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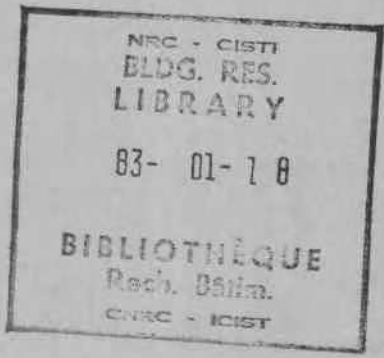
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THE INFLUENCE OF SNOW COVER ON THE GROUND THERMAL REGIME

by L. E. Goodrich

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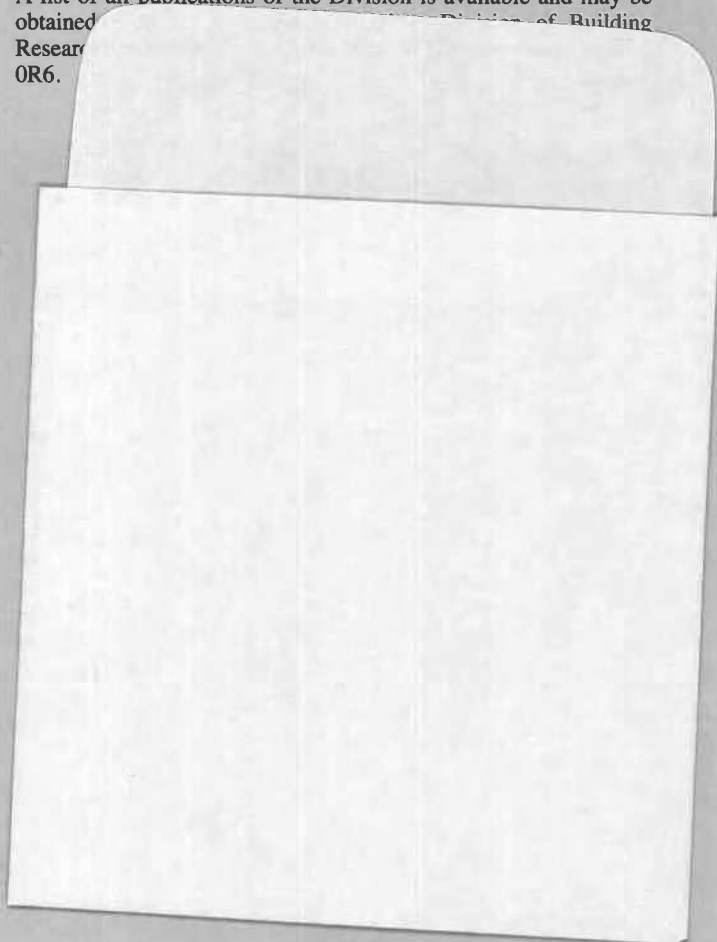
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The influence of snow cover on the ground thermal regime

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This paper presents the results of a numerical study of the effects of snow cover on long-term, periodic, steady-state equilibrium ground temperatures. It is shown that mean annual ground temperatures decrease with depth when the soil thermal conductivity is greater in the frozen than in the unfrozen phase. For permafrost conditions the increase in mean annual ground temperatures due to seasonal snow cover is augmented significantly when soil latent heat is present. In seasonal frost cases the calculated depth of frost penetration is extremely sensitive to details of the snow cover buildup. In permafrost cases calculated mean annual temperatures are extremely sensitive to the assumptions made in treating the snow cover. In either case, because it is difficult to model snow cover accurately, the reliability of ground thermal regime computations is adversely affected.

Keywords: ground thermal regime, ground temperatures, soil temperatures, numerical model, finite difference, snow cover.

Cet article présente les résultats d'une étude numérique sur les effets d'un couvert de neige sur les températures du sol en régime permanent, périodique, à long terme. On montre que les températures moyennes annuelles décroissent avec la profondeur lorsque la conductivité thermique du sol est plus grande en phase gelée qu'en phase non gelée. En conditions de permafrost, l'augmentation des températures annuelles moyennes due à une couverture de neige saisonnière est accrue de façon importante si la chaleur latente de sol est présente. Dans les cas de gel saisonnier la profondeur de pénétration du gel calculée est très sensible aux détails de la formation du couvert de neige. Dans le cas de permafrost, les températures annuelles moyennes calculées sont très sensibles aux hypothèses faites dans le traitement du couvert de neige. Dans les deux cas, la difficulté de modéliser précisément le couvert de neige nuit à la fiabilité des calculs de régime thermique du sol.

Mots-clés: régime thermique du sol, température du sol, modèle numérique, différences finies, couvert de neige.

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Introduction

The influence of seasonal snow cover on the annual ground thermal regime has long been recognized. Snow cover is an insulator by comparison with most natural ground surface materials and, since it is present only during the cold part of the annual temperature cycle, the net effect over a year is to raise mean annual ground temperatures, often by several degrees.

This paper describes a numerical study of snow-ground thermal interactions and presents a series of graphical results that illustrate the effect on the long-term ground thermal regime of variations in snow and soil properties. These calculations originally formed a part of the author's Ph.D. thesis (Goodrich 1976), work done in the early 1970s. Although a number of field studies had been reported at that time (Gold 1963, 1964, 1967), theoretical computations of the effects of snow cover on ground temperatures had not progressed beyond the simplified analysis presented by Lachenbruch (1959). This analysis neglected soil latent heat as well as changes of thermal properties associated with freezing and thawing of the ground. In addition, the snow cover was represented as a simple constant thermal resistance that was switched on and off according to the season.

Gilpin and Wong (1976) extended the analytical

description of Lachenbruch to include a sinusoidal time dependence for the thermal properties of the surface. In addition, a simplified numerical model was used to examine the effects of introducing soil latent heat.

The parameter sensitivity study presented here was made with a somewhat more elaborate numerical model, which has been described in detail elsewhere (Goodrich 1974, 1978). A brief description is given in Appendix B. Conclusions similar to those of Gilpin and Wong (1976) are reached regarding the effects of soil latent heat. A number of additional aspects of the snow-ground thermal interaction are discussed.

Model assumptions

Model calculations were made for periodic steady-state conditions assuming a sinusoidal dependence of surface temperature with a period of 1 year, applied to the snow surface when snow cover was present and to the bare ground surface during the remainder of the year. The boundary condition at depth was represented by a constant heat flux whose value, 0.063 W/m^2 , corresponded to the geothermal heat flux. The lower calculation boundary was located at 15 m; calculations showed that even in the worst case this depth was adequate to ensure no significant effect on temperatures (errors $< 0.01^\circ\text{C}$) in the first 5 m below the surface. The snow

TABLE 1. Material properties assumed in the calculations

| Material | k_f Frozen thermal conductivity (W/(m·K)) | C_f Frozen volumetric heat capacity (MJ/(m ³ ·K)) | k_t Thawed thermal conductivity (W/(m·K)) | C_t Thawed volumetric heat capacity (MJ/(m ³ ·K)) | ρ Bulk density (t/m ³) | W Water content (mass basis) (%) | L_y Volumetric latent heat (MJ/m ³) |
|-----------------------|---|--|---|--|---|--|--|
| Snow | $k = 2.9\rho^2$ | 2.09 ρ | — | — | 0.25 or variable | — | — |
| Fine-grained soil | 2.21 | 1.92 | 1.13 | 2.87 | 1.3 | 35 | 152.0 |
| Coarse-grained soil | 3.01 | 1.86 | 2.19 | 2.40 | 1.75 | 15 | 87.7 |
| Organic surface layer | 1.20 | 2.30 | 0.40 | 3.89 | 0.17 | 500 | 283.9 |

cover mass was assumed to build up linearly to a maximum value, remaining constant thereafter until melting in the spring. In most of the calculations snow density was assumed constant but in others provision was made to simulate snow cover densification using a numerical solution of the viscous compaction formulation of Kojima (1966). Thermal properties were related to snow density using Abel's empirical formula (Bader and Kuroiwa 1962). Snow melting was accommodated using a degree-day approach. Further information on the snow cover model assumptions is given in Appendix A.

In most cases the soil was represented as a single homogeneous layer with distinct frozen and thawed thermal properties. Values chosen approximated the empirical results of Kersten (1949). No attempt was made to account for variations in soil thermal properties accompanying seasonal changes in moisture content at shallow depths. Latent heat release or absorption accompanying soil freezing and thawing was included, assuming that the phase change took place completely at the freezing point. No attempt was made to accommodate either frost heaving or thaw settlement.

These assumptions constitute a considerable idealization when compared with real snow covers and soil conditions. Inclusion of soil freezing over a temperature range and/or of moisture redistribution and geometric changes accompanying frost heaving or thaw settlement would evidently lead to different numerical values of certain computed results such as frost or thaw depths and to curvature of the mean annual ground temperature profiles. More realistic models for the snow cover would have an even greater effect on these quantities. The calculations presented here are primarily intended to illustrate the major features of snow-ground thermal interactions in general. The conclusions reached are essentially qualitative, and it is believed that the model is appropriate for this purpose.

Material properties used in the calculations are listed in Table 1. The results of calculations made for a total of 11 seasonal frost cases and 12 permafrost cases are summarized graphically in Figs. 1–10. In addition

Tables 2 and 3 give some numerical results. In the figures the sections labelled (a) show the ground surface temperature as a function of time (Julian calendar). During snow-free periods this is identical to the assumed sinusoidal surface temperature. With snow cover the curve represents the computed snow-ground interface temperature. Sections (b) show the depth of freezing or thawing computed directly by the model. Sections (c) indicate the assumed or computed snow depth and density history. Finally, Sections (d) show the computed mean annual ground temperature as functions of depth and also indicate the maximum-minimum temperature envelopes.

Seasonal frost cases

Two standard cases were computed, assuming a homogeneous fine-grained soil with properties as listed in Table 1. The sinusoidal surface temperature had an annual mean of 5°C with amplitude 15°C. For run 1 the ground surface remained bare throughout the year. For run 2 a snow cover of constant density (0.25 t/m³) was assumed to begin building up on day 300, 5 days after the onset of freezing, and continuing for 95 days until a maximum 0.5-m depth was achieved. Spring melt was computed using a melt factor of 5 kg·m⁻²·day⁻¹·K⁻¹. The remaining nine cases consider variations of snow cover parameters, surface temperature amplitude, snow melt rate, and soil properties.

Results for run 1 are indicated by the solid curves in Fig. 1. The curvature of the mean annual temperature profile within the active zone corresponding to the maximum depth of seasonal frost should be noted. This thermal rectification effect is a consequence of the difference in frozen and thawed soil conductivities. Such effects have received little attention in the ground thermal regime literature. In the remainder of this paper the term thermal offset is used to indicate the difference between the mean annual ground temperature at the bottom of the active zone and that at the ground surface. The sign of the thermal offset is negative whenever the frozen soil conductivity value exceeds the thawed value.

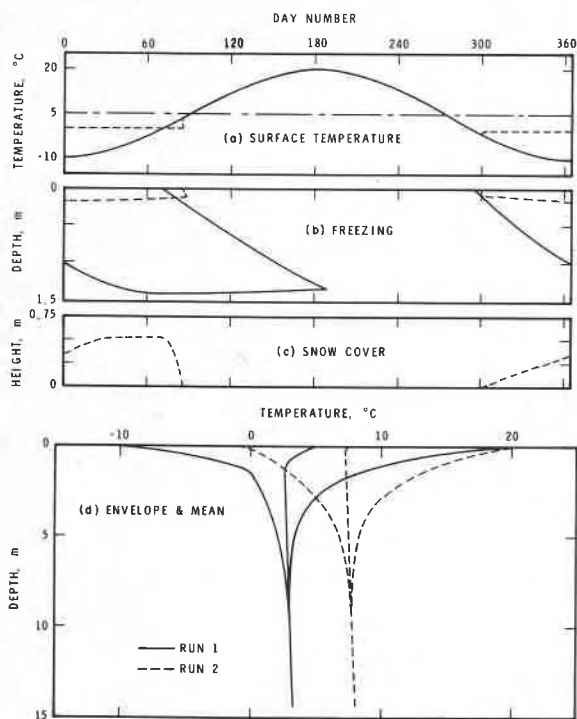


FIG. 1. Seasonal frost: standard snow cover compared with bare surface.

The magnitude of the offset is determined, for given ground surface temperature conditions, by the ratio of frozen to thawed conductivity and by the depth of the active zone.

The effectiveness of the standard snow cover in inhibiting frost penetration may be seen in Fig. 1b, run 2. Note that nearly half the total frost depth is achieved in the brief freezing period just prior to the inception of the snow cover buildup. In Fig. 1d it may be seen that the presence of the snow cover has displaced the mean annual ground surface temperature by 2.3°C while, because of the reduction in thermal offset, temperatures at depth have been increased by an additional 2.2°C . Figure 1d also indicates that, while snow cover reduces the amplitude of annual temperature variations at the surface, at depth the amplitude may be *increased* relative to that for bare surface conditions. This is a consequence of the liberation of soil latent heat during the winter. With latent heat release, wintertime cooling within the soil (viz., the magnitude of temperature decrease below the mean) is diminished. Owing to the greater intensity of freezing, this effect is greatest for bare surface conditions.

Reducing the maximum thickness of snow cover can be expected to reduce mean annual ground temperatures while increasing the predicted frost depths. The results obtained (run 3) using a maximum snow cover thickness

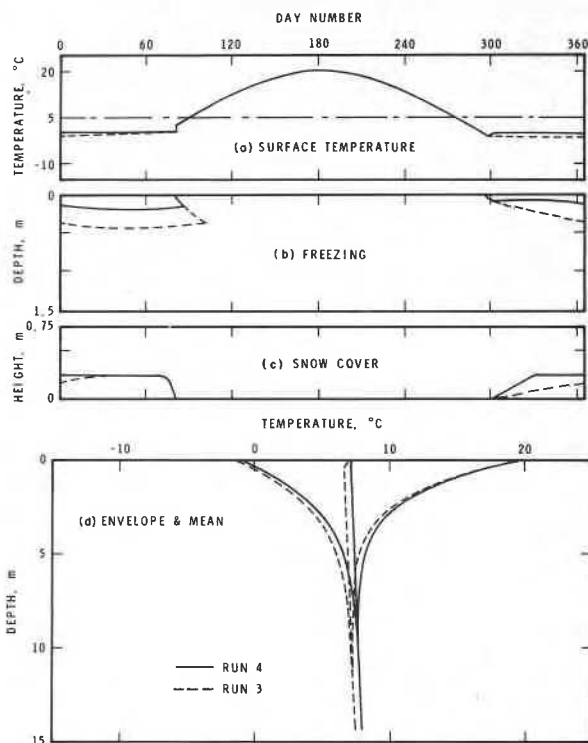


FIG. 2. Seasonal frost: shallow snow cover—slow compared with rapid snow buildup.

of 0.25 m are shown by the dashed curves in Fig. 2. The maximum frost depth achieved is more than twice that obtained with the 0.5-m snow cover and occurs somewhat later in the season. The mean annual ground surface temperature is, however, decreased by only 0.3°C from the value obtained in run 2.

The solid curves in Fig. 2 show the result (run 4) of reducing the assumed snow cover buildup period to 30 days. The maximum predicted frost penetration is less than half that found in run 3. More than half of the increase in mean annual ground temperatures over those of run 3 is attributable to the decrease in thermal offset. At the ground surface, however, the mean annual temperature is nearly the same in both cases.

As pointed out by Lachenbruch (1959) the effect of seasonal snow cover in warming the ground is increased if the surface temperature amplitude is increased. This arises because, due to the presence of the insulating snowpack, summertime increases in ground surface temperatures are not compensated for by identical decreases in the winter. Calculations were made for conditions similar to those of run 2, except that the surface temperature amplitude was increased from 15 to 17°C . The results, summarized in Table 2, show the mean annual temperature to be 0.5°C warmer than in run

TABLE 2. Summary of calculated results for seasonal frost cases

| Run No. | Soil type | Maximum snow height (m) | Comments | Maximum frost depth (m) | Minimum ground surface temp. (°C) | Mean annual ground surface temp. (°C) | Thermal offset (°C) | Total profile displacement from bare surface case (°C) | Ground surface freezing index (°C·day) | Ground surface thawing index (°C·day) |
|---------|-----------------------|-------------------------|-----------------------------------|-------------------------|-----------------------------------|---------------------------------------|---------------------|--|--|---------------------------------------|
| 1 | Fine | 0 | | 1.40 | -10.0 | +5.0 | -2.3 | 0 | 931 | 2761 |
| 2 | Fine | 0.5 | Standard snow cover | 0.18 | -1.12 | 7.35 | -0.1 | 4.6 | 48 | |
| 3 | Fine | 0.25 | Shallow snow cover | 0.43 | -1.53 | 7.08 | -0.4 | 4.1 | 153 | |
| 4 | Fine | 0.25 | Rapid snow cover buildup | (0.08*) 0.18 | (-1.12*) -0.44 | 7.38 | ~-0.1 | 4.7 | 44 | |
| 5 | Fine | 0.5 | Increased surface temp. amplitude | 0.27 | -1.94 | 7.83 | ~-0.3 | 5.0 | 92 | |
| 6 | Fine | ~0.5 | Compacting snow cover | (0.07*) ~0 | -1.12 | 7.46 | ~0 | 4.8 | 4 | |
| 7 | Fine | 0.5 | Melt factor reduced | 0.18 | -1.12 | 7.26 | ~-0.1 | 4.5 | 49 | |
| 8} | Coarse | 0 | Standard snow cover | 1.99 | -10.0 | 5.0 | -0.95 | 0 | 931 | |
| 9} | | 0.5 | | 0.20 | (-1.12*) | 7.35 | ~-0.05 | 3.3 | 44 | |
| 10} | Organic layer on fine | 0 | Standard snow cover | 1.27 | -10.0 | 5.0 | -3.54 | 0 | 931 | 2761 |
| 11} | | 0.5 | | 0.11 | -1.12 | 7.32 | -0.3 | 4.3 | 54 | 2733 |

*Early maximum prior to onset of snow cover.

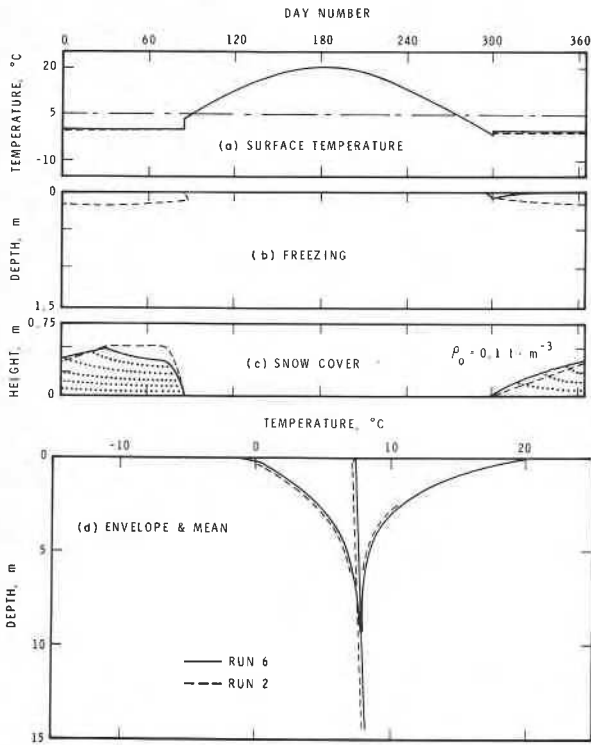


FIG. 3. Seasonal frost: compacting snow cover compared with standard snow cover.

2 and this is the greatest difference from standard for all the conditions examined.

Figure 3 (solid curves) shows results obtained assuming a snow cover that densifies as the season progresses (Appendix A). A constant accumulation rate lasting for the first 95 days of the snow season was assumed. During this period, snow at the surface remains at nearly the initial density (0.1 t/m^3). Within the snowpack the density increases with depth and with time. The dotted lines in Fig. 3c indicate layers of equal snow mass. They also represent the layer boundaries used for both the temperature and snow compaction computations. The calculations indicate that, following the onset of snow cover, the frost line rises and the ground surface remains essentially at the freezing point throughout the snow season. Figure 3d shows that the mean annual surface temperature is nearly identical to that of run 2 (dashed curves), whereas the difference at depth arises because of differences in thermal offset.

To test the sensitivity of the thermal regime to differences in the rate of spring melt, in run 7 the melt factor was reduced to one half the standard value, yielding a snow melt period of 20 days compared with 14 days in run 2. This resulted in an insignificant ($<0.1^\circ\text{C}$) lowering of mean annual ground temperatures, whereas other aspects of the thermal regime remained

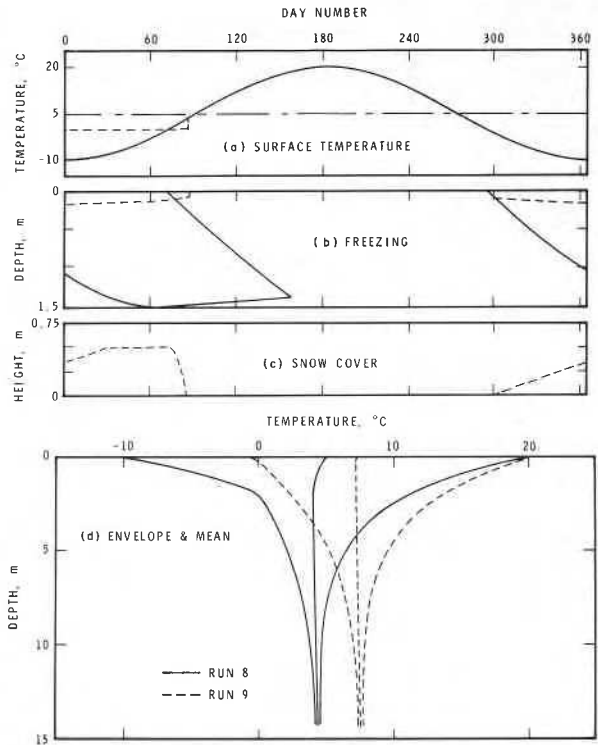


FIG. 4. Seasonal frost: coarse-grained soil with and without snow cover.

unchanged from the standard snow cover case (see Table 2).

Runs 8 and 9 were made assuming the ground to be a coarse-grained material with properties as indicated in Table 1. Comparison of Fig. 4b with Fig. 1b shows that, although the frost penetration for bare surface conditions is nearly 2 m in the coarse-grained soil, the thaw rate also is correspondingly rapid, with the result that the ground is fully thawed a month earlier than in the fine-grained soil case. Figure 4d shows a thermal offset of only -1°C for bare surface conditions, which is consistent with the smaller ratio of frozen to thawed conductivity. At the same time, because of the higher conductivity values and lower moisture contents, the amplitude of seasonal temperature fluctuations at depth is much greater than for the fine-grained soil. The dashed curves in Fig. 4 show the results obtained under standard snow cover conditions. Although the frozen conductivity is considerably greater and the volumetric latent heat much less than for the fine-grained soil, and ground surface freezing indices are similar, the maximum frost penetration is only slightly greater than that obtained in run 2. The tendency for the frost line to rise in midwinter is also more marked in coarse-grained soil. Both effects can be attributed to the increased role of ground heat storage.

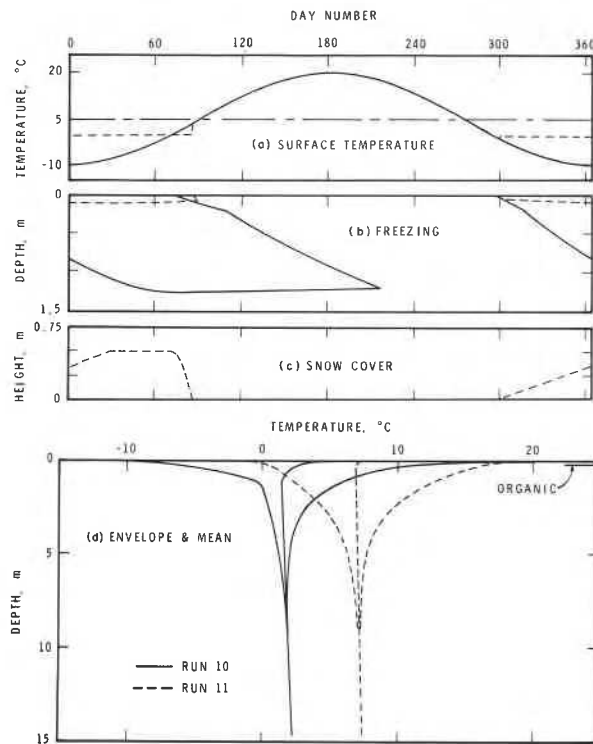


FIG. 5. Seasonal frost: organic surface layer.

Figure 5b shows the effectiveness of an organic surface layer in reducing frost and thaw penetration rates. The very large thermal offset associated with the high ratio of frozen to thawed conductivity ($k_f/k_t = 3$) of the organic material may be seen in Fig. 5d. Of a total thermal offset of -3.5°C , -2.25°C occurs across the 0.2-m thick organic layer. As a result, deep ground temperatures are 1.7°C cooler than in the standard case (run 1).

Results with snow cover are shown by the dashed curves in Fig. 5. Although the mean annual surface temperature is essentially identical to that of run 2, the total increase in mean temperature at depth brought about by the addition of snow cover is 5.4°C . More than half of this increase is associated with the reduction in thermal offset. It may also be seen in Fig. 5d that, at depth, the amplitude of seasonal temperature variations is again *greater* with snow cover than without it.

Permafrost cases

Calculations were made for soil and snow cover conditions similar (except as noted) to those assumed in the seasonal frost cases. The snow cover was assumed to begin building up on day 250, five days after the start of freezing conditions, and to continue at a constant rate until day 30, when a maximum depth of 0.5 m was

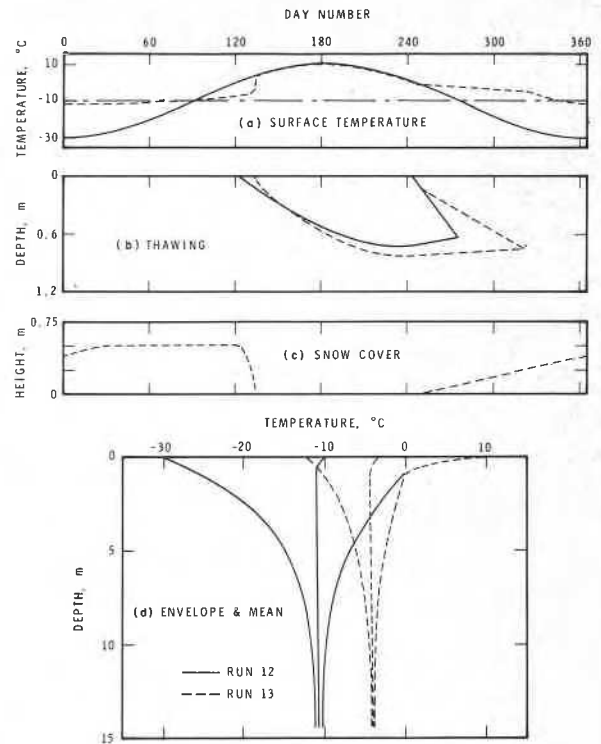


FIG. 6. Permafrost: standard snow cover compared with bare surface.

reached. The surface temperature was assumed to have a mean value of -10°C with an amplitude of 20°C .

The thermal regime for bare surface conditions, run 12, is shown by the solid lines in Fig. 6. The thermal offset obtained is about half that found in the seasonal frost case. The maximum thaw depth in this case is also about half the frost depth obtained in run 1.

The dashed curves in Fig. 6 show results obtained for the standard snow cover conditions, run 13. The behaviour of the snow-ground interface temperature is noteworthy. During the autumn the ground surface temperature cannot descend much below freezing until the active layer has completely refrozen. This effect results from the combination of soil latent heat and the presence of the insulating snow cover. Once freezeback is complete, temperatures in the active layer drop rapidly and, in contrast with seasonal frost cases where latent heat is released throughout the winter, relatively cool temperatures may eventually be obtained. As Fig. 6d shows, the addition of snow cover results in a substantial increase in the mean annual surface temperature without significantly changing the thermal offset. At the same time, the width of the maximum-minimum temperature envelope is reduced sharply.

Results obtained with the maximum snow cover

thickness reduced to 0.25 m are shown as run 14 in Fig. 7. Compared with run 13 there is a reduction of about 30 days in the time required to refreeze the active layer completely in the autumn and, taken together with the reduced insulation associated with the thinner snow cover, this permits colder ground surface temperatures to be maintained for a longer period. In consequence, the mean annual ground surface temperature is 3.6°C colder than that obtained in run 13. Corresponding to the lower mean annual temperatures, the width of the maximum–minimum temperature envelope is also considerably greater.

Reducing the assumed snow cover buildup period to 30 days makes a large change in the mean annual temperature. The results, shown as run 15 in Fig. 7, indicate an increase in the mean annual ground temperature of nearly 4°C by comparison with run 14.

The consequences of a small change in assumed surface temperature amplitude (increased from 20 to 22°C) are summarized in Table 3, run 16. In spite of the colder winter snow surface temperatures the minimum ground surface temperature is more than 1°C warmer than that calculated in run 13. The mean annual ground surface temperature is -2°C, which is nearly 1.5°C warmer than for the standard case.

Figure 8 (solid curves) shows the thermal regime

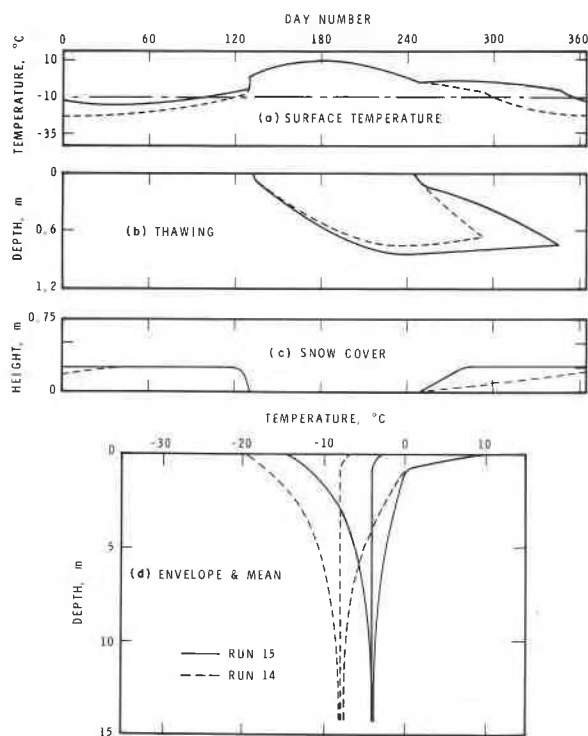


FIG. 7. Permafrost: rapid snow cover buildup, shallow snow depth.

predicted assuming a snow cover that densifies during the season (run 17). The snow cover model and parameter assumptions used were similar to those of run 6. Compared with the standard case (dashed curves) the thermal resistance of the compacting snow cover is much higher during the critical autumn period. The effect on mean annual temperatures is sufficiently great that the permafrost is predicted to melt entirely.

Results obtained with the melt factor taken as one half of the value used in the standard case showed that, as in the seasonal frost case, the snow melt rate has only a slight effect on the thermal regime. The maximum thaw depth was diminished by 3% and mean annual surface temperatures were reduced by 0.3°C (Table 3).

Using the standard 0.5-m snow cover but, assuming a buildup period reduced to 30 days, the mean annual ground temperature is raised to +1°C (Table 3), and the permafrost is predicted to disappear entirely. This result underlines the significance of autumn snow conditions in determining mean annual ground temperatures in permafrost.

Calculations for coarse-grained soil with (run 20) and without (run 21) seasonal snow cover are shown in Fig. 9. By comparison with the fine-grained soil (Fig. 6b), there is both a deeper maximum thaw penetration and a more marked tendency for the thaw plane to rise in late

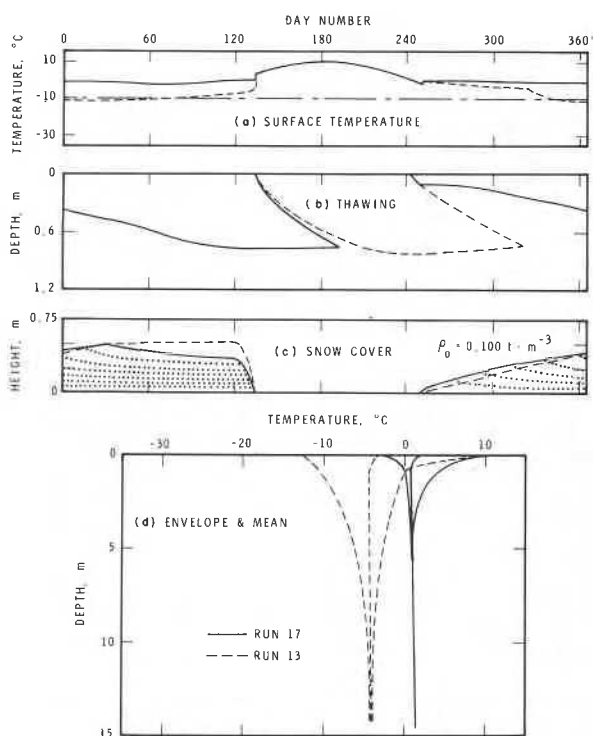


FIG. 8. Permafrost: compacting snow cover compared with standard snow cover.

TABLE 3. Summary of calculated results for permafrost cases

| Run No. | Soil type | Maximum snow height (m) | Comments | Maximum thaw or frost depth (m) | Minimum ground surface temp. (°C) | Mean annual ground surface temp. (°C) | Thermal offset (°C) | Total profile displacement from bare surface case (°C) | Ground surface freezing index (°C·day) | Ground surface thawing index (°C·day) |
|---------|-----------------------|-------------------------|-----------------------------------|---------------------------------|-----------------------------------|---------------------------------------|---------------------|--|--|---------------------------------------|
| 12 | Fine | 0 | | 0.74 | -30.0 | -10.0 | -1.1 | 0 | 4458 | 798 |
| 13 | Fine | 0.5 | Standard snow cover | 0.84 | -12.25 | -3.42 | -1.1 | 6.6 | 2027 | |
| 14 | Fine | 0.25 | Shallow snow cover | 0.76 | -19.68 | -7.04 | -1.0 | 3.0 | 3362 | |
| 15 | Fine | 0.25 | Rapid snow cover buildup | 0.85 | -14.54 | -3.14 | -1.0 | 6.9 | 1933 | |
| 16 | Fine | 0.5 | Increased surface temp. amplitude | 0.98 | -11.03 | -2.01 | -1.3 | 7.8 | 1713 | |
| 17 | Fine | ~0.5 | Compacting snow cover | 0.77 (frost) | -2.43 | +1.39 | -0.7 | 11.8 | 262 | |
| 18 | Fine | 0.5 | Melt factor reduced | 0.81 | -12.59 | -3.78 | ~-1.05 | 6.3 | 2128 | |
| 19 | Fine | 0.5 | Rapid snow cover buildup | 0.95 (frost) | -2.81 | +1.09 | -0.9 | 11.3 | 374 | |
| 20} | Coarse | 0 | Standard snow cover | 1.32 | -30.00 | -10.0 | -0.56 | 0 | 4458 | |
| 21} | | 0.5 | | 1.57 | -10.22 | -2.61 | -0.54 | 7.4 | 1728 | |
| 22} | Organic layer on fine | 0 | Standard snow cover | 0.42 | -30.00 | -10.0 | -1.37 | 0 | 4458 | 798 |
| 23} | | 0.5 | | 0.49 | -14.47 | -4.78 | -1.33 | 5.3 | 2522 | 772 |

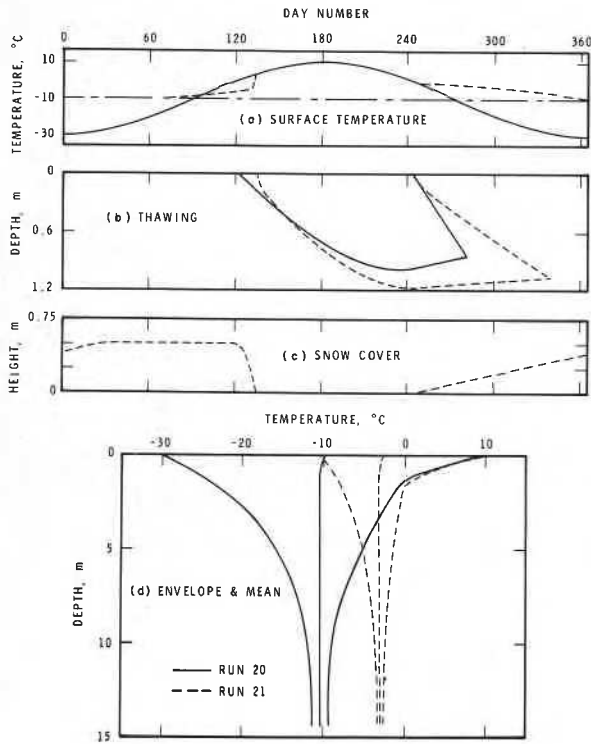


FIG. 9. Permafrost: coarse-grained soil with and without snow cover.

summer prior to freezeback. Even though the volumetric latent heat assumed for the coarse-grained soil is much less than that assumed for the fine-grained soil a longer time is required for complete refreezing of the active layer. With the standard snow cover, the predicted maximum thaw depth increases by nearly 20% over that for the bare surface. This is nearly twice the increase obtained for the fine-grained soil. Figure 9d shows only a slight curvature of the mean annual temperature profile and a thermal offset of only $\sim -0.5^{\circ}\text{C}$ is predicted. The mean annual ground surface temperature with snow cover is nearly 1°C warmer than that calculated for the fine-grained soil.

Figure 10 shows results (runs 22 and 23) obtained when a 0.2-m organic layer was added to the surface, while all other parameters remained as in the standard cases. Thaw retardation caused by the organic layer is evident in Fig. 10b, and total thaw depths achieved both with and without snow cover are about one half of those calculated for the standard cases (Fig. 6b). The time required for freezeback is also shorter even though the organic layer causes a large reduction in the freezing and thawing rates. The mean annual profile exhibits a thermal offset of -1.4°C , almost all of which is associated with the organic layer. With seasonal snow

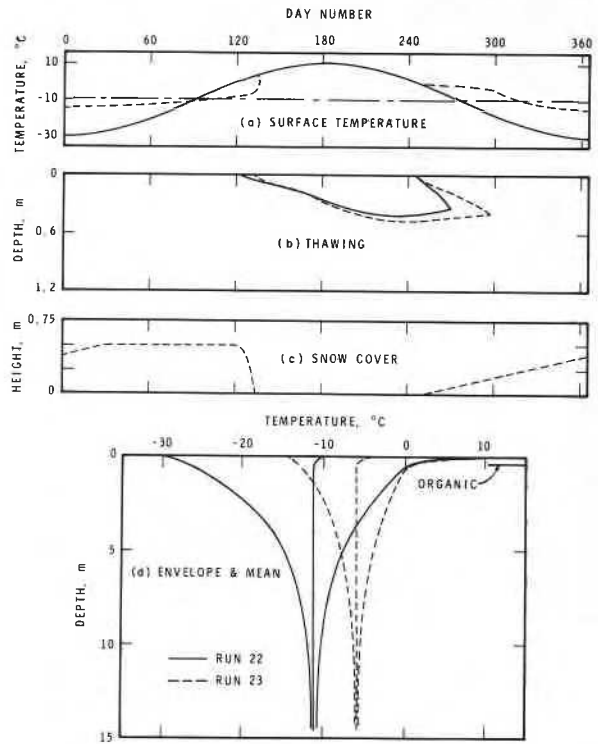


FIG. 10. Permafrost: organic surface layer.

cover the overall effect of the insulating but wet organic layer is to lower mean annual ground surface temperatures. The value obtained in run 23 is approximately 1.4°C colder than in the standard case (run 13). For bare surface conditions, the amplitude of the maximum–minimum temperature envelope is slightly reduced over that of the standard case (run 12). With snow cover, however, the situation reverses and the amplitude is *greater* when the organic layer is present (Fig. 10d), than without it (Fig. 6d).

Discussion and conclusions

Mean annual ground temperatures may be increased by several degrees in the presence of snow cover. In seasonal frost cases, for given surface temperature conditions, the increase is controlled primarily by the duration of the snow cover and is insensitive to parameters governing the snow cover buildup. In permafrost cases, on the other hand, mean annual ground temperatures are strongly dependent on snow cover parameters. The maximum snow depth as well as snow conditions in the early winter are particularly critical to the mean annual ground temperature in permafrost regions.

For ground materials with temperature-dependent thermal conductivities the mean annual temperature

profile at equilibrium may show a thermal offset of up to 3 or 4°C for typical seasonal frost and permafrost conditions. The thermal offset is negative if $k_f > k_t$, as is commonly the case, and increases in value with increasing ratio k_f/k_t . The thermal offset will also increase with active layer depth. In seasonal frost cases the decrease in ground temperatures that would result from the elimination of the snow cover is accompanied by an additional decrease associated with the thermal offset. In the cases considered this effect accounted for from ~25–65% of the total profile displacement. In permafrost cases, by contrast, the value of the thermal offset is practically not affected by the existence of seasonal snow cover.

In seasonal frost conditions both the insulating snow cover and the soil latent heat combine to maintain the snow-ground interface temperature at values near freezing during the entire winter. The mean annual ground surface temperature is not, however, greatly affected by the amount of soil latent heat available. The snow cover alone is sufficient to maintain winter ground surface temperatures near freezing and thus ensure that the annual mean value is essentially determined by the summer regime.

In permafrost conditions the effect of latent heat release combined with an insulating snow cover during freezeback of the active layer results in an additional increase in mean ground temperatures over and above that due directly to the snow cover. In contrast with the seasonal frost case, mean annual ground temperatures in permafrost with snow cover may be several degrees warmer than in the absence of latent heat. The sensitivity of mean temperatures to other parameters is also increased by the presence of latent heat.

In seasonal frost cases, although the frost penetration depth is controlled by the usual combination of soil properties and surface temperatures, with seasonal snow cover the frost penetration is very strongly conditioned by the thermal resistance of the snow pack. Autumn penetration may represent a large fraction of the total frost depth achieved, and details of the autumn period are critical to determining subsequent frost penetration. In addition, frost penetration is strongly retarded by ground heat storage when snow cover is present.

In permafrost cases the effect of snow cover on summer thaw depths, although small, is not entirely negligible. Compared with bare surface conditions, increases of up to 13% were calculated for fine-grained soil and 19% for coarse-grained soil. In most of the permafrost cases the maximum thaw depth is reached some time prior to the onset of refreezing. This tendency is most noticeable in those cases having the coldest mean ground temperatures.

The width of the maximum–minimum temperature envelope is narrower the closer the mean annual temperature profile is to the freezing point. This is most

evident in the permafrost results. In most of these cases the effect is partly due to the presence of snow cover, but even with bare surfaces the envelope width at depth narrows as the mean temperature approaches the freezing point for otherwise similar conditions. The reason is that latent heat effects dominate for a proportionately greater fraction of the annual cycle.

In contrast with permafrost conditions where the amplitude of annual temperature fluctuations at depth decreases when snow cover is present, in seasonal frost conditions the inclusion of snow cover can actually increase the amplitude of ground temperature fluctuations. The calculations also suggest that, contrary to some statements (cf Brown 1970), the spring snow melt rate has only a very minor effect on mean annual ground temperatures.

Implications for numerical model calculations

In seasonal frost problems with latent heat and seasonal snow cover, the mean annual ground surface temperature is insensitive to details of the snow cover treatment. In addition, ground thermal properties including latent heat have almost no effect on mean ground surface temperatures (assuming the summer regime is unaffected). The quality of numerical predictions of mean ground surface temperatures will depend principally on the ability of the model to treat the summer regime.

The accuracy of prediction of mean ground temperatures at depth will, however, depend also on the correctness of the predicted thermal offset. The thermal offset is strongly determined by the temperature dependence of the soil thermal conductivity and depends as well on the actual freezing depth and duration. When modeling the effects of removing snow covers, the uncertainties associated with predicting the thermal offset may, in seasonal frost problems, significantly affect the reliability of the prediction of ground temperatures at depth.

The depth of frost under snow cover is, on the other hand, highly sensitive to the assumptions made in treating the snow cover and depends critically on the treatment of the autumn period. Unfortunately, it is during this period that the snow cover is most difficult to model. As a result, prediction of frost depths under snow covers may be very unreliable even in the simplest case where the problem is not further complicated by moisture redistribution and frost heaving.

In permafrost problems the snow cover thermal resistance cannot easily be modeled with accuracy and yet mean annual ground temperatures are critically dependent on this factor. In addition, soil latent heat not only increases mean annual ground temperatures, but also increases the sensitivity to other factors such as snow cover, soil thermal conductivity, and surface temperature amplitude. Mean annual temperatures in

permafrost are much more sensitive to all these factors than is the case for seasonal frost conditions and it is doubtful that reliable calculations can be made. Summer thaw depths, on the other hand, are only slightly affected by snow cover and the reliability of prediction depends primarily on the treatment of the summer regime.

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Appendix A—Snow cover evolution

A simplified model of snow cover densification based on the work of Kojima (1966) was used in two of the

runs. The model assumes that vertical compaction of a snow layer is controlled by the weight of the overlying snow layers.

The stress acting at depth Z below the snow surface is

$$[A.1] \quad \sigma = \int_0^{Z(t,t_0)} \rho(Z, T, t_0) dZ = \int_{t_0}^t A(t) dt$$

where $Z(t, t_0)$ = depth below the surface (at time t) of snow deposited at time t_0 , $\rho(Z, t, t_0)$ = snow density, and $A(t)$ = accumulation rate.

Assuming constant mass for a snow layer, the vertical strain rate is given by

$$[A.2] \quad \dot{\epsilon} = -\frac{\partial Z / \partial t}{Z} = \frac{\partial \rho / \partial t}{\rho}$$

According to Kojima (1966) a linear stress-strain rate relationship is appropriate for seasonal snow cover

$$[A.3] \quad \dot{\epsilon} = \sigma / \eta$$

where η is an empirically defined compactive viscosity factor related to snow density and temperature by

$$[A.4] \quad \eta = \eta_0 e^{qp} e^{A/RT}$$

In this equation η_0 and q are empirical coefficients whose value is dependent on snow type, A is the activation energy for snow, R is the universal gas constant, and T the absolute temperature.

Equations [A.2]–[A.4] can be approximated by the central difference expression

$$[A.5] \quad \frac{\rho_j^{m+1} - \rho_j^m}{\Delta t} = \frac{1}{\eta_0 \exp(A/RT_j^m)} \times \frac{\frac{1}{2}(\rho_j^{m+1} + \rho_j^m)\sigma^{m+\frac{1}{2}}}{\exp(q\frac{1}{2}(\rho_j^{m+1} + \rho_j^m))}$$

where ρ_j^m is the density of layer j at time $m\Delta t$.

Solving [A.5] by iteration and using

$$[A.6] \quad \Delta Z_j^{m+1} = \frac{\Delta Z_j^m \rho_j^m}{\rho_j^{m+1}}$$

to calculate the thickness of each layer, the snow density profile can be determined for successive time steps. Further details have been given by Goodrich (1976).

Appendix B—Numerical model for conduction heat transfer with phase change

The Fourier heat conduction equation in one dimension,

$$[B.1] \quad c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right)$$

is approximated by the centred time finite difference equation for layered systems

$$\begin{aligned}
 \text{[B.2]} \quad & (c_{i-1}\Delta x_{i-1} + c_i\Delta x_i)(T_i^{m+1} - T_i^m)/\Delta t \\
 & = \frac{k_{i-1}}{\Delta x_{i-1}} (T_{i-1}^{m+1} - T_i^{m+1} + T_{i-1}^m - T_i^m) \\
 & \quad + \frac{k_i}{\Delta x_i} (T_{i+1}^{m+1} - T_i^{m+1} + T_{i+1}^m - T_i^m)
 \end{aligned}$$

where i and m are space and time indices, $\Delta x_i = x_{i+1} - x_i =$ width of i th element, $c_i =$ volumetric heat capacity of i th element, $k_i =$ thermal conductivity of i th element, and $T_i^m =$ temperature of node i at time $m\Delta t$.

Freezing and thawing are treated by a front tracking method which is capable of good accuracy for problems with phase change at a fixed temperature. The boundary conditions at the moving phase boundary are

$$\text{[B.3]} \quad T(Z(t), t) = 0$$

and

$$\text{[B.4]} \quad k_a \frac{\partial T}{\partial x} \Big|_Z - k_b \frac{\partial T}{\partial x} \Big|_Z = \pm L_v \frac{dZ}{dt}$$

where a and b refer to the two phases (frozen or unfrozen), L_v is the latent heat per unit volume of soil, and Z is the position of the phase interface. Equation [B.4] is approximated by the finite difference equation.

$$\begin{aligned}
 \text{[B.5]} \quad & \pm L_v(Z^{m+1} - Z^m)/\Delta t = \frac{1}{2}k_a \frac{(T_p^{m+1} + T_p^m)}{\bar{Z} - x_p} \\
 & \quad + \frac{1}{2}k_b \frac{(T_{p+1}^{m+1} + T_{p+1}^m)}{x_{p+1} - \bar{Z}}
 \end{aligned}$$

In [B.5] the subscript p designates the element within

which the phase plane is located, Z^m and Z^{m+1} are the positions of the phase plane at times $m\Delta t$ and $(m+1)\Delta t$ respectively, and

$$\bar{Z} = \frac{1}{2}(Z^m + Z^{m+1})$$

The simultaneous solution of [B.2] and [B.5] is achieved by using both upper and lower triangulation. This makes it possible to eliminate both T_p^{m+1} and T_{p+1}^{m+1} so that [B.5] contains only one unknown and can be solved easily for Z^{m+1} . Once Z^{m+1} has been found the forward and backward substitution equations can be used to evaluate the nodal temperatures at time level $m+1$. This procedure yields considerably more accurate results than can be achieved by the more usual apparent heat capacity methods of treating phase change. At the same time the only additional calculation required over that normally needed to solve [B.4] by Gaussian elimination is that involved in the solution of [B.5]. As a result, the method is highly efficient. Equation [B.5] requires modification to allow the phase plane to move across the element boundaries X_p and X_{p+1} . Further modification of the method is required in order to treat cases involving more than one phase plane. In the calculations presented here this was done by noting that at a phase interface the heat flux from within the region bounded by two phase planes diminishes rapidly since the temperatures in this zone quickly approach the freezing point. Thus the corresponding heat flux term on the right-hand side of [B.5] can be ignored without serious loss of accuracy and the progress of the two phase planes can be treated independently. Further details of this method and the modifications required may be found in the paper by Goodrich (1978).