Geophys. J. R. astr. Soc. (1978) 55, 203–216

The interaction of surge and tide in the North Sea and River Thames

D. Prandle and J. Wolf *Institute of Oceanographic Sciences, Bidston Observatory, Merseyside*

Received 1978 April 10; in original form 1977 October 3

Summary. Although the tendency for surge peaks in the Thames to occur on rising tide has been recognized for some time, no satisfactory physical explanation has been presented. The phenomenon almost certainly results from non-linear interaction between tide and surge and it is the mechanism of this interaction which is examined in the present study.

A statistical analysis of surges recorded at 10 ports located along the east coast of Britain demonstrated the development of interaction as surges propagate southwards. This analysis showed that surges tend to develop a peak on the rising tide in the Thames irrespective of the phase relationship between tide and surge in the northern North Sea.

A one-dimensional model of the River Thames was used to examine how surge-tide interaction varied for surges of differing types. In order to identify the mechanics of interaction, a new modelling technique was developed involving two models, one of tidal propagation and one of surge propagation, operated simultaneously with cross-linkages in the form of perturbation terms providing the effects of interaction. By this means it was shown that quadratic friction is the dominant interaction mechanism in the Thames.

1 Introduction

Storm surges represent one of the most menacing natural disasters that threaten the British Isles. This threat is particularly severe in the region of the Thames Estuary and it is in this area that the phenomenon of surge-tide interaction is most evident. Following the disastrous surge of 1953 January there appeared a series of papers on the subject of surge-tide interaction in the Thames by Proudman (1955a, 1957), Doodson (1956) and Rossiter (1961). However, these papers do not present a complete explanation of the underlying mechanics of the phenomenon and it is the purpose of the present paper to investigate further and clarify ideas.

The paper consists of three main parts: the first involves a statistical analysis of recorded surges aimed at establishing the observed characteristics of surge-tide interaction. The second part uses a numerical model to examine the response of the Thames to surges of different types. The third part extends the modelling approach with the objective of identifying the separate components of interaction.

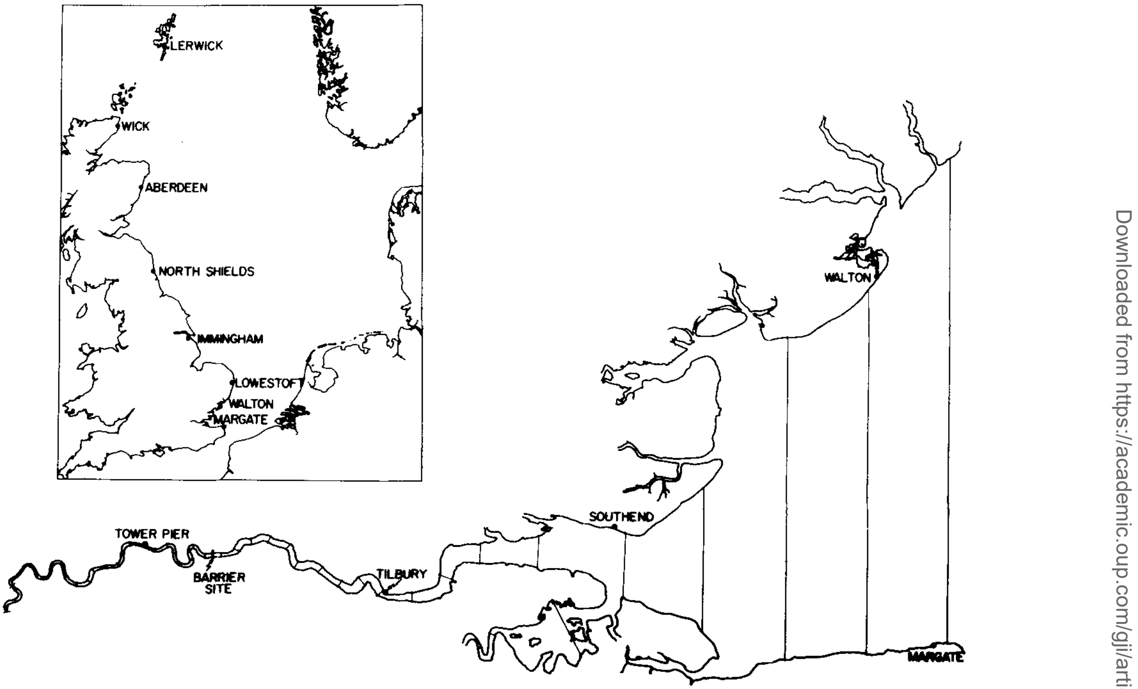

Figure 1. Map of the North Sea and River Thames.

2 Analysis of recorded surge data

An analysis of recorded surge data was carried out for the following ports: Lerwick, Wick, Aberdeen, North Shields, Lowestoft, Walton, Southend, Tilbury and Tower Pier (Fig. 1). The analysis employed 5 years of hourly recordings for the years 1969 to 1973.

A storm surge is defined as a variation in sea level resulting from the action of meteorological forces. Thus the hourly surge heights were calculated as follows:

$$R_t = O_t - P_t - M \qquad (1)$$

where R_t is the residual elevation of the sea surface at time t, O_t the recorded elevation, P_t the predicted astronomical tidal height, and M the annual mean of O_t.

Fig. 2 shows the mean surge level (positive and negative surges are treated separately throughout this analysis) at each port for surges occurring at four tidal phases namely: (a) rising tide, HT – 3.5 to – 2.5 hr; (b) high tide, HT – 0.5 to + 0.5 hr; (c) falling tide, HT + 2.5 to + 3.5 hr and (d) low tide, HT – 6.5 to – 5.5 hr (HT denotes high tide). The divergence of the four curves is a measure of the magnitude of the interaction at each location. Clearly, interaction can be detected as far northwards as Wick; it then increases continuously as far as Tower Pier with the effect of increasing surge height on the rising tide (Doodson 1929). An exception to this continuous increase is the small decrease between North Shields and Lowestoft discussed by Wolf (1978). Fig. 2 also shows that the mean values of the negative surges are less than the mean values for the positive surges – this asymmetry is discussed further in Section 2.

The results in Fig. 2 refer to mean surge values over the 5-yr sample period. Similar curves were obtained considering larger surges only, and it was deduced that interaction increases proportionately with surge height. To test this deduction further, statistics for selected surge peaks were examined.

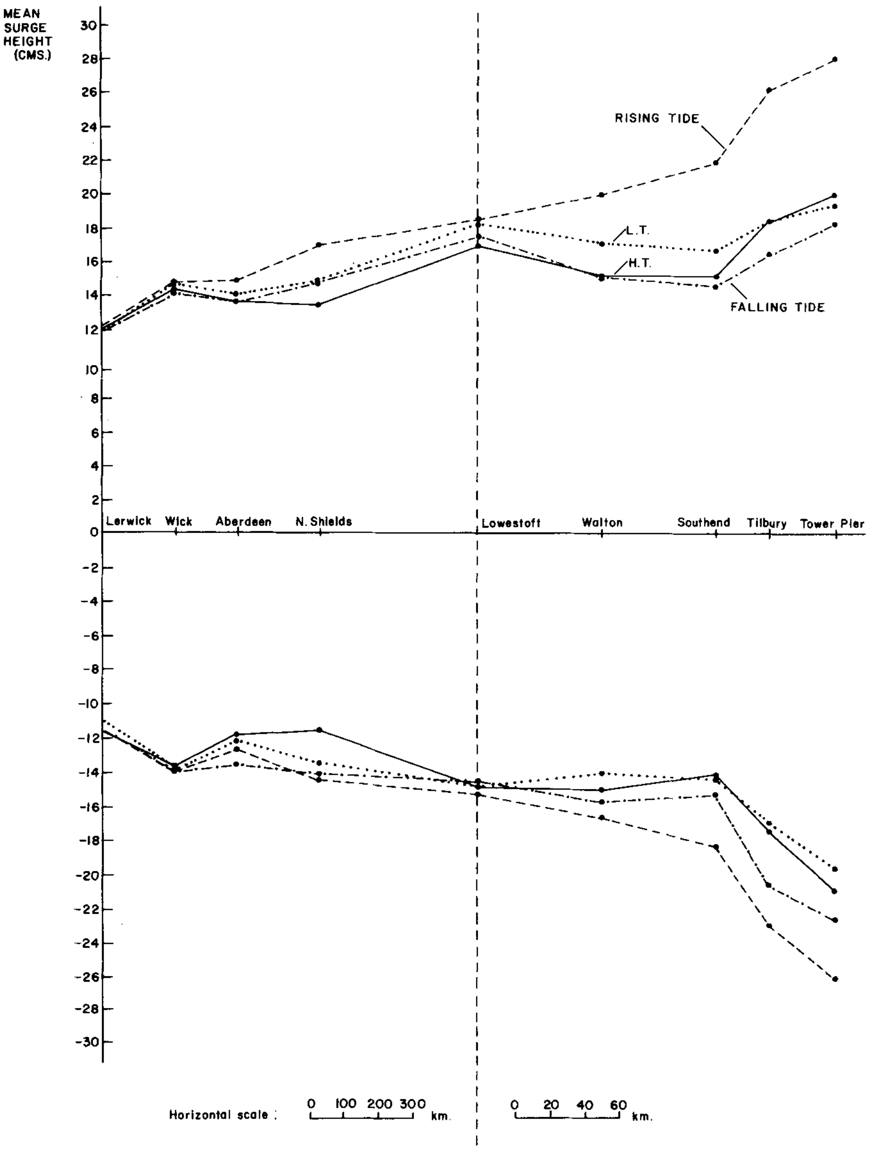

Figure 2. Recorded mean surge level for four tidal phases.

Discrete surges were first identified at Southend subject to the condition of a minimum height of 15 cm persisting for at least 3 hr. A particular surge event was then included for subsequent analysis only if corresponding peaks could be traced at all other ports. Using this approach, 55 positive and 35 negative surges were identified over the 5-yr period. Fig. 3 shows the amplification of these individual surge peaks between Lowestoft and Tower Pier; each point in the scatter diagram represents a particular surge event and the straight line indicates the best least-squares fit. These results do not support the inference sometimes drawn from the work of Rossiter (1961) that large surges are amplified less than smaller ones as they propagate into the Thames. On the contrary, the results are not at variance with the suggestion of a linear relationship between the magnitude of interaction and the associated surge height.

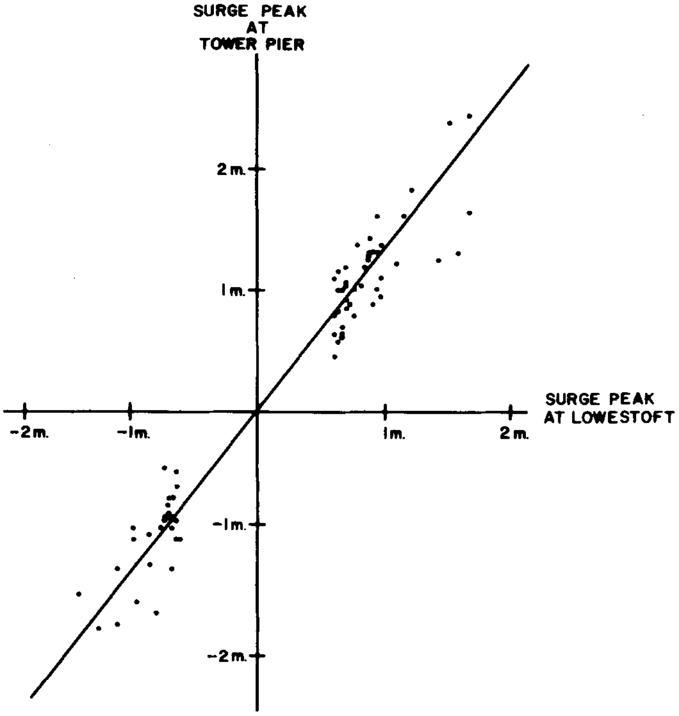

Figure 3. Amplification of surge peaks.

Fig. 4 shows, at each location for these discrete surges, the percentage of surge peaks occurring at different tidal phases. Clearly, relative to other tidal stages, a large number of surge peaks occur on the rising tide at Southend whereas the same surges traced back to Lerwick or Wick show a more or less random phase distribution. This is an important result demonstrating that surges tend to develop a peak on the rising tide in the Thames irrespective of the phase relationship between surge and tide in the northern North Sea.

2 Response characteristics of the River Thames

The numerical model developed by Prandle (1975) was used to examine the response of the River Thames to a range of surge conditions. The one-dimensional equations of motion used in the model were as follows:

$$\frac{\partial u}{\partial t}+g\frac{\partial Z}{\partial x}+\frac{K|u|u}{(D+Z)}=0 \tag{2}$$

and

$$B\frac{\partial Z}{\partial t}+\frac{\partial}{\partial x}\left\{(D+Z)Bu\right\}=0 \tag{3}$$

where x is the horizontal axis measured along the length of the channel, t time, B the breadth of the channel at the level of the water surface, D depth of the channel below a horizontal datum at approximately mean water level, Z elevation of the water surface above the same horizontal datum, u velocity in the x direction – mean value over a cross-section, K a friction coefficient.

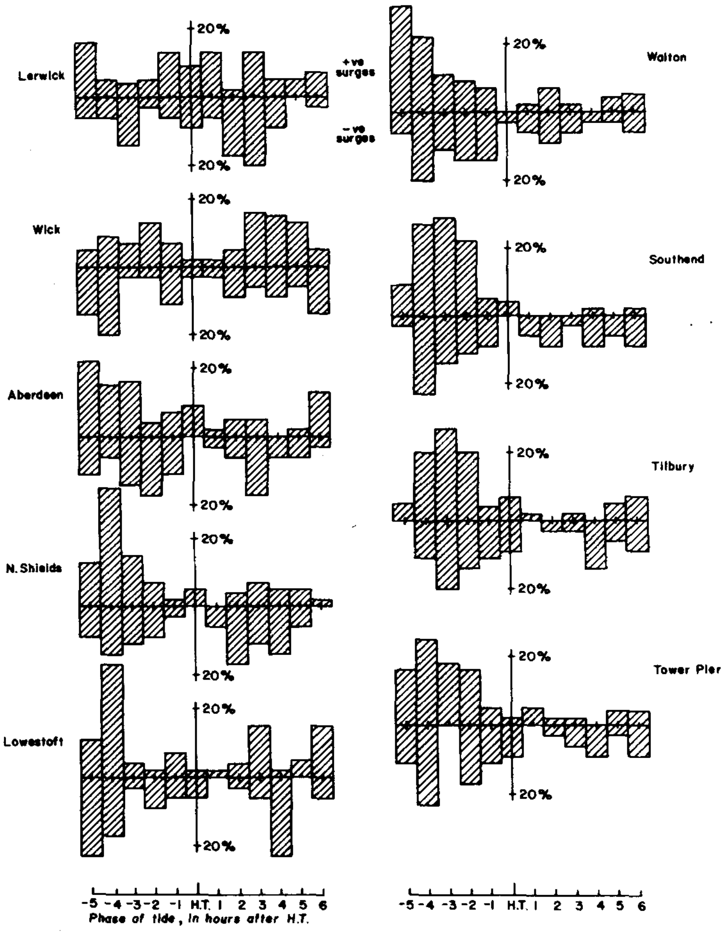

Figure 4. Frequency distribution of surge peaks as a function of tidal phases.

At the open-sea boundary of the model (Fig. 1) the surface elevation, $Z_B(t)$, was specified as a combination of tide plus surge in the following manner:

$$Z_B(t) = 180 \sin \omega t + H + j \frac{A}{2} \left(1 - \cos \left\{ \frac{2\pi}{P} (t - \theta) \right\} \right) \tag{4}$$

where $j = 1$ for $\theta < t < \theta + P$ and $j = 0$ at all other times.

The first term represents a tide of amplitude 180 cm and semi-diurnal period. The remainder represents a surge which may include a variation in mean sea level H, but in the main comprises a sinusoidal profile of total height A and duration P; θ represents the phase of the surge relative to the tide. The model was operated with a range of values of these surge parameters including, in each case, a complete range of phase differences θ. Surge values were obtained by subtracting the results for a simulation of tide alone from the values for tide plus surge.

Fig. 5 shows surge values computed at Tower Pier for a surge at the mouth of height $A = 180$ cm, duration $P = 25$ hr and with $H = 0$. Fig. 6 shows corresponding values of total water level for the same series of model runs. The horizontal axis in these figures represents the effect of varying θ. Time is measured along the vertical axis with the times of high tide

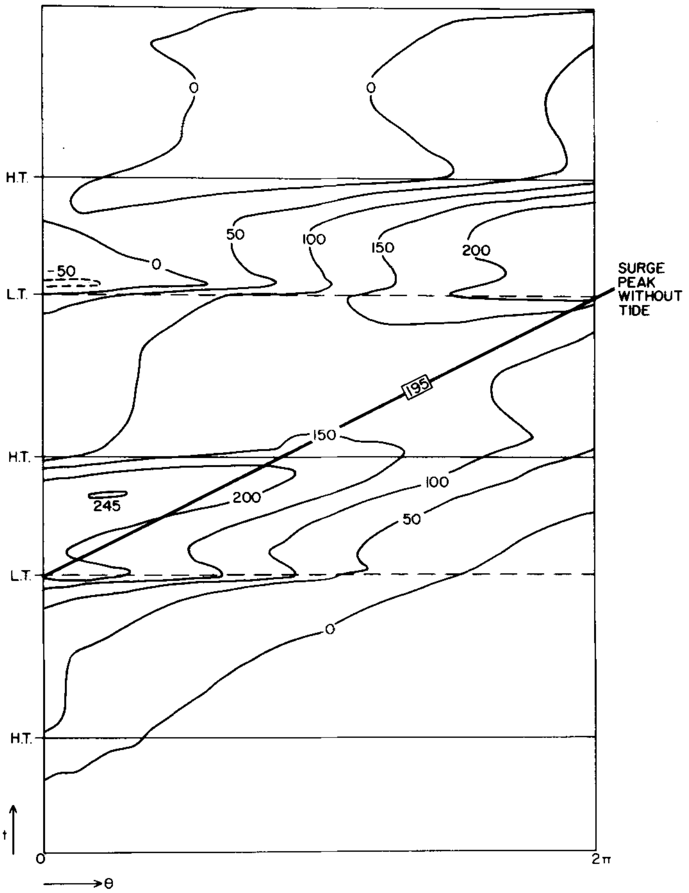

Figure 5. Computed residuals at Tower Pier: diurnal surge. Contours show surge height in cm.

(HT) and low tide (LT) indicated. Contour values along any vertical strip give the time profile of the surge for a particular θ. This is a new way of presenting variations in surge height, demonstrating the effect of surge-tide interaction. Thus, without this interaction, all contours would be parallel to the double straight line shown, the latter representing the surge peak without tidal influence.

Fig. 5 shows that, over all values of θ, the maximum surge peak of 245 cm occurs when the peak of the surge is on the rising tide. The value of this particular peak is about 25 per cent larger than the value of 195 cm obtained for surges propagating alone, thus demonstrating that interaction can increase surge heights appreciably. This contrasts with reduced surge heights at other tidal stages, shown in the figure.

Fig. 6 shows that maximum water levels tend to occur at high tide irrespective of the phase relationship, θ, between tide and surge at the mouth. The highest maximum water level, for all θ, is produced when the surge peak occurs around the time of high tide. This maximum level of 500 cm includes a surge value of only about 200 cm (Fig. 5) – considerably less than the largest surge value of 245 cm.

Diagrams similar to Figs 5 and 6 were produced for a range of surge input parameters A, P and H. Using these results, Fig. 7 shows the maximum surge height at Tower Pier, taken over all θ, plotted against surge duration P, for $A = \pm 60, \pm 120$ and ± 180 cm. Each continu-

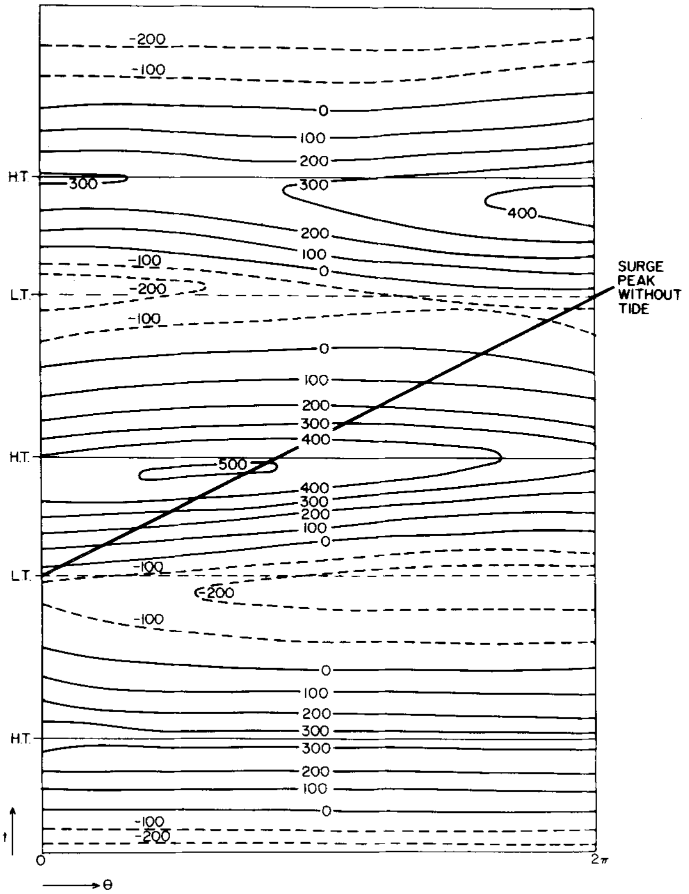

Figure 6. Computed total water levels at Tower Pier: diurnal surge. Contours show surge height in cm.

ous line connects results for surges having the same height A prescribed at the mouth. Values shown for zero frequency refer to surges generated by changes in mean sea level, H, at the mouth. The dashed lines show, for comparison, corresponding results obtained for the case of surge propagating alone and hence the divergence between corresponding dashed and continuous lines is a measure of the magnitude of surge-tide interaction. These results indicate that the amplification of maximum surge levels between the mouth and Tower Pier is, to a first approximation, independent of the surge height at the mouth. However, it is evident from the curves that this amplification is highly dependent on the duration of the surge, amplification increasing with decreasing surge duration. Comparison with the equivalent results for surge alone shows that interaction can increase surge heights significantly; particularly so for negative surges and for surges of short duration.

Fig. 8 shows a similar plot to that of Fig. 7 but the values shown in this case refer to the surge levels occurring at the time of maximum (or minimum) total water level. Manifestly, at the time of maximum (or minimum) level interaction reduces surge height. Interaction is more effective in reducing negative surges at times of minimum water level than reducing positive surges at times of maximum water level. This is in accordance with recorded water level statistics (Cartwright 1968) and with the earlier deduction from Fig. 3.

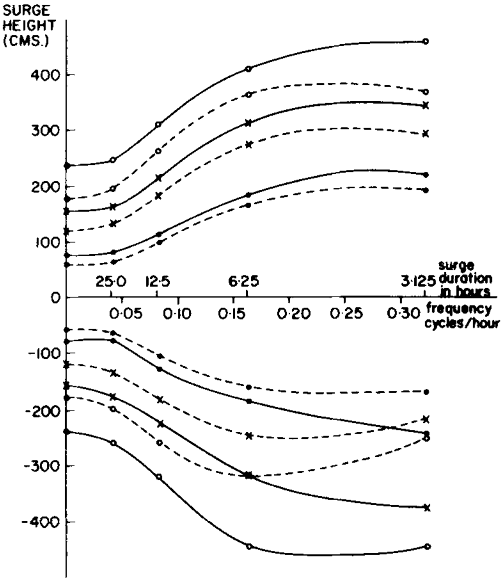

Figure 7. Response characteristics (1): Tower Pier. ---- maximum surge height, surge alone; —— maximum surge height, surge plus tide; surge height at mouth (*A*): ● 60 cm, x 120 cm, o 180 cm.

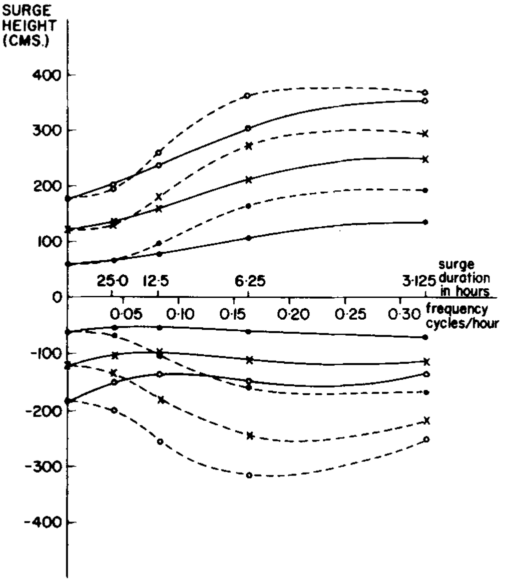

Figure 8. Response characteristics (2): Tower Pier. ---- maximum surge height, surge alone; —— surge height at maximum or minimum water level; surge height at mouth (*A*): ● 60 cm, x 120 cm, o 180 cm.

3 Mechanics of interaction in the Thames

In an attempt to gain an understanding of the mechanics of interaction, a technique was developed to simulate tide and surge separately while introducing interaction between the two phenomena in the form of perturbation terms. This involved two models, of the type used in Section 2, operating simultaneously, one of tidal propagation and the other of surge propagation – these models are referred to subsequently as 'parallel models'.

3.1 PARALLEL MODELS

Simplified equations for river flow, expressed in terms of cross-sectional transport Q, are (Dronkers 1969):

$$\frac{1}{B}\frac{\partial Q}{\partial t}+g(D+Z)\frac{\partial Z}{\partial x}+K|u|u=0, \tag{5}$$

$$B\frac{\partial Z}{\partial t}+\frac{\partial Q}{\partial x}=0, \tag{6}$$

where it is assumed that breadth B does not vary with Z.

For the tidal model let $Z=Z_{\mathrm{T}}$, $Q=Q_{\mathrm{T}}$, $u=u_{\mathrm{T}}$; for the surge model: $Z=Z_{\mathrm{S}}$, $Q=Q_{\mathrm{S}}$, $u=u_{\mathrm{S}}$ and for the combined simulation of tide plus surge: $Z=Z_{\mathrm{C}}$, $Q=Q_{\mathrm{C}}$, $u=u_{\mathrm{C}}$. Then:

$$Z_{\mathrm{C}}=Z_{\mathrm{T}}+Z_{\mathrm{S}}, \qquad Q_{\mathrm{C}}=Q_{\mathrm{T}}+Q_{\mathrm{S}} \tag{7}$$

and approximately, for small values of Z/D

$$u_{\mathrm{C}}=u_{\mathrm{T}}+u_{\mathrm{S}}. \tag{8}$$

The assumption made in the parallel models is that (5) and (6) may be separated out into a tidal motion governed by:

$$\frac{1}{B}\frac{\partial}{\partial t}Q_{\mathrm{T}}+g(D+Z_{\mathrm{T}}\underline{+Z_{\mathrm{S}}})\frac{\partial Z_{\mathrm{T}}}{\partial x}+K|u_{\mathrm{T}}\underline{+u_{\mathrm{S}}}|u_{\mathrm{T}}=0, \tag{9}$$

and

$$B\frac{\partial Z_{\mathrm{T}}}{\partial t}+\frac{\partial Q_{\mathrm{T}}}{\partial x}=0, \tag{10}$$

and a surge motion governed by

$$\frac{1}{B}\frac{\partial Q_{\mathrm{S}}}{\partial t}+g(D+\underline{Z_{\mathrm{T}}}+Z_{\mathrm{S}})\frac{\partial Z_{\mathrm{S}}}{\partial x}+K|\underline{u_{\mathrm{T}}}+u_{\mathrm{S}}|u_{\mathrm{S}}=0, \tag{11}$$

and

$$B\frac{\partial Z_{\mathrm{S}}}{\partial t}+\frac{\partial Q_{\mathrm{S}}}{\partial x}=0. \tag{12}$$

Boundary conditions in the tidal motion are: $Z=Z_{\mathrm{T}}$ at the mouth and $Q_{\mathrm{T}}=0$ at the head, and similarly for the surge motion $Z=Z_{\mathrm{S}}$ at the mouth, $Q_{\mathrm{S}}=0$ at the head.

In operating the tidal model with (9) and (10), the surge parameters Z_{S} and u_{S} which appear in equation (9) are evaluated from the simultaneous operation of the surge model, while in operating the surge model with (11) and (12) the tidal parameters Z_{T} and u_{T} which appear in (11) are obtained from the concurrently-running tidal model. Using this parallel model technique to simulate a number of recorded surge events in the Thames, it has been shown that the results from the separate simulations of tide and surge can be combined to give values in close agreement with results obtained from the simulation of tide and surge combined – so that the approach satisfies the conditions (7) and (8). It seems reasonable, therefore, to regard the additional terms underlined in equations (9) and (11) as representing the interaction between tide and surge. The magnitudes of the four interaction terms are all proportional to a product of tidal amplitude and surge amplitude. Hence for a tide of

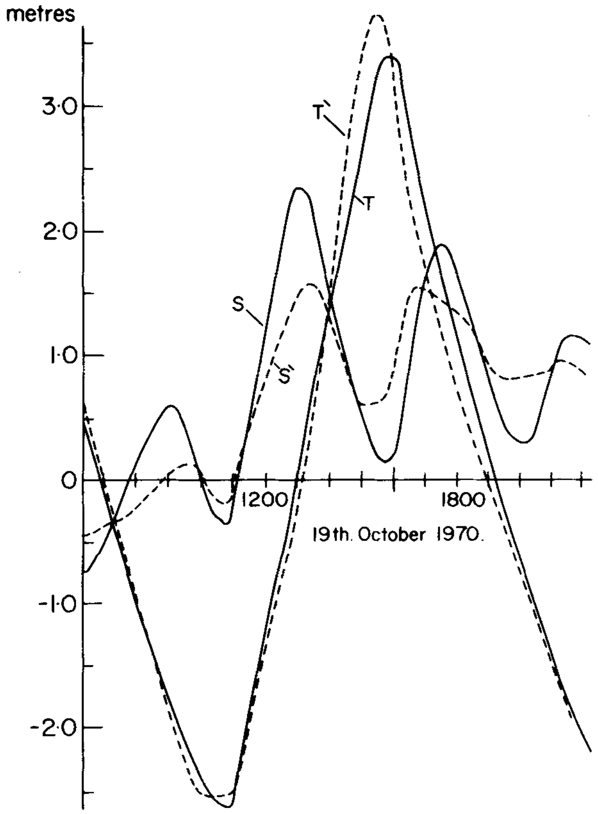

Figure 9. Interaction due to shallow water and quadratic friction: Tower Pier. —— tide alone (T), surge alone (S); ---- tide (T′) and surge (S′) modified by interaction.

approximately constant amplitude, the interaction at any particular phase of the tide is linearly proportional to the surge amplitude. Conversely, for a surge of a given magnitude, at any particular location, the degree of interaction is directly proportional to the tidal range at that location; this latter result supports similar deductions made by Proudman (1955a); Keers (1968); Cartwright (1968) and Banks (1974).

3.2 SIMULATION OF THE SURGE OF 1970 OCTOBER 18–21

In the application of the above technique, equation (2) was used rather than equation (5). Thus in simulating tidal motions, shallow-water interactions due to the surge were introduced by modifying D to become $D = D + Z_S$ and quadratic friction interactions by modifying $|u_T|u_T$ to $|u_T + u_S|u_T$ – corresponding modifications were introduced into the model of surge propagation to account for interaction with the tide.

The results shown in Figs 9, 10 and 11 for Tower Pier refer to the storm of 1970 October 18–21.

These have been shown to be representative of surge-tide interactions over a wide range of conditions.

The models incorporated the predicted tide and the recorded surge at Margate as boundary conditions at the mouth. Computer runs were carried out for:

(1) model of tide alone (T);

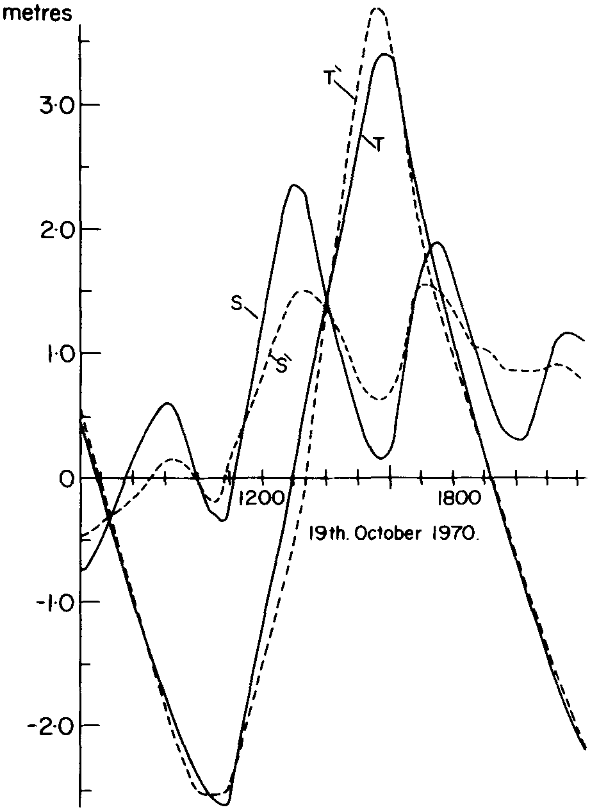

Figure 10. Interaction due to shallow water: Tower Pier. —— tide alone (T), surge alone (S); ---- tide (T') and surge (S') modified by interaction.

(2) model of surge alone (S);

(3) model of tide with interaction from model (4) due to shallow water and quadratic friction (T');

(4) model of surge with interaction from model (3) due to shallow water and quadratic friction (S').

The elevations at Tower Pier computed from these four models are shown in Fig. 9. The difference between the curves for T and T' in the figure represents the modification of tidal propagation due to interaction with the surge and similarly the difference between the curves for S and S' represents the modification of the surge due to the tide. The figure shows that the amplitude of surge peaks are appreciably reduced by interaction and that the phases of both tide and surge are modified by up to about 20 min.

Similar results were obtained for models (1), (2), (3) and (4) with the interactions introduced into models (3) and (4) limited to the terms involving quadratic friction. The results for T' and S' were found to be almost identical to the values shown in Fig. 9, so that it may be deduced that the major source of interaction in the Thames is due to the quadratic friction term.

Fig. 10 shows a similar set of results with interactions in models (3) and (4) limited to the shallow-water terms. The results show that the phase of the surge is advanced when the tide rises above mean water level and retarded for low tidal levels. A similar advance in the tidal phase can be seen for positive surge levels. However, the amplitude of both tide and surge

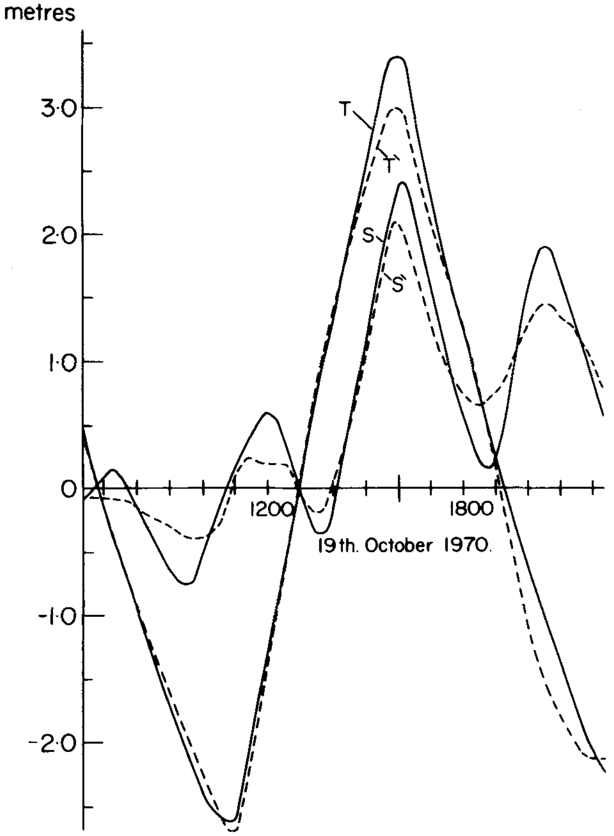

Figure 11. Interaction for a surge peak coincided with high tide: Tower Pier. —— tide alone (T), surge alone (S); ---- tide (T') and surge (S') modified by interaction.

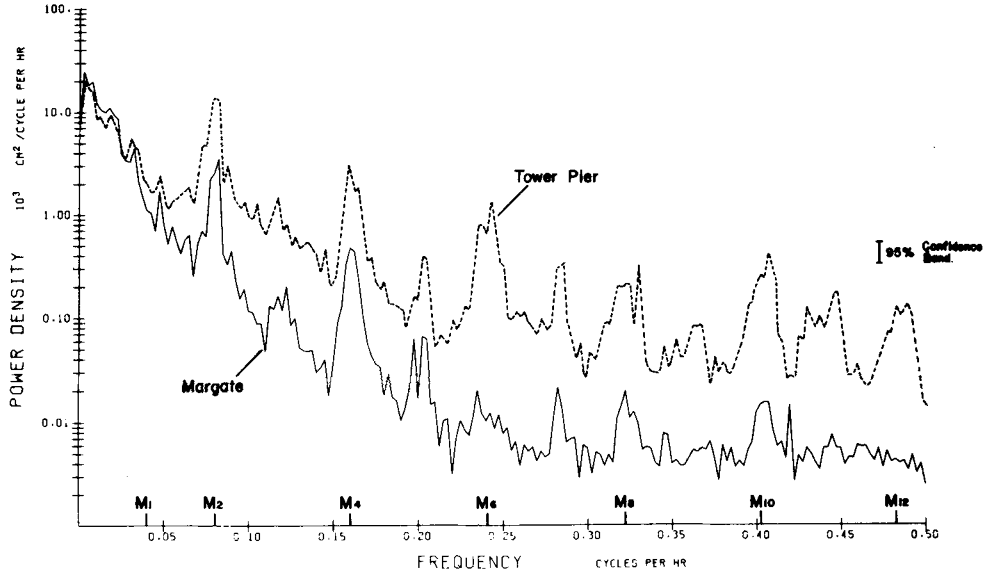

Figure 12. Spectra of recorded surges.

remains almost unchanged. The magnitude of these phase changes amounts to as much as 20 min for both tide and surge. For a tidal amplitude of 300 cm this phase advance produces an increase in surge height of 50 cm on the rising tide, which underlines the importance of this mechanism as noted by Rossiter (1961).

Results obtained by fictitiously delaying the phase of the surge by 3 hr at the mouth of the model are shown in Fig. 11. With both shallow water and frictional interaction taken into account, this figure is to be compared with Fig. 9 describing the original conditions. Although interaction now decreases both the surge and tidal levels at high water, the maximum total residual, $S' + T' - T$, coincides with high tide – demonstrating that, given the appropriate conditions between tide and surge at the mouth, it is possible for the surge peak actually to occur on high tide.

4 Summary and conclusions

Surge levels are, in general, amplified progressively as surges propagate southwards in the North Sea; the amplification in any particular case is largely independent of the initial surge height but is sensitive to the shape or time profile of the surge. Similarly the degree of interaction between surge and tide increases progressively between Wick and Tower Pier with one exception in the region of Lowestoft.

On reaching the Thames, statistical analysis shows that surge peaks tend to occur on the rising tide, moreover it has been shown that surges tend to develop a peak on the rising tide in the Thames irrespective of the original phase relationship between tide and surge in the northern North Sea. Model studies have shown that, for any surge event, maximum sea levels will generally occur at high tide. However, the greatest sea levels one may expect will occur when a surge peak actually coincides with high tide (Fig. 6). These studies also demonstrated an important asymmetry existing between maximum and minimum levels (Figs 2 and 8).

A method of resolving the processes involved in interaction has been developed using two models operated simultaneously, one of tidal propagation and the other of surge propagation. Interaction between these models was introduced by cross-linkage terms: the form of these terms showed that interaction is proportional to a product of surge height and tidal amplitude. In this way it was found that the principal interaction mechanism in the Thames is due to the quadratic friction term.

In an attempt to relate the results from the statistical analysis and the modelling studies, a spectral analysis was carried out on 342 days of simultaneous hourly recordings of surge heights at Tower Pier and Margate, where Margate represents conditions at the mouth of the Thames. The spectra show that the ratio of surge heights between Tower Pier and Margate increases for surges of shorter duration as deduced from model results (Figs 7 and 8). A significant enhancement of the spectrum at Tower Pier occurs at the tidal frequencies of M_2, M_4, M_6 etc. This enhancement may be related to the residual component arising from modification of the tidal phase due to interaction with the surge (Section 3). The origins of this additional surge component explains why surge heights in the Thames vary in such a consistent fashion with tidal phase and why, at certain tidal phases, interaction increases surge heights.

One difficulty in interpreting the results from this study is that the modelling approach only considered interaction within the Thames whereas the statistical approach showed that appreciable interaction occurs in the open sea regions. Interaction outside of the Thames has since been studied in a further paper (Prandle & Wolf 1978), however, it is appropriate to consider here the question: can, in reality, a large surge occur on high tide in the Thames?

It has been shown that this is possible in the model (Figs 5 and 11) given appropriate conditions at the mouth of the Estuary. The evidence from observation is that the situation can occur if a locally-generated surge component, becoming increasingly effective between rising and high tide, is superimposed on a large surge propagating from the north in the usual way. Two examples in which large surge peaks occurred close to tidal high water are the surge of 1928 January 5–8 (Corkan 1948; Doodson 1929) and the surge of 1965 December 9–10 (Synnott 1966). The significant feature of these storms is that they both travelled rapidly across the North Sea approximately during the time between the preceding tidal high water and the high water on which the surge peak occurred in the Thames.

Acknowledgments

The authors wish to thank Dr N. S. Heaps for his support and encouragement in the course of this study. Thanks are also due to other members of staff at I.O.S. Bidston, namely Sheila Brown and David Blackman of the Tidal Computation Section for supplying tide and surge data, and Robert Smith for preparation of the diagrams.

The work described in this paper was funded by a Consortium consisting of the Natural Environment Research Council, the Ministry of Agriculture Fisheries and Food and the Departments of Energy, Environment and Industry.

References

Banks, J. E., 1974. A mathematical model of a river-shallow sea system used to investigate tide, surge and their interaction in the Thames–southern North Sea region, *Phil. Trans. R. Soc. Lond. A*, **275**, 567–609.

Cartwright, D. E., 1968. A unified analysis of tides and surges round north and east Britain, *Phil. Trans. R. Soc. Lond. A*, **263**, 1–55.

Corkan, R. H., 1948. *Storm surges in the North Sea*, US hydrogr. Off., Misc. 15072, **1**, 174 pages, and **2**, 166 pages.

Doodson, A. T., 1929. Report on Thames floods, *Geophys. Mem. Lond.*, No. 47, 1–26.

Doodson, A. T., 1956. Tides and storm surges in a long uniform gulf, *Proc. R. Soc. Lond. A*, **237**, 325–343.

Dronkers, J. J., 1969. Tidal computations for rivers, coastal areas, and seas, *Proc. Am. Soc. Civ. Engnrs*, **95** HY1, 29–77.

Keers, J. F., 1968. An empirical investigation of interaction between storm surge and astronomical tide on the east coast of Great Britain, *Dt. Hydrogr. Z.*, **21**, 118–125.

Prandle, D., 1975. Storm surges in the southern North Sea and River Thames, *Proc. R. Soc. Lond. A*, **344**, 509–539.

Prandle, D. & Wolf, J., 1978. *Surge-tide interaction in the southern North Sea*, Ninth Liege Colloquium, Hydrodynamics of Estuaries and Fjords, University of Liege, 1977 May 2–6.

Proudman, J., 1955a. The propagation of tide and surge in an estuary, *Proc. R. Soc. Lond. A*, **231**, 8–24.

Proudman, J., 1955b. The effect of friction on a progressive wave of tide and surge in an estuary, *Proc. R. Soc. Lond. A*, **233**, 407–418.

Proudman, J., 1957. Oscillations of tide and surge in an estuary of finite length, *J. Fluid Mech.*, **2**, 371–382.

Rossiter, J. R., 1961. Interaction between tide and surge in the Thames, *Geophys. J. R. astr. Soc.*, **6**, 29–53.

Synnott, J. N. N., 1966. *25th meeting, Advisory Committee on Oceanographic and Meteorological Research, 22 June 1966*, Unpublished paper, No. 25 (8). Report by the STWS on the Storm Surge of 1965 December 9–10.

Wolf, J., 1978. The interaction of tide and surge in a semi-infinite uniform channel, with application to surge propagation down the east coast of Britain, *Appl. math. model.*, in press.