Netherlands School of Economics

ECONOMETRIC INSTITUTE

Report 6505

THE INFORMATION VALUE OF DEMAND EQUATIONS AND PREDICTIONS

by H. Theil and Robert H. Mnookin

May 10, 1965

Preliminary and confidential

THE INFORMATION VALUE OF DEMAND EQUATIONS AND PREDICTIONS[1]

by H. Theil and Robert H. Mnookin[2]

Contents

1. INTRODUCTION AND SUMMARY

The objective of classical demand theory is to describe, for some commodity i, $i = 1, \ldots, n$, the quantity bought q_i as a function of income m and prices $p_1, \ldots, p_n$. Income m is identified with total expenditure $\Sigma\, p_i q_i$. If we succeed in performing this task, the value shares $w_i = p_i q_i / m$ are described as well-defined functions of m and the p's. Each of these shares should be nonnegative; their sum should be 1.

We shall never succeed in performing this task completely, since there will be unexplained residuals in all demand equations. An obvious question then is: If our success is not 100 per cent, how great is it? How great is the success if we compare it with naive methods, such as no-change extrapolation, which do not use any sophisticated demand theory at all? Also, it should be remembered that the usefulness of demand equations is frequently limited by imperfect forecasts of income and price changes. The only thing which classical demand theory

[1] The authors are indebted to Mr. A.P. Barten for his comments on this paper and for his willingness to put his data at their disposal, and to Mr. J. Boas for the programming of the computations.

[2] The article was written while R.H. Mnookin was a visitor at the Econometric Institute as a Fulbright grantee.

has to say about these variables is that it considers them to be exogenous. So there is the additional question: What remains of the value of a demand equation when imperfect exogenous estimates are substituted?

The purpose of this article is to present a measure, based on information theory, to evaluate the merits of one demand equation and of a system of such equations. The order of discussion is as follows. We start in Section 2 with a decomposition of value share changes and consider the volume part of that decomposition in Section 3. This volume part is the dependent variable of the demand equation as specified in a recent publication of one of the authors [4], which was in turn largely based on [1]. The specification of Section 3 is in algebraic terms. We proceed numerically in Section 4, which deals with the data and the coefficient values. The evaluation criterion used is the information inaccuracy, which is explained in Section 5. The later sections deal with alternative prediction methods. Section 6 considers no-change extrapolations, Section 7 presents forecasts based on the demand model and on perfect as well as imperfect income and price estimates. It turns out that, when all income and price changes are predicted perfectly, the demand model reduces the average information inaccuracy in the prewar and postwar period by about 50 per cent. The rest is to be ascribed to the disturbances of the demand equations. When the change in real income is predicted perfectly but those in relative prices are predicted to vanish, the success is obviously less but still of some importance. However, when the income and price predictions are based on simple autoregression schemes, the results are scarcely better than those of naive no-change extrapolations. This is shown in Section 8.

The last section deals with the expected value of the information inaccuracy due to the random variability of the coefficient estimates and the disturbances of the demand equations. For this purpose the inaccuracy is approximated by a quadratic expression, so that variances and covariances can be used. It appears that the variances of the disturbances of the demand equations account for about 80-90 per cent of the expected information inaccuracy, and the sampling variances and covariances of the coefficients of these equations for only 10-20 per cent. Among the latter variances those of the income coefficients are more important than those of the price coefficients.

2. THE DECOMPOSITION OF VALUE SHARE CHANGES

Our approach is mainly in terms of value shares, $w_i = p_i q_i / m$, where p_i is the price and q_i the quantity bought of the i^{th} commodity and m income or total expenditure. In particular, it is in terms of <u>changes</u> in value shares in view of the demand equations that will be discussed in Section 3. An infinitesimal change dw_i can be decomposed as follows:

$$(2.1) \qquad dw_i = w_i d(\log q_i) + w_i d(\log p_i) - w_i d(\log m)$$

where log stands for natural logarithm. For finite changes we apply the following approximation:

$$(2.2) \qquad w_i - (w_i)_{-1} \approx \frac{w_i + (w_i)_{-1}}{2} [\log q_i - \log (q_i)_{-1}] + \frac{w_i + (w_i)_{-1}}{2} [\log p_i - \log (p_i)_{-1}] - \frac{w_i + (w_i)_{-1}}{2} [\log m - \log m_{-1}]$$

where the subscript -1 indicates that the value of the previous period is considered. It will prove convenient to use an explicit subscript t for time and to simplify the notation by writing

$$(2.3) \qquad w^*_{it} = \frac{w_{it} + w_{i,t-1}}{2} \qquad D = \Delta(\log \quad)$$

Hence w^*_{it} stands for the average of the i^{th} value share in t and the preceding period, while D is the operator of taking the change in the natural logarithm (the log-change). Then (2.2) is reduced to

$$(2.4) \qquad w_{it} - w_{i,t-1} \approx w^*_{it} Dq_{it} + w^*_{it} Dp_{it} - w^*_{it} Dm_t$$

The last two terms are taken as exogenous in demand theory. The first is the dependent variable of the demand equation that will be discussed in the next section.

3. THE DEMAND MODEL

The demand equations are assumed to be of the following form:

$$(3.1) \qquad w^*_{it} Dq_{it} = B_i D\bar{m}_t + \sum_{j=1}^{n} C_{ij} D\bar{p}'_{jt} + u_{it}$$

the various terms of which will be discussed in the following seven steps:[3]

(1) The left-hand variable, being the first term of the right-hand side of the decomposition (2.4), can be interpreted as the volume component of the change in the i^{th} value share.

---

[1] For details see [4].

(2) The coefficient B_i is the marginal value share $\partial(p_i q_i)/\partial m$. It is assumed to be constant, which implies that Engel curves are approximated linearly. This is restrictive, but probably not too serious, given the moderate changes in real income revealed by our data.

(3) The term $D\bar{m}_t$ is the log-change in real income:

$$D\bar{m}_t = Dm_t - Dp_t \tag{3.2}$$

$$Dp_t = \sum_{i=1}^{n} w^*_{it} Dp_{it} \tag{3.3}$$

This implies that the log-change in the cost of living price index is defined as a weighted average of the log-changes in the individual prices, the weights being the value share averages w^*_{it} in the current and the preceding period. It can be shown that this kind of weighting ensures that we have a local quadratic approximation to the change in the "true" index.

(4) The C_{ij} are coefficients of relative prices. It can be shown that they form an $n \times n$ matrix $[C_{ij}]$ which is equal to the inverse U^{-1} of the Hessian matrix of the underlying utility function, pre- and post-multiplied by a diagonal matrix. [The specification (3.1) is based on the ordinary procedure of maximizing this function subject to the budget constraint $\Sigma\, p_i q_i = m$.] When utility is "additive" (see [2]) we can write the function as

$$u(q_1, \ldots, q_n) = \sum_{i=1}^{n} u_i(q_i)$$

in which case the marginal utility of the i^{th} commodity depends only on q_i, $i = 1, \ldots, n$. Hence the second-order cross derivatives of the utility function are then all zero, so that U is diagonal and the same applies to U^{-1} and $[C_{ij}]$. In the empirical part of this paper we shall confine ourselves to that special case, which means that each (i^{th}) demand equation contains only one relative-price term $C_{ii} D\bar{p}'_{it}$.

(5) The term $D\bar{p}'_{jt}$ is the log-change in the relative price of the j^{th} commodity:

$$D\bar{p}'_{jt} = Dp_{jt} - Dp'_t \tag{3.4}$$

$$Dp'_t = \sum_{i=1}^{n} B_i Dp_{it} \tag{3.5}$$

This means that we do not deflate prices by the cost of living index but by the "marginal" price index (see [3] whose log-change is obtained from the log-changes in the individual prices by using as weights corresponding marginal (instead of average) value shares.

(6) The last term u_{it} is a disturbance, which is assumed to have certain statistical properties. These will be discussed in Section 9.

(7) The coefficients B_i, C_{ij} are subject to certain constraints. One is

$$\sum_{i=1}^{n} B_i = 1 \tag{3.6}$$

Another is that $[C_{ij}]$ os a symmetric matrix; this is, however, irrelevant if we proceed with a diagonal matrix, as we shall do. The third is

$$\sum_{j=1}^{n} C_{ij} = \varphi B_i \qquad \varphi = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} \tag{3.7}$$

In words: The sum of the price coefficients of each demand equation is proportional to the marginal value share. In our case of a diagonal $[C_{ij}]$ this means that the ratio C_{ii}/B_i is equal to φ, which is the income flexibility (the reciprocal of the income elasticity of the marginal utility of income) and is independent of i.

4. THE DATA

We shall work with four commodity groups: Food (i = 1), Vice or pleasure goods (i = 2), Durables (i = 3), and Remainder (i = 4). The data, supplied by A.P. Barten, refer to the Netherlands in the period 1921-1939, 1948-1963; details are given in the Appendix of this paper. We shall consider three periods. The first is the prewar period and consists of 18 observations, starting with the log-changes in 1921/22 and ending with those of 1938/39. The second is the war transition, which consists of only one observation. Here t should be interpreted as 1948, t - 1 as 1939. The third is the postwar period, which consists of 15 observations, the first being 1948/49 and the last 1962/63.

The estimation procedure of the coefficients of the demand equation (3.1) is not the objective of the present paper; we refer to a forthcoming publication by A.P. Barten. Several preliminary results are available, however, which induced us to use the following values:

$$\begin{array}{ll} B_1 = 0.2 & C_{11} = -0.08 \\ B_2 = 0.1 & C_{22} = -0.04 \\ B_3 = 0.4 & C_{33} = -0.16 \\ B_4 = 0.3 & C_{44} = -0.12 \end{array} \tag{4.1}$$

Hence $\varphi = \Sigma C_{ii} = -0.4$, which means that the marginal utility of income decreases by 1 per cent when income goes up by $2\frac{1}{2}$ per cent, prices remaining constant. The B values can be judged conveniently when we divide them by the corresponding value shares (the w's), so that we obtain the income elasticities of the various commodity groups. For all data combined the four average value shares are 0.29, 0.10, 0.24, and 0.37, so that on the basis of these averages the B's of (4.1) imply income elasticities of about 0.7, 1.0, 1.6, 0.8 of Food, Vice, Durables, and Remainder, respectively.

5. A BIT ABOUT INFORMATION THEORY

It will be clear that the demand specification (3.1) is particularly suitable for the prediction of value share changes. We have to predict the log-changes in real income and relative prices, possibly - if we can - the disturbance u_{it} as well, which gives an estimate of $w_{it}^{*}Dq_{it}$. We add to this the estimate of $w_{it}^{*}Dp_{it} - w_{it}^{*}Dm_{t}$, which gives the value share change according to (2.4). By adding this predicted change to last year's value share $w_{i,t-1}$ we obtain a forecast $\hat{w}_{it}$ of w_{it}.

We shall consider several alternative forecasts of this type in the next sections. At this stage it is sufficient to know that, in one way or another, we have obtained forecasts $\hat{w}_{it}$ which satisfy

$$(5.1) \qquad \hat{w}_{it} \geq 0 \quad \text{each } i \text{ and } t \qquad\qquad \sum_{i=1}^{n} \hat{w}_{it} = 1 \quad \text{each } t$$

The question that will be considered here is: Is there an obvious manner to evaluate the quality of such forecasts?

To answer this question we start by observing that (5.1) and the analogous condition on the observed w_{it} imply that we can regard each set of n value shares (predicted as well as observed) as a complete set of probabilities. The forecasts are the "prior" probabilities; at some point of time a message comes in, which states what the value shares actually are and which thus changes the prior probabilities $\hat{w}_{it}$ into "posterior" probabilities w_{it}. The information content of such a message is defined in information theory as

$$(5.2) \qquad I_t = \sum_{i=1}^{n} w_{it} \log \frac{w_{it}}{\hat{w}_{it}}$$

which is always positive unless $w_{it} = \hat{w}_{it}$ for each i (perfect forecasts), in which case $I_t = 0$. The larger the differences between w_{it} and $\hat{w}_{it}$, the worse the forecasts are and the larger the information content of the message on the realization is. Therefore, I_t is called the <u>information inaccuracy</u> of the forecasts $\hat{w}_{1t}, \ldots, \hat{w}_{nt}$ with respect to the corresponding realizations $w_{1t}, \ldots, w_{nt}$ (see [6]).

We shall work with natural logarithms in (5.2), not with logarithms to the base 2 as is customary in most applications of information theory. The reason is that we already worked with natural logarithms in the decomposition (2.1). We shall present average information inaccuracies,

$$\bar{I} = \frac{1}{T} \sum_{t=1}^{T} I_t \tag{5.3}$$

both prewar (T = 18) and postwar (T = 15). It will be noted that the simple additive form of $\bar{I}$ implies that, when additional observations for later years become available, they can be combined very easily with the earlier data.

6. NAIVE MODELS

The simplest prediction method amounts to assuming that there will be no changes in income, prices, and quantities from one year to the next. This amounts to the no-change extrapolation

$$\hat{w}_{it} = w_{i,t-1} \tag{6.1}$$

for which we can compute (5.2) and (5.3). The results are presented on the first line of Table 1, which contains the average information inaccuracy $\bar{I}$ for the prewar and postwar period and the single inaccuracy value of the war transition. It appears that the two averages are of the order of one twentieth of one per cent, while the war transition value is more than ten times larger. This is qualitatively understandable, given that the composition of the consumer's basket in 1948 differs rather substantially from that of 1939.

It is also clear that the extrapolation method (6.1) requires the availability of the value shares in the year preceding the prediction year. Such data are frequently available only after some time lag, so that it is worthwhile to consider also the extrapolation method

$$\hat{w}_{it} = w_{i,t-2} \tag{6.2}$$

This amounts to assuming that, when year t is predicted at the end of year t - 1, the most recent data are those of year t - 2. The corresponding average information inaccuracies of the prewar and postwar period are presented on the third line of Table 1. Since they cannot be based on the first observation (1921/22 and 1948/49) they should be compared with the average inaccuracies of (6.1) which do not include that first year. The latter values are presented on the second

TABLE 1. INFORMATION INACCURACIES OF NO-CHANGE EXTRAPOLATIONS

Forecast $\hat{w}_{it}$	Prewar	Postwar	War
		Four commodity groups	
$w_{i,t-1}$	396	556	6082
Same, first observation excluded	369	451	
$w_{i,t-2}$	765	1386	
		Food	
$w_{i,t-1}$	121	148	1155
Same, first observation excluded	102	153	
$w_{i,t-2}$	279	442	
		Vice	
$w_{i,t-1}$	26	45	2019
Same, first observation excluded	22	46	
$w_{i,t-2}$	38	102	
		Durables	
$w_{i,t-1}$	244	377	3007
Same, first observation excluded	221	324	
$w_{i,t-2}$	274	969	
		Remainder	
$w_{i,t-1}$	161	204	1831
Same, first observation excluded	170	94	
$w_{i,t-2}$	525	410	

Note. All figures are to be multiplied by 10^{-6}

line. The average information inaccuracy for (6.2) is two to three time as large as for (6.1). It is also seen that deleting the first observation reduces the $\bar{I}$ of (6.1), particularly in the postwar period. This is due to the rather sizable value share changes in 1921/22 and 1948/49.

The first three lines of Table 1 are based on I_t as defined in (5.2) for $n = 4$. They deal with the complete decomposition $w_{1t}, \ldots, w_{nt}$. It is also possible to consider only one commodity group by concentrating on one value share w_{it} and its complement $1 - w_{it}$. This amounts to combining all commodity groups other than the i^{th}.[4] Since $1 - \hat{w}_{it}$ is the forecast of $1 - w_{it}$, the resulting information inaccuracy is

$$(6.3) \qquad I_{it} = w_{it} \log \frac{w_{it}}{\hat{w}_{it}} + (1 - w_{it}) \log \frac{1 - w_{it}}{1 - \hat{w}_{it}}$$

[4] It is equally possible to make any other combinations, such as $w_{1t} + w_{2t}$ and $w_{3t} + w_{4t}$, but this will not be pursued here.

and its average over T observations:

$$\bar{I}_i = \frac{1}{T} \sum_{t=1}^{T} I_{it} \tag{6.4}$$

The results are shown in Table 1. They too indicate that extrapolation from t - 2 leads to results that are considerably worse than extrapolating from t - 1. The figures differ rather substantially for the four different i values. However, all figures for the individual commodity groups have in common that they are smaller than the corresponding figure in the first three rows, which deals with all four groups simultaneously. This, in fact, is generally true, because we have

$$I_{it} \leq I_t \tag{6.5}$$

which can be shown as follows. The difference between the two I's is

$$I_t - I_{it} = \sum_{j \neq i} w_{jt} \log \frac{w_{jt}}{\hat{w}_{jt}} - (1 - w_{it}) \log \frac{1 - w_{it}}{1 - \hat{w}_{it}}$$

$$= \sum_{j \neq i} w_{jt} \left[\log \frac{w_{jt}}{\hat{w}_{jt}} - \log \frac{1 - w_{it}}{1 - \hat{w}_{it}} \right]$$

$$= (1 - w_{it}) \sum_{j \neq i} \frac{w_{jt}}{1 - w_{it}} \log \frac{\dfrac{w_{jt}}{1 - w_{it}}}{\dfrac{\hat{w}_{jt}}{1 - \hat{w}_{it}}}$$

Hence $I_t - I_{it}$ is equal to $1 - w_{it}$ multiplied by a conditional information inaccuracy, the condition being that the i^{th} commodity is disregarded. Assuming that $w_{it} < 1$, we conclude that (6.5) holds with the strict inequality sign except when

$$\frac{\hat{w}_{jt}}{1 - \hat{w}_{it}} = \frac{w_{jt}}{1 - w_{it}} \quad \text{for each } j \neq i$$

in which case $I_{it} = I_t$. This limiting case implies that for each commodity $j \neq i$ there is perfect prediction of the amount spent on that commodity when this amount is measured a fraction of what remains of income after subtraction of what is spent on the i^{th} commodity.

7. THE DEMAND MODEL SUPPLEMENTED BY DIRECT INCOME AND PRICE PREDICTIONS

We now turn from naive no-change extrapolations to more sophisticated procedures based on demand equations and on income and price predictions. One should expect that such a procedure would be most successful when the log-changes in income and prices are all predicted perfectly. Going back to (2.4) and (3.1), we conclude that the only source of error is then the disturbance u_{it} of the demand equation, which is put equal to zero instead of its true value.[5] Hence the prediction method amounts to

$$\hat{w}_{it} = w_{it} - u_{it} \tag{7.1}$$

Note that it is assumed here implicitly that the value shares of year t - 1 are known. This seems to be rather obvious in the present context, since the demand equation (3.1) describes only what happens during the transition from t - 1 to t.[6]

The four-group inaccuracy values of the method (7.1) are shown on the second line of Table 2 below the corresponding values of the extrapolation method (6.1), which have been taken from Table 1. It turns out that the former values are about one half of the corresponding latter values in the prewar and postwar period, and about three quarters for the war transition. Hence knowledge of all demand equations and of all income and price changes enables us to reduce the average information inaccuracy of the no-change extrapolations by about 50 per cent in the periods before and after the war. This knowledge is also useful for the description of the war transition, but not as useful (only 25 per cent). The table shows further that similar statements can be made for the individual commodity groups, although these are characterized by some variability. The Food value of (7.1) exceeds that of (6.1) for the war transition; the same applies to the average Vice value of the prewar period.

5 Note that we have ≈ in (2.4), which implies that the right-hand side of that equation does not add up to zero exactly when summed over i. This implies, in turn, that the sum of the forecasts (7.1) over i is not exactly 1, but only approximately. Whenever this is the case for any type of prediction, we have raised or lowered the n forecasts proportionally so that they do add up to 1. (The sum of the u_{it} over i is related to the information difference component, which is generally small; see [4].)

6 It will be noticed that the w^*_{it} by which the log-changes are multiplied in (2.4) is not really known, because it is the average of the past value $w_{i,t-1}$ (which is assumed to be known) and the value w_{it} which is to be predicted and which is, therefore, unknown. This procedure could be refined in the following iterative manner. First, replace w^*_{it} in (2.4) and (3.1) by $w_{i,t-1}$, which leads to a forecast $\hat{w}_{it}$ of w_{it}. Then take the average of this $\hat{w}_{it}$ and $w_{i,t-1}$ and use this as the substitute for w^*_{it}, after which a new forecast $\hat{w}_{it}$ is computed, and so on. However, this would make sense only if one predicts over a longer time span than one year, because the effect of replacing w^*_{it} by $w_{i,t-1}$ is otherwise almost negligible.

(Footnote continued on page 11)

TABLE 2. INFORMATION INACCURACIES OF DEMAND MODELS BASED ON DIRECT INCOME AND PRICE PREDICTIONS

Forecast $\hat{w}_{it}$	Prewar	Postwar	War
		Four commodity groups	
(6.1)	396	556	6082
(7.1)	203	272	4613
(7.2)	271	414	9971
		Food	
(6.1)	121	148	1155
(7.1)	73	76	4573
(7.2)	68	116	1980
		Vice	
(6.1)	26	45	2019
(7.1)	34	22	397
(7.2)	27	44	2102
		Durables	
(6.1)	244	377	3007
(7.1)	89	160	430
(7.2)	129	232	6326
		Remainder	
(6.1)	161	204	1831
(7.1)	84	125	1114
(7.2)	158	186	2878

Note. All figures are to be multiplied by 10^{-6}

The ordinary demand analyst must be expected to predict below the level of (7.1), because his income and price predictions will not be perfect. Perhaps the relative price change predictions are the most difficult ones. So let us adopt a macroeconomic point of view by assuming that the demand analyst confines himself to the prediction of the change in real income and assumes that there are no changes in relative prices. Hence $D\bar{p}'_{jt}$ is predicted to be zero for each j and t. The disturbance u_{it} is also predicted to be zero. We assume that the change in real income is predicted perfectly. Hence $w^*_{it}Dq_{it}$ as defined in (3.1) is predicted to be $B_i D\bar{m}_t$. For the other two terms in the right-hand side of (2.4) we write

$$w^*_{it}Dp_{it} - w^*_{it}Dm_t = w^*_{it}(Dp_{it} - Dp_t) - w^*_{it}(Dm_t - Dp_t)$$

(Footnote 6 continued)

We did compute the information inaccuracy of the approximation error implied by replacing w^*_{it} by $w_{i,t-1}$ in the right-hand side of (2.4), which turned out to be of the order of 1 per cent of the corresponding no-change extrapolation values. The maximum inaccuracy reductions of the more interesting forecasts are of the order of 50 per cent.

The price deals with relative prices ($Dp_{it} - Dp_t$) and is therefore predicted to be zero. The income term is $-w^*_{it}D\bar{m}_t$, which is predicted perfectly. We conclude that the "real income" prediction of value share changes amounts to

$$\hat{w}_{it} = w_{i,t-1} + (B_i - w^*_{it})D\bar{m}_t \tag{7.2}$$

This means that the i^{th} value share is predicted to increase when real income increases if the marginal value share exceeds the average share, i.e., if the income elasticity is larger than 1.

The results are shown in Table 2. As one would have expected, the information inaccuracies are mostly between those of the no-change extrapolation method (6.1) and the "complete" demand method (7.1). The war transition is a major exception, which is primarily due to Durables. This, in turn, was due to the substantial increase in the relative price of Durables from 1939 to 1948, which was only partly compensated by a decrease in quantity.

8. THE DEMAND MODEL SUPPLEMENTED BY AUTOREGRESSIVE INCOME AND PRICE PREDICTIONS

We shall now assume that no direct income and price predictions are available. We suppose, however, that there exists some knowledge of the autoregressive nature of the income and price changes. Consider

$$D\bar{m}_t - \mu = \rho(D\bar{m}_{t-1} - \mu) + \varepsilon_t \tag{8.1}$$

where μ is the long-run average of the log-change in real income, ρ some nonnegative constant less than 1, and ε_t a random variable with zero mean. We shall put $\mu = 0.02$ and experiment with alternative ρ values. The observed average log-change in real income over all 18 prewar and 15 postwar observations is 0.019.

We shall use a similar scheme for relative prices:

$$D\bar{p}_{it} = \rho D\bar{p}_{i,t-1} + \varepsilon_{it} \qquad D\bar{p}_{it} = Dp_{it} - Dp_t \tag{8.2}$$

$$D\bar{p}'_{it} = \rho D\bar{p}'_{i,t-1} + \varepsilon'_{it} \qquad D\bar{p}'_{it} = Dp_{it} - Dp'_t \tag{8.3}$$

Hence we consider two different sets of relative prices, one of which ($D\bar{p}'_{it}$) we already met in the demand equation (3.1) and the other ($D\bar{p}_{it}$) will be needed to handle the price term of (2.4). The ε_{it} and ε'_{it} are regarded as random variables with zero mean; hence the long-run average of the log-change in each relative price is supposed to vanish. To simplify the procedure we shall work with the same parameter ρ in (8.1), (8.2) and (8.3).

Let us rewrite (2.4) as follows:

$$w_{it} - w_{i,t-1} \approx w^*_{it}Dq_{it} + w^*_{it}(Dp_{it} - Dp_t) - w^*_{it}(Dm_t - Dp_t)$$

$$= w^*_{it}Dq_{it} + w^*_{it}D\bar{p}_{it} - w^*_{it}D\bar{m}_t$$

On combining this with the demand equation (3.1) we conclude that $(B_i - w^*_{it})D\bar{m}_t$ is the part of the i^{th} value share change which is to be attributed to the change in real income. Using (8.1) we have

$$(B_i - w^*_{it})D\bar{m}_t = (B_i - w^*_{it})[(1 - \rho)\mu + \rho D\bar{m}_{t-1}] + \varepsilon_t$$

which is estimated from the data of year t - 1 by putting $\varepsilon_t = 0$. Furthermore, we have two price terms. One of these is $w^*_{it}D\bar{p}_{it}$, which we can estimate by $\rho w^*_{it}D\bar{p}_{i,t-1}$, using (8.2). The other is the price term $C_{ii}D\bar{p}'_{it}$ of the demand equation (3.1), which we may estimate by $\rho C_{ii}D\bar{p}'_{i,t-1}$, using (8.3). The two price term estimates combined are therefore

$$\rho(w^*_{it}D\bar{p}_{i,t-1} + C_{ii}D\bar{p}'_{i,t-1}) \approx \rho(w_{it} + C_{ii})D\bar{p}_{i,t-1}$$

where the ≈ sign is based on the approximation of $D\bar{p}'_{i,t-1}$ by $D\bar{p}_{i,t-1}$. The indices Dp_t and Dp'_t are close to each other as is shown in the Appendix (Table 6). We could also have approximated in the opposite direction ($D\bar{p}_{i,t-1}$ by $D\bar{p}'_{i,t-1}$), but the coefficient of $D\bar{p}_{i,t-1}$ exceeds on the average that of $D\bar{p}'_{i,t-1}$ in absolute value, since $\Sigma\, w^*_{it} = 1$ and $\Sigma\, C_{ii} = \varphi = -0.4$.

On combining these various components we obtain the following autoregressive prediction of the value shares:

$$\hat{w}_{it} = w_{i,t-1} + (B_i - w^*_{it})[(1 - \rho)\mu + \rho D\bar{m}_{t-1}] + \rho(C_{ii} + w^*_{it})D\bar{p}_{i,t-1} \tag{8.4}$$

The μ term of the right-hand side implies that the i^{th} value share is subject to an upward trend if the income elasticity of the i^{th} commodity is larger than 1. This is understandable, because that particular term has to do with the long-term increase in real income. The expression in square brackets is a weighted average of last year's log-change in real income and the long-run average log-change μ. If last year's value $D\bar{m}_{t-1}$ exceeds μ, this is a prima facie (autoregressive) indication that this year's value $D\bar{m}_t$ also exceeds μ, so that the effect just described becomes more pronounced. The relative price term has a coefficient $\rho(C_{ii} + w^*_{it})$ which is usually positive. This implies that, if the relative price of the i^{th} commodity increased last year, the i^{th}

TABLE 3. INFORMATION INACCURACIES OF DEMAND MODELS BASED ON AUTOREGRESSIVE INCOME AND PRICE PREDICTIONS

Forecast $\hat{w}_{it}$	Prewar	Postwar
	Four commodity groups	
Extrapolation (6.1)	369	451
Autoregressive forecast (8.4), ρ = 0	430	463
0.2	397	446
0.4	386	438
0.6	399	442
0.8	434	455
	Food	
Extrapolation (6.1)	102	153
Autoregressive forecast (8.4), ρ = 0	92	155
0.2	91	148
0.4	104	146
0.6	130	148
0.8	171	153
	Vice	
Extrapolation (6.1)	22	46
Autoregressive forecast (8.4), ρ = 0	23	46
0.2	24	48
0.4	25	51
0.6	28	54
0.8	31	59
	Durables	
Extrapolation (6.1)	221	324
Autoregressive forecast (8.4), ρ = 0	284	334
0.2	271	318
0.4	265	308
0.6	267	305
0.8	278	308
	Remainder	
Extrapolation (6.1)	170	94
Autoregressive forecast (8.4), ρ = 0	200	101
0.2	165	96
0.4	140	95
0.6	124	96
0.8	118	101

Note. All figures are to be multiplied by 10^{-6}.

value share is predicted to increase. Evidently, the price effect via the quantity term is outweighed by the direct price effect on the value share change. We have a negative price coefficient in (8.4) only if $C_{ii} + w^*_{it} < 0$, which in view of $C_{ii} = \varphi B_i$ is equivalent to $B_i/w^*_{it} > -1/\varphi = 2\frac{1}{2}$. In words: The income elasticity of the i^{th} commodity must be larger in absolute value than the income elasticity of the marginal utility of income; i.e. the commodity must be a real "luxury."

The results of the prediction method (8.4) for some alternative ρ values are presented in Table 3, together with those of the no-change extrapolation method (6.1). [The figures presented refer to the prewar and postwar period excluding the first year, because the $D\bar{m}_{t-1}$ and $D\bar{p}_{i,t-1}$ data are not available for that year.] The outcomes make us sadder but also wiser. There is no gain at all compared with no-change extrapolation in the prewar period, whatever ρ we care to choose, which is probably due to the fact that $\mu = 0.02$ overestimates the increase in real income during that period. [The no-change extrapolation assumes $\mu = 0$, of course, which is about as good an approximation to the observed average prewar log-change.] There is a minor inaccuracy decrease from the extrapolation value in the postwar period (for which a larger μ value than 0.02 would have been more accurate), provided that we choose ρ appropriately. For both periods the best ρ value is around 0.4. The picture of the individual commodity groups varies somewhat, but it is not essentially different.

The autoregressive achievements are therefore rather modest. Given the fairly positive results of the real income predictions of the previous section, we must conclude that - as far as the present evidence goes - it is essential that one have forecasts of real income changes which are more accurate than those afforded by this simple autoregressive approach.

9. THE EXPECTED INFORMATION INACCURACY DUE TO THE RANDOM VARIABILITY OF COEFFICIENTS AND DISTURBANCES

Up to this point we assumed that the true values of the coefficients of the demand eautions (the B's and C's) are known. This will normally not be the case; what we usually have is a set of point estimates and an estimated covariance matrix. The implications of the estimation procedure can also be evaluated along informational lines, although the logarithmic criterion is difficult to adjust to the quadratic estimation criterion which is implied by the use of variances and covariances. We can, however, expand the natural logarithm of $\hat{w}_{it}/w_{it}$ according to powers of the ratio $(\hat{w}_{it} - w_{it})/w_{it}$. The leading nonzero term is quadratic:

$$I_t \approx \frac{1}{2} \sum_{i=1}^{n} \frac{(\hat{w}_{it} - w_{it})^2}{w_{it}} \tag{9.1}$$

The expansion converges when $\hat{w}_{it}$ is positive and smaller than $2w_{it}$. Actually, all of our forecasts are close to the corresponding realization, because even the no-change extrapolations have very small relative errors. Therefore, the quadratic approximation (9.1) may be regarded to be sufficiently accurate.

Let us take the expectation of both sides of (9.1):[7]

---

[7] We disregard here the random nature of the right-hand denominator (w_{it}) of (9.1). This is of minor importance, however, since the random component of w_{it}, given $w_{i,t-1}$, is the disturbance u_{it} of the demand equation whose root-mean-square is very small compared with the expectation of w_{it}; see (9.4) below.

$$(9.2) \qquad \mathcal{E} I_t \approx \tfrac{1}{2} \sum_{i=1}^{n} \frac{\mathcal{E}(\hat{w}_{it} - w_{it})^2}{w_{it}}$$

We shall now evaluate the expectation in the right-hand numerator under the assumption of perfect income and price predictions. Writing $\hat{B}_i$ and $\hat{C}_{ii}$ for the point estimates of B_i and C_{ii}, respectively, we then have

$$\hat{w}_{it} = w_{i,t-1} + \hat{B}_i D\bar{m}_t + \hat{C}_{ii} D\bar{p}'_{it} + w^*_{it} Dp_{it} - w^*_{it} Dm_t$$

$$w_{it} \approx w_{i,t-1} + B_i D\bar{m}_t + C_{ii} D\bar{p}'_{it} + u_{it} + w^*_{it} Dp_{it} - w^*_{it} Dm_t$$

We subtract, square and obtain

$$\begin{aligned}(\hat{w}_{it} - w_{it})^2 \approx\ & (D\bar{m}_t)^2(\hat{B}_i - B_i)^2 + (D\bar{p}'_{it})^2(\hat{C}_{ii} - C_{ii})^2 + u_{it}^2 \\ & + 2D\bar{m}_t D\bar{p}'_{it}(\hat{B}_i - B_i)(\hat{C}_{ii} - C_{ii}) \\ & - 2D\bar{m}_t(\hat{B}_i - B_i)u_{it} - 2D\bar{p}'_{it}(\hat{C}_{ii} - C_{ii})u_{it}\end{aligned}$$

Let us assume that $\hat{B}_i$ and $\hat{C}_{ii}$ are unbiased estimates; let us also make the (classical) assumption that $D\bar{m}_t$ and $D\bar{p}'_{it}$ are fixed (nonstochastic) numbers. Then, after taking the expectation, we conclude that the first term on the right is $(D\bar{m}_t)^2$ multiplied by the variance of $\hat{B}_i$, that the second is $(D\bar{p}'_{it})^2$ multiplied by the variance of $\hat{C}_{ii}$, and that the fourth is $2D\bar{m}_t D\bar{p}'_{it}$ multiplied by the covariance of $\hat{B}_i$ and $\hat{C}_{ii}$. We assume also that the disturbances u_{it} are random with zero mean and variance σ_i^2 (independent of t) and that they are uncorrelated with $\hat{B}_i$ and C_{ii}.[8] Then the expectation of the third term is σ_i^2 and that of the last two terms is zero. Hence:

$$\begin{aligned}\mathcal{E}(\hat{w}_{it} - w_{it})^2 \approx\ & (D\bar{m}_t)^2 \operatorname{var} \hat{B}_i + (D\bar{p}'_{it})^2 \operatorname{var} \hat{C}_{ii} \\ & + 2\, D\bar{m}_t D\bar{p}'_{it} \operatorname{cov}(\hat{B}_i, \hat{C}_{ii}) + \sigma_i^2\end{aligned}$$

On substituting this into (9.2) and averaging over time, so that we obtain the expected value of the average inaccuracy, we find

[8] Note that we do not have to assume that the disturbances are uncorrelated over time. [If they are correlated, however, we can improve on the prediction method (7.1) by taking the correlation pattern and past disturbance values into account.]

$$\mathcal{E}\bar{I} \approx \frac{1}{2T}\sum_{t=1}^{T}(D\bar{m}_t)^2\sum_{i=1}^{n}\frac{\operatorname{var}\hat{B}_i}{w_{it}} + \frac{1}{2T}\sum_{t=1}^{T}\sum_{i=1}^{n}\frac{(D\bar{p}'_{it})^2\operatorname{var}\hat{C}_{ii}}{w_{it}} + \frac{1}{T}\sum_{t=1}^{T}D\bar{m}_t\sum_{i=1}^{n}\frac{D\bar{p}'_{it}\operatorname{cov}(\hat{B}_i,\hat{C}_{ii})}{w_{it}} + \frac{1}{2T}\sum_{t=1}^{T}\sum_{i=1}^{n}\frac{\sigma_i^2}{w_{it}} \tag{9.3}$$

The first three terms on the right represent jointly the effect of the random variation of the demand function coefficient estimates on the expected value of the average information inaccuracy $\bar{I}$. The fourth represents the effect of the disturbances of the demand equation. Each of the first three terms deals with one aspect of the random variation of the coefficient estimates: the first with the variances of the marginal value shares, the second with the variances of the price coefficients, the third with the covariance of $\hat{B}_i$ and $\hat{C}_{ii}$ in each demand equation. Note that covariances of coefficients and disturbances of different demand equations do not occur.

The result (9.3) shows that its computation requires the knowledge of several variances and covariances. We shall estimate the variances σ_i^2 of the disturbances by the mean squares of the 18 + 25 = 33 prewar and postwar observations on the u_{it} which are implied by the B's and C's of (4.1). This gives

$$\begin{aligned} \sigma_1^2 &= 3214 \times 10^{-8} \\ \sigma_2^2 &= 491 \times 10^{-8} \\ \sigma_3^2 &= 4644 \times 10^{-8} \\ \sigma_4^2 &= 4441 \times 10^{-8} \end{aligned} \tag{9.4}$$

To specify the variances and covariances of the B's and C's we start by interpreting the values of (4.1) as unbiased point estimates. Next, we shall specify a covariance matrix of the C's. The preliminary computations mentioned in Section 4 suggest the following matrix:

$$V = 10^{-4}\begin{bmatrix} 4 & 2 & 4 & 3 \\ 2 & 9 & 4 & 4 \\ 4 & 4 & 16 & 8 \\ 3 & 4 & 8 & 16 \end{bmatrix} \tag{9.5}$$

The diagonal elements of V determine the standard errors of the $\hat{C}$'s, which take the following values (in brackets):

$$\hat{C}_{11} = -0.08\ (0.02) \qquad \hat{C}_{33} = -0.16\ (0.04)$$
$$\hat{C}_{22} = -0.04\ (0.03) \qquad \hat{C}_{44} = -0.12\ (0.04)$$

This implies that $\hat{C}_{22}$ does not differ significantly from zero. Furthermore, since $\varphi = \Sigma\, C_{ii}$, we have

$$\operatorname{var} \hat{\varphi} = \sum_{i=1}^{n} \sum_{j=1}^{n} \operatorname{cov} (\hat{C}_{ii}, \hat{C}_{jj}) = 95 \times 10^{-4}$$

$$\operatorname{cov} (\hat{\varphi}, \hat{C}_{ii}) = \sum_{j=1}^{n} \operatorname{cov} (\hat{C}_{ii}, \hat{C}_{jj})$$

This result implies that $\hat{\varphi} = -0.4$ has a standard error of almost 0.1. This standard error tends to be on the high side due to the positive values of the covariances of the $\hat{C}$'s.

We see from (9.3) that variances and covariances involving $\hat{B}$'s are also needed. These will be evaluated on the basis of a large-sample approximation. We have $dB_i/B_i = dC_{ii}/C_{ii} - d\varphi/\varphi$ in view of $B_i = C_{ii}/\varphi$. If we interpret differentials as sampling errors, square both sides and take the expectation, we obtain

$$\frac{\operatorname{var} \hat{B}_i}{B_i^2} = \frac{\operatorname{var} \hat{C}_{ii}}{C_{ii}^2} + \frac{\operatorname{var} \hat{\varphi}}{\varphi^2} - 2\, \frac{\operatorname{cov} (\hat{C}_{ii}, \hat{\varphi})}{C_{ii}\varphi}$$

apart from terms of higher order of smallness. The variance of $\hat{B}_i$ is then approximated by substituting point estimates for the coefficients in the various denominators. This leads to the following standard errors (in brackets):

$$\hat{B}_1 = 0.2\ (0.04) \qquad \hat{B}_3 = 0.4\ (0.06)$$
$$\hat{B}_2 = 0.1\ (0.06) \qquad \hat{B}_4 = 0.3\ (0.06)$$

Finally, the covariance of $\hat{B}_i$ and $\hat{C}_{ii}$ is obtained by multiplying both sides of $dB_i/B_i = dC_{ii}/C_{ii} - d\varphi/\varphi$ by dC_{ii}, which gives

$$\frac{\operatorname{cov} (\hat{B}_i, \hat{C}_{ii})}{B_i} = \frac{\operatorname{var} \hat{C}_{ii}}{C_{ii}} - \frac{\operatorname{cov} (\hat{C}_{ii}, \varphi)}{\varphi}$$

This completes the derivation of the ingredients which are necessary for the breakdown of $\mathcal{E}\bar{I}$ as defined in (9.3). The numerical results for both periods are presented on the first six lines of Table 4. They indicate that about 80 to 90 per cent of the total expected inaccuracy is due to the disturbance variances, both prewar and postwar.

TABLE 4. DECOMPOSITION OF THE EXPECTED VALUE OF AVERAGE INFORMATION INACCURACIES

Breakdown of inaccuracy	Prewar	Postwar
	Four commodity groups	
Total expected inaccuracy	278	299
Due to disturbances	243	232
Due to coefficients	36	66
due to variances of income coefficients	31	57
due to variances of price coefficients	8	6
due to covariances	-3	3
	Food	
Total expected inaccuracy	82	86
Due to disturbances	77	79
Due to coefficients	5	8
due to variance of income coefficient	3	7
due to variance of price coefficient	1	1
due to covariance	1	0
	Vice	
Total expected inaccuracy	50	70
Due to disturbances	30	27
Due to coefficients	21	44
due to variance of income coefficient	20	36
due to variance of price coefficient	4	3
due to covariance	-3	5
	Durables	
Total expected inaccuracy	145	139
Due to disturbances	134	120
Due to coefficients	11	20
due to variance of income coefficient	8	15
due to variance of price coefficient	2	3
due to covariance	1	2
	Remainder	
Total expected inaccuracy	101	110
Due to disturbances	94	98
Due to coefficients	7	13
due to variance of income coefficient	7	14
due to variance of price coefficient	3	2
due to covariance	-3	-3

Note. All figures are to be multiplied by 10^{-6}

This suggests that our limited knowledge of the demand function coefficients is not very serious compared with that of the disturbances. The contributions of the variances of the marginal value shares are four to nine times larger than those of the variances of the price coefficients in spite of the fact that the standard errors of the former coefficients, when measured as fractions of the point estimates, are smaller than the corresponding fractions of the latter coefficients. This must be ascribed to the greater importance of the log-changes in real income relative to those in relative prices. The covariance contributions are small and not of the same sign in the two periods.

For individual commodity groups the derivation is as follows. We start by considering (9.1), which takes the following form in the case of I_{it}:

$$\frac{1}{2}\frac{(\hat{w}_{it}-w_{it})^2}{w_{it}}+\frac{1}{2}\frac{(1-\hat{w}_{it}-1+w_{it})^2}{1-w_{it}}=\frac{1}{2}\frac{(\hat{w}_{it}-w_{it})^2}{w_{it}(1-w_{it})}$$

The further derivation is completely analogous; for the expected value of the average $\bar{I}_i$ we obtain:

$$\mathcal{E}\bar{I}_i \approx \frac{\text{var}\,\hat{B}_i}{2T}\sum_{t=1}^{T}\frac{(D\bar{m}_t)^2}{w_{it}(1-w_{it})}+\frac{\text{var}\,\hat{C}_{ii}}{2T}\sum_{t=1}^{T}\frac{(D\bar{p}'_{it})^2}{w_{it}(1-w_{it})}$$
$$+\frac{\text{cov}\,(\hat{B}_i,\hat{C}_{ii})}{T}\sum_{t=1}^{T}\frac{D\bar{m}_t D\bar{p}'_{it}}{w_{it}(1-w_{it})}+\frac{\sigma_i^2}{2T}\sum_{t=1}^{T}\frac{1}{w_{it}(1-w_{it})} \qquad (9.6)$$

This result shows that the one-commodity values $\mathcal{E}\bar{I}_i$ depend only on the variances and the covariance of the coefficients and disturbances of the corresponding (i^{th}) demand equation. The empirical breakdown is shown in Table 4, which reveals that the picture is largely the same as that of all commodities combined. Vice is an exception to the extent that the coefficient contribution to $\mathcal{E}\bar{I}_2$ has the same order of magnitude as the disturbance contribution.

REFERENCES

[1] A.P. Barten, "Consumer Demand Functions under Conditions of Almost Additive Preferences." Econometrica, Vol. 32 (1964), pp. 1-38.

[2] H.S. Houthakker, "Additive Preferences." Econometrica, Vol. 28 (1960), pp. 244-257.

[3] V. Rajaoja, A Study in the Theory of Demand Functions and Price Indexes. Societas Scientiarum Fennica, Helsinki. 1958.

[4] H. Theil, "The Information Approach to Demand Analysis," Econometrica, Vol. 33 (1965), pp. 67-87.

[5] H. Theil, "Simultaneous Estimation of Complete Systems of Demand Equations." Mimeographed lecture notes, Center for Mathematical

Studies in Business and Economics, The University of Chicago. 1964.

[6] C.B. Tilanus and H. Theil, "The Information Approach to the Evaluation of Input-Output Forecasts." Report 6409 of the Econometric Institute of the Netherlands School of Economics, 1964. To be published in <u>Econometrica</u>.

APPENDIX

The price and volume log-changes Dp_{it} and Dq_{it} are given in Table 5. Their construction by A.P. Barten can be briefly described as follows. From various sources, both published and unpublished, prices and total expenditure series are constructed for 99 basic commodities before the war, and for 108 after the war. Price indices for the four major groups are defined as follows:

$$\text{(A.1)} \qquad Dp_{it} = \sum_{k \in S_i} \frac{\frac{1}{2}(w_{(k)t} + w_{(k)t-1})}{w^*_{it}} Dp_{(k)t} \qquad i = 1, \ldots, 4$$

where S_i is the set of all basic commodities which are part of the i^{th} aggregate, $Dp_{(k)t}$ the log-change in the price of the k^{th} basic commodity, and $w_{(k)t}$ the share of that commodity in the total expenditure on all four major groups. The volume log-change of each basic commodity is defined as the log-change in the expenditure on this commodity minus the log-change in its price, after which Dq_{it} for each major group is derived in a manner similar to (A.1), the two p's being replaced by q's. [Note that the volume figures are all per capita, constructed by dividing expenditures by the mid-year population.] The following survey gives a minor-group idea of the composition of the major group:

<u>Food</u>: Groceries, Dairy products, Vegetables anf fruits, Meat, Fish and Bread

<u>Vice</u>: Tobacco products, Confectionary and ice cream, Beverages

<u>Durables</u>: Clothing and other textiles, Footwear, Household durables, Other durables

<u>Remainder</u>: Water, light and heat, House rent, Services and other commodities.

The all-commodity aggregates Dm_t, Dp_t, Dp'_t are presented in Table 6. It appears that there are only five observations which show a discrepancy between Dp_t and Dp'_t of about 1 or 2 per cent - disregarding the war transition, of course. Table 6 contains also the disturbances u_{it} of the four demand equations. The second-order moment matrix

$$\left[\frac{1}{T} \sum_t u_{it} u_{jt}\right]$$

takes the following values for the prewar and postwar periods (when multiplied by 10^6):

$$\begin{bmatrix} 32 & -1 & -8 & -22 \\ & 6 & -6 & 1 \\ & & 31 & -18 \\ & & & 39 \end{bmatrix} \qquad \begin{bmatrix} 33 & 2 & -10 & -20 \\ & 4 & -8 & 2 \\ & & 65 & -41 \\ & & & 51 \end{bmatrix}$$

respectively, and the following value for all 33 prewar and postwar observations combined:

$$(A.2) \qquad \begin{bmatrix} 32 & 0 & -9 & -21 \\ & 5 & -7 & 1 \\ & & 46 & -28 \\ & & & 44 \end{bmatrix}$$

The computations of Section 9 are based on the diagonal elements of the last matrix, see (9.4). This procedure of using adjusted figures obtained from the sample period is somewhat asymmetric compared with the procedure of the B's and C's, for which we used round members. This objection can be met as follows. A theoretical model has been developed in [5], according to which - under additive preference conditions - the variance of u_{it} is of the form $kB_i(1 - B_i)$ and the covariance of u_{it} and u_{jt} is $- kB_iB_j$. If we specify $k = 2 \times 10^{-4}$, this gives the following theoretical covariance matrix (multiplied by 10^6):

$$(A.3) \qquad \begin{bmatrix} 32 & -4 & -16 & -12 \\ & 18 & -8 & -6 \\ & & 48 & -24 \\ & & & 42 \end{bmatrix}$$

The correspondence between (A.2) and (A.3) is rather close. This holds particularly for the variances, which are the only elements of the covariance matrix which are needed for (9.3) and (9.6). The variance of the Vice equation is the main exception, since the theoretical value in (A.3) is three or four times as large as the observed value in (A.2). If we would use the theoretical value, the exception mentioned at the end of the text would vanish.

The observed and predicted value shares of the four commodity groups are given in Tables 7 through 10.

TABLE 5. LOG-CHANGES IN PRICE AND QUANTITY OF FOUR COMMODITY GROUPS

	Dp_{1t}	Dp_{2t}	Dp_{3t}	Dp_{4t}	Dq_{1t}	Dq_{2t}	Dq_{3t}	Dq_{4t}
1921/22	-1629	-652	-1349	-281	944	-23	1756	104
1922/23	-475	-123	-965	-82	23	-346	-394	-178
1923/24	57	23	41	-13	-111	215	-49	-90
1924/25	331	-86	51	-118	-569	-147	-162	357
1925/26	-687	-637	-713	-88	469	856	467	-44
1926/27	-359	-34	-55	53	251	-16	553	58
1927/28	94	-58	8	74	202	354	246	340
1928/29	-16	-487	-7	25	-117	204	257	265
1929/30	-650	-121	-799	-131	223	201	619	422
1930/31	-1279	-226	-658	-283	311	-258	-394	93
1931/32	-1473	-621	-1176	-320	235	-653	-63	-235
1932/33	-111	-499	-783	-310	-380	-241	245	-9
1933/34	47	-157	-265	-227	-269	-388	-819	-46
1934/35	-371	-542	-337	-287	21	22	-253	-153
1935/36	-97	-281	-919	-376	-142	156	1058	232
1936/37	693	120	724	205	-65	115	-251	-110
1937/38	421	38	425	2	26	313	-738	87
1938/39	-128	37	518	31	443	456	1063	305
1939/48	7957	9148	11,019	5173	-2058	-322	-2656	921
1948/49	591	871	267	378	648	193	1386	-312
1949/50	1163	378	927	536	212	71	182	13
1950/51	758	898	1409	958	187	-177	-1027	-105
1951/52	401	111	-948	377	84	89	-262	-159
1952/53	-125	-61	-159	-32	496	465	573	424
1953/54	352	229	86	659	424	369	1245	173
1954/55	127	46	-29	342	119	274	1186	551
1955/56	394	-74	-71	293	310	822	1233	437
1956/57	474	696	92	637	-244	297	-10	-43
1957/58	-210	387	-68	507	245	-139	-331	-105
1958/59	183	-17	-7	124	155	364	464	256
1959/60	-101	-32	152	457	332	402	1063	386
1960/61	202	47	80	245	422	594	733	141
1961/62	295	83	90	302	244	352	573	308
1962/63	332	130	114	396	340	487	971	345
Average:								
prewar	-313	-261	-347	-118	83	46	175	78
postwar	322	246	125	412	263	298	532	154

Note. All figures are to be multiplied by 10^{-4}. The prewar averages are based on the 18 observations 1921/22 through 1938/39, the postwar averages on the 15 observations 1948/49 through 1962/63.

TABLE 6. LOG-CHANGES IN TOTAL EXPENDITURE AND IN PRICE INDICES AND DISTURBANCES OF DEMAND EQUATIONS

	Dm_t	Dp_t	Dp'_t	u_{1t}	u_{2t}	u_{3t}	u_{4t}	$\sum_{i=1}^{4} u_{it}$
1921/22	-255	-1019	-1015	110	-64	66	-106	5.5
1922/23	-609	-426	-518	47	2	-93	46	2.3
1923/24	-33	26	26	-22	26	15	-18	0.4
1924/25	-42	69	42	-139	-8	8	139	1.2
1925/26	-152	-475	-513	71	44	-54	-62	-0.6
1926/27	117	-111	-81	12	-22	42	-31	-0.2
1927/28	328	52	39	12	2	-57	43	0.0
1928/29	97	-42	-47	-61	-13	12	63	0.3
1929/30	-55	-444	-501	-23	-5	-56	85	-0.1
1930/31	-641	-651	-626	35	-9	-100	74	-0.2
1931/32	-965	-861	-923	42	-38	-13	8	-0.7
1932/33	-451	-376	-478	-60	-15	35	39	-0.5
1933/34	-454	-153	-180	4	-3	-66	66	-0.3
1934/35	-452	-343	-349	26	5	-6	-25	0.0
1935/36	-111	-400	-528	-63	-6	39	31	0.2
1936/37	338	445	502	18	5	26	-50	0.1
1937/38	134	215	259	37	26	-93	30	0.4
1938/39	606	88	195	1	-18	69	-51	0.4
1939/48	6854	7722	8465	-441	83	123	222	-12.4
1948/49	861	459	425	108	-2	181	-238	48.9
1949/50	920	803	802	65	-21	24	-63	4.6
1950/51	739	1018	1092	83	2	-121	33	-2.4
1951/52	-76	17	-174	90	30	-154	40	6.2
1952/53	395	-94	-104	53	2	-66	8	-2.6
1953/54	914	379	326	25	-19	56	-61	1.0
1954/55	713	153	121	-76	-32	59	49	-0.5
1955/56	840	188	131	-20	7	38	-25	-0.1
1956/57	393	450	392	-51	48	-28	32	0.0
1957/58	77	145	121	55	3	-89	29	-1.2
1958/59	384	92	69	-6	5	-11	12	0.1
1959/60	725	176	174	-42	-22	53	10	-0.2
1960/61	589	169	151	32	14	18	-63	0.7
1961/62	588	220	194	-1	-6	-7	14	0.0
1962/63	806	274	244	-10	-10	37	-17	0.1
Average:								
prewar	-144	-245	-261	3	-5	13	16	0.5
postwar	591	297	264	20	0	-1	-16	3.6

See note below Table 5.

TABLE 7. OBSERVED AND PREDICTED VALUE SHARES FOR FOOD

Year	Observed	Forecasts Section 7		Forecasts (8.4)				
		(7.1)	(7.2)	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$	$\rho = 0.8$
1921	3374	.	.	.	.	.	.	.
22	3235	3126	3274	.	.	.	.	.
23	3283	3236	3258	3209	3165	3120	3076	3031
24	3275	3297	3290	3257	3262	3268	3273	3279
25	3212	3352	3289	3250	3258	3266	3274	3282
26	3191	3119	3173	3188	3208	3228	3247	3267
27	3120	3109	3165	3168	3154	3141	3127	3114
28	3111	3098	3090	3098	3086	3075	3063	3051
29	3040	3102	3096	3089	3089	3089	3089	3089
30	2929	2952	3002	3020	3023	3025	3027	3029
31	2835	2800	2928	2912	2888	2885	2871	2858
32	2759	2717	2844	2819	2797	2775	2753	2730
33	2749	2809	2765	2744	2724	2703	2682	2662
34	2814	2810	2772	2733	2746	2759	2772	2785
35	2842	2816	2823	2797	2813	2829	2845	2860
36	2806	2870	2819	2826	2830	2834	2837	2841
37	2888	2870	2815	2789	2798	2807	2816	2825
38	2980	2943	2896	2870	2887	2904	2921	2938
39	2894	2894	2931	2961	2976	2990	3005	3020
1948	2678	3115	2963	.	.	.	.	.
1949	2732	2637	2650	.	.	.	.	.
50	2854	2791	2723	2716	2718	2719	2721	2722
51	2915	2831	2879	2836	2852	2869	2885	2901
52	3074	2986	2925	2895	2895	2895	2895	2895
53	3070	3016	3022	3053	3073	3092	3111	3130
54	3027	3003	3014	3049	3041	3034	3026	3019
55	2890	2966	2973	3008	3000	2991	2983	2975
56	2851	2871	2833	2873	2865	2857	2849	2842
57	2805	2856	2856	2835	2835	2835	2835	2835
58	2794	2738	2810	2789	2793	2797	2800	2804
59	2781	2787	2771	2778	2767	2757	2746	2736
60	2647	2689	2742	2767	2769	2771	2772	2774
61	2656	2624	2620	2634	2619	2604	2589	2575
62	2643	2644	2632	2643	2641	2639	2637	2635
63	2608	2618	2610	2631	2631	2631	2631	2632

Note. All figures are to be multiplied by 10^{-4}.

TABLE 8. OBSERVED AND PREDICTED VALUE SHARES FOR VICE

Year	Observed	Forecasts Section 7		Forecasts (8.4)				
		(7.1)	(7.2)	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$	$\rho = 0.8$
1921	948	.	.	.	.	.	.	.
22	909	973	953	.	.	.	.	.
23	922	919	907	911	915	920	924	929
24	947	921	921	923	925	927	929	930
25	929	937	946	948	948	947	947	947
26	964	920	931	930	928	926	924	921
27	948	971	965	965	963	961	959	957
28	945	943	950	949	950	951	953	954
29	910	923	946	947	946	944	943	942
30	922	928	913	912	907	902	898	893
31	937	946	923	924	927	930	933	936
32	908	946	936	938	943	947	951	955
33	883	897	908	910	912	913	914	915
34	875	878	879	885	882	880	877	874
35	869	864	873	877	876	874	872	871
36	867	873	872	871	869	866	863	860
37	858	853	866	870	870	871	871	871
38	877	851	857	861	858	854	851	847
39	867	886	884	880	878	876	874	871
1948	1052	967	864	.	.	.	.	.
1949	1073	1080	1049	.	.	.	.	.
50	1024	1046	1072	1072	1077	1082	1087	1092
51	1022	1020	1024	1023	1018	1013	1007	1002
52	1051	1021	1022	1021	1021	1021	1020	1020
53	1052	1050	1048	1050	1049	1049	1049	1048
54	1019	1038	1050	1051	1051	1051	1051	1052
55	980	1012	1019	1019	1017	1015	1013	1010
56	971	964	981	980	979	978	977	975
57	1031	983	971	971	967	964	960	957
58	1049	1046	1031	1030	1033	1036	1038	1041
59	1045	1039	1047	1048	1051	1054	1057	1060
60	1008	1030	1043	1044	1043	1041	1039	1038
61	1013	1000	1008	1008	1005	1003	1000	997
62	998	1004	1013	1013	1012	1010	1008	1007
63	979	989	999	998	996	995	993	991

Note. All figures are to be multiplied by 10^{-4}.

TABLE 9. OBSERVED AND PREDICTED VALUE SHARES FOR DURABLES

Year	Observed	Forecasts Section 7		Forecasts (8.4)				
		(7.1)	(7.2)	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$	$\rho = 0.8$
1921	2343	.	.	.	.	.	.	.
22	2495	2430	2463	.	.	.	.	.
23	2315	2409	2466	2527	2539	2551	2564	2576
24	2321	2306	2305	2349	2325	2301	2277	2254
25	2305	2296	2302	2354	2346	2337	2329	2320
26	2283	2337	2360	2339	2327	2315	2304	2292
27	2372	2330	2321	2317	2316	2316	2315	2315
28	2354	2411	2417	2404	2407	2410	2412	2415
29	2390	2379	2377	2387	2388	2389	2391	2392
30	2360	2416	2453	2422	2421	2419	2418	2416
31	2265	2366	2362	2394	2393	2392	2391	2390
32	2204	2217	2247	2300	2294	2288	2281	2275
33	2185	2150	2190	2240	2223	2206	2189	2172
34	2052	2118	2128	2223	2204	2186	2168	2150
35	2024	2030	2030	2091	2069	2048	2026	2004
36	2075	2036	2080	2063	2050	2038	2026	2014
37	2103	2077	2054	2113	2107	2102	2096	2090
38	2011	2104	2087	2142	2134	2127	2119	2111
39	2217	2148	2109	2049	2042	2035	2027	2020
1948	2544	2418	2076	.	.	.	.	.
1949	2753	2585	2599	.	.	.	.	.
50	2806	2783	2768	2778	2777	2777	2776	2775
51	2708	2828	2771	2831	2831	2832	2832	2833
52	2421	2576	2695	2737	2734	2731	2728	2725
53	2425	2490	2498	2452	2420	2388	2356	2325
54	2528	2472	2507	2456	2463	2470	2478	2485
55	2643	2584	2608	2557	2559	2561	2563	2565
56	2730	2691	2729	2670	2674	2679	2683	2688
57	2646	2674	2722	2756	2760	2765	2769	2773
58	2524	2612	2636	2674	2658	2641	2625	2608
59	2542	2553	2566	2553	2540	2527	2515	2502
60	2669	2615	2618	2569	2569	2569	2569	2569
61	2729	2711	2724	2695	2703	2712	2720	2729
62	2749	2756	2775	2754	2757	2760	2763	2766
63	2827	2790	2814	2774	2774	2774	2774	2774

Note. All figures are to be multiplied by 10^{-4}.

TABLE 10. OBSERVED AND PREDICTED VALUE SHARES FOR REMAINDER

Year	Observed	Forecasts Section 7		Forecasts (8.4)				
		(7.1)	(7.2)	ρ = 0	ρ = 0.2	ρ = 0.4	ρ = 0.6	ρ = 0.8
1921	3336	.	.	.	.	.	.	.
22	3362	3470	3309	.	.	.	.	.
23	3481	3436	3369	3353	3381	3409	3436	3464
24	3457	3476	3484	3472	3488	3505	3521	3537
25	3554	3415	3463	3447	3448	3449	3450	3451
26	3562	3623	3536	3543	3537	3531	3525	3520
27	3560	3591	3549	3551	3567	3582	3598	3614
28	3590	3547	3544	3548	3556	3564	3572	3580
29	3660	3597	3581	3577	3577	3577	3577	3577
30	3788	3703	3632	3645	3649	3654	3658	3662
31	3963	3889	3787	3771	3782	3793	3805	3816
32	4128	4120	3973	3942	3966	3991	4015	4040
33	4184	4144	4137	4105	4142	4179	4215	4252
34	4260	4195	4221	4159	4167	4175	4183	4191
35	4265	4290	4274	4235	4242	4250	4257	4264
36	4252	4221	4229	4240	4251	4262	4273	4285
37	4150	4200	4265	4228	4224	4220	4217	4213
38	4132	4102	4160	4127	4121	4115	4109	4103
39	4021	4072	4076	4110	4105	4099	4094	4089
1948	3726	3500	4097	.	.	.	.	.
1949	3442	3698	3702	.	.	.	.	.
50	3316	3381	3437	3434	3428	3422	3417	3411
51	3354	3321	3326	3310	3298	3287	3276	3265
52	3454	3416	3358	3346	3350	3353	3357	3361
53	3453	3444	3432	3445	3458	3471	3484	3497
54	3425	3486	3430	3445	3445	3445	3445	3445
55	3487	3438	3400	3416	3425	3433	3441	3450
56	3448	3473	3456	3477	3482	3486	3491	3495
57	3518	3487	3451	3439	3438	3437	3436	3435
58	3634	3605	3522	3507	3517	3527	3537	3547
59	3633	3621	3616	3622	3642	3662	3682	3702
60	3676	3665	3597	3619	3619	3619	3619	3619
61	3602	3665	3649	3663	3672	3681	3690	3699
62	3610	3596	3579	3590	3590	3591	3591	3592
63	3586	3603	3578	3598	3599	3600	3602	3603

Note. All figures are to be multiplied by 10^{-4}.