The information available to a moving observer from specularities

Andrew Zisserman
Robotics Research Group
Department of Engineering Science
University of Oxford
Oxford OX1 3PJ

Peter Giblin
Department of Pure Mathematics
The University of Liverpool
Liverpool L69 3BX

Andrew Blake
Robotics Research Group
Department of Engineering Science
University of Oxford
Oxford OX1 3PJ

We examine the information available from the motion of specularities (highlights) due to known movements by the viewer. In particular two new results are presented. Firstly, we show for local viewer movements the concave/convex surface ambiguity can be resolved without knowledge of the light source position. Secondly, we investigate what further geometrical information is obtained under extended viewer movements, from tracked motion of a specularity. We show the reflecting surface is constrained to coincide with a certain curve. However, there is some ambiguity - the curve is a member of a one-parameter family. Fixing one point uniquely determines the curve.

1 Introduction

One of the aims of computer vision is to extract concise surface descriptions from several images of a scene. The descriptions can be used for the purposes of object recognition and also for geometric reasoning (such as collision avoidance). Stereo vision determines depths at surface features (such as edges and creases), often only sparsely distributed. It cannot yield full information on surface shape.

Ambiguity arises when the distribution of surface elements is very sparse. This is common with smooth, especially man-made, objects. Typically the only visible surface features will be contours, (steps or creases) between adjacent surfaces. Apart from at contours, surface shading varies smoothly, making stereo correspondence difficult. But judicious analysis of surface shading can considerably augment the geometric information obtained directly from stereo vision.

Ikeuchi [1] has described methods of finding the surface normal (and hence the depth) at every point of a smooth surface by using the shading and a bounding stereoscopically viewed contour. However, "shape from shading" algorithms of this type depend on having precise photometric information about the light source and surface reflectance properties. This is not possible, except in a strictly controlled environment.

An alternative is to use more qualitative methods which do not return the depth at every point [2]. There are a number of shading cues which can provide robust and reliable information about surface shape and source position. For example, specularities (highlights), shadow boundaries and self-shadowing can yield considerable information on local surface curvature and source position. Similarly extremal boundaries constrain the local curvature of the surface (specifically the Gaussian curvature) and extrema of intensity can be related to surface characteristics [3].

In this paper we address the question:

> *What information is available from observing the movement of specular points in two or more images for known viewer motion?*

Here we are not concerned with the detection of specularities [4,5,6] and no surface reflectance characteristics are assumed [5,6,7,8] other than the simple mirror condition. Where necessary we assume a point light source. This is not a restriction because if the source has finite extent then the brightest point of the specularity can be used. We do not use the information contained in the shape or intensity profile of the specularity [8,9,10]. Section 2 is a brief review of two approaches for deriving surface shape from the movement of specularities.

Two new results are presented: In section 3 we describe a simple test which resolves the convex/concave ambiguity. All that is required is two views of the scene and an estimate of viewer-surface distance. No knowledge of the light source position, or surface slant is needed. Ex-

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amples are given in section 4. In section 5 we show that extended viewer movements (where the specular point is tracked through many images) constrain the reflecting surface to coincide with one member of a one-parameter family of curves. If the curve passes through a known point on the surface, then the ambiguity is removed and the curve uniquely determined.

2 Motion of specular points

Koenderink and Van Doorn [3] give a qualitative description of the movement of specularities as the vantage point changes by considering the Gauss map of the surface. Using this analysis it is clear that the velocity of the specularity is less if the curvature is high (it depends on the Weingarten map of the surface - the differential of the Gauss map), so that "(specularities) tend to cling to the strongly curved parts"; and also that specularities are created or annihilated in pairs at parabolic lines on the surface, and move transversely to the lines at their moment of creation. This approach is valid provided the distance of the viewer from the surface is much greater than the largest radius of curvature.

Local metric information (constraints on the surface curvature) can be obtained from 2 views [9] provided the position of the light source and of a surface feature (close to the reflecting point) are known. The 2 views might be a stereo pair or from a moving monocular observer. In either case the baseline is assumed known and surface features can be matched using the epipolar constraint. The specularity will move relative to these surface features. The constraints on surface curvature are contained in the *Specular Motion Equation* [9]. This is a linearised relation between the change $\mathbf{x} = (x_1, x_2)$ in the position of the specularity in the tangent plane of the surface, and the (small) viewer movement $\mathbf{d} = (d_1, d_2, d_3)$. The linear system can be expressed as [11]

$$2V(MH - \kappa_{VL}I)\mathbf{x} = \mathbf{w}, \quad (1)$$

where

$$\mathbf{w} = (-d_1 + d_3 \tan\sigma, -d_2)^T,$$

$$M = \begin{pmatrix} \sec\sigma & 0 \\ 0 & \cos\sigma \end{pmatrix},$$

and

$$\kappa_{VL} = \frac{1}{2}\left(\frac{1}{V} + \frac{1}{L}\right)$$

The coordinate frame is the local normal frame, where the origin lies on the surface (at the reflecting point), and the z-axis is along the local surface normal. It is arranged so incident and reflected rays are in the xz plane, and the movement of the specularity is in the xy plane - the local tangent plane. The angle of reflectance is σ. Vectors $\mathbf{V}$ and $\mathbf{L}$ are vectors from the origin to the viewer and light source ($V = \|\mathbf{V}\|$ and $L = \|\mathbf{L}\|$). The Hessian matrix H is the matrix of second partial derivatives of the surface $z(x, y)$,

$$H = \begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix}$$

In the normal coordinate frame the eigen-values of H are the principal curvatures of the surface.

This linear approximation is valid if the baseline is relatively short, that is, when

$$\|\mathbf{d}\| \ll \|\mathbf{V}\| \cos\sigma,$$

and provided the surface does not focus incoming rays to a point or line close to the centre of projection.

3 Local viewer movement

We describe a simple test, making minimal assumptions, for distinguishing between convex and concave surfaces. Loosely, we show that on a convex surface the specularity *moves with* the viewer (sympathetic motion), whereas on a concave surface the movement is (in general) *against* the viewer motion (contrary motion). You can convince yourself that this is true by looking at specular reflections in the front and back surface of a spoon. The terms "moves with" and "moves against" the viewer motion are made precise in the following theorem (which is proved in the appendix)

Theorem 1 *If H is negative definite (surface locally convex elliptic) then $\mathbf{d}_\perp.\mathbf{x}_\perp \geq 0$. If H is positive definite (surface locally concave elliptic) and the smallest principal curvature κ satisfies $\kappa > \sec\sigma\kappa_{VL}$ then $\mathbf{d}_\perp.\mathbf{x}_\perp \leq 0$.*

Here, $\mathbf{d}_\perp$ and $\mathbf{x}_\perp$ are the projections of the vectors $\mathbf{d}$ and $\mathbf{x}$ onto the plane perpendicular to $\mathbf{V}$. Their calculation is described below. Because other surface shapes are possible (for example hyperbolic) where the scalar product could have either sign, it is the corollaries that provide the useful tests:

1. If $\mathbf{d}_\perp.\mathbf{x}_\perp < 0$ ("contrary motion") then the surface *is not* convex.

2. If $\mathbf{d}_\perp.\mathbf{x}_\perp > 0$ ("sympathetic motion") then the surface *is not* concave, unless one of the principal curvatures is less than $\sec\sigma\kappa_{VL}$.

The first test shows it is always possible to determine if a surface is not convex.

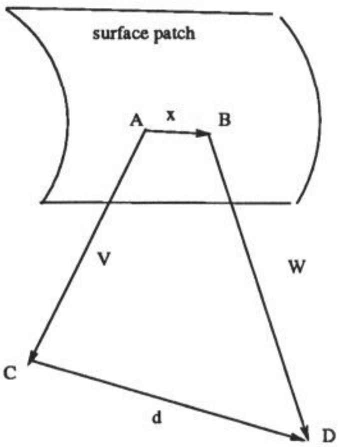

Figure 1: *Specularity Geometry. The viewer moves between points C and D. The specularity moves (in the tangent plane of the surface) between A and B. The vectors* $\mathbf{V}$ *and* $\mathbf{W}$ *are the reflected rays at A and B.*

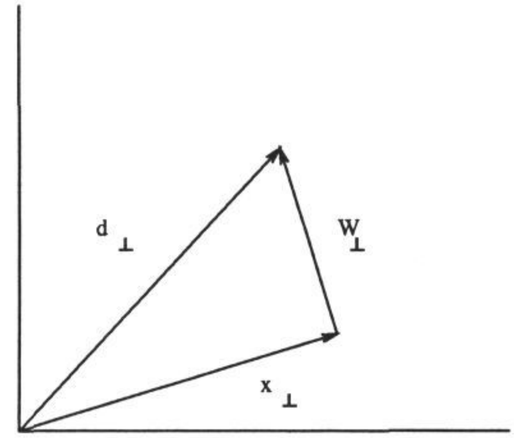

Figure 2: *Projected vectors in the plane perpendicular to* $\mathbf{V}$*. It is clear that if the magnitude of* $\mathbf{W}$ *(and hence* $\mathbf{W}_\perp$*) varies, the scalar product* $\mathbf{d}_\perp.\mathbf{x}_\perp$ *can have either sign.*

Calculation of $\mathbf{d}_\perp.\mathbf{x}_\perp$

Using the geometry shown in figure 1, we have the simple vector cycle:

$$\mathbf{V} + \mathbf{d} - \mathbf{W} - \mathbf{x} = 0$$

The projected vectors are

$$\begin{aligned} \mathbf{d}_\perp &= \mathbf{d} - (\mathbf{d}.\hat{\mathbf{V}})\hat{\mathbf{V}} \\ \mathbf{x}_\perp &= \mathbf{d}_\perp - \mathbf{W}_\perp \\ &= \mathbf{d}_\perp - W\mathbf{U} \end{aligned} \tag{2}$$

where $\mathbf{U} = \hat{\mathbf{W}} - (\hat{\mathbf{W}}.\hat{\mathbf{V}})\hat{\mathbf{V}}$, and $\hat{\mathbf{V}}$ indicates a unit vector. Hence,

$$\mathbf{d}_\perp.\mathbf{x}_\perp = |\mathbf{d}_\perp|^2 - W(\mathbf{d}_\perp.\mathbf{U}) \tag{3}$$

To calculate the scalar product then, involves only knowing the viewer motion ($\mathbf{d}$), the directions of the reflected ray in each view ($\hat{\mathbf{V}}$ and $\hat{\mathbf{W}}$), and an estimate of the surface distance W. The important point is the test does not require any knowledge of the light source position, or the surface slant. The estimate of the viewer-surface distance can be obtained, for example, from a nearby surface feature (whose position can be measured using binocular stereo). It is worth noting that in the coordinate frame defined by the triad $\{\hat{\mathbf{d}}_\perp, \hat{\mathbf{V}} \wedge \mathbf{d}_\perp, \hat{\mathbf{V}}\}$ the length $\mathbf{d}_\perp.\mathbf{x}_\perp$ is the epipolar (horizontal) disparity.

4 Errors in convex/concave test

The test only involves the sign of the scalar product. However, the magnitude can be used to gauge the immunity to errors.

The important question is how sensitive is the sign of the scalar product to the estimate of W? It is clear from equation (2) and figure 2 that the scalar product will *always* change sign eventually as W varies. From (3) the sign change occurs at W_0, where

$$\begin{aligned} W_0 &= \frac{|\mathbf{d}_\perp|^2}{\mathbf{d}_\perp.\mathbf{U}} \\ &= \frac{|\mathbf{d}_\perp|^2}{\hat{\mathbf{W}}.\mathbf{d}_\perp} \end{aligned}$$

The error in this estimate is given by

$$\delta W_0 = \frac{\partial W_0}{\partial \hat{\mathbf{V}}}\delta\hat{\mathbf{V}} + \frac{\partial W_0}{\partial \hat{\mathbf{W}}}\delta\hat{\mathbf{W}}$$

where $\delta\hat{\mathbf{V}}$ = (error in $\mathbf{V}$ perpendicular to $\hat{\mathbf{V}}$)/$||\mathbf{V}||$, and it is assumed that the error in $\mathbf{d}$ is negligible.

Provided the estimate of W is outside the range ($W_0 \pm \delta W_0$) we can be confident in the sign of the scalar product.

Results of convex/concave test

The results of applying this test to the image pairs shown in figures 3-5 are tabulated in table 1.

For the computation, the specularities are detected using Brelstaff's specularity detector and matcher [4]; the estimate of W is obtained from the distance to a surface feature close to the specularity. This distance is determined by the PMF binocular stereo algorithm [12]. The error estimates for $\mathbf{V}$ and $\mathbf{W}$ are based on an error of $\pm\frac{1}{2}$ pixel in localising the brightest point of the specularity in the image. In all cases (see table 2) the estimate of W falls outside the region ($W_0 \pm \delta W_0$) so we can be confident that the sign of the scalar product is correct.

Table 1: Scalar product and interpretations.

Figure	Description	$\mathbf{d}_\perp.\mathbf{x}_\perp$	Interpretation
3	convex ellipsoid	0.024991	not concave
4	concave ellipsoid	-0.021369	not convex
5	beach ball	0.001497	not concave

Table 2: Viewer-surface distance estimate and safety margins.

Figure	W	W_0	δW_0
3	10.574994	10.799026	0.037323
4	10.632490	10.447312	0.028527
5	0.900474	0.964750	0.000453

The first two pairs of stereo images (figures 3 and 4) are computer generated, using a narrow field of view to exaggerate disparities. This makes it easy to see relative displacement of specularities. Figure 3 shows a convex surface with a stereoscopically visible specularity and nearby surface markings. Note that the displacement of the specularity (relative to surface markings) is in the same direction as the relative displacement of the optical centres of left and right views. That is the motion of the specularity is sympathetic ($\mathbf{d}_\perp.\mathbf{x}_\perp > 0$). Figure 4, however, shows a concave surface and, as expected, the relative displacement of the specularity is reversed, opposing the motion of the viewer ($\mathbf{d}_\perp.\mathbf{x}_\perp < 0$). Figure 5 is a real image of a beach ball (convex), and again the motion of the specularity is sympathetic ($\mathbf{d}_\perp.\mathbf{x}_\perp > 0$).

5 Global viewer movement

In this section we describe what information is available from extended or continuous viewer movements, where the information from many views is combined.

We prove the following theorem:

Theorem 2 *Given a source of light S (or a direction of light from infinity), and for each point r on a curve in R^3, given the direction of a reflected ray through r (the reflection being from an unknown reflecting surface M) then this determines a unique curve m (the locus of the reflecting points) on M provided one point on m is known.*

The proof is given in the appendix.

Thus, given a fixed light source and surface, and observing the direction of the reflected rays as the viewer moves, uniquely determines a curve on the reflecting surface. Without a known point r there is ambiguity, as the curve is only determined by the directions of the reflected rays to lie in a one parameter family. If a point is known on the curve, the ambiguity is removed and the curve uniquely determined. The required point r could be found where the curve m crosses an edge, whose position is known from binocular stereo.

The surface normal is also known along m. This places strong constraints on the local surface curvature. Furthermore, by making a second movement that crosses the original path (so that the paths are transverse, and both transverse to the reflected rays) we obtain transverse curves on the reflecting surface. At the point where these curves cross the surface curvature (principal curvatures and direction of principal axis) is completely determined. We are currently exploring the type and extent of the constraints placed on the surface by curves of this type.

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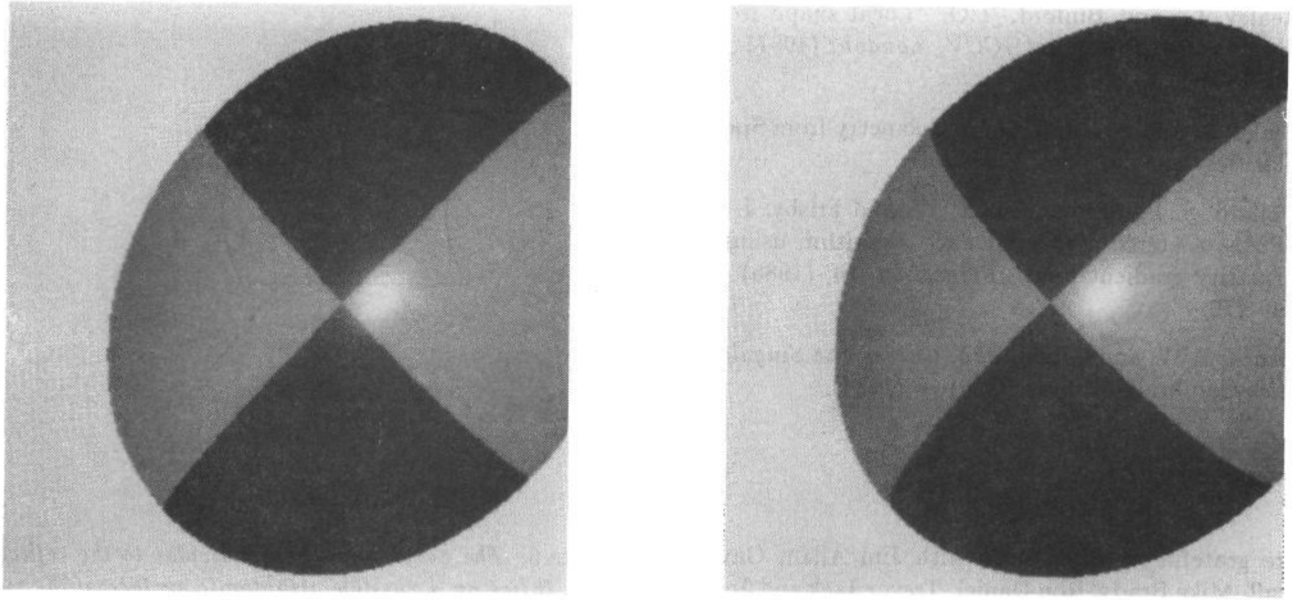

Figure 3: *Convex ellipsoid with surface markings (artificial images).*

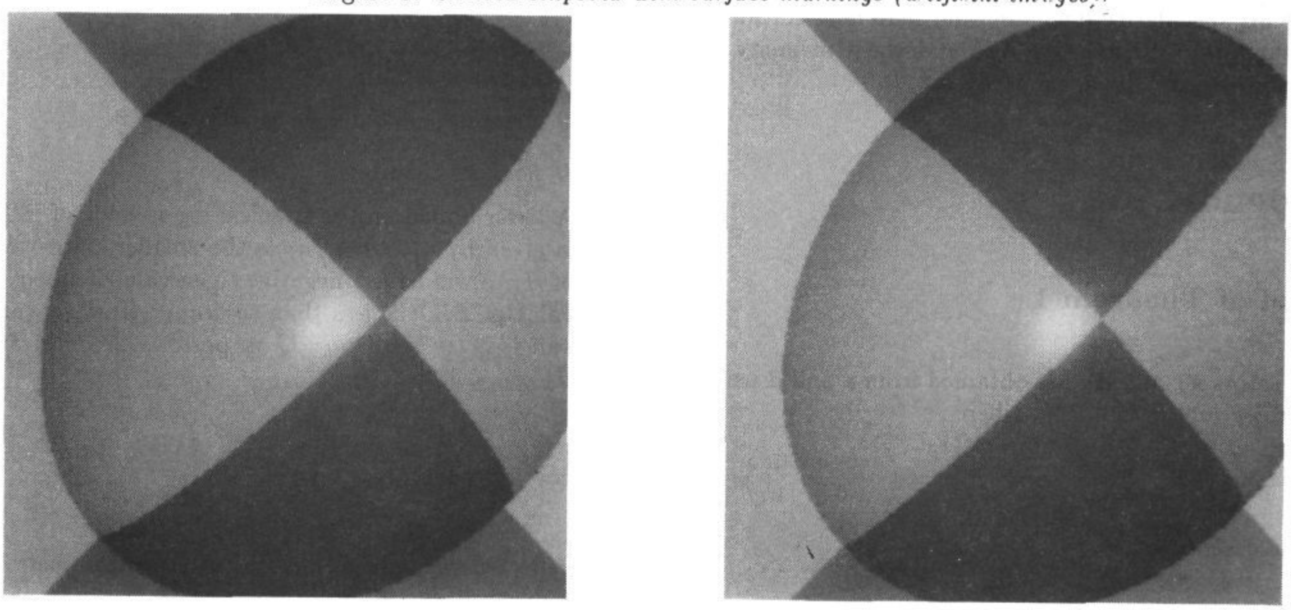

Figure 4: *Concave surface of an ellipsoid with surface markings (artificial images).*

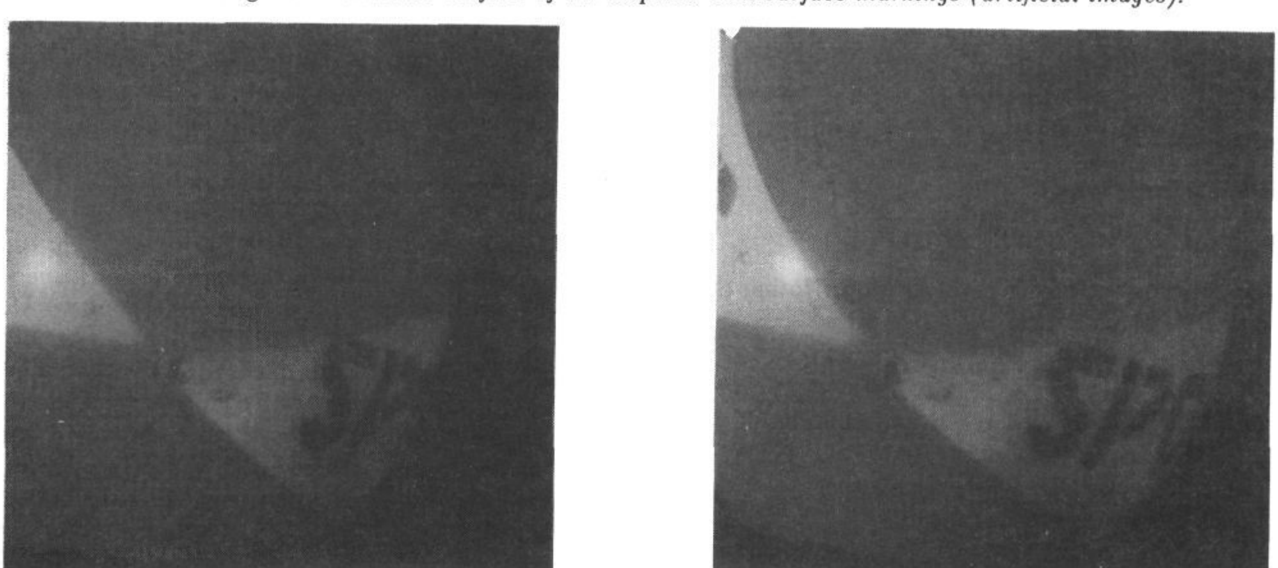

Figure 5: *Beach ball of 12cm radius.*

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Appendix

Proof of Theorem 1

The vectors $\mathbf{x}_\perp$ and $\mathbf{d}_\perp$ are obtained from $\mathbf{x}$ and $\mathbf{d}$ using the projection matrix P^{-1}, where

$$P = \begin{pmatrix} \sec\sigma & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad P^{-1} = \begin{pmatrix} \cos\sigma & 0 \\ 0 & 1 \end{pmatrix}$$

We have

$$\mathbf{x}_\perp = P^{-1}\mathbf{x} \qquad \mathbf{d}_\perp = -P^{-1}\mathbf{w}$$

Using the Specular Motion Equation (1)

$$P^{-1}2V(MH - \kappa_{VL}I)P(P^{-1}\mathbf{x}) = -\mathbf{d}_\perp$$

and hence

$$\mathbf{d}_\perp.\mathbf{x}_\perp = -2V(P^{-1}\mathbf{x})^T \left[P^{-1}(MH - \kappa_{VL}I)P\right](P^{-1}\mathbf{x})$$

This is a quadratic form

$$\mathbf{d}_\perp.\mathbf{x}_\perp = -2V(P^{-1}\mathbf{x})^T Q(P^{-1}\mathbf{x})$$

The sign depends on the eigen-values of Q. Noting that $M = \cos\sigma P^2$

$$Q = \cos\sigma PHP - \kappa_{VL}I$$

Now, $\det(PHP) = \sec^2\sigma \det(H)$, and considering the trace of PHP or noting that $(0,1)PHP(0,1)^T = (0,1)H(0,1)^T$, proves PHP is positive (negative) definite as H is positive (negative) definite.

We consider the two cases:

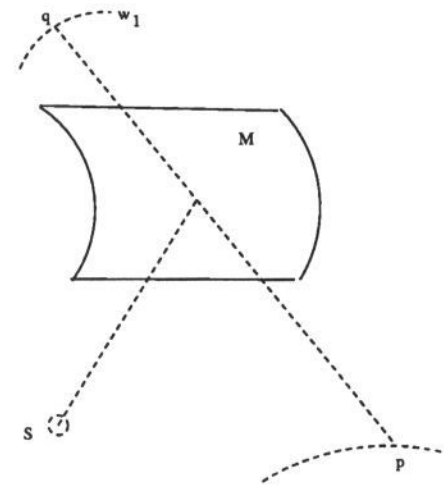

Figure 6: *The curve w_1 is perpendicular to the reflected rays. It lies on a possible orthotomic surface of the mirror M relative to the light source S.*

1. H negative definite

   Q is negative definite, hence $\mathbf{d}_\perp.\mathbf{x}_\perp \geq 0$.

2. H positive definite

   A lower bound on the smallest eigen-value of PHP is given by κ_1, where κ_1 is the smaller eigen-value of H. Q will have a negative eigen-value if an eigen-value of PHP is less than $\sec\sigma\kappa_{VL}$. However, provided $\kappa_1 > \sec\sigma\kappa_{VL}$, Q is positive definite, and $\mathbf{d}_\perp.\mathbf{x}_\perp \leq 0$.

Proof of Theorem 2[1]

The reflected rays are all normal to the orthotomic W of M relative to S (The orthotomic [13] is the locus of reflections of S in tangent planes to M). Choose any point q on the reflected ray (see figure 6). There is a unique piece of curve w_1 through q perpendicular to all the reflected rays through points p. This curve w_1 is on a possible orthotomic surface W_1 through q.

Taking perpendicular bisector planes of segments joining S to points t of w_1 gives a 1-parameter family of planes which are tangent to a possible mirror M_1. For each t the intersection of the perpendicular bisector plane with the reflected ray through t determines a point on M_1.

Hence, the choice of q determines a curve m_1 on a possible mirror M_1. Changing q will change w_1 and hence m_1, so q can be adjusted until m_1 passes through a known point on M. Thus, provided such a known point exists the curve and the surface normals along the curve are determined.

[1] The proof is slightly modified if the light source is at infinity.