pore pressure–induced horizontal deformation has a small effect on pressure diffusion induced by atmospheric loading.

CONCLUSIONS

The static response of the water level in a well to areally extensive deformation under confined conditions provides both a measure of well strain sensitivity and a means to measure in situ formation material properties. In the absence of fluid flow influences, formations can be expected to be sensitive to induced horizontal deformation such as that produced by Earth tides and tectonic strain if they are relatively stiff and are of low porosity. Open wells can be expected to be sensitive to atmospheric loading if they tap formations which are relatively stiff and are of high porosity.

Although knowledge of the static-confined response of wells is useful, it is not always observable. In three of the wells examined in this paper (GD, TF, and JC) the response observed must be corrected to give the static-confined response. If fluid flow influences water well response, it is possible to significantly underestimate the static-confined sensitivity of a well to deformation. Estimates of formation material properties directly based on the observed response of a well to deformation can be in error.

Under homogeneous conditions the influence of these imposed deformations on pore pressure diffusion can be readily described by the use of simple one-dimensional diffusion equations involving source terms proportional to the undrained response of the formation to deformation. The diffusivity which governs pore pressure response to applied areal strain is identical to that which governs fluid mass diffusion. The diffusivity which governs pore pressure response to applied loads is slightly lower, owing to the influence of pore pressure on horizontal deformation under conditions of loading. From a practical standpoint, however, the diffusivity which governs fluid mass diffusion is essentially the same as that which governs pressure diffusion due to loading. Even if one assumes that the lateral deformation factor H is unity, the difference between these diffusivities for the wells examined here is of the order of 20%.

If the static-confined response of a well can be observed or inferred, use of water well response to atmospheric loading and Earth tides can be expected to provide only approximate values of matrix compressibility, porosity, and specific storage. The values determined from well response may be in error as much as 50%. While estimates of porosity which have such a potential for error are likely of limited utility, rough estimates of matrix compressibility and specific storage are often of use to geophysicists and hydrologists.

APPENDIX A: INFLUENCE OF LAYERING ON THE STRAINS PRODUCED BY A SURFACE LOAD

If the near surface of the Earth possessed homogeneous material properties, areally extensive surface loading would produce areal strains at typical well depths which were equal to the vertical strain [*Farrell*, 1972]. There are, however, many conditions where there are large contrasts in formation elastic properties at shallow crustal depths. For example, basins with alluvial fill of high compressibility may be underlain at shallow crustal depths by bedrock which has a low compressibility. We examine the influence of such layering on the areal strains produced by atmospheric loading, through the use of a very simple model. We idealize the atmospheric load as a uniform pressure source over a radius a. The basin is assumed to consist of a layer of thickness h and undrained compressibility $\bar{\beta}_1$ underlain by a homogeneous half-space with undrained compressibility $\bar{\beta}_2$.

The stresses and strains produced under the center of the uniform surface load can be readily determined from the solution given by *Burmister* [1945] for a load with a distribution $-mJ_0(mr)$ where r is the radial distance and m is a constant.

Assuming that the Poisson's ratios for the layer and the half-space are the same, the principal stresses, σ_z, σ_r, and σ_θ

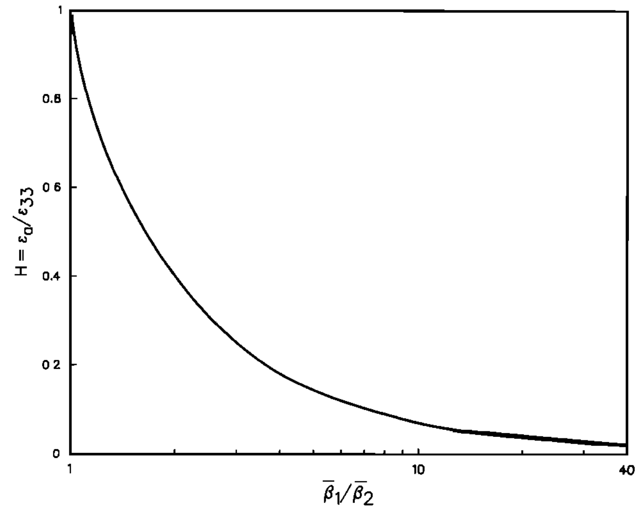

Fig. A1. Ratio of areal strain ε_a to vertical strain ε_{33} as a function of the ratio of the undrained compressibility of an overlying layer $\bar{\beta}_1$ to that of the elastic half-space $\bar{\beta}_2$. Solution is for the overlying layer beneath the center of a disc shape load. The undrained Poisson's ratio ν_u for both the layer and the half-space is 0.35.

in the surface layer produced beneath the center of the atmospheric load as a function of dimensionless height above the interface, $y = z/h$, are given by

$$\sigma_z = \frac{a}{h}\int_0^\infty J_1\left(\frac{a}{h}x\right)\{[p_{a1} - p_{c1}(1 - 2\nu_u - xy)]e^{-x(1-y)}$$
$$+ [n_{a1} - n_{c1}(1 - 2\nu_u - xy)]\, e^{-x(3-y)}$$
$$+ [p_{b1} + (1 - 2\nu_u + xy)]e^{-x(1+y)}$$
$$+ [n_{b1} + n_{d1}(1 - 2\nu_u + xy)]e^{-x(3+y)}\}\Delta^{-1}\, dx \qquad (A1a)$$

$$\sigma_r = \sigma_\theta = \frac{a}{2h}\int_0^\infty J_1\left(\frac{a}{h}x\right)\{[p_{a1} + p_{c1}(1 + 4\nu_u + xy)]e^{-x(1-y)}$$
$$+ [n_{a1} + n_{c1}(1 + 4\nu_u + xy)]e^{-x(3-y)}$$
$$+ [p_{b1} - (1 + 4\nu_u - xy)]e^{-x(1+y)}$$
$$+ [n_{b1} - n_{d1}(1 + 4\nu_u - xy)]e^{-x(3+y)}\}\Delta^{-1}\, dx \qquad (A1b)$$

where

$$p_{a1} = [j(1 - 4\nu_u)(1 + 2x) - 1]/2 \qquad (A2a)$$

$$p_{c1} = j(1 + 2x) \qquad (A2b)$$

$$n_{a1} = ji(4\nu_u - 2x)/2 \qquad (A2c)$$

$$n_{c1} = -ji \qquad (A2d)$$

$$n_{b1} = [-i + j(1 - 4\nu_u)(1 - 2x)]/2 \qquad (A2e)$$

$$n_{d1} = -j(1 - 2x) \qquad (A2f)$$

$$\Delta = 1 - (i + j + 4jx^2)e^{-2x} + jie^{-4x} \qquad (A2g)$$

$$j = \frac{1 - \bar{\beta}_1/\bar{\beta}_2}{1 + (3 - 4\nu_u)\bar{\beta}_1/\bar{\beta}_2} \qquad (A2h)$$

$$i = \frac{(3 - 4\nu_u)(1 - \bar{\beta}_1/\bar{\beta}_2)}{3 - 4\nu_u + \bar{\beta}_1/\bar{\beta}_2} \qquad (A2i)$$

Equations (A1*a*) and (A1*b*) can be solved using numerical quadrature. Once the stresses are determined, the ratio of areal strain to vertical strain, H, can be readily obtained.

In Figure A1, the influence of layering on the ratio H is examined for the case where the undrained Poisson's ratio ν_u of the formations is 0.35 and the ratio a/h is 1000. Major continental atmospheric loading fronts have radii of the order of 1000 km [*Rabbel and Zschau*, 1985], so the model roughly describes the influence of layering on atmospherically induced crustal deformation when the sediment layer has a thickness of 1 km.

Numerical evaluation of (A1*a*) and (A1*b*) indicates that with the above geometry the ratio H is essentially independent of dimensionless height y. The ratio H is, however, very sensitive to the ratio of compressibilities, $\bar{\beta}_1/\bar{\beta}_2$. When $\bar{\beta}_1/\bar{\beta}_2$ is equal to 1, the solution is identical to that given by *Timoshenko and Goodier* [1970, p. 406] for a homogeneous half-space. The results indicate that the ratio H is roughly inversely proportional to $\bar{\beta}_1/\bar{\beta}_2$ and areal strain in the upper layer is strongly controlled by areal strain in the half-space.

Appendix B: Finding the Static-Confined Response of Wells

The static-confined response of the wells to atmospheric loading and Earth tides was inferred by fitting the observed response of these wells to theoretical models which describe the influence of water table drainage on well sensitivity. The observed response was determined from cross-spectral analysis of the water level time series against atmospheric pressure and the theoretical Earth tide.

The wells examined showed high coherence (greater than 0.85) between water level and air pressure from 0.08 to 2 c/day, and between water level and the theoretical tide at the peak tidal frequencies. The transfer function between water level and atmospheric load was fit to a theoretical solution which is governed by (24) and describes the influence of water table drainage on water well response [*Rojstaczer*, 1988*a*, *b*]. The barometric efficiency at which the theoretical solution showed fluid flow influences to be negligible was taken to be the static-confined barometric efficiency. Fuller descriptions are given elsewhere [*Rojstaczer*, 1988*a*, *b*; RR, 1988].

The static-confined areal strain sensitivities were determined iteratively. First, the fit of the atmospheric load transfer function was used to determine the vertical hydraulic diffusivity $c\Upsilon$ (equation (24)); the estimated barometric efficiency and observed M_2 tidal sensitivity then gave estimates of the material properties, and from these the specific storages S_a and S_s could be found. Given $c\Upsilon$ and S_a (and assuming $H = 1$), the hydraulic conductivity is $k = c\Upsilon S_a$; this hydraulic conductivity then gave an estimate of the hydraulic diffusivity $c = k/S_s$. This estimate of c was inserted into a theoretical solution to (22) which describes the influence of water table drainage on well response to Earth tides [*Rojstaczer*, 1988*a*; RR, 1988]. The degree of attenuation or amplification of response indicated by the theoretical solution was used to obtain a new estimate of the static-confined areal strain sensitivity. A new estimate of the material properties and the specific storages S_a and S_s was then made on the basis of the new M_2 sensitivity, and the process was repeated until convergence was achieved. Further details on inferring strain response can be found elsewhere [*Rojstaczer*, 1988*a*].

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