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The interaction of nonlinear gravity waves with fixed and floating structures

C. Haack, V. Schlegel, O. Mahrenholtz
Ocean Engineering Section II, Technical University Hamburg-Harburg, D-2100 Hamburg 90, Germany

ABSTRACT

The paper presents the formulation of a numerical wave channel (NWC) for the investigation of the dynamics of free floating bodies in nonlinear gravity waves. A boundary element (BE) approach for two-dimensional configurations is introduced using cubic spline approximations for the free surface discretization and a double node concept for the modeling of contact points between structures and the fluid. The unknown time-dependent and nonlinear boundary conditions on the free surface are evaluated by a time-stepping procedure. In addition, this initial value problem is applicable to the equations of motion of free floating bodies. In this case the right hand sides are the external forces, calculated by integrating the pressure distribution on the submerged surfaces at every time step. Here, the unknown time derivatives of the velocity potential of the fluid have to be derived e.g. by a finite difference scheme or, as proposed here, by a polynomial approximation. The advantages of this procedure are minimal discretization expenses for typical test configurations and a time domain solution, taking into account the fully nonlinear boundary conditions. Several applications of this approach are presented and discussed.

INTRODUCTION

To analyze fluid structure interactions experiments with floating bodies are typically carried out in wave channels. Besides the measurement equipment and test objects experiments require free capacities of laboratory facilities. In order to reduce costs and to allow the analysis of various different test configurations, the development of reliable and sufficient NWC models is of great interest. In the first place the numerical treatment of the considered problem requires an efficient and reliable computation scheme for the solution of the flow problem. Compared with other methods the boundary element method (BEM) offers several advantages for this specific application due to the following significant items:

- in general we have to consider large, arbitrarily shaped domains for practical purposes which cause large discretization expenses for other methods (e.g. finite elements or finite differences),
- all quantities of interest — either given or unknown — are localized on the boundary itself (see problem formulation),
- the discretization of the geometry shows an extreme curvature especially on the free surface and at the intersections to floating bodies,
- for the implementation of fixed or floating structures only the submerged surfaces of the structures have to be taken into account.

PROBLEM FORMULATION

In order to solve the fluid flow problem the following usual assumptions are made:

incompressible fluid: this is valid for the considered interaction of floating bodies with gravity waves and is therefore no restriction for the mentioned applications. Compressibility of the fluid has to be taken into account in the case of e.g. earthquake induced shock waves, ref. Antes [1].

irrotational flow: this is not the case with real fluids, especially in the vicinity of fixed or floating structures. But in general the friction induced rotation of the flow can be neglected due to low dynamical viscosity of water and to low relative velocities of gravity waves and floating structures.

This allows to introduce the potential flow concept, described by the Laplace equation:

$$\operatorname{div} \mathbf{u} = \operatorname{div} \operatorname{grad} \Phi = \nabla^2 \Phi = 0 \ , \tag{1}$$

where $\mathbf{u}$ is the fluid velocity and Φ the corresponding velocity potential. Beyond this the equation of motion of the fluid particles can be reduced to the Bernoulli equation. Written in the general form it states

$$\begin{aligned} \frac{d\Phi}{dt} &= \frac{\partial \Phi}{\partial t} + \mathbf{v}\,\mathrm{grad}\Phi\,, \\ &= -g\,y - \frac{1}{\rho}p - \frac{|\mathbf{u}|^2}{2} + \mathbf{v}\,\mathbf{u}\,, \end{aligned} \tag{2}$$

with g the gravitational acceleration, y the vertical position of the considered point, ρ the density of the fluid and p the pressure. In this general form one has to distinguish between

$\mathbf{u}$: the velocity vector of a fluid particle in the flow and
$\mathbf{v}$: the velocity vector of a point moved arbitrarily through the fluid.

As mentioned before the fluid flow problem is solved by transforming the Laplace equation (1) with a direct method into an integral equation of the form

$$C(\mathbf{x})\,\Phi(\mathbf{x}) = \int_\Gamma \mathbf{u}_n(\xi)\,G(\mathbf{x},\xi) - \Phi(\xi)\,\frac{\partial\,G(\mathbf{x},\xi)}{\partial\,\mathbf{n}(\xi)}\,d\gamma(\xi)\,. \tag{3}$$

Field and source points are denoted by $\mathbf{x}$ and ξ, the fundamental solution of the problem by G and C is a constant with the property $C(\mathbf{x}) = 1/2$, if Γ is smooth and $\mathbf{x} \in \Gamma$.
This formulation is discretized by a BE approach and results in the approximation

$$(\mathbf{C} + \mathbf{H})\,\tilde{\Phi}(\mathbf{x}) = \mathbf{F}\,\tilde{\mathbf{u}}_n(\mathbf{x})\,, \tag{4}$$

where the vectors $\tilde{\Phi}$ and $\tilde{\mathbf{u}}_n$ consist of the ansatz-functions with regard to the different types of elements (e.g. linear, quadratic, splines, etc.). The matrices $\mathbf{C}$, $\mathbf{H}$ and $\mathbf{F}$ are defined by:

$$\left.\begin{aligned} \mathbf{C} &= \left[C_{i,j}\right]\,, \\ \mathbf{H} &= \left[\int_\Gamma \varphi_i(\xi)\,\frac{\partial\,G(\mathbf{x}_j,\xi)}{\partial\,\mathbf{n}(\xi)}\,d\gamma(\xi)\right]\,, \\ \mathbf{F} &= \left[\int_\Gamma \varphi_i(\xi)\,G(\mathbf{x}_j,\xi)\,d\gamma(\xi)\right]\,, \end{aligned}\right\}\quad \begin{aligned} i &= 1\ldots n_\varphi\,, \\ j &= 1\ldots n_q \end{aligned} \tag{5}$$

where n_φ depends on the order of the ansatz-functions φ_i and n_q gives the number of elements.
Considering a mixed boundary value problem the known and unknown boundary conditions in equation (4) have to be rearranged to get them in the form

$$\mathbf{A}\,\mathbf{y} = \mathbf{b}\,, \tag{6}$$

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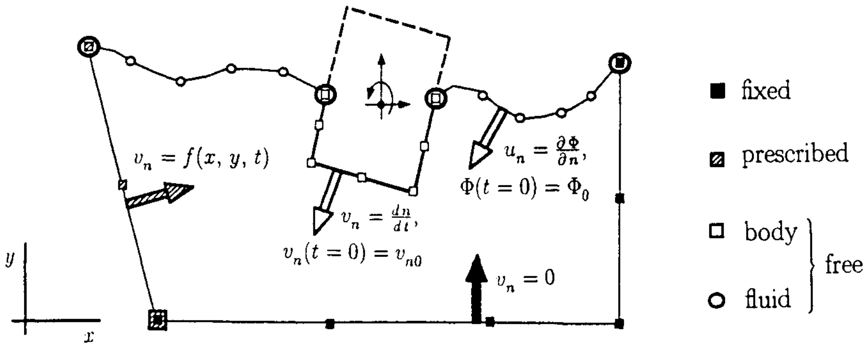

Figure 1: Boundary conditions

as a set of linear equations, where **A** is in general a dense and unsymmetric matrix. The right hand sides **b** are given by the boundary conditions:

$$\begin{aligned} \Phi &: \text{ on the free surfaces of the fluid and} \\ \mathbf{u}_n = \mathbf{v}_n &: \text{ on all other boundaries.} \end{aligned}$$

In Figure 1 the considered domain and the different types of boundaries and boundary conditions are characterized by using different kinds of symbols for the nodes of the discretization:

a) **fixed boundaries:** e.g. bottom or walls of the NWC, where the velocity component in the normal direction to the boundary vanishes, i.e.

$$v_n = 0 \,. \tag{7}$$

b) **arbitrarily moved boundaries:** this kind of boundary is necessary to describe facilities like wave generators, where a prescribed motion of the boundary is given by a function f in space and time by

$$v_n = f\,(x,\, y,\, t) \,. \tag{8}$$

c) **free floating bodies:** at the boundaries of the submerged parts of rigid bodies the time-dependent normal velocity is given by the time derivative of the normal direction

$$v_n = \frac{dn}{dt} = h(t)\,, \tag{9}$$

and is evaluated by means of the equations of motion of the rigid body.

d) free surfaces of the fluid: the free surfaces are described by the fluid particles themselves. At these boundaries we assume that the time-dependent velocity potential $\Phi(t)$ is given, thus the normal velocity of a fluid particle can be written as

$$u_n = \frac{\partial\,\Phi(t)}{\partial n} = h(t)\,, \tag{10}$$

and is part of the BE solution.

In order to determine the time-dependent boundary conditions c) and d) an additional initial value problem has to be set up and solved. The motion of the fluid particles at the free surfaces is described by a Lagrangian formulation, with $\mathbf{v} = \mathbf{u}$ in equation (2). The equations of motion for the fluid particles yield in fully nonlinear form:

$$\begin{aligned} \frac{D\mathbf{x}}{Dt} &= \nabla\Phi = \mathbf{u}\,, \\ \frac{D\Phi}{Dt} &= -g\,y - \frac{1}{\rho}p - \frac{|\mathbf{u}|^2}{2}\,. \end{aligned} \tag{11}$$

Here, the position of the fluid particles on the free surfaces is described by the vector $\mathbf{x} = [x_{fl}\ \ y_{fl}]^T$. The initial condition is given by $\Phi\,(t=0) = \Phi_0$. The location and the velocity of a rigid body or a multibody system are derived by the standard vector differential equations of motion:

$$\frac{d}{dt}\underbrace{\left[\begin{array}{c} \mathbf{x}_1 \\ \hline \mathbf{x}_2 \end{array}\right]}_{\mathbf{x}} = \underbrace{\left[\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline -\mathbf{M}^{-1}(\mathbf{K}+\mathbf{N}) & -\mathbf{M}^{-1}(\mathbf{D}+\mathbf{G}) \end{array}\right]}_{\mathbf{A}} \mathbf{x} + \underbrace{\left[\begin{array}{c} \mathbf{0} \\ \hline \mathbf{M}^{-1}\mathbf{h} \end{array}\right]}_{\mathbf{b}}\,. \tag{12}$$

In this state space representation the vector of the state variables is denoted by $\mathbf{x}$, $\mathbf{I}$ is the matrix identity, $\mathbf{M}$ the mass matrix, $\mathbf{K}$ the matrix of the conservative, $\mathbf{N}$ the matrix of the nonconservative, $\mathbf{D}$ the matrix of the damping and $\mathbf{G}$ the matrix of the giroscopic forces. The function of excitation is given by the vector $\mathbf{h}\,(t)$. In the case of a single floating body the generalized coordinates are

$$\mathbf{x}_1 = [x_s\ \ y_s\ \ \alpha_s]^T \quad \text{and} \quad \mathbf{x}_2 = [\dot{x}_s\ \ \dot{y}_s\ \ \dot{\alpha}_s]^T\,, \tag{13}$$

with respect to the center of mass of the floating body and with the initial values

$$\mathbf{x}_2\,(t=0) = [\dot{x}_0\;\dot{y}_0\;\dot{\alpha}_0]^T \quad \text{and} \quad \dot{\mathbf{x}}_2\,(t=0) = [\ddot{x}_0\;\ddot{y}_0\;\ddot{\alpha}_0]^T\,. \tag{14}$$

The excitation $\mathbf{h}\,(t)$ of the floating bodies contains the vectors of the external forces $\mathbf{f}_e$ and the external torques $\mathbf{t}_e$. They are calculated from the time-dependent pressure distribution $p(t)$ on the submerged surfaces of the floating bodies:

$$\mathbf{f}_e = \int_{\Gamma_{sub}} p(t)\,\mathbf{n}\,d\gamma\,, \tag{15}$$

$$\mathbf{t}_e = \mathbf{d}_s \times \mathbf{f}_e\,, \tag{16}$$

where $\mathbf{n}$ is the normal vector and $\mathbf{d}_s$ is the distance vector from the submerged part of the boundary to the center of mass of the floating body. The unknown time-dependent pressure on the submerged surface is gained by transforming equation (2) into the form

$$p\,(t) = \rho\left(-g\,y - \frac{|\mathbf{u}|^2}{2} - \frac{d\,\Phi}{d\,t} + \mathbf{v}\,\mathbf{u}\right)\,. \tag{17}$$

NUMERICAL IMPLEMENTATION

As mentioned before the two-dimensional fluid flow problem is solved by a direct BEM. The considered domain is discretized by one-dimensional finite elements and a double-node concept allows both, arbitrary shaped boundaries as well as the transitions of different kinds of boundary conditions (a, b, c or d). A spline formulation is used for the discretization of free surfaces in order to provide the calculation of tangential derivatives in element coordinates. With this the velocity of the fluid particles on the free surface becomes

$$\mathbf{u} = \sqrt{{u_n}^2 + {u_t}^2}\,. \tag{18}$$

All computations of singular integrals of the matrices $\mathbf{H}$ and $\mathbf{F}$ in (5) utilize special analytical formulations with regard to the considered element type. The boundary integral formulation (3) is the model of a steady state flow problem. In order to analyze even time-dependent, transient problems the ordinary differential equations (11) and (12) with nonlinear r.h.s. have to be taken into account. These two sub-problems are condensed to a general initial value problem

$$\frac{d\,\mathbf{x}}{d\,t} = f\;(\mathbf{x}, t)\;, \quad \mathbf{x}(t=0) = \mathbf{x}_0 \tag{19}$$

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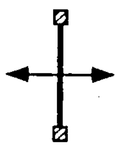
and is solved by an explicit predictor-corrector scheme, starting with a higher order Runge-Kutta method. The accuracy and stability of this procedure is improved by evaluating the integral equation of the flow problem (3) at every intermediate time step (ref. Zandbergen et al. [5]). This procedure already yields good results for nonlinear steady state and breaking waves, see [3].

The evaluation of the pressure distribution (17) requires an approximation of the time derivative of the potential $\frac{d\Phi}{dt}$. This is implemented by a second order polynomial approximation of $\Phi(t)$ and achieves best agreement for the considered applications. Especially, if the pressure distribution is of interest at all Neumann-type boundaries the BE approach offers another advantage (ref. Vinje and Brevig [4]). Due to the fact that the Laplace equation is valid even for time derivatives of the potential $\dot{\Phi} = \frac{d\Phi}{dt}$, equation (4) can be applied to

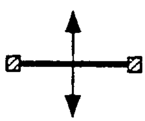
$$[\mathbf{C}+\mathbf{H}]\begin{bmatrix} \tilde{\dot{\Phi}} \\ \vdots \\ \tilde{\dot{\Phi}} \end{bmatrix} = \mathbf{F}\begin{bmatrix} \tilde{\dot{\mathbf{u}}}_n \\ \vdots \\ \tilde{\dot{\mathbf{u}}}_n \end{bmatrix}, \tag{20}$$

and just results in an additional right hand side of the linear equations (6).

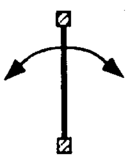
The main feature of a NWC is a controlled excitation of the free surface and therefore of floating bodies. Such a wave generator is numerically implemented by a given function (8) at a Neumann-type boundary. If a sinusoidal excitation is assumed, either translational or rotational, the time-dependent position vector $\mathbf{x}(t)$ and the boundary conditions $v_n(t)$ are now described by:



$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} x_0 + A\,sin(\omega t + \psi) \\ y \end{bmatrix} \\ v_n(t) &= \omega\, A\, cos(\omega t + \psi) \end{aligned} \tag{21}$$



$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} x \\ y_0 + A\,sin(\omega t + \psi) \end{bmatrix} \\ v_n(t) &= \omega\, A\, cos(\omega t + \psi) \end{aligned} \tag{22}$$



$$\begin{aligned} \alpha(t) &= \alpha_0 + A\,sin(\omega t + \psi) \\ \mathbf{x}(t) &= r\begin{bmatrix} sin(\alpha(t)) \\ cos(\alpha(t)) \end{bmatrix} \\ v_n(t) &= r\,\dot{\alpha}(t) = r\,\omega\, A\, cos(\omega t + \psi) \end{aligned} \tag{23}$$

with amplitude A, angular velocity $\omega = 2\,\pi/T$ = const., phase shift ψ and distance r to the hinge of the flap.

RESULTS

The proposed numerical implementation allows a flexible application of a NWC to analyze fluid structure interactions. In order to verify the model different test configurations were set up. The results are shown and discussed exemplarily.

Example 1: Horizontally moved piston-type wave generator

In Figure 2 a sketch of this problem is given, where d = 1 m and ℓ = 2 m. The free surface is excited on the left hand side by a piston-type wave generator with a given function, see equation (21). The amplitude is A = 0.3 m, the frequency of the excitation Ω = 0.5 Hz and the phase shift is $\psi = -\pi/2$. This means, the sinusoidal motion of the flap starts at the minimum position (ref. Figure 3 and 4). On the right hand side the wave tank is bounded by a fixed vertical wall. Figure 3 shows the time evolution of the free surface for $t = 0 \ldots 1.38$ s, with $\Delta t = 0.06$ s. Both, the runup of the wave on the fixed wall and on the piston as well as the movement of the free surface wave crest is evident. The runup behavior is more clearly given in Figure 4. Here, the time-dependent position of the free surface is depicted for the contact points at the piston and at the fixed wall and compared with the movement of the piston in x-direction. Due to the propagating wave crest an increasing phase shift between excitation $x(t)$ and wave elevation $y(t)$ at the piston is obviously. A similar experiment with a constant horizontal velocity of the piston is discussed in detail by van Daalen [2].

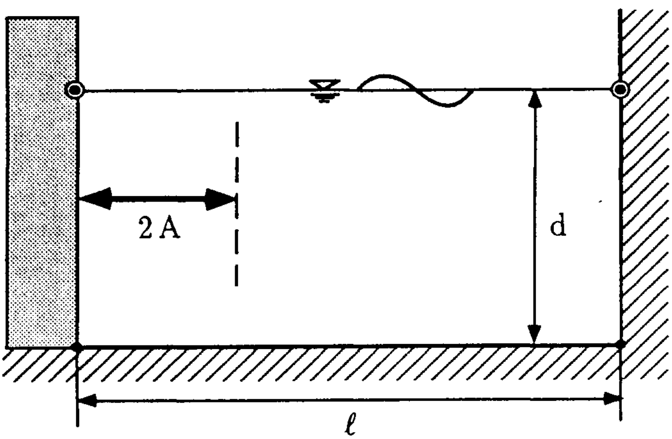


Figure 2: Definition sketch, ex. 1

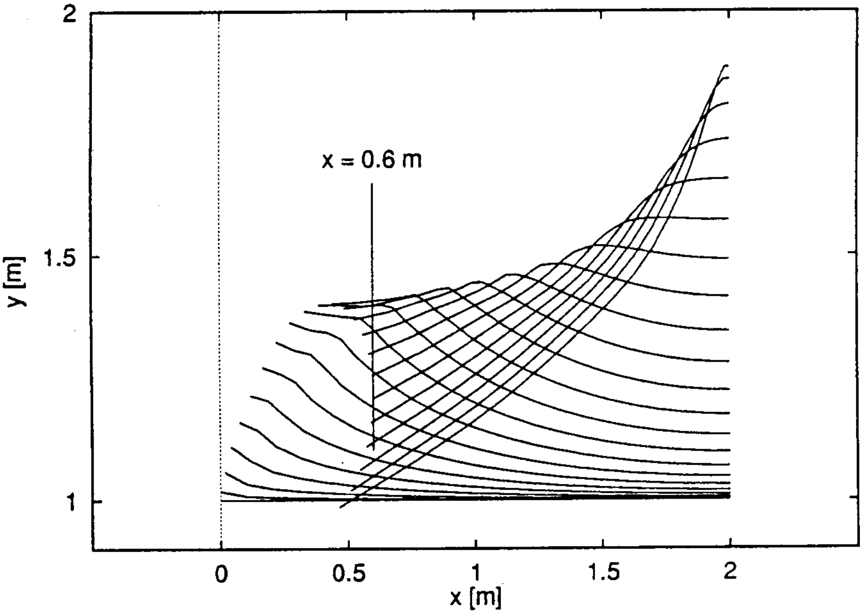


Figure 3: Time evolution, $t = 0 \ldots 1.38$ s

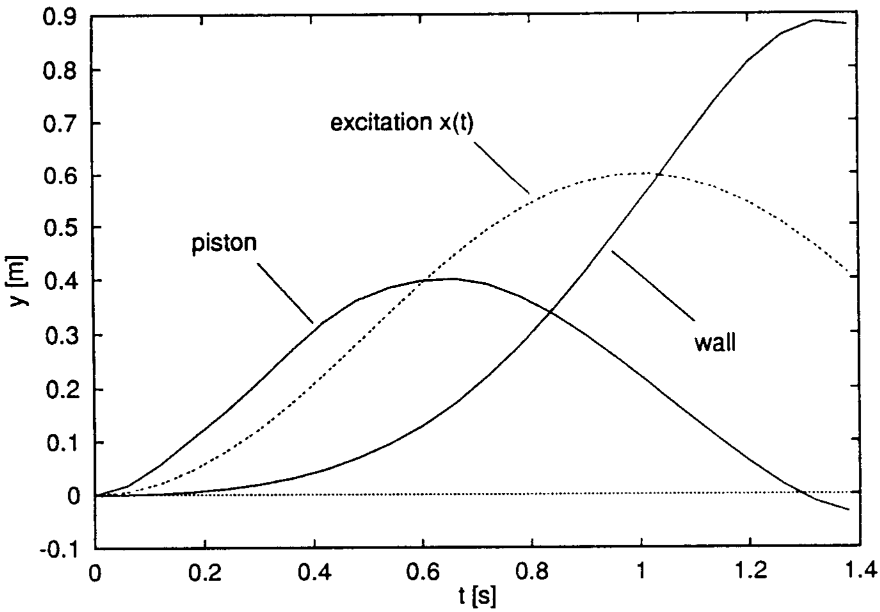


Figure 4: Runup on the piston and on the wall compared with the motion of the piston $x(t)$

Example 2: Vertically moved piston-type wave generator

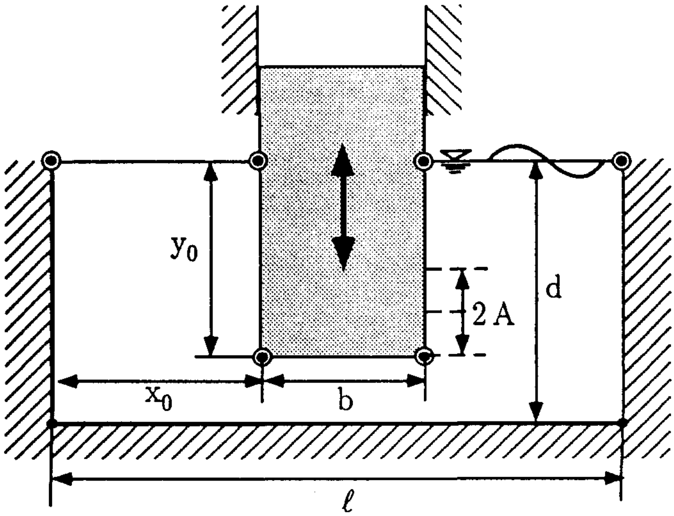


Figure 5: Definition sketch, ex. 2

This test configuration was chosen in order to compute the hydrodynamic force acting on the bottom of a piston-type wave generator. In Figure 5 the definition sketch is given, with $A = 0.2\,m$, $b = 1\,m$, $d = 1\,m$, $\ell = 8\,m$, $x_0 = 5\,m$, $y_0 = 0.8\,m$ and the frequency of the excitation is $\Omega = 1$ Hz, ref. to equation (22). For this nonsymmetric problem with a divided free surface the forces are computed by integrating the pressure distribution on the considered surface, i.e. the bottom of the wave generator. In order to compare the influence of the hydrostatic and the nonlinear hydrodynamic terms equation (17) was split and computed by parts. The results of this procedure are depicted in Figure 6. For this configuration, with $A = 0.2\,m$ and $\Omega = 1$ Hz the resultant force acting on the piston-type wave generator depends obviously on the hydrodynamic part. The intense changes and greater amplitude in the hydrodynamic terms dominate the pressure distribution although the absolute value of the hydrostatic part, that just depends on the vertical position of the piston, is on a much higher level.

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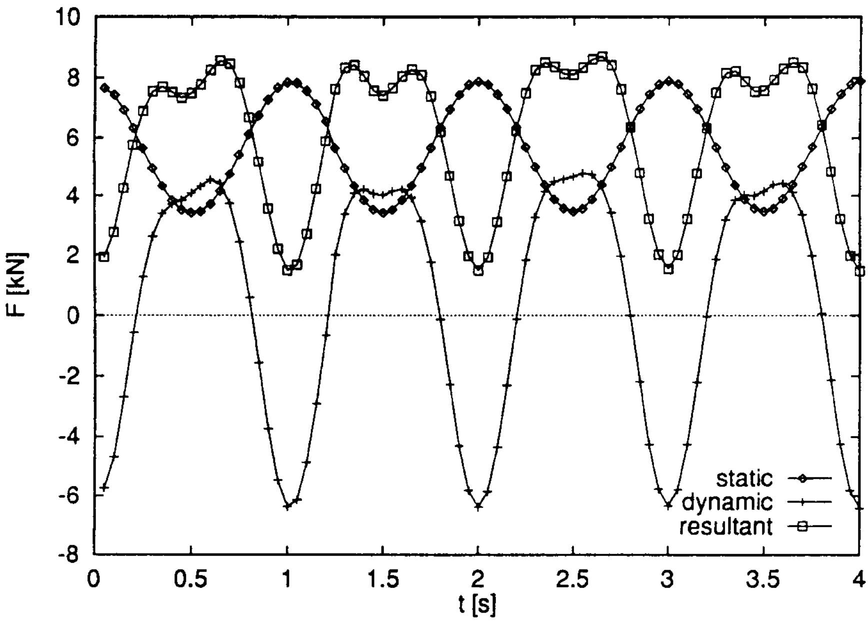

Figure 6: Hydrostatic, hydrodynamic and resultant force

Example 3: NWC with a flap-type wave generator and a single free floating body

Figure 7 depicts a sequence of a NWC animation of the considered problem with a free floating body. The free surface is excited on the left hand side by a flap-type wave generator with a given time-function (23). On the right hand side the wave tank is bounded by a fixed vertical wall. The ratio of waterdepth to tank-length is 1/10 and the ratio of body-intersected surface to the tank-length is 1/20. In Figure 8 the change of the shape of the boundary during the first ten time-steps ($\Delta t = 0.2\,\mathrm{s}$) is depicted. It clearly shows the initially starting drift motion of the body in the x-direction. In order to describe the time-dependent, transient behavior of the free floating body it is more convenient to use the time histories as in Figure 9 and 10. The increasing roll amplitudes can be seen in Figure 9 and especially the drift-motion of the free floating body due to the nonlinear description of the free surface flow is evident in Figure 10.

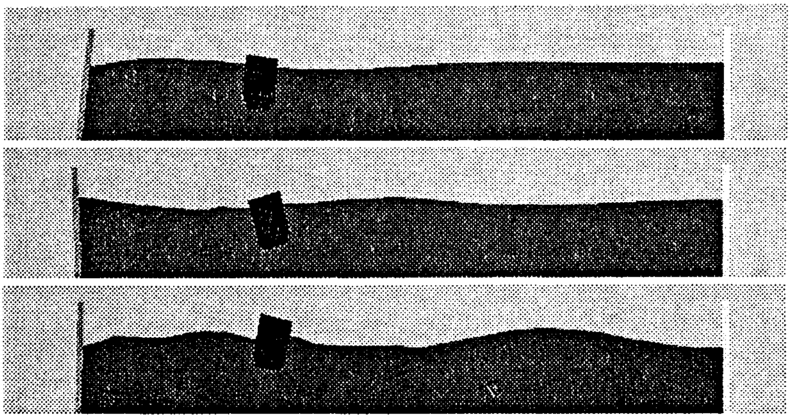

Figure 7: Animation; NWC with free floating body

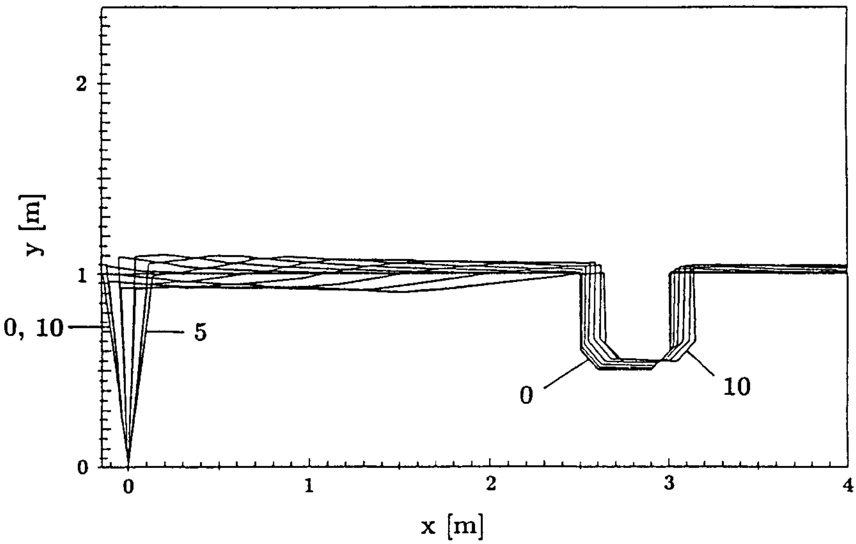

Figure 8: Time evolution of the flap-type wave generator and the floating body motion

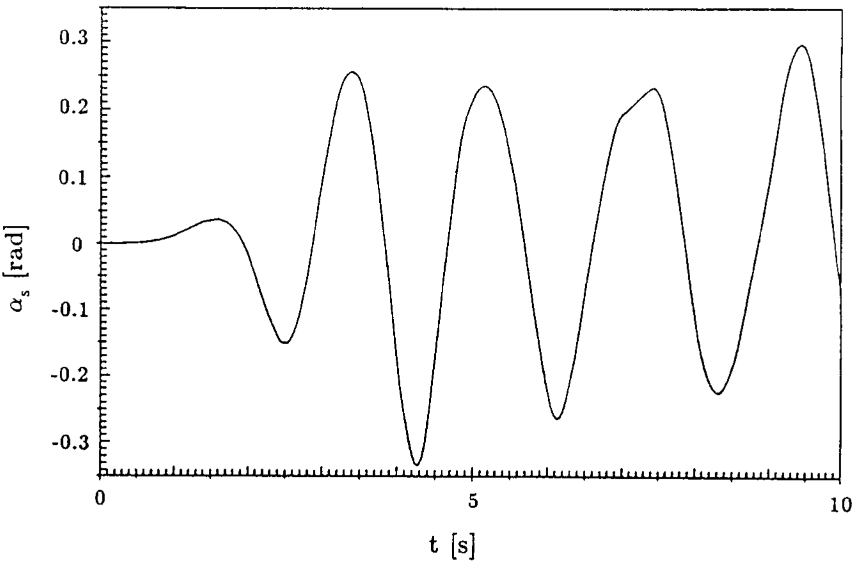

Figure 9: Roll angle α_s versus time

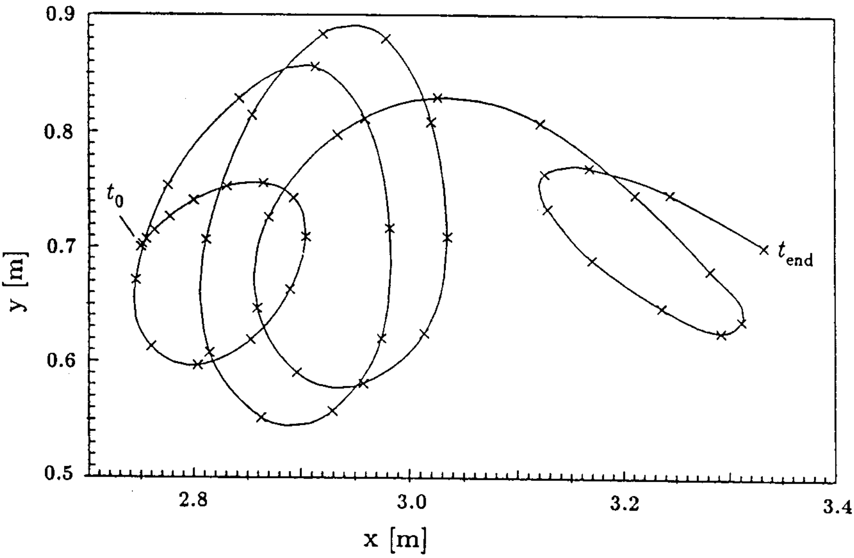

Figure 10: Plane motion of the center of mass

CONCLUSIONS

The paper presents a boundary element formulation to simulate fluid structure interaction problems with nonlinear gravity waves. Because all quantities of interest are located on the boundary itself and with regard to the recent developments in boundary element methods (BEM) this is a most efficient approach with minimal discretization expenses. The major advantage of a boundary element formulation is the direct numerical implementation of the equations of motion in an explicit form, both for the fluid particles on the free surface and on one ar more floating bodies. A time-stepping procedure allows the treatment of time-dependent and nonlinear conditions. In particular this proposal provides the extension of the model to a numerical wave channel (NWC) concept in order to analyze in time domain single or multibody systems in nonlinear gravity waves. Some typical test configurations for the proposed method are described, the results are shown and discussed. These NWC applications with flap- or piston-type wave generators obviously show the drift motion of floating bodies in wave tanks and the transient behavior of roll and heave motion.

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