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THE INFORMATIVE SAMPLE SIZE FOR DYNAMIC MULTIPLE EQUATION SYSTEMS WITH MOVING AVERAGE ERRORS.

by Harry H. Tigelaar

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The problem considered here, is that of finding suitable conditions for dynamic economic systems that exclude the existence of observationally equivalent structures. Here observational equivalence refers to equality of distributions or first and second moments of a small finite sample from the observable process. It is shown, that under these conditions we may act as if the lagged endogenous variables are nonrandom exogenous variables, when global identifiability is investigated.

1. INTRODUCTION.

A LARGE CLASS of econometric models can be represented by an equation system of the form

$$(1) \qquad \sum_{k=0}^{p} A_k y_{t-k} + Bx_t = \sum_{j=0}^{q} C_j \varepsilon_{t-j} \qquad (t=0, \pm 1, \ldots),$$

where $\{y_t\}$ and $\{\varepsilon_t\}$ are m-variate stochastic processes, $\{x_t\}$ is a *nonrandom* sequence of k-vectors, A_k and C_j are $m \times m$ matrices and B is an $m \times k$ matrix. The integers p and q are supposed to be a priori known, although the highest order of lag may be less than p (or q) in some equations (we only consider the case $p \geq 1$). The last m_0 equations of the system ($m_0 \leq m - 1$) are identities, i.e. equations with known coefficients and zero errors. Using partitioned matrices and -vectors, the model (1) can be written

$$(2)\quad \sum_{k=0}^{p} \begin{array}{c} \left[\begin{array}{c:c} A_k^{(11)} & A_k^{(12)} \\ \hdashline A_k^{(21)} & A_k^{(22)} \end{array}\right] \\ (m-m_0) \quad (m_0) \end{array} y_{t-k} + \begin{array}{c} \left[\begin{array}{c} B^{(1)} \\ \hdashline B^{(2)} \end{array}\right] \\ (k) \end{array} x_t = \sum_{j=0}^{q} \begin{array}{c} \left[\begin{array}{c:c} C_j^{(1)} & 0 \\ \hdashline 0 & 0 \end{array}\right] \\ (m-m_0)\ (m_0) \end{array} \begin{array}{c} \left[\begin{array}{c} \tilde{\varepsilon}_{t-j} \\ \hdashline 0 \end{array}\right] \\ (1) \end{array} \begin{array}{c} (m-m_0) \\ (m_0) \end{array} ,$$

where the integers between brackets indicate the dimensions of the partitioning. We shall use a similar partitioning for the generating functions $A(z) = \Sigma_{k=0}^{p} A_k z^k$ and $C(z) = \Sigma_{j=0}^{q} C_j z^j$. Thus we have $A^{(11)}(z)$, $C^{(1)}(z)$ etc. For notational convenience, all variables and coefficients are allowed to take complex values and we shall transposition of a matrix or vector always combine with complex conjugation, which shall be denoted by an asterisk. This does not complicate the discussion or affect the results.

The process $\{\tilde{\varepsilon}_t\}$ is a zero-mean white noise process with nonsingular covariance matrix $\Omega_{\tilde{\varepsilon}} = E\, \tilde{\varepsilon}_t \tilde{\varepsilon}_t^*$. Thus $\Omega_\varepsilon = E\, \varepsilon_t \varepsilon_t^*$ can be partitioned as

$$(3)\qquad \Omega_\varepsilon \quad \begin{bmatrix} \Omega_{\tilde{\varepsilon}} & 0 \\ 0 & 0 \end{bmatrix} \begin{array}{c} (m-m_0) \\ (m_0) \end{array} \quad .$$

We shall assume that the observable process $\{y_t\}$ has stationary covariances, i.e. $E(y_t - Ey_t)(y_s - Ey_s)^*$ only depends on t-s.

The identifiability problems for models related to the type described above have been treated by several authors e.g. Deistler [1 , 2], Hannan [3 , 4] and Hatanaka [5] , under various conditions. They all have in common, however, that observational equivalence is defined in terms of the infinite dimensional distribution of the observable process. In this paper we shall restrict our attention to first and second-order properties of a finite sample (as small as possible).

Let $\mu_t = Ey_t$ and $\Gamma_s = E(y_t - \mu_t)(y_{t-s} - \mu_{t-s})^*$, $(s,t = 0, \pm 1, \pm 2, \ldots)$. *Then we shall say that the sample size T is second order informative for a parameter* Θ, *if* Θ *can be determined uniquely from* $\mu_1, \ldots, \mu_T$ *and* $\Gamma_0, \Gamma_1, \ldots, \Gamma_{T-1}$. (For a detailed treatment of this concept see

[7 , ch.1]). As a consequence, the question of identifiability of the spectral measure (or equivalently the sequence $\Gamma_0, \Gamma_1, \ldots$) is not trivial anymore, and our starting point is to obtain a second-order informative sample size for it.

We shall assume that the model (1) is complete i.e that A_0 is nonsingular. Premultiplication with A_0^{-1} yields the reduced form

$$(4) \qquad \sum_{k=0}^{p} P_k y_{t-k} + A_0^{-1} B x_t = \sum_{j=0}^{q} Q_j \eta_{t-j} \qquad (t=0, \pm 1, \ldots),$$

where $P_0 = I_m$ (the $m \times m$ unit matrix), $P_k = A_0^{-1} A_k$, $Q_j = A_0^{-1} C_j A_0$ and $\eta_t = A_0^{-1} \varepsilon_t$ (the introduction of $Q_j = A_0^{-1} C_j A_0$ instead of $A_0^{-1} C_j$ will turn out to be useful later on). Notice, that $\Omega_\eta = A_0^{-1} \Omega_\varepsilon A_0^{-1*}$ is not necessarily of the form (3).

Clearly, $P_1, \ldots, P_p$ play exactly the same role as $A_0^{-1} B$ does in the non-dynamic model ($p = q = 0$), *provided that* $P_1, \ldots, P_p$ *and* $A_0^{-1} B$ *are identified*[1]. Therefore the major part of this paper is devoted to obtain an informative sample size for $(P_1, \ldots, P_p, A_0^{-1} B)$. Again for notational convenience, we shall say that a sample is informative for the holomorphic function $\varphi(z)$, if it is informative for the sequence of its Taylor coefficients. In particular we are interested in informative sample sizes for the polynomials $P(z) = A_0^{-1} A(z)$ and $Q(z) = A_0^{-1} C(z) A_0$, and for the rational function $A^{-1}(z) C(z)$.

2. INFORMATIVE SAMPLES FOR THE SPECTRAL MEASURE

Since the spectral measure of the process $\{y_t\}$, or equivalently the covariance function Γ_s does not depend on B and x_t, we can without loss of generality put $x_t = 0$. The reduced form is then an m-variate mixed autoregressive moving average model (ARMA p, q)). We need the following lemma.

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1). This statement shall be made more precise in the final section.

LEMMA 1: *Let U and $\tilde{U}$ be arbitrary $m \times m$ matrices. Then there exist $m \times m$ matrices $S_0, S_1, \ldots, S_m$, such that*

$$U^{m+1} = \sum_{k=0}^{m} S_k U^k \quad , \quad \tilde{U}^{m+1} = \sum_{j=0}^{m} S_j U^j \quad ,$$

PROOF: See [7], lemma 3.2.7.

THEOREM 1: *If $\det A(z)$ is supposed to have no zeros inside the unit circle, then the sample size $q + (m+1)p$ is second-order informative for the spectral measure.*

PROOF: Since every m-variate ARMA (p, q) model can be written as an mp-variate ARMA (1, q) model, we can without loss of generality take $p = 1$. We must, however, keep in mind that a sample of size s from the ARMA (1,q) process corresponds to a sample of size $s + p - 1$ from the original ARMA (p, q) process (*sample size* is interpreted as the number of points in time the process under consideration is observed). Thus we consider the m-variate ARMA (1, q) model

$$(5) \qquad y_t - Uy_{t-1} = \sum_{k=0}^{q} Q_k \eta_{t-k} \quad (t = 0, \pm 1, \ldots),$$

with covariance function Γ_s. Suppose we have an alternative specification satisfying the conditions of the theorem,

$$(6) \qquad y_t - \tilde{U}y_{t-1} = \sum_{k=0}^{q} \tilde{Q}\eta_{t-k} \qquad (t = 0, \pm 1, \ldots),$$

with covariance function $\tilde{\Gamma}_s$. Then we have to prove the implication

$$[\Gamma_s = \tilde{\Gamma}_s \text{ for } |s| \leq q + m] \Rightarrow [\Gamma_s = \tilde{\Gamma}_s \text{ for all } s].$$

Since also $\det P(z)$ ($= \det (I_m - Uz)$) has all its zeros outside the unit circle, the model (5) has a one-sided moving average representation, which implies that the covariance functions Γ_s and $\tilde{\Gamma}_s$ satisfy

(7) $\Gamma_s = U\,\Gamma_{s-1}$, $\tilde{\Gamma}_s = \tilde{U}\,\tilde{\Gamma}_{s-1}$, $(s=q+1,\ q+2,\ \ldots)$.

Suppose we have $\Gamma_s = \tilde{\Gamma}_s$ for $|s| \leq q + m$. Then we have, using (7) and lemma 1

$$\Gamma_{q+m+1} - \tilde{\Gamma}_{q+m+1} = (U^{m+1} - \tilde{U}^{m+1})\Gamma_q = \sum_{k=0}^{m} S_k (U^k - \tilde{U}^k)\Gamma_q =$$

$$= \sum_{k=0}^{m} S_k (\Gamma_{q+k} - \tilde{\Gamma}_{q+k}) = 0 \quad ,$$

and so, by induction $\Gamma_s = \tilde{\Gamma}_s$ for all s.
Thus the sample size $q + m + 1$ is second order informative for the spectral measure of an m-variate ARMA (1,q) process. Hence, for an m-variate ARMA (p,q) process the sample size $q + mp + 1 + p - 1 = q + (m+1)p$ is second-order informative.
This completes the proof.

The spectral density matrix of the process $\{y_t\}$ is well known to be

(8) $$f(\lambda) = \frac{1}{2\pi}\ P^{-1}(e^{-i\lambda})Q(e^{-i\lambda})\Omega_\eta\ Q^*(e^{-i\lambda})P^{-1*}(e^{-i\lambda}) \quad (-\pi < \lambda \leq \pi),$$

thus if we can prove the factorization (8) to be unique, the sample size $q + (m+1)$ is also second-order informative for the reduced form i.e. for $(P(z),\ Q(z),\ \Omega_\eta)$. For nonsingular Ω_η, conditions where given by HANNAN [3]. In the next sections we shall give conditions for the case that we have identities.

3. CONDITIONAL IDENTIFIABILITY AND INFORMATIONAL INDEPENDENCE

Let $\theta \in \Theta$ denote some aggregate of unknown parameters of the model, and suppose $\theta = (\phi,\ \psi)$, where $\phi \in \Phi$ and $\psi \in \Psi$. If $R_n(\theta)$ denotes the covariance structure of a sample of size n from the observable process, then we shall say that ϕ is second-order identifiable *conditional on* ψ, if we have the implication

$$\left.\begin{array}{l}\phi_1 \neq \phi_2 \\ \psi_1 = \psi_2\end{array}\right\} \Rightarrow R_n(\Theta_1) \neq R_n(\Theta_2) \qquad \begin{array}{l}(\Theta_i = (\phi_i, \psi_i) \in \theta \\ \quad i = 1,2.)\end{array}$$

The sample size n is then called second-order informative conditional on ψ. It is easily seen that it implies identifiability of ϕ if ψ is identified. We shall say that the sub-aggregates ϕ and ψ of Θ are *informationally independent*, if all observational equivalence classes are of the form $U \times V$, where $U \subset \Phi$ and $V \subset \Psi$. Notice, that it is not necessary that $\theta = \Phi \times \Psi$. (For a more general and formal treatment of these concepts, see [7 , ch. 1] .) We have

LEMMA 2: *If ϕ is identifiable conditional on ψ, and ϕ and ψ are informationally idependent, then ϕ is identifiable.*

PROOF: Let $\phi_1 \neq \phi_2$, and $\Theta_i = (\phi_i, \psi_i) \in \theta$, $i = 1,2$. If $\psi_1 = \psi_2$ then we have $R_n(\Theta_1) \neq R_n(\Theta_2)$ by the conditional identifiability of ϕ . If $\psi_1 \neq \psi_2$ and $R_n(\Theta_1) = R_n(\Theta_2)$, then Θ_1 and Θ_2 are observational equivalent, and by informational independence we have $\Theta_3 = (\phi_2, \psi_1) \in \theta$ and $R_n(\Theta_1) = R_n(\Theta_3)$. This contradicts the conditional identifiability of ϕ and proves the lemma.

It turns out, that, under rather natural conditions, A_0 and $(P(z), Q(z), \Omega_\eta)$ are informationally independent. Thus, by lemma 2, it suffices to prove the identifiability of $(P(z), Q(z), \Omega_\eta)$ conditional on A_0.

4. THE IDENTIFIABILITY CONDITIONS.

For the identities in the model, we make the following assumptions.

ASSUMPTIONS (a) $A_0^{(22)}$ is nonsingular

(b) degree $[A^{(21)}(z), A^{(22)}(z)] \leq p-1$

(c) $A^{(22)}(z)$ is proper [2].

REMARK: Although assumptions (b) and (c) are not always fulfilled in practice, it is often possible to transform the model so that they are. (see the example in the final section).

The range of the unknown part of $A(z)$, $B(z)$ and Ω_ε must be restricted, in order to avoid observational equivalent structures. The following conditions are natural generalizations of those given by HANNAN for the case $m_0 = 0$.
In the ramainder of this paper we shall write p_0 for degree $[A^{(21)}(z), A^{(22)}(z)]$, and $r[\ .\ ]$ for the rank of a matrix.

CONDITIONS:

(A) $\det A(z) \neq 0, \quad |z| \leq 1$

(B) $\det C^{(1)}(z) \neq 0, \ |z| < 1$

(C) $r[A(z), C(z)] = m \quad$ for all $z \in C$

(D) $r[A_p^{(11)}, C_q^{(1)}] = m - m_0$

(E) degree $[A^{(12)}(z)] \leq p_0$

Since $A_0^{(22)}$ is nonsingular, the model can be transformed into a model satisfying $A_0^{(12)} = 0$. Clearly this has no effect on the reduced form, and so the identifiability of the reduced form under the conditions (A) - (E) is equivalent to identifiability under the extra condition

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2). A polynomial matrix is proper if it has a nonsingular leading coefficient matrix.

(F) $A_0^{(12)} = 0$.

Let $\Theta = (\phi, \psi)$, where $\phi = A_0$ and $\psi = (P(z), Q(z), \Omega_\eta)$. We shall say that the range θ of Θ is *maximal* if it is the set of all pairs (ϕ,ψ) satisfying the conditions (A) - (E), and *sub-maximal* if it is the set of all pairs (ϕ,ψ) satisfying the conditions (A) - (F)[3].

LEMMA 3: *If θ is sub-maximal, then ϕ and ψ are informationally independent.*

PROOF: Let Φ and Ψ denote the range of ϕ and ψ respectively. We shall first prove that $\theta = \Phi \times \Psi$. Let $\phi \in \Phi$ and $\psi \in \Psi$ be arbitrary. If $\phi = \tilde{A}_0$ and $\psi = (P(z), Q(z), \Omega_\eta) = (A_0^{-1} A(z), A_0^{-1} C(z)A_0, A_0^{-1}\Omega_\varepsilon A_0^{-1*})$ then we must show that $\tilde{A}_0 A_0^{-1} A(z)$ and $\tilde{A}_0 A_0^{-1} C(z)A_0\tilde{A}_0^{-1}$ satisfy conditions (A) - (E), and that $\tilde{A}_0 A_0^{-1} \Omega_\varepsilon A_0^{-1*}\tilde{A}_0^{*}$ has zeros in the last m_0 rows and columns. This follows easily from the fact that

$$A_0 A_0^{-1} = \left[\begin{array}{c|c} T & 0 \\ \hline 0 & I_{m_0} \end{array}\right] \qquad ,$$

Since $A_0^{(12)} = \tilde{A}^{(12)} = 0$ and $[A_0^{(21)}, A_0^{(22)}] = [\tilde{A}_0^{(21)}, \tilde{A}_0^{(22)}]$.
Finally, if (ϕ_1, ψ_1) and (ϕ_2, ψ_2) are (second-order) observational equivalent, also (ϕ_1, ψ_2) and (ϕ_2, ψ_1) belong to that equivalence class since the spectral measure (and so the covariance structure of any sample) only depends on ψ. Hence ϕ and ψ are informationally independent.

The following lemma, which was proved in [7 , p. 100] shall also be used to obtain the main result of this paper.

LEMMA 4: *Under the conditions (A) - (E) the polynomials $A(z)$ and $C(z)$ are uniquely determined by A_0 and the rational function $A^{-1}(z)C(z)$.*

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3). If C(z) is a priori known, we may drop the conditions (B) and (D).

If $C(z)$ is an a priori known polynomial (e.g. if $q = 0$) we shall assume that it has a determinant not identically zero. (This assumption takes over the role of condition (B)). In that case we may need the following variant of lemma 4:

LEMMA 4': *Under the conditions (A), (C) and (E) the polynomial $A(z)$ is uniquely determined by A_0 and the rational function $A^{-1}(z)\ C(z)$.*

The proof is a simple version of that of lemma 4 and shall be omitted. Notice that the lemma is non-trivial since $C(z)$ has zeros in the last m_0 rows and columns.

5. INFORMATIVE SAMPLES FOR THE REDUCED FORM.

In this section we shall first prove that there exists a finite set of covariances $\Gamma_0, \Gamma, \ldots, \Gamma_n$, that identifies $(P(z), Q(z), \Omega_\eta)$ uniquely. Therefore we put $x_t = 0$ for all t and return to the general case later on.

THEOREM 2: *Under the conditions (A) - (E) the sample size $q + (m+1)p$ is is second-order informative for the reduced form.*

PROOF: Since we consider the reduced form, we can without loss of generality suppose that the range of $(A_0, P(z), Q(z), \Omega_\eta)$ is *sub-maximal*. Let A_0 be fixed, and suppose that $(A(z), C(z), \Omega_\varepsilon)$ and $(\tilde{A}(z), \tilde{C}(z), \tilde{\Omega}_\varepsilon)$ are (second-order) observational equivalent (with respect to the sample size $q + (m+1)p$). Then it follows from theorem 1 that also the spectral density matrices coïncide which implies

$$(9) \qquad A^{-1}(e^{-i\lambda})C(e^{-i\lambda})\Omega_\varepsilon C^*(e^{-i\lambda})A^{-1*}(e^{-i\lambda}) =$$

$$= \tilde{A}^{-1}(e^{-i\lambda})\tilde{C}(e^{-i\lambda})\tilde{\Omega}_\varepsilon\ \tilde{C}^*(e^{-i\lambda})\tilde{A}^{-1*}(e^{-i\lambda}), \quad (-\pi<\lambda\leq\pi).$$

Putting $W(z) = A(z)\tilde{A}^{-1}(z)$ we obtain from (9)

(10) $$C(e^{-i\lambda})\Omega_\varepsilon C^*(e^{-i\lambda}) = W(e^{-i\lambda})\tilde{C}(e^{-i\lambda})\Omega_\varepsilon\tilde{C}^*(e^{-i\lambda})W^*(e^{-i\lambda}),\quad (-\pi<\lambda\leq\pi)$$

Partitoning $W(z)$ according to the identities, and using the facts that $C(z)$, $\tilde{C}(z)$, Ω_ε and $\tilde{\Omega}_\varepsilon$ have zeros in the last m_0 rows and columns, and $[A^{(21)}(z), A^{(22)}(z)] = [\tilde{A}^{(21)}(z), \tilde{A}^{(22)}(z)]$, with rank m_0 for almost all z, it follows from (10) and the definition of $W(z)$ that we Must have $W^{(21)}(z) = 0$ for all z and $W^{(22)}(z) = I_{m_0}$ for all z. But then (10) implies

(11) $$C^{(1)}(e^{-i\lambda})\Omega_{\tilde{\varepsilon}}C^{(1)*}(e^{-i\lambda}) =$$

$$= W^{(11)}(e^{-i\lambda})\tilde{C}^{(1)}(e^{-i\lambda})\tilde{\Omega}_{\tilde{\varepsilon}}\,\tilde{C}^{(1)*}(e^{-i\lambda})W^{(11)*}(e^{-i\lambda}),\quad (-\pi < \lambda \leq \pi).$$

As $A_0 = \tilde{A}_0$ we have $W(0) = I_m$ and so $W^{(11)}(0) = I_{m-m_0}$.
Since also $C^{(1)}(z)$, $\tilde{C}^{(1)}(z)$ and $W^{(11)}(z)$ are nonsingular inside the unit circle, (11) implies $\Omega_{\tilde{\varepsilon}} = \tilde{\Omega}_{\tilde{\varepsilon}}$ and $C^{(1)}(z) = W^{(11)}(z)\,\tilde{C}^{(1)}(z)$. (see [7 , Lemma 3.2.1]). But then we also have $C(z) = W(z)\,\tilde{C}(z)$ or equivalently $A^{-1}(z)C(z) = \tilde{A}^{-1}(z)\tilde{C}(z)$.
Hence, by lemma 4 it follows that $A(z) = \tilde{A}(z)$ and $C(z) = \tilde{C}(z)$. Thus the sample size $q + (m+1)p$ is second-order informative for $(A(z), C(z), \Omega_\varepsilon)$ *conditional* on A_0. Hence it is also second-order informative for $(P(z), Q(z), \Omega_\eta)$ conditional on A_0. Since A_0 and $(P(z), Q(z), \Omega_\eta)$ are informationally independent by lemma 3, it follows from lemma 2 that the sample size $q + (m+1)p$ is second-order informative for $(P(z), Q(z), \Omega_\eta)$.

In the case that $C(z)$ is a priori known, we can prove (using lemma 4' instead of lemma 4):

THEOREM 2': *Under the conditions (A), (C) and (E) the sample size $q + (m+1)p$ is second-order informative for the reduced form.*

6. IDENTIFICATION OF STRUCTURAL EQUATIONS : EXAMPLE.

In this section we shall suppose that the model (1) is *non-collinear* i.e. that there exists an integer n_0 such that $r[\Sigma_{t=1}^{n_0} x_t x_t^*] = k$.

THEOREM 3: *If the sample size n is second-order informative for $P(z)$, then the sample size max $\{n, n_0\}$ is second-order informative for $A_0^{-1}B$.*

PROOF: Since the model is non-collinear, the sample size n_0 is second-order informative for $A_0^{-1}B$ conditional on $P(z)$. The result follows then from the fact that the sample size n is second-order informative for $P(z)$.

Ignoring the MA-part for the moment, we shall say that the i-th structural equation is identified if the i-th row of $[A(z), B]$ is identified. Let M_i be the matrix, consisting of the columns of $[A_0, A_1, \ldots, A_p, B]$ having zeros in the i-th row. Then the well known rank condition for the structural form states that the i-th row of $[A(z), B]$ is uniquely determined by $[P(z), A_0^{-1}B]$ if $R[M_i] = m-1$.
Thus if the rank condition is satisfied for $i = 1, \ldots, m-m_0$, and the reduced form is identified, then $A(z)$ and B are identified and so are $C(z)$ and Ω_ε.

To illustrate the results we shall derive an informative sample size for the model of KLEIN (see e.g. [6, p.412]). In the notation of the preceding sections it is:

$$y_{t,1} = \beta_0 + \beta_1 y_{t,5} + \beta_2 y_{t-1,5} + \beta_3 (y_{t,3} + x_{t,4}) + \varepsilon_{t,1}$$

$$y_{t,2} = \beta_0' + \beta_1' y_{t,5} + \beta_2' y_{t-1,5} + \beta_3' y_{t-1,6} + \varepsilon_{t,2}$$

$$y_{t,3} = \beta_0'' + \beta_1'' y_{t,4} + \beta_2'' y_{t-1,4} + \beta_3'' x_{t1} + \varepsilon_{t,3}$$

$$y_{t,4} = y_{t,1} + y_{t,2} - x_{t,2}$$

$$y_{t,5} = y_{t,4} - y_{t,3} - x_{t,3}$$

$$y_{t,6} = y_{t-1,6} + y_{t,2}$$

Clearly assumptions (b) and (c) are not fulfilled. When we lag the second equation once, substract it from the second equation and substitute $y_{t-1,6} - y_{t-2,6} = y_{t-1,2}$ we obtain

$$y_{t,2} = (1+\beta_3')y_{t-1,2} + \beta_1'(y_{t,5}-y_{t-1,5}) + \beta_2'(y_{t-1,5}-y_{t-2,5}) +$$

$$+ \varepsilon_{t,2} - \varepsilon_{t-1,2} \quad .$$

Since we eliminated $y_{t,6}$, we may now skip the six-th equation. Notice that the constant term has disappeared and that a MA-part is introduced with known coefficients. The generating functions are now

$$A(z) = \left[\begin{array}{ccc|cc} 1 & 0 & -\beta_3 & 0 & -(\beta_1+\beta_2 z) \\ 0 & 1-(1+\beta_3')z & 0 & 0 & -\{\beta_1'-(\beta_1'+\beta_2')z-\beta_2'z^2\} \\ 0 & 0 & 1 & -(\beta_1''+\beta_2''z) & 0 \\ \hline -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right]$$

and

$$C^{(1)}(z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The next step is to transform the system in order to satisfy condition (E), which means that we must eliminate the lagged values of $y_{t,4}$ and $y_{t,5}$. Straight forward calculation yields the following system

$$y_{t,1} = \beta_0 + \beta_2(y_{t-1,1}+y_{t-1,2}) + \beta_3 y_{t,3} - \beta_2 y_{t-1,3} +$$

$$+ \beta_1 y_{t,5} - \beta_2(x_{t-1,2}+x_{t-1,3}) + \beta_3 x_{t,4} + \varepsilon_{t,1}$$

$$y_{t,2} = (\beta_2'-\beta_1')y_{t-1,1} - \beta_2' y_{t-2,1} + (1-\beta_1' + \beta_2'+\beta_3')y_{t-1,2} - \beta_2' y_{t-2,2} +$$

$$+ (\beta_1'-\beta_2')y_{t-1,3} + \beta_2' y_{t-2,3} + \beta_1' y_{t,5} + (\beta_1'-\beta_2')x_{t-1,2} +$$

$$+ \beta_2' x_{t-2,2} + (\beta_1'-\beta_2')x_{t-1,3} + \beta_2' x_{t-2,3} + \varepsilon_{t,2} - \varepsilon_{t-1,2}$$

$$y_{t,3} = \beta_0'' + \beta_2''(y_{t-1,1}+y_{t-1,2}) + \beta_1'' y_{t,4} + \beta_3'' x_{t,1} - \beta_2'' x_{t-1,2} + \varepsilon_{t,3}$$

$$y_{t,4} = y_{t,1} + y_{t,2} - x_{t,2}$$

$$y_{t,5} = y_{t,4} - y_{t,3} - x_{t,3} \quad .$$

Let $\tilde{A}(z)$ denote the coefficient generating function for the y's. Then

$$\tilde{A}(z) = \left[\begin{array}{ccc|cc} 1-\beta_2 z & -\beta_2 z & -\beta_3+\beta_2 z & 0 & -\beta_1 \\ (\beta_1'-\beta_2')z+\beta_2' z^2 & 1-(1-\beta_1'+\beta_2'+\beta_3')z+\beta_2' z^2 & (\beta_2'-\beta_1')z-\beta_2' z^2 & 0 & -\beta_1' \\ -\beta_2'' z & -\beta_2'' z & 1 & -\beta_1'' & 0 \\ \hline -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right] .$$

Note, that also lagged values of the x's occur. When we put

$$\tilde{x}_t = \begin{pmatrix} 1 \\ x_{t,1} \\ x_{t,2} \\ x_{t-1,2} \\ x_{t-2,2} \\ x_{t,3} \\ x_{t-1,3} \\ x_{t-2,3} \\ x_{t,4} \end{pmatrix} ,$$

the corresponding matrix of coefficients is

$$B = -\left[\begin{array}{ccccccccc} \beta_0 & 0 & 0 & -\beta_2 & 0 & 0 & -\beta_2 & 0 & \beta_3 \\ 0 & 0 & 0 & \beta_1'-\beta_2' & \beta_2' & 0 & \beta_1'-\beta_2' & \beta_2' & 0 \\ \beta_0'' & \beta_3'' & 0 & -\beta_2'' & 0 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{array}\right] .$$

In order that the transformed model is non-collinear, we must have observations on the x's for at least 11 periods.

When we restrict the values of the coefficients such that det $\tilde{A}(z) \neq 0$, $|z| \leq 1$, and $r[\tilde{A}(z), C(z)] = 5$ (which is equivalent to $|\beta_1'| + |\beta_3'| > 0$), then it follows from theorems 2' and 3 that the sample size 13 is second-order informative for the reduced form of the transformed system. The rank conditions for the structural equations of the transformed system are:

$$r|M_1| = r\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta_2' & 0 & 0 & 0 \\ 0 & -\beta_1'' & 0 & \beta_3'' & 0 & 0 \\ -1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 \end{bmatrix} = 4 \Leftrightarrow |\beta_1''| + |\beta_3''| > 0 \ ,$$

$$
r[M_2] = r \begin{bmatrix} 1 & -\beta_3 & 0 & \beta_0 & 0 & 0 & 0 & \beta_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\beta_1'' & \beta_0'' & \beta_3'' & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} = 4 ,
$$

$$
r[M_3] = r \begin{bmatrix} 1 & 0 & -\beta_1 & \beta_2 & 0 & 0 & 0 & \beta_3 \\ 0 & 1 & -\beta_1' & \beta_2'-\beta_1' & \beta_2' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} = 4 ,
$$

where equal columns are only written down once and we skipped columns of zeros. Only the first condition restricts the range of some coefficients. Thus the sample size 13 is now second-order informative for the parameters of the transformed system. Apart from β_0', the parameters of the original system can be uniquely determined from the transformed system; hence they are also identified. On the other hand, β_0' is easily seen to be identified conditional on all other parameters and so the original system is also identified.

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