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# The Integrated Design of Fuzzy Collision-Avoidance and $H_\infty$-Autopilots on Ships

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Collision avoidance remains the most important concern for ships at sea. Despite the electronic equipment now fitted on ships to support the mariner, expert experience is still essential when a ship is in danger of colliding with the others. To include these experts' experiences to resolve the problems of collision, we have designed a fuzzy collision-avoidance expert system that includes a knowledge base to store facts and rules, an inference engine to simulate experts' decisions and a fuzzy interface device. Either a quartermaster or an autopilot system can then implement the avoidance action proposed in the research. To perform the task of collision-avoidance effectively, a robust autopilot system using the state space $H_\infty$ control methodology has been designed to steer a ship safely for various conditions at sea in performing course keeping, course-changing and route-tracking more robustly. The integration of fuzzy collision-avoidance and $H_\infty$autopilot systems is then proposed in this paper.

KEY WORDS

1. Collision avoidance. 2. Autopilots. 3. Automation.

1. INTRODUCTION. In recent years, shipping has rapidly developed in marine nations to meet growing economic demands. In order to remedy the shortage of personnel and to improve the safety of navigation, vessel systems are becoming more and more automatic and intelligent. In this paper, we use the concepts of the fuzzy set theory and fuzzy inference method to design a fuzzy collision-avoidance system. To fit time-varying environments for ship navigation, static obstacles with no prior position information and moving ships with unknown trajectories are considered in this study. Fuzzy logic is applied to guide a ship from a starting point toward the target trajectory without colliding with any obstacles or other ships. Intuitive actions of human beings are modelled into fuzzy rules such that the ship has the capability, like human beings, of avoiding obstacles or other moving ships. These fuzzy rules can be dynamically weighted according to the proximity of the found obstacles or target ships. Furthermore, the proposed approach can also be used for navigation of multiple ships, with few modifications to the original algorithm.

When a ship navigates at the sea, the influence of ship speed, the depth of water and the draft of ship will cause changes in its dynamic properties. Besides, the influence of the currents, winds and waves will cause extra inputs to this system. These uncertain factors may also make a closed-loop system unstable. To eliminate the ill effects of these uncertain factors, we applied $H_\infty$ theory to design an auto-pilot in this paper to find an optimal control law such that the system still has certain robustness

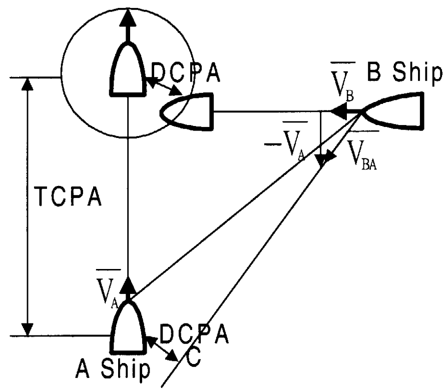


Figure 1. Graphical interpretation of DCPA and TCPA.

under the worst exogenous input, while it keeps the closed loop system stable and ensuring a certain degree of accuracy in tracking target trajectories. This paper is organized as follows: Section 2 illustrates the development of ship collision-avoidance, ship and obstacle safety domains, traffic separation schemes and avoid actions. Section 3 describes the applications of fuzzy theory and basic configuration of fuzzy logic. Section 4 proposes the design process of an $H_\infty$ autopilot, including the theorem for obtaining the state space solution of $H_\infty$ optimal control. Section 5 presents the computer simulation results for the proposed integrated system, which is a combination of fuzzy collision-avoidance system and the $H_\infty$ autopilot system, to demonstrate the feasibility of the proposed integrated system. Finally, a summary and main conclusion of this study are described in Section 6.

## 2. COLLISION-AVOIDANCE OF SHIPS.

Benefiting from the development of modern science, high technology is now widely used in the field of navigation. Satellite navigation and communication systems have been successfully applied to minimize the problems faced by the sailor and, as a result, the problem of collision-avoidance becomes relatively more important. Moreover, collision accidents are increasing as ships increase in size, in speed and in number. The problem of collision-avoidance has thus become an urgent issue.

For the above reasons, the major maritime countries of the world have given a great deal of attention to this problem. One solution to the problem is to establish navigation regulations, to strengthen traffic control, and to improve the technical level of seafarer training, as well as to study collision-avoidance systems.

To obtain the maximum economic benefit, newly built ships are tending to be get bigger and to be operated more automatically. COLREGS (The International Regulations for Preventing Collisions at Sea) are the legal provisions to co-ordinate the behaviour of ships when there is a risk of collision at sea. However, many of these rules are qualitative and can only be used after quantifying the situation. In practice, this causes some difficulties for sailors in the implementation of the COLREGS. For this reason, many researchers in western maritime countries began to study the quantitative methods in the early fifties and sixties. Their results established some specific terms concerning ship collision such as the distance at the closest point of approach (DCPA) and the time to reach the closest point of approach (TCPA).

Figure 1 describes the concepts of DCPA and TCPA. It can be used to determine the possibility of collision between two ships and the remaining time for taking collision-avoidance action to avoid the risk. Note that to avoid a possible collision, the DCPA and TCPA must be considered simultaneously. The appraisal index proposed by Kearon (1997) is the weighted sum of the squares of DCPA and TCPA:

$$\lambda_i = (a\,DCPA_i)^2 + (b\,TCPA_i)^2$$
$$i = 1, 2, 3, \ldots .n,$$

where:

$\lambda_i$ is the appraisal index,
$a$, $b$ are weightings,
$i$ is the number of target ships.

When $\lambda_i$ reaches a preset threshold value, collision-avoidance action must be taken. Statistic analysis of experimental data is explored by Holmes as an alternative method to appraise the danger of ship collision. Shimizu uses the method of fuzzy reasoning and fuzzy control to establish a model for determining the time needed for taking collision-avoidance action. This research uses basic fuzzy theory to design a fuzzy collision-avoidance system, which is an effective one. In recent years, because of the increase of international trade, the current commodities and the flat-top building for oil exploration have made traffic more complex at the sea. In order to resolve the increased danger of collision, researchers have begun to use the concepts of 'Ship and Obstacle Safety Domain Theory.' The definition of a ship domain proposed by Goodwin is: 'The surrounding affective waters that the navigator of a ship wants to keep clear of other ships or fixed objects.' Basically the shape or the size of a ship safety domain is affected by the following factors:

(a) Physical factors: the size of ships, traffic density and relative speed, etc.,
(b) Environment factors: weather, visibility, etc., and
(c) Psychology factors: navigator's work record, etc.

As to the size of a ship safety domain, researchers in many countries have different results, but they have the same viewpoint, that is 'encounters by ships travelling in different directions have different safety domains because of the difference of their sizes, speeds, relative positions and directions.' The safety domain defined by Goodwin (1975) can be divided into three sectors:

Sector 1, starboard sector: $0 \leqslant \theta \leqslant 112\cdot5$
Sector 2, port sector: $247\cdot5 < \theta < 360$
Sector 3, astern sector: $112\cdot5 < \theta < 247\cdot5$

The radius of each sector in Dover Strait and North Sea is listed in Table 1.

Table 1. The value of each sector's radius in different waters (nm).

| | Sector 1 | Sector 2 | Sector 3 |
|---|---|---|---|
| Dover Strait | 0·82 | 0·75 | 0·10 |
| North Sea | 0·85 | 0·70 | 0·45 |

Because this ship safety domain is not continuous, it will increase the complexity in computer simulation. Thus, Davis *et al*. (1982) improves upon Goodwin's safety domain to make the domain boundary continuous. Then, the Japanese scholar Fuji (Toyoda and Fuji, 1971) completed an experiment on ship domains for large, middle and small ships, and he realized that a ship's safety domain has connection with its length and that the domain is not symmetric. The starboard sector is the largest one, the port sector is the next and the astern sector is the smallest one. However, the size of the astern sector increases as a ship increases in length. Thus, considering a conservative ship safety domain, a circle with radius eight times the ship's length is used to describe a ship's safety domain in this research. If other ships enter the ship's safety domain, the proposed fuzzy collision-avoidance system will provide advice to avoid possible collision.

In recent years, there has been a considerable increase in the number of structures used for offshore oil exploitation. Thus, an obstacle safety domain should also be built to avoid collision. The obstacle's size and shape, the depth of surrounding water and the draft of the ship determine the safety domain size of an obstacle. For simulation convenience, we select a circle with radius $1 \cdot 2$ times the obstacle's length radius as our obstacle's safety domain.

The first traffic separation schemes in the world were introduced in the Dover Strait in 1967. A traffic separation scheme is mainly composed of a separation line, a separation zone and a course borderline. For various types of terrain, the separation lines and separation zones can be designed as follows:

(a) Use natural obstacles and geographic features to separate ships travelling in opposite directions.
(b) Utilize the inshore traffic zone to separate the ships travelling in opposite directions.
(c) Construct a fan-shaped zone for a port to separate the course lane.
(d) Construct a ring-shaped separation zone close to conjunction points.

The width of a course lane in open waters is set to be about 5-3-5, i.e., both sides of a course fairway have a width of 5 miles respectively and a width of 3 miles is used as the centre separation zone, and can be up to 20-10-20 (Gung, 1990).

According to the COLREGS, when a ship encounters others at sea, the burden or the privilege ship can be identified by the strategy listed in Table 2 (Leo, 1979).

Table 2. The adopted strategy of the privilege ship and the burden ship.

| | Heading- on encounter | Crossing encounter | Overtaking encounter | By- overtaking encounter |
|---|---|---|---|---|
| Privilege ship | | V | | V |
| Burden ship | V | V | V | |

In any potential collision situation, the navigator faces two questions; do I face a collision of collision? If so, should I take avoiding actions and what actions should be taken while considering all vessels in the vicinity? When an encounter involves a risk of collision (i.e., the DCPA is less than the radius of the ship's safety domain), the actions should be taken to avoid the collision. In Figure 1, ship A is the burden ship and ship B is the privileged one. If both ships keep their current course and speed, ship B will pass the CPA with a relative speed $V_{BA}$. Since the DCPA in this case is less than the distance needed for safety, the navigator of ship A must take action and he has two choices; he can either turn right to obtain sufficient DCPA before passing the CPA or, he can change speed. At a slower speed $V_A'$, the encounter situation is as shown in Figure 2 and will be safe in the sense that the DCPA is greater than or equal

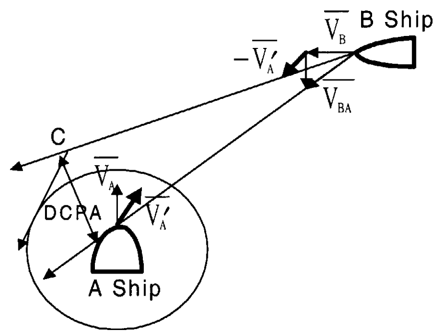


Figure 2. Graphical interpretation of avoiding action.

to the safety distance. For the cases of crossing traffic lanes in a TSS, other regulations should be followed; this is explained in references Gung (1990) and Leo (1979). To offer proper advice to avoid collision between ships, the collision-avoidance system is designed based on Fuzzy set theory and is described in the next section.

3. FUZZY COLLISION-AVOIDANCE SYSTEM. During the past decade, fuzzy logic control has emerged as one of the most active and fruitful areas for research in the application of fuzzy set theory, fuzzy logic and fuzzy reasoning (Kearon, 1977). Since fuzzy reasoning can be done in linguistic ways, which can effectively simplify the complexity in modelling system dynamics especially for non-linear and ill-defined systems like ships, we used fuzzy logic to design the ship collision-avoidance system described in this paper. The basic operation of a fuzzy set can be illustrated as follows:

(a) When U (the universe of discourse) is discrete, a fuzzy set A can be represented as:

$$A = \frac{\mu_A(\chi_1)}{\chi_1} + \frac{\mu_A(\chi_2)}{\chi_2} + \ldots + \frac{\mu_A(\chi_n)}{\chi_n},$$

where: $(\mu_A(\chi_i)/\chi_i)$ represents the relationship of the generic element $\chi_i$ of U and its grade of membership $\mu_A(\chi_i)$ (Lee, 1999, and Lin and Pun (1994)).

(b) Fuzzy intersection; the membership function $\mu_c(x)$ of the intersection $A \cap B$ is defined for all $\mu \in U$ by:

$$\mu_C(\chi) = \min\{\mu_A(\chi), \mu_B(\chi)\} = \mu_A(\chi) \wedge \mu_B(\chi)$$

(c) Fuzzy union; the membership function $\mu_c(x)$ of the union $A \cup B$ is defined for all $\mu \in U$ by:

$$\mu_C(\chi) = \max\{\mu_A(\chi), \mu_B(\chi)\} = \mu_A(\chi) \vee \mu_B(\chi)$$

(d) Fuzzy complement; he membership function $\mu_{\bar{A}}(x)$ of the complement of a fuzzy set A is defined for all $\mu \in U$ by:

$$\mu_{\bar{A}}(\chi) = 1 - \mu_A(\chi)$$

(e) Fuzzy relation; if $A$ and $B$ are fuzzy relation in $x*y$ and $y*z$, respectively, the composition of $A$ and $B$ is a fuzzy relation denoted by $A \circ B$ and the membership function $\mu_C(x, z)$ of the composition $A$ and $B$ is defined by:

$$\mu_C(x, z) = \mu_{A \circ B}(x, z) = \sup\{\min[\mu_A(x, y), \mu_B(y, z)]\},$$

$$or\ c_{ij} = \vee_k \{a_{ik} \wedge b_{kj}\}$$

Based on the above fuzzy operation concepts, the basic configuration of a fuzzy logic controller (FLC) is proposed and shown in Figure 3, which comprises four

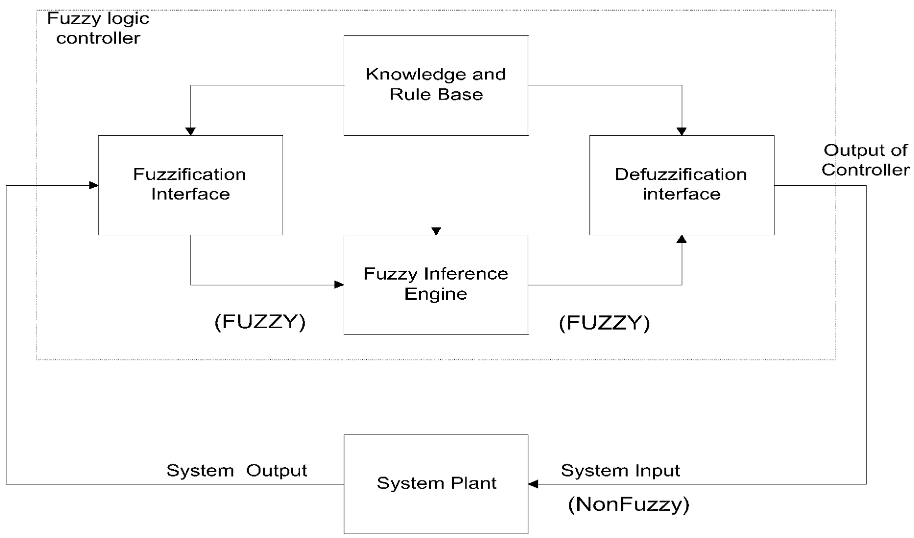


Figure 3. Basic configuration of fuzzy logic controller.

principal components: a fuzzification interface, a knowledge base, an inference engine and a de-fuzzification interface. The main functions of these four components can be described as follows:

(a) The fuzzification interface involves the following functions:
   (i) It receives the state variables from the plant.
   (ii) It transfers the range of values of input variables into corresponding universes of discourse.
   (iii) It performs the function of fuzzification that converts input data into suitable linguistic values.
(b) The knowledge base consists of a 'data base' and a 'linguistic control rule base':
   (i) The database provides necessary definitions that are used to define linguistic control rules and fuzzy data manipulation in an FLC.
   (ii) The rule base characterizes the control policy and control goals of the domain experts by means of a set of linguistic control rules.

(c) The inference engine is the most important kernel, and it is the decision-making centre of a FLC, which is designed by simulating a human thinking model.
(d) The de-fuzzification interface performs the following functions:
   (i) It yields a non-fuzzy control action from an inferred fuzzy control action.
   (ii) It converts the range of values of output variables into corresponding universes of discourse.

Fuzzification is related to the vagueness and imprecision in a natural language. It is a subjective valuation to transform measurement data into valuation of a subjective value. Hence it can be defined as a mapping from an observed input space to labels of fuzzy sets in a specified input universe of discourse. Since the data manipulation in a FLC is based on fuzzy set theory, fuzzification is necessary and desirable at an early stage. In fuzzy control applications, the observed data are usually crisp. A natural and simple fuzzification approach is to convert a crisp value $X_0$ into a fuzzy singleton $A$ within the specified universe of discourse. That is, the membership function $\mu_A(x)$ of $A$ is equal to 1 at the point $X_0$ and zero at other places.

A fuzzy system is characterized by a set of linguistic statements based on expert knowledge. The expert knowledge is usually as 'if-then' rules, which are easily implemented by fuzzy conditional statements in fuzzy logic. Fuzzy control rules have the form of fuzzy conditional statements that relate the state variables in the antecedent and process control variables in the consequence. Many experts have found that fuzzy control rules provide a convenient way to express their domain knowledge. This explains why most FLC are based on the knowledge and experience that are expressed in the language of fuzzy 'if- then' rules. The general form of the fuzzy control rules in the case of two-input single-output systems is:

IF $x$ is $A_1$ and $y$ is $B_1$ THEN $z$ is $C_1$
IF $x$ is $A_2$ and $y$ is $B_2$ THEN $z$ is $C_2$
…
IF $x$ is $A_n$ and $y$ is $B_n$ THEN $z$ is $C_n$

Where $x$, $y$ and $z$ are linguistic variables representing the process state variable and control variable, respectively? $A_n$, $B_n$ and $C_n$ are the linguistic values of the linguistic variables $x$, $y$ and $z$ in the universe of discourse $U$, $V$, and $W$. In what follows, we consider some useful properties of the FLC inference engine (Lee, 1990; Wan and Ho, 1992 and Klir and Folger, 1988).

*Theorem 1*

$$(A', B') \circ \bigcup_{i=1}^{n} R_i \Rightarrow \mu_{C'}(z) = (\mu_{A'}(x), \mu_{B'}(y)) \circ \max_{x,y,z}(\mu_{R_1}(x,y,z), \ldots, \mu_{R_n}(x,y,z))$$

$$= \sup_{x,y} \max_{x,y,z}\{\min[(\mu_{A'}(x), \mu_{B'}(y)), \mu_{R_1}(x,y,z)], \ldots, \min[(\mu_{A'}(x), \mu_{B'}(y)), \mu_{R_n}(x,y,z)]\}$$

$$= \max_{x,y,z}\{[(\mu_{A'}(x), \mu_{B'}(y)) \circ \mu_{R_1}(x,y,z)], \ldots, [(\mu_{A'}(x), \mu_{B'}(y)) \circ \mu_{R_n}(x,y,z)]\}$$

$$\Rightarrow = \bigcup_{i=1}^{n} C'_i = \bigcup_{i=1}^{n} (A', B') \circ R_i$$

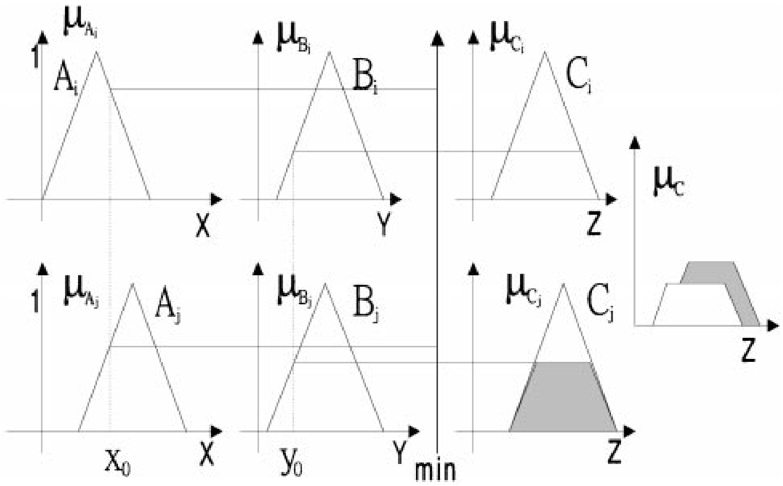


Figure 4. Graphical interpretation of fuzzy inference under minimum rule.

*Theorem 2*

For the intersection operation of fuzzy sets, the minimum and the product methods are formulated as follows:

If $\mu_{A_i \times B_i} = \mu_{A_i} \wedge \mu_{B_i}$ then

$$(A', B') \circ (A_i \textit{ and } B_i \Rightarrow C_i) = [A' \circ (A_i) \Rightarrow C_i] \cap [B' \circ (B_i \Rightarrow C_i)]$$

If $\mu_{A_i \times B_i} = \mu_{A_i} \times \mu_{B_i}$ then

$$(A', B') \circ (A_i \textit{ and } B_i \Rightarrow C_i) = [A' \circ (A_i \Rightarrow C_i)] \times [B' \circ (B_i \Rightarrow C_i)]$$

The above two formulae imply that we need to make a combination of the membership function operation and the logic operation. Because $A_i$ and $B_i \Rightarrow C_i$ is not easily operated, we partition it into two parts and evaluate them separately.

*Theorem 3*

If the inputs are fuzzy singletons, namely, $A' = x_0$, $B' = y_0$, based on the minimum operation and the product operation rules, we have the following four different operations.

$$\begin{array}{l} \alpha_i^{\wedge} \wedge \mu_{C_i}(z) \\ \alpha_i^{\wedge} \cdot \mu_{C_i}(z) \\ \alpha_i^{\cdot} \wedge \mu_{C_i}(z) \\ \alpha_i^{\cdot} \cdot \mu_{C_i}(z) \end{array} \quad \text{where:} \quad \begin{array}{l} \alpha_i^{\wedge} = \mu_{A_i}(x_0) \wedge \mu_{B_i}(y_0) \\ \alpha_i^{\cdot} = \mu_{A_i}(x_0) \cdot \mu_{B_i}(y_0) \end{array}$$

The above theorems explain the process of fuzzy inference. Figure 4 gives a graphic interpretation of Theorem 3 in terms of minimum operation rule, while Figure 5 offers a graphic interpretation of Theorem 3 in terms of product operation rule.

Basically, de-fuzzification is a mapping from a space of fuzzy control actions defined over an output universe of discourse into a space of non-fuzzy control actions. It is employed because a crisp control action is required in many practical

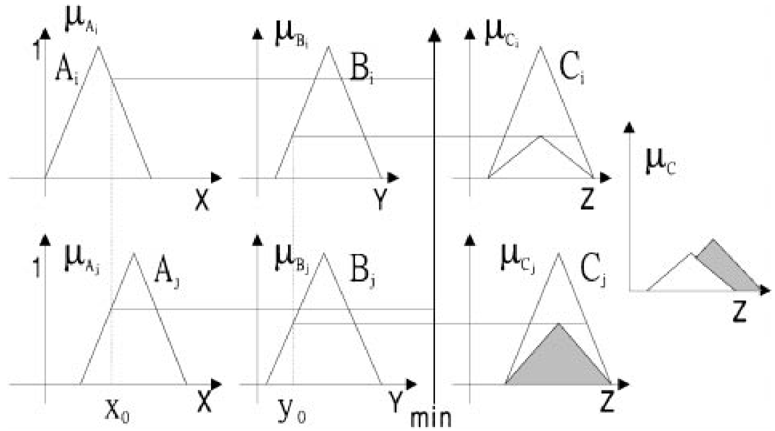


Figure 5. Graphical interpretation of fuzzy inference under product rule.

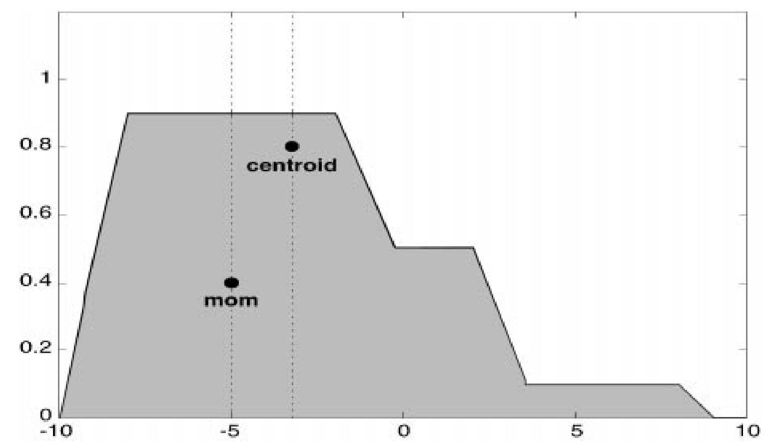


Figure 6. The general output of fuzzy controller.

applications. At present, the commonly used de-fuzzification strategies may be described by the method of the centre of area or the mean of maximum (Lee, 1990):

(a) *The Centre of Area Method* (*COA*). The widely used COA strategy generates the centre of gravity of the possibility distribution of a control action. In the case of a discrete universe, this method yields:

$$z_{COA} = \frac{\sum_{i=1}^{n} \mu_C(z_i) z_i}{\sum_{i=1}^{n} \mu_C(z_i)}.$$

The notation $n$ in the above equation is the number of quantitative levels of the output.

(b) *The Mean of Maximum Method* (*MOM*). The MOM strategy generates a control action that represents the mean value of all local control actions whose

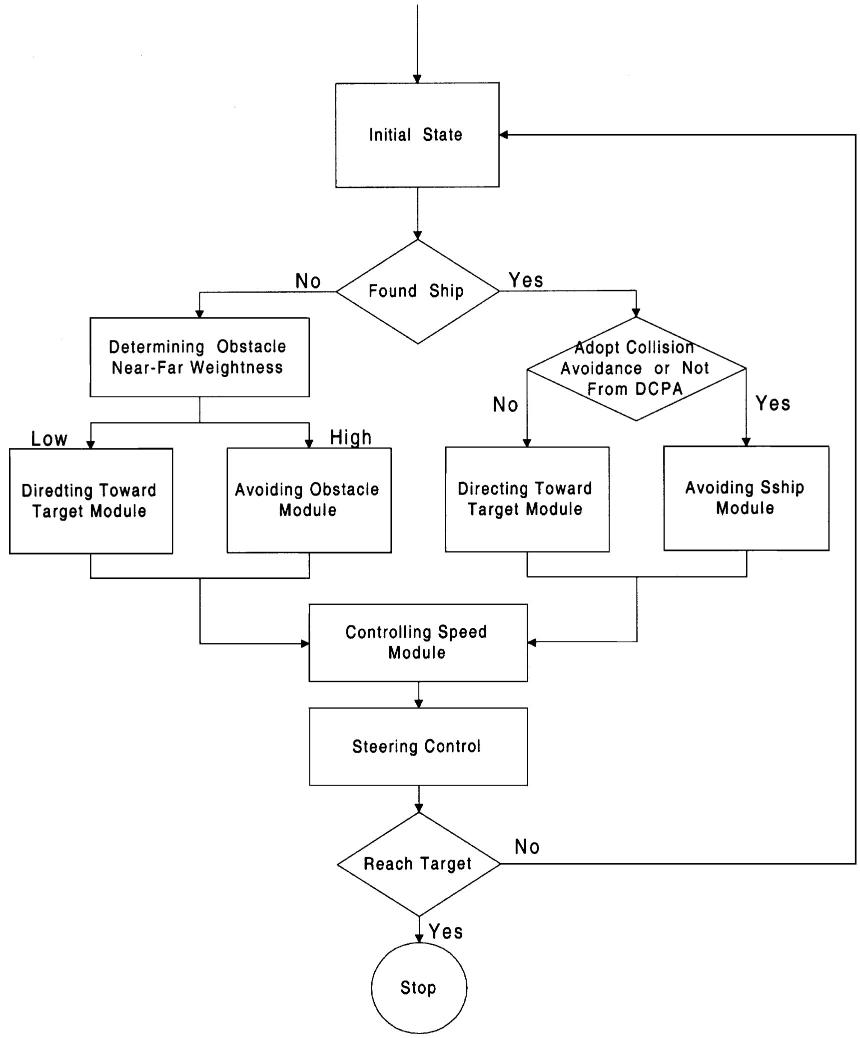


Figure 7. The flow chart of fuzzy collision avoidance.

membership functions reach the maximum. In the case of a discrete universe, the control action may be expressed as:

$$z_{MOM} = \sum_{i=1}^{m} \frac{z_i}{m}$$

In the above equation, $z_i$ is the support value. At this value, the membership function reaches the maximum value $\mu_C(z_i)$, and $m$ is the number of the support values. Figure 6 shows a graphic interpretation of the various defuzzification strategies mentioned above.

These fuzzy approach skills are now applied to design a ship collision-avoidance system, which is composed of five fuzzy-controlled modules: the detecting obstacle or ship near-far module, the keeping away from the static obstacle module, the avoiding encountering ship module, the tracking target-course module and the ship speed-control module. The fuzzy rules of each of the control modules are derived from human being's intuitive methods of adopting collision-avoidance actions and how they apply the above fuzzy theorems and operation processes.

When one ship encounters obstacles or other ships, its radar will detect whether they are in left front, right front or directly in front of the ship. The data obtained from radar are then the inputs of the detecting obstacle or ship near-far module. The output of this module is the nearness degree of the found objects, which is used to determine each of the weights of the keeping away from static obstacle module, the avoiding encountering ship module, the tracking target-course module and the speed-control module. If one of the obstacles or target ships detected by radar is closer to the ship, it will have a larger weight. When the value is greater than a preset threshold for any of these modules, which is decided by the ship safety domain and the fuzzy rules mentioned before, the module would then be initiated to take action to avoid a possible collision. The design process of fuzzy collision-avoidance is illustrated in Figure 7.

When own ship is in the danger of colliding with obstacles or encountering ships, the fuzzy collision-avoidance system proposed above will advise a proper avoidance action to resolve the risk. The avoidance action can then be implemented by either a quartermaster or an autopilot system. The objective of $H_\infty$ control problem is to obtain an $H_\infty$ optimal control law such that the transfer function between the exogenous input and the controlled output is minimum while keeping the closed-loop system stable. In the next section, we will design an autopilot with $H_\infty$ theory to ensure that the ship can avoid collision while keeping a good performance even under the worst exogenous input. The avoidance action advised by the fuzzy collision-avoidance system can be implemented by $H_\infty$ autopilot system. In other words, the integration of fuzzy collision-avoidance and $H_\infty$ autopilot systems will be proposed below.

## 4. DESIGN OF $H_\infty$ AUTOPILOT.

The earlier autopilots are of mechanical construction and are only able to provide simple control actions. The PID-type controllers were introduced to replace the mechanical devices; these electrical and electronic equipments make the autopilots more flexible but have to be adjusted manually when the ship's dynamics or disturbances change. Katebi and Byrne (1988) used adaptive control theory to develop the adaptive auto-pilot system whose parameters can be adjusted automatically when the ship's situation or disturbances change. A lot of research has been undertaken in the field of artificial intelligence (AI) and its application to control problems, such as the development of the AI knowledge-based and expert systems, the neural-networked and self-organizing fuzzy controllers.

However, when a ship navigates at sea, its performance may be influenced by its own speed, its draft, the depth of water, the encountering situations and the surrounding currents, winds and waves, etc., which cause extra inputs to the system dynamics. These uncertain factors may also cause the system to become unstable. For this reason, the $H_\infty$ autopilot system is a feasible alternative because of its excellent disturbance rejection capability.

$H_\infty$ control theory considers the worst-case of the inputs. In other words, it ensures a good performance of the closed-loop system even in the worst situation. $H_\infty$ norm in the frequency domain is defined as:

$$\|G_1\|_{H_\infty} = \sup_{\omega} {}^{\sigma}{}_{\max}|G_1(j\omega)| \tag{1}$$

$G_1(s)$: Transfer Function's $= j\omega$.

The objective of the $H_\infty$ control problem is to obtain an $H_\infty$ optimal control law such that the transfer function between exogenous input and controlled output is minimum while keeping the whole closed loop system stable. The 'Standard Problem' is considered in the state space form as follows:

$$\begin{aligned} \dot{X}(t) &= AX(t) + B_2 u(t) + B_1 w(t) \\ Y(t) &= C_2 X(t) + D_{21} w(t) \\ Z(t) &= C_1 X(t) + D_{12} u(t) \end{aligned} \tag{2}$$

In (2), $X(t) \in R^n$, $Y(t) \in R^p, u(t) \in R^m$, $w(t) \in R^r$ and $Z(t) \in R^l$ denote the state, the measurement output, the control law, the exogenous input and the control output, respectively. We suppose that $u(t) = KX(t)$, where $K$ is the controller constant gain matrix. The exogenous input $w(t)$ typically consists of reference inputs, disturbance, and sensor noises. The components of the controlled output $z(t)$ are tracking errors, control efforts, etc. The objective of the $H_\infty$ control problem is to obtain an $H_\infty$ optimal control law $u(t) \in L_2[0, \infty)$ such that for any exogenous input $w(t)$ in a pre-specified ball $\Omega$ of $L_2[0, \infty)$, the controlled output $\|z(t)\|_2$ is minimum, i.e., the transfer function between exogenous input and controlled output is minimum in $L_2[0, \infty)$. By defining $\Omega$ in the normalized form:

$$\Omega = \{w(t) | w(t) \in L_2[0, \infty), \|W(t)\|_2 \leqslant 1\}, \tag{3}$$

the theorem proposed by Hwang (1993) can now be applied.

*Theorem 1*. For a plant in the standard form of (2), suppose $(A, B_2)$ is controllable, $(C_2, A)$ is observable, $D_{12}^T D_{12} = I$, $D_{12}^T C_1 = 0$ (the orthogonal assumption). Then, the $H_\infty$ optimal control law u(t) minimizing $\|Z\|_2$ under the worst exogenous input in a pre-specified ball in $L_2[0, \infty)$ is given by:

$$u(t) = -B_2^T K_1 X(t) \tag{4}$$

where $K_1$ is the positive definite solution of the Algebraic Riccatic Equation (ARE):

$$A^T K_1 + K_1 A + K_1 (B_1 B_1^T - B_2 B_2^T) K_1 + C_1^T C_1 = 0. \tag{5}$$

If (4) and (5) are under the assumption of a white Gaussian input, $u(t)$ is given by:

$$u(t) = -B_2^T K X(t), \tag{6}$$

where $K$ is the positive definite solution of the ARE:

$$A^T K + KA - KB_2 B_2^T K + C_1^T C_1 = 0. \tag{7}$$

However, in many cases it is difficult to form the standard problem satisfying the orthogonal condition, $D_{12}^T C_1 = 0$. Therefore, to facilitate use of the $H_\infty$ approach, the orthogonal assumption must be removed. For this reason, the following theorem leading to a more general solution of the time-varying $H_\infty$ optimal control problem is developed.

*Theorem 2*. For a plant in the standard form of (2), suppose $(A, B_2)$ is controllable, $(C_2, A)$ is observable, $D_{12}^T D_{12} = I$, then, the $H_\infty$ optimal state feedback control law $u(t)$ minimizing $\|Z\|_2$ under the worst exogenous input in a pre-specified ball in $L_2[0, \infty)$ is given by:

$$u(t) = -(B_2^T K_1 + D_{12}^T C_1) X(t) \tag{8}$$

where $K_1$ is the positive definite solution of the ARE:

$$\begin{aligned} 0 = {} & (A - B_2 D_{12}^T C_1)^T K_1 + K_1 (A - B_2 D_{12}^T C_1) + K_1 (B_1 B_1^T - B_2 B_2^T) K_1 \\ & + C_1^T (I - D_{12} D_{12}^T)(I - D_{12} D_{12}^T) C_1 \end{aligned} \tag{9}$$

For a linear time-invariable system $P(s)$ expressed in the following form:

$$\begin{aligned} \dot{x}_S(t) &= A_S x_S(t) + B_S u(t) + G_S d(t) \\ y(t) &= C_S x_S(t) \end{aligned} \tag{10}$$

It can easily be transformed into a standard form as shown in (2) by the formulation methods mentioned in Hwang (1993). The above theorems can then be applied to obtain an optimal controller for the proposed autopilot.

Based on the above strategy, the system block diagram of the proposed $H_\infty$ autopilot system is designed and shown in Figure 8. By integrating the proposed fuzzy collision-avoidance and $H_\infty$ autopilot systems, the avoidance action advised by fuzzy collision-avoidance system can then be implemented by $H_\infty$ autopilot system.

5. COMPUTER SIMULATION RESULTS. In this section, the methodology developed in the previous sections is applied to an oil tanker (Cheng, 1994), whose system parameters are known, so as to prove the feasibility of the proposed integrated system. The ship is of length 331 m, width 52 m, mould depth 26 m, mould draft 20 m and weight 285944 tons.

The dynamic equation of the ship at 15 knots is:

$$\begin{aligned} \dot{x}_S(t) &= A_S x_S(t) + B_S u(t) + G_S(t) D_S(t) \\ y_S(t) &= C_S x_S(t) \end{aligned}$$

where:

$$A_s = \begin{bmatrix} -0.2019*10^{-1} & -0.1199*10^{2} & 0 \\ -0.3679*10^{-4} & -0.3996*10^{-1} & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{11}$$

$$\begin{aligned} B_s &= \begin{bmatrix} 0.1415*10^{0} \\ -0.9786*10^{-3} \\ 0 \end{bmatrix} \\ C_s &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \\ G_s &= \begin{bmatrix} 7.6703*10^{-4} \sin 4t & 0 \\ 0 & 1.1069*10^{-5} \sin 6t \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{12}$$

The state vector $x_s = [v\ r\ \phi]^T$ represents the sway velocity, the yaw angle velocity and the yaw angle of the ship, respectively. The control input $u(t)$ denotes the rudder angle of the ship. Disturbance $D_S(t)$ contains the sidelong force and yaw moment

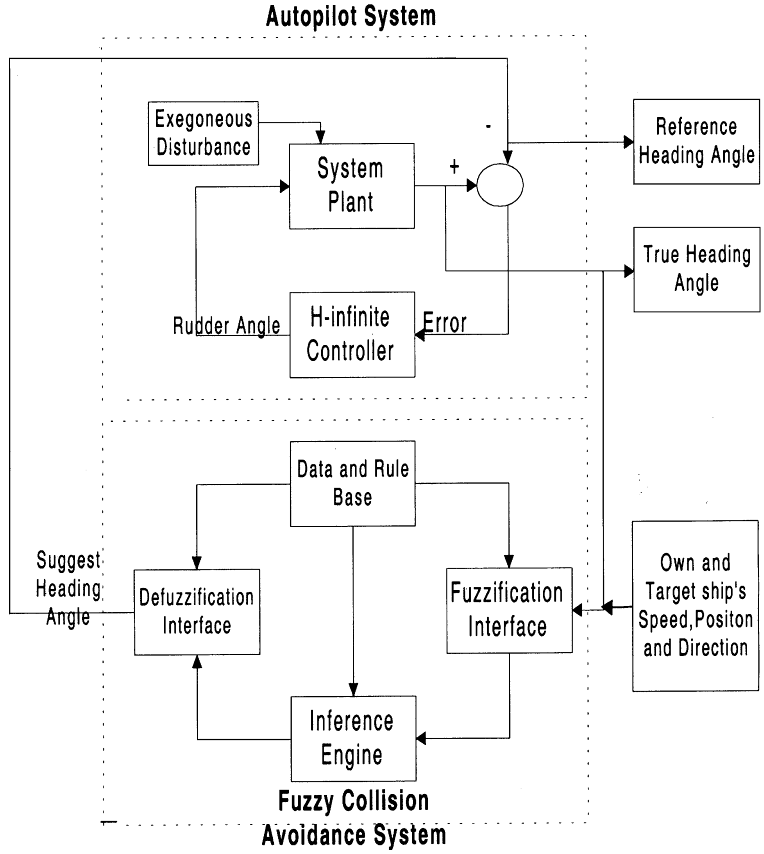


Figure 8. Diagram of the integration of fuzzy collision-avoidance and $H_\infty$ autopilot system.

induced by sea waves. In simulation, the disturbance is chosen as a unit step input. The dynamic equation of the rudder can be expressed as:

$$\frac{\delta}{\delta_C} = \frac{1}{1+\tau s},$$

where $\delta$ is the output rudder angle, $\delta_C$ is the input rudder signal of the steering system, and $\tau$ is the time constant of the system. Therefore, the plant, which includes the ship's model and the steering system, can be rearranged as:

$$\begin{bmatrix} \dot{x}_S(t) \\ \dot{\delta}(t) \end{bmatrix} = \begin{bmatrix} A_S & B_S \\ 0 & -\dfrac{1}{\tau} \end{bmatrix} \begin{bmatrix} x_S(t) \\ \delta(t) \end{bmatrix} + \begin{bmatrix} G_S(t) \\ 0 \end{bmatrix} D_S(t) + \begin{bmatrix} 0 \\ \dfrac{1}{\tau} \end{bmatrix} \delta_C(t)$$
$$y(t) = [C_S \; 0] \begin{bmatrix} x_S(t) \\ \delta(t) \end{bmatrix} \tag{12}$$

To study the course tracking performance of the ship, the servo compensator and the weighting function are chosen as $(A_C, B_C, C_C) = (-0{\cdot}001, 850, 1)$ and $(A_3, B_3, C_3) = (-0{\cdot}001, 0{\cdot}22, 1)$, respectively. To ensure good disturbance-rejection capability for all kinds of exogenous inputs, other than pure white Gaussian signals, the proposed $H_\infty$ autopilot formulation is used in this example. It is assumed that the positions, the

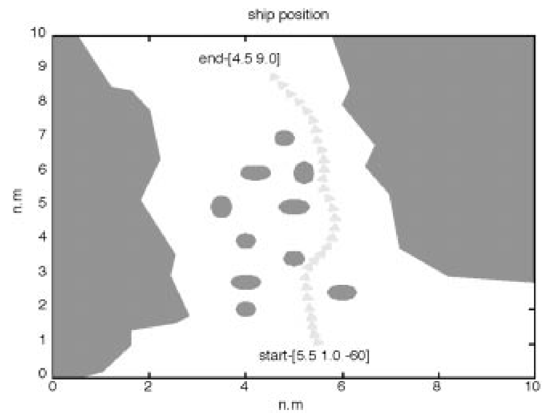


Figure 9. Suggested path for avoiding obstacles (case 1).

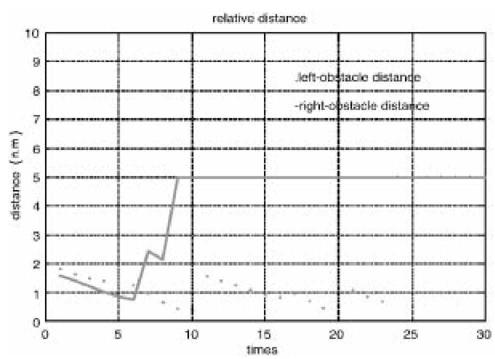


Figure 10. Relative distance from the obstacles for case 1.

heading angles and the speeds of own ship and the encountering ships can be obtained from the on-line outputs of a Radar, a Loran-C, a GPS, a Doppler Log, an electromagnetic log or an automatic radar plotting apparatus (ARPA). When a ship navigates at sea, there are numerous encountering cases, fifty of which are discussed in detail by Imazu. To demonstrate the feasibility of the proposed integrated system, three typical and complex encountering cases among these fifty situations are explored. They represent the encountering cases of avoiding obstacles, crossing a single lane and crossing double lanes, which are shown in Figures 9, 11, and 13 respectively. They are denoted as the case 1, the case 2 and the case 3 in these figures.

For the encountering obstacles, the computer simulation results reveal that the proposed fuzzy collision-avoidance system advises a dotted-line ship path as shown in Figure 9. Figure 10 gives the relative distance between the ship and obstacles, which clearly indicates that the system can effectively avoid a possible collision with these obstacles. The path advised by the proposed fuzzy collision-avoidance system is then executed by the $H_\infty$ auto-pilot, which achieves a perfect route-tracking as shown in Figure 15 under the surrounding disturbances while the details of the path tracking errors are given in Figure 16.

The encountering case shown in Figure 11 shows how the fuzzy collision-avoidance system instructs the ship 1 to avoid the collision with the ships 2 and 3 in crossing a single lane. The relative position between the encountering ships shown in Figure 12

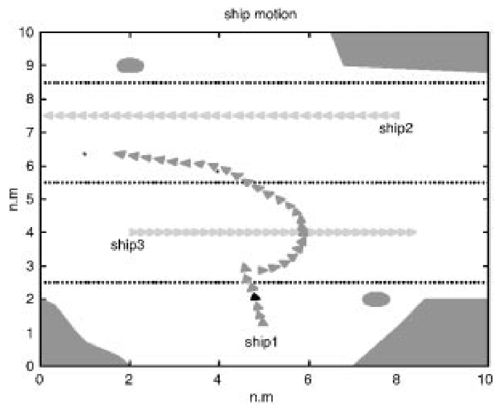


Figure 11. Suggested path to avoid collision in the case of crossing a single alley (case 2).

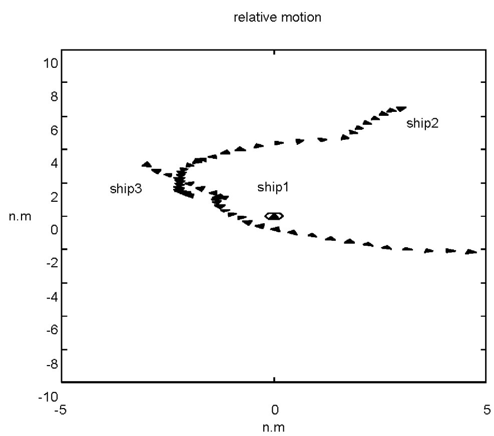


Figure 12. Simulation of crossing single alley for case 2.

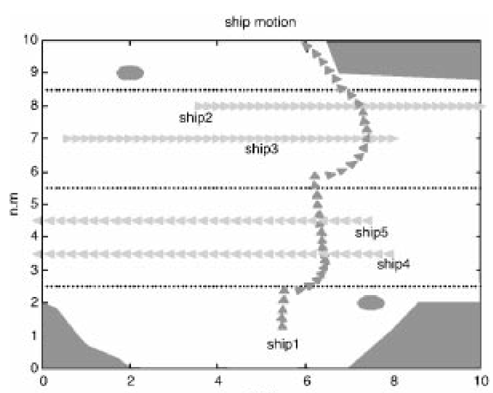


Figure 13. Suggested path to avoid collision in the case of crossing double alleys (case 3).

indicates that the avoidance action is successful. By applying the $H_\infty$ autopilot to the system, Figure 17 shows that an excellent tracking result can be achieved even under a persistent disturbance mentioned before. The tracking errors of the suggested path are small and are shown in Figure 18.

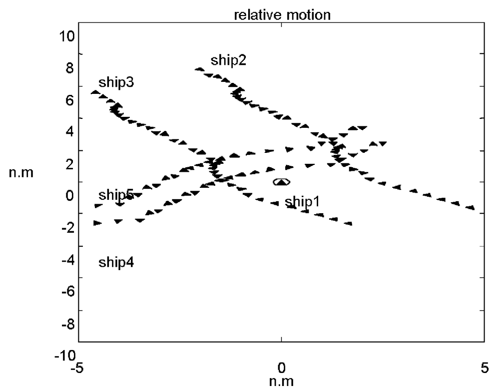


Figure 14. Relative position between the encountering ships for case 3.

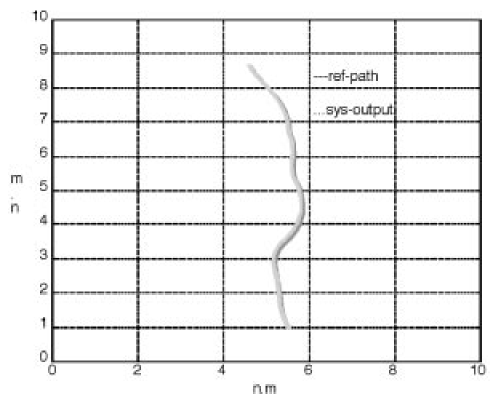


Figure 15. Path tracking executed by $H^\infty$-autopilot for case 1.

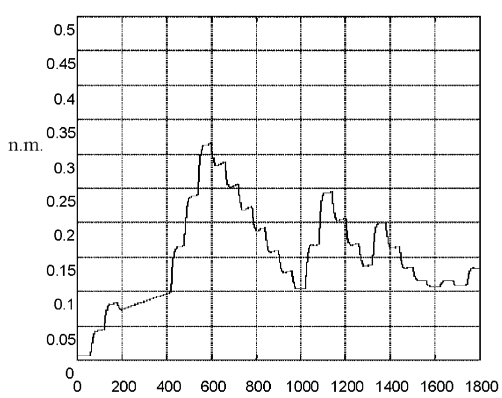


Figure 16. The tracking errors in case 1.

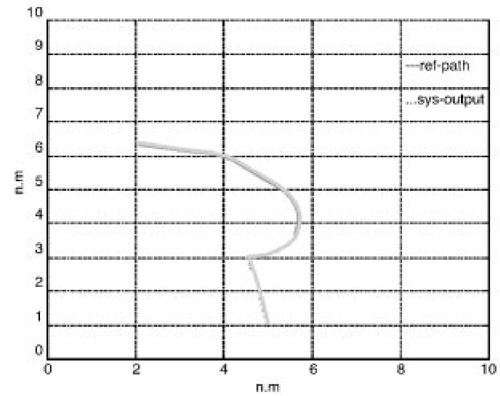


Figure 17. Path tracking executed by $H^{\infty}$-autopilot for case 2.

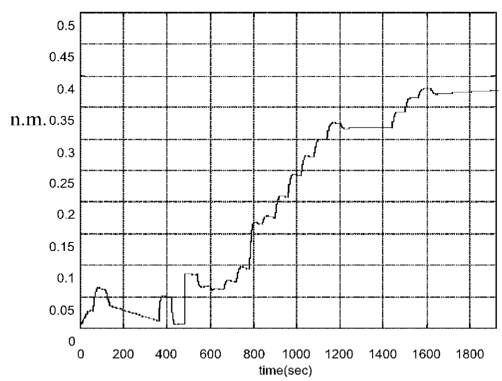


Figure 18. Tracking errors in case 2.

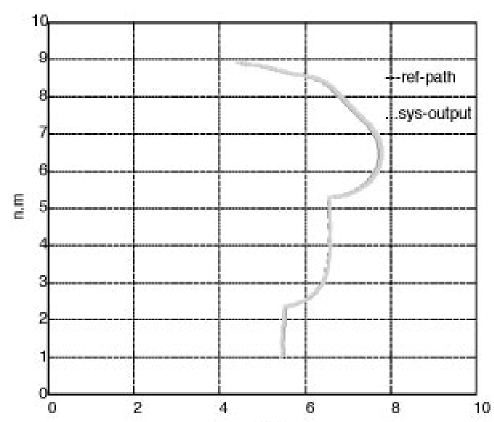


Figure 19. Path tracking executed by $H^{\infty}$-autopilot for case 3.

In Figure 13, ship 1 meets four ships and two obstacles when it intends to cross the double lanes. The navigating path suggested by the fuzzy collision-avoidance system is shown in Figure 13, which demonstrates its feasibility of the fuzzy collision-

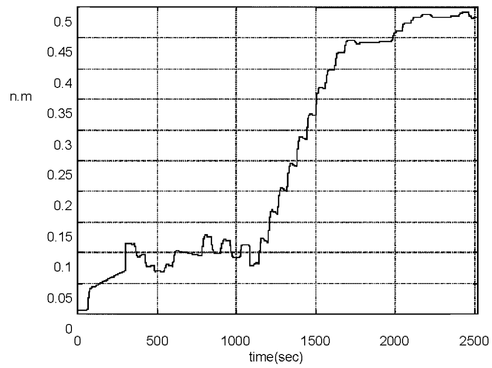


Figure 20. Tracking errors in case 3.

avoidance system in Figure 14. The path advised by the proposed fuzzy collision-avoidance system is then executed by the $H_\infty$ autopilot, which can still achieve a good route-tracking precision as shown in Figure 19 even under various surrounding disturbances. The details of the path tracking errors are given in Figure 20.

6. CONCLUSION. As international trade increases, vessels tend to be larger and to be operated more automatically, yet collision accidents are also increasing as ships increase in size, in speed and in number. The problem of collision-avoidance, therefore, becomes an urgent issue. The fuzzy collision-avoidance system, cooperating with navigators and explored in this paper, could be used to avoid collisions between ships. When own ship is in the danger of colliding with obstacles or target ships, the fuzzy collision-avoidance system will advise a proper avoidance action to resolve the risk. Either a quartermaster or an autopilot system can then implement the avoidance action.

The objective of $H_\infty$ control problem is to obtain an $H_\infty$ optimal control law such that the transfer function between the exogenous input and the controlled output is minimum while keeping the closed-loop system stable. In this research, we designed an autopilot with $H_\infty$ theory to ensure that the closed-loop system still has certain robustness under the worst exogenous input.

The integration of fuzzy collision-avoidance and $H_\infty$ autopilot systems is proposed in this paper. The avoidance action advised by fuzzy collision-avoidance system can then be implemented by $H_\infty$ autopilot system.

Computer simulation results reveal that, with the aid of the integration of fuzzy collision-avoidance system and $H_\infty$ auto-pilot system designed in this paper, ships can undertake a proper avoidance action to avoid the risk of collision at the right time and can track the desired path within allowable range to reach the target port.

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