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The Intergenerational State: Education and Pensions*

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ABSTRACT

When credit markets to finance investment in human capital are missing, the competitive equilibrium allocation is inefficient. When generations overlap, this failure can be mitigated by properly designed social arrangements. We show that public financing of education and public pensions can be designed to implement an intergenerational transfer scheme supporting the complete market allocation. Neither the public financing of education nor the pension scheme we consider resemble standard ones. In our mechanism, via the public education system, the young borrow from the middle aged to invest in human capital. They pay back the debt via a social security tax, the proceedings of which finance pension payments. When the complete market allocation is achieved, the rate of return implicit in this borrowing-lending scheme should equal the market rate of return.

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1. Introduction

A well-established tradition argues that government policies and, in particular, most institutions comprising the welfare state are justified by the inability of decentralized markets to deliver a Pareto efficient allocation. This approach is positive and normative at once. It explains the existence of certain arrangements as cooperative remedies to allocational inefficiencies, and it provides guidance to the optimal design of such institutions.

We adopt a normative stance, and study the role of public education and public pensions. We build a simple dynamic environment in which the lack of a specific credit market leads to a suboptimal accumulation of human capital. By construction, public financing of education is desirable. Introducing a scheme for the public financing of education cannot, by itself, restore the complete market allocation. An additional institutional arrangement, closely resembling a public pension system, is also needed. Further, we show that a simple, but so far altogether ignored, link between the two systems must hold. Such link is captured by the risk-adjusted equality between the two rates of return (implicit in the public financing of education and pensions) and the market rate of return on capital. After characterizing the optimal intergenerational arrangement we briefly discuss practical ways of implementing it.

In an earlier paper, Becker and Murphy (1988) argue that the welfare state serves purposes previously served by intra-family arrangements, and which are instrumental for implementing efficient intergenerational allocations. We examine this conjecture in a specific dynamic general equilibrium context. Young generations would like to accumulate productive human capital, but are unable to finance it via credit markets. Middle age individuals would like to diversify their retirement portfolios by investing in the human capital of younger people, but financial instruments to do so are unavailable. Parental altruism and within family arrangements may replace the missing financial instruments, thereby greatly reducing the scope for public intervention. In principle, we share this view; nevertheless, two considerations should be taken into account. Efficiency is achieved only when parents fully internalize the utility of all future generations, and are not credit constrained in turn. The ability of within family arrangements to replicate the complete market allocation (e.g. Kotlikoff and Spivak (1981)) may be severely affected by enforcement problems.

The model we study is very simplified, but its main implications are robust to the addition of more realistic features. In particular, introducing population growth and a realistic number of periods of life would leave the results unaltered. Adding some form of parental (or filial, as in Boldrin and Jones (2002)) altruism would modify the quantitative but not the qualitative prescriptions, unless one adopts the fully dynastic model of familial relations proposed in Barro and Becker (1989). Adding uncertainty, in the form of unexpected shocks to the productivity of the two kinds of capital, would most likely strengthen our normative prescriptions on the grounds of portfolio diversification. This is akin to the point already made by Merton (1983) in a different context, but with similar implications for policy. Finally, and aside from the redistributive concerns this may or may not create for public policy, the introduction of heterogeneity within a generation would also not alter our main prescription.

Neither Becker and Murphy (1988) nor we are the first to argue that a link between public education and public pensions does or should exist. Pogue and Sgontz (1977) make this point in the context of a simple model of social security taxation. While they do not fully develop the dynamic implications of their argument, nor bring it to the data, they stress that "the investment incentive provided by [pay-as-you-go payroll tax] financing is for *collective* investment by each generation in capital that will enhance the income of persons who will be working during the generation's years of retirement" (p. 163, italics in original). Richman and Stagner (1986) also argue, albeit even more informally, that the very existence of a pay-as-you-go pension system should generate an incentive for the older cohorts to invest in the younger ones. Further, a very large demographic, sociological, and anthropological literature has long argued that such intergenerational links (within the family, the clan, the village, or the entire society) are critical for understanding both fertility choices and parental investments in children. Caldwell (1978) and Nugent (1985) are recent references, while Neher (1971) is a very early economic paper in which fertility choices are linked to the parental desire to draw a pension when old.

In recent years, other authors have addressed a more general but closely related issue in the context of the overlapping generations model. That is: If current generations are selfish, why should they invest in assets that are valuable only to future generations? Symmetrically, what does lead the young generations to transfer resources to the old ones who will not be around tomorrow? A paper by Kotlikoff, Persson and Svensson (1988) is an earlier reference: they cast the problem in terms of time-consistency of the optimal policy. The solution proposed involves a social contract which is "sold" by the old to the young generation in exchange for tax revenues. Boldrin (1992, see p. 31) and Boldrin and Rustichini (2000) analyze public education and public pensions, respectively. In the first case, education is publicly financed because it increases the future productivity of private physical capital, which provides the old generation with a channel to collect (part of) the return on their investment. In the second case, pensions are paid because they allow the working generation to act as a "monopolist" in the supply of savings, and therefore earn a higher total return on its investment. Subgame perfectness is used to show that an equilibrium with social security can be sustained. Rangel (1999) and Conley (2001) reach the following general conclusion: Establish intergenerational arrangements such that future payoffs accruing to generations not yet born at the time the investment was made are transferred backward to the generation which made the investment. Rangel (1999) derives an interesting theory of "backward" and "forward" public goods on the basis of these premises. He uses game theoretical arguments, not dissimilar from those used in earlier versions of this paper, to show that an equilibrium exists in which all generations play a trigger strategy guaranteeing that the appropriate amount of (backward) public goods is purchased. While Rangel's argument is developed in the context of a stationary exchange economy, it can be generalized to one with production and endogenous growth. Conley (2001) shows that when the public goods in question are durable and there is land, the Tiebout solution of providing the public goods locally achieves the efficient allocation. Finally, Bellettini and Berti Cerroni (1999) also use an overlapping generations model with production to argue that the existence of pay-as-you-go pensions which are financed by labor income taxation may not necessarily reduce growth. They do so by introducing public capital in the production function and using game theoretical arguments to show that, when pensions are financed by taxes on future labor income, there exists a subgame perfect equilibrium in which investment in the public good and economic growth are higher than otherwise.

While the positive predictions of our model may prove valuable to understanding the historical origins of public education and public pensions, it is on the normative prescriptions that we like to put our emphasis. Should the public education and the public pension systems be designed according to the simple rules presented here? We believe they should. Would this be practically feasible? We discuss three possible implementations, all of which use fairly traditional tools of public policy: taxes, subsidies, transfers, and public debt. Our empirical analysis of the Spanish data shows that, indeed, the intergenerational flows implied by our criteria would not be very different from those that the current system generates and could, therefore, be implemented without generating major resistance from the affected parties.

2. The Basic Model

2.1. Complete Markets

Consider an overlapping generations economy in which agents live for three periods. Within each generation individuals are homogeneous, and, to simplify, each generation has a constant size of one. Adding population growth would not alter the results, while the case of stochastic fertility and mortality rates is considered in Boldrin and Montes (2003a).

Physical capital, k_t, and human capital, h_t, are owned, respectively, by the old and the middle-age individuals. Output of the homogeneous commodity is $y_t = F(h_t, k_t)$, where $F(h, k)$ is a constant returns to scale neoclassical production function. Young agents are born with an endowment h_t^y of basic knowledge, which is an input in the production of their future human capital $h_{t+1} = h(d_t, h_t^y)$. With d_t we denote the physical resources invested in education. We assume that competitive markets exist in which young agents can borrow such resources. The function $h(d, h^y)$ is also a constant returns to scale neoclassical production function. During the second period of life, individuals work and carry out consumption-saving decisions. When old, they consume the total return on their savings. We assume agents draw utility from (c_t^m, c_{t+1}^o), denoting consumption when middle age and old, respectively. Neither consumption when young, nor leisure, nor the welfare of descendants affects lifetime utility. Adding such considerations would only increase the notational burden without contributing additional insights.

Let the homogeneous commodity be the numeraire. Output y_t is allocated to three purposes: aggregate consumption ($c_t = c_t^m + c_t^o$), accumulation of physical capital for next period (k_{t+1}), and investment in education (d_t). Human capital and physical capital are purchased by firms at competitive prices equal, respectively, to $w_t = F_1(h_t, k_t)$ and $1 + r_t = F_2(h_t, k_t)$ (subscripts of functions indicate partial derivatives). Aggregate saving finances investment in physical and human capital ($s_t = k_{t+1} + d_t$), accruing a total return equal to $(1 + r_{t+1})s_t = R_{t+1}s_t$.

The life-cycle optimization problem for an agent born in period $t - 1$ is

$$U_{t-1} = \max_{d_{t-1}, s_t} \left\{ u(c_t^m) + \delta u(c_{t+1}^o) \right\} \tag{2.1}$$

subject to:

$$\begin{aligned}
&0 \leq d_{t-1} \leq \frac{w_t h_t}{R_t} \\
&c_t^m + s_t + R_t d_{t-1} \leq w_t h_t \\
&c_{t+1}^o \leq R_{t+1} s_t \\
&h_t = h(d_{t-1}, h_{t-1}^y).
\end{aligned}$$

First-order conditions simplify to:

$$u' \left[w_t h(d_{t-1}, h_{t-1}^y) - s_t - R_t d_{t-1} \right] = \delta R_{t+1} u' \left[s_t R_{t+1} \right] \tag{2.2a}$$

$$\left[w_t h_1(d_{t-1}, h_{t-1}^y) - R_t \right] = 0. \tag{2.2b}$$

The first condition is the usual equality between the interest factor and the marginal rate of substitution in consumption. The second equates the private return from investing in human capital to the cost of financing it via the credit market.

A *Competitive Equilibrium* is defined by (2.2) and:

$$F(h_t, k_t) = c_t + s_t \tag{2.3a}$$

$$F_1(h_t, k_t) = w_t \tag{2.3b}$$

$$F_2(h_t, k_t) = R_t \tag{2.3c}$$

$$s_t = d_t + k_{t+1}. \tag{2.3d}$$

Given a sequence $\{h_t^y\}_{t=0}^{\infty}$, one can solve equations (2.2) and (2.3) for (d_t, h_{t+1}, k_{t+1}), $t = 0, 1, \ldots,$ to obtain a dynamic system $\Phi : (d_{t-1}, h_t, k_t) \mapsto (d_t, h_{t+1}, k_{t+1})$. Given initial conditions (d_{-1}, h_0, k_0), Φ induces the equilibrium path $\{(d_t, h_{t+1}, k_{t+1})\}_{t=0}^{\infty}$.

In our setting, the equilibrium rental-wage ratio R/w is a decreasing function of the factor intensity ratio $x = k/h$; that is,

$$\frac{R}{w} = \frac{f'(x_t)}{f(x_t) - x_t f'(x_t)} = \frac{R(x_t)}{w(x_t)} = \omega(x_t)$$

where $f(x) = F(1, k/h)$. Without loss of generality, the algebra leading from (2.2) and (2.3) to Φ can be simplified by means of three technical assumptions.

Assumption 1 The function $h : \Re_+^2 \mapsto \Re_+$ is smooth. The function $g : \Re_+^2 \mapsto \Re_+$ satisfying $h_1[g(x, h^y), h^y] - \omega(x) = 0$ exists, is well defined, and continuous.

Assumption 2 The function $u : \Re_+ \mapsto \Re_+$ is strictly increasing, strictly concave, and smooth. Given numbers $I \geq 0$, $R \geq 0$, the function $V(I - z, Rz) = u(I - z) + \delta u(Rz)$ is such that $\arg\max_{0 \leq z \leq I} V(I - z, Rz) = S(R, I)$ has the form $S(R, I) = s(R) \cdot I$, with $s(\cdot)$ monotone increasing.

Assumption 3 For all $t = 0, 1, 2, \ldots$, the endowment h_t^y satisfies $h_t^y = \mu h_t$, $\mu > 0$.

Under these hypotheses, tedious but straightforward algebra shows that, given d_{t-1}, the two-dimensional implicit function problem

$$
\begin{aligned}
h_{t+1} - h\big[g(x_{t+1}, h_t), h_t\big] &= 0 \\
s\big[R(x_{t+1})\big]\big[w(x_t)h_t - R(x_t)d_{t-1}\big] - k_{t+1} - g(x_{t+1}, h_t) &= 0
\end{aligned}
$$

has a well-defined solution:

$$
h_{t+1} = \Phi^1(h_t, k_t) \tag{2.4a}
$$

$$
k_{t+1} = \Phi^2(h_t, k_t). \tag{2.4b}
$$

Standard methods can be used to show that, given (h_t, k_t) and d_{t-1}, the equilibrium choice of (h_{t+1}, k_{t+1}) is unique and induces an efficient allocation of resources in period t. This amounts to *static efficiency*: In each period aggregate savings are allocated to equalize rates of return between the investments in physical and human capital. Dynamic efficiency is subtler. It requires that, given (d_{-1}, h_0, k_0), there exists no feasible path $\left\{(\hat{k}_t, \hat{h}_t)\right\}_{t=0}^{\infty}$ which delivers more consumption than the competitive equilibrium during some periods without requiring less consumption during any other period. In our setting, one can use the characterization of dynamically efficient paths obtained by Cass (1972). To apply the original argument one must account for the possible unboundedness of consumption paths, which requires normalizing all variables by a factor growing at the balanced growth rate.[1] Under our assumptions, the technology set is a convex cone and unbounded paths are feasible. They are an equilibrium if the utility function allows for enough intertemporal elasticity of substitution in consumption. In this case, the dynamic

[1] Technical details are available from the authors upon request.

system (2.4) does not have any fixed point

$$\begin{aligned} h^* &= \Phi^1(h^*, k^*) \\ k^* &= \Phi^2(h^*, k^*) \end{aligned}$$

other than that the origin and equilibria converge to a (unique) balanced growth path characterized by a constant growth rate and a constant ratio $x^* = k^*/h^*$. We illustrate our results through a simple example.

Example Let $u(c) = \log c$, $F(h,k) = A \cdot k^\alpha h^{1-\alpha}$, and $h(d, h^y) = B \cdot \lambda(h^y) d^\beta$, $\alpha \in (0,1)$, $\beta \in (0,1)$, $A \geq 1$, $B \geq 1$, with $\lambda : \Re_+ \mapsto \Re_+$ is continuous and monotone increasing. Manipulating the first-order conditions yields

$$s_t = \frac{\delta}{1+\delta}\left[w_t h_t - (1+r_t) d_{t-1}\right]$$

$$d_{t-1} = \frac{\beta(1-\alpha)}{\alpha} k_t.$$

Setting $\frac{\beta(1-\alpha)}{\alpha} = \gamma$ and using the market-clearing condition for saving and investment gives

$$d_{t-1} = \frac{\gamma s_{t-1}}{1+\gamma}.$$

Aggregate saving is therefore equal to

$$s_t = \left[A \frac{\delta(1-\alpha)(1-\beta)}{1+\delta}\right]\left[k_t^\alpha h_t^{1-\alpha}\right]$$

which implies that

$$k_{t+1} = A\eta \left[k_t^\alpha h_t^{1-\alpha}\right] \tag{2.5a}$$

$$h_{t+1} = B\lambda(h_t^y)(A\gamma\eta)^\beta \left[k_t^\alpha h_t^{1-\alpha}\right]^\beta \tag{2.5b}$$

where $0 < \eta = \frac{\delta}{1+\delta}\frac{(1-\alpha)(1-\beta)}{1+\gamma} < 1$. Now let $h_t^y = h_t$. Different functional forms for $\lambda(\cdot)$ yield different patterns of long-run behavior. One, none, or more than one interior steady states may exist and may be asymptotically either stable or unstable. Similarly, balanced growth may or may not be an equilibrium. A convenient specification is $\lambda(h) = h^{1-\beta}$. Then the dynamic system (2.5) reads

$$k_{t+1} = A\eta \left(k_t^{\alpha} h_t^{1-\alpha}\right) \tag{2.6a}$$

$$h_{t+1} = B(A\gamma\eta)^{\beta} \left(k_t^{\alpha\beta} h_t^{1-\alpha\beta}\right). \tag{2.6b}$$

The only rest point of (2.6) is the origin. The ray

$$x^* = \frac{k_t}{h_t} = \left[\frac{A\eta}{B\left(A\gamma\eta\right)^{\beta}}\right]^{\frac{1}{1-\alpha(1-\beta)}} \tag{2.7}$$

in the (h_t, k_t) plane defines a balanced growth path. Straightforward algebra shows that for all initial conditions $(h_0, k_0) \in \Re^2_+$, iteration of (2.6) leads (h_t, k_t) to the ray x^*.

Along the balanced growth path, the two stocks of capital expand (or contract) at the factor

$$1 + g^* = A\eta \left[\frac{B(A\gamma\eta)^{\beta}}{A\eta}\right]^{\frac{1-\alpha}{1-\alpha(1-\beta)}}$$

which is larger than one (i.e. there is unbounded growth) when

$$\eta > \frac{1}{A} \cdot \left[\frac{1}{B^{1/\beta}\gamma}\right]^{(1-\alpha)}.$$

A sufficient condition for the equilibrium path to be dynamically efficient is that the gross rate of return on capital be larger than or equal to one plus the growth rate of output. With linearly homogeneous production functions, the rate of return on capital is determined by the factor intensity ratio. Hence we need

$$(1 + g^*) < \alpha A \left(x^*\right)^{-(1-\alpha)}.$$

The latter reduces to $\alpha > \eta$, which is equivalent to

$$\frac{(1-\alpha)(1-\beta)}{\alpha + \beta(1-\alpha)} < \frac{1+\delta}{\delta}.$$

For reasonable values of α and β, the latter is satisfied, as long as $\delta > 0$.

2.2. Equilibrium When Credit Markets Are Missing

In reality, credit markets financing education investments are rare. The reasons for such a lack of privately provided credit are various and widely studied. (See, for example, Becker (1975) for a classical discussion; Kehoe and Levine (2000) for a more recent one.) In our model, a lack of borrowing opportunities for the young generation implies that $d_t = 0$ for all t and, therefore, that $h_{t+1} = h(0, h_t^y)$. This makes the complete market allocation (CMA, from now on) unachievable and, by eliminating investment in human capital, leads the economy to an inefficient equilibrium. The specific properties of such an equilibrium would depend upon the assumptions one is willing to make about $h(0, h^y)$. This is not our concern here. Our interest lies, instead, with the CMA as a theoretical benchmark and with the class of intergenerational transfer policies that are capable of replicating it when credit markets to finance education are unavailable. We now turn to this issue.

3. Introducing the Intergenerational State

Consider the situation in which d_t is constrained to zero in all periods. In general, condition (2.2b) is violated and $F_2(h_t, k_t) = R_t < w_t h_1(0, h_{t-1})$ holds. Profitable investment opportunities, which cannot be exploited, exist in the educational sector. Too much is invested in the physical stock of capital, the k/h ratio is too high, and the rate of return on capital is too low with respect to the benchmark case. The allocation is inefficient: The young could increase their lifetime income by borrowing in order to accumulate human capital, and the middle age could increase their retirement income by shifting some savings from k_t to d_{t-1}, but both are prevented from doing so.

Apparently, such inefficiencies can be erased, and the CMA restored by a simple policy of taxing the middle age an amount equal to d_t, to be spent in financing the education of the young. It turns out that, in general, this statement is not correct, and that a more sophisticated kind of public policy is required to fully restore CMA and efficiency. More precisely

Proposition 1. *When credit markets for investment in human capital are missing, a policy that taxes the middle age a lump sum amount d_t and uses it to finance the education of the young is associated to a competitive equilibrium with the following properties:*

(i) the allocation it induces is different from the CMA;

(ii) it may be inefficient, both in the static and the dynamic sense;

(iii) when it achieves efficiency, it makes the initial generations worse off than under the CMA.

We proceed to proving these three statements. Assume, then, that the government levies a lump-sum tax on the middle-age to finance education for the young. Assume that the initial conditions (k, h) are the same as in the case of complete markets, and that, in each period t, an amount d_t, equal to the one chosen under complete markets, is transferred from middle-age to young people. It is simple to see why this policy cannot restore the CMA: after the transfer is implemented individuals have the correct amount of human capital but have "too much" disposable income when middle age and "too little" income when old, relative to the CMA. This leads them to an investment in physical capital which is, generally, too high. To be precise we compare a representative generation living in a world with incomplete markets where an "education financing only" policy of the kind just described is implemented with one living in a world with complete markets. To lighten notation we describe the case of a balanced growth path, but the argument applies in the general case. Consider a middle age person during an arbitrary period t, when the economy is growing at the factor $G = 1 + g$ and the return on capital is $R = 1 + r$. The public education policy requires a transfer of Gd from the middle age individuals to the young agents during the current period. Let the total labor income of the middle age agent be W. The endowment of the agent when middle age and old is $Z = [(W - Gd), 0]$, as reported in Figure 1. The budget constraints read: $c^m + s = W - Gd$ for this period and $c^o = Rs$ for the next. The agent chooses $s \geq 0$, yielding a consumption pair (c^m, c^o) on the intertemporal budget line $c^o = R(W - Gd - c^m)$, also reported in Figure 1. To help the intuition, let the utility function be separable and logarithmic, with discount factor δ. Then we have

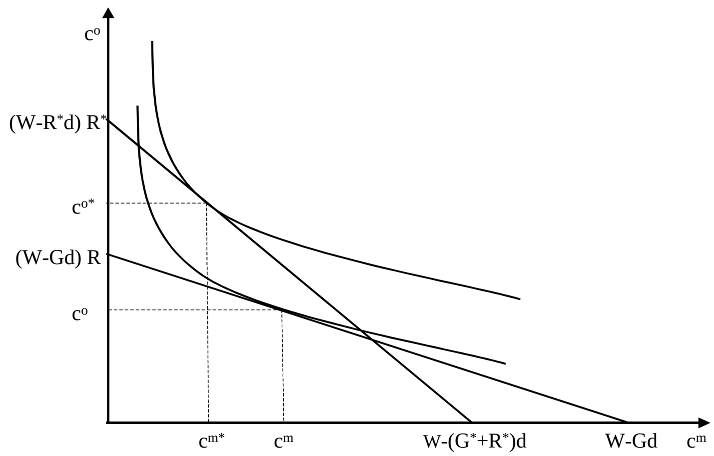
$$c^m = \frac{W - Gd}{1 + \delta}; \qquad c^o = R\delta \frac{W - Gd}{1 + \delta};$$

and

$$s = \delta \frac{W - Gd}{1 + \delta}.$$

Next, consider the same agent in a world with complete markets. That is, she borrowed d in the previous period and will have to pay back R^*d plus lend G^*d to the young in the current; in exchange she will receive a payment equal to R^*G^*d next period. Under this arrangement her endowment position is $Z^* = [(W - (G^* + R^*)d), R^*G^*d]$. Here and in what follows we use starred symbols to denote the CMA quantities. We show later that, indeed, $R^* \geq R$ and $G^* \leq G$, with strict inequalities holding in general. Given initial conditions (k, h) identical to the previous case, she picks s^*, i.e. physical capital for next period, yielding a consumption pair (c^{m*}, c^{o*}) on the intertemporal budget line $c^{o*} = R^*[W - (R^* + G^*)d - c^{m*}] + R^*G^*d$, $c^{m*} \leq W - (R^* + G^*)d$. This is also reported in Figure 1.

Figure 1



Again, in the case of logarithmic separable utility we have

$$c^{m*} = \frac{W - R^* d}{1+\delta}; \qquad c^{o*} = R^* \delta \frac{W - R^* d}{1+\delta};$$

and

$$s^* = \delta \frac{W - R^* d}{1+\delta} - G^* d.$$

Here, as in the general case, $s^* \leq s$ holds. This implies that $R^* \geq R$ and that $G^* \leq G$, because returns are decreasing, and the balanced growth rate is an increasing function of the investment rate. This shows that the two allocations are different. It also shows the circumstances under which the "education financing only" policy may fail to achieve efficiency. When returns on physical capital are decreasing, restriction $(2.2b)$, which assures an efficient allocation of aggregate investment between k and h, and which is automatically satisfied by s^*, cannot be met if $s > s^*$. Too much investment in physical capital is taking place and the conditions for static efficiency are violated. Hence, the allocation is not only different from the CMA one, it is also inefficient. This can be avoided by choosing

d_t in such a way that (2.2b) is satisfied in equilibrium. For given initial conditions (k, h), the argument just given implies that satisfying (2.2b) under the education-financing-only policy requires picking $d_t > d^*$. Decreasing returns then imply that $k_{t+1} = s_t$ would still exceed $k^*_{t+1} = s^*_t$ and $R < R^*$ would again hold. This has two implications. First, the new rate of return on capital may be so low that the dynamic efficiency condition is now violated; this may happen, in particular, because the new balanced growth rate is higher than the one that obtains in the CMA, and $R > G$ is required for dynamic efficiency. Second, even if the latter condition is verified and efficiency, both static and dynamic, obtains, the first few generations will be worse off under this policy than under the CMA.

To understand the sources of this intergenerational redistribution, we compare the lifetime utility of the first middle age generation under the two arrangements (CMA and efficient education financing) when its initial debt toward the old is zero. This guarantees that the utility of the first old generation is the same in the two settings. In the CMA environment, the endowment of the first middle age generation is $Z^* = (W - d^*, R^*d^*)$, while in the efficient education financing environment we have $Z = (W - d, 0)$. As we have shown, $d^* \leq d$ holds and Z^* strictly dominates Z. Further, $s > s^*$ and so $R^* > R$. Hence the set of feasible (c^m, c^o) is strictly larger in the CMA than in the efficient education financing environment, which implies that the generation of people that are middle age in period $t = 0$ is strictly worse off in the second environment. Depending on parameter values, this may be true for a (finite) number of generations after that. Consider now the general case in which the initial debt $d_{-1} > 0$. In this case, when d_{-1} is very large it is possible that the first old generation bears all the burden of the intergenerational transfer, while all other generations are better off under the education-financing-only policy. In any case, an intergenerational redistribution of welfare takes place as the education-financing-only policy always leads to "too much" investment and "too little" consumption for the first few generations. This induces a higher growth rate and, therefore, may benefit generations in the far future, as they enter life with a higher initial endowment of k and h than otherwise. But this occurs at the cost of reducing the welfare of the initial generations.

The solution to this overinvestment problem is simple: In each period, middle-age individuals must pay back their debt to the old people who, via the public education system, lent them the money in the first place. In such a way, old people will be collecting the amount $R_{t+1}(k_{t+1} + d_t)$ as in the CMA, and the incentive to overinvest in k_{t+1} will disappear. Notice that, while the $R_{t+1}k_{t+1}$ comes from privately issued financial securities, the portion $R_{t+1}d_t$ corresponds to an intergenerational transfer mediated by the government. A mechanism that taxes the working middle age and transfers the proceeds to the old retirees is needed. What is crucial, though, is that in this scheme the two intergenerational transfers are not independent but, instead, are tied together by a rate of return restriction.

3.1. Publicly Financed Education and Pay-As-You-Go Pensions

Consider the following scheme. In each period t, two lump-sum taxes are levied to finance two transfers. Both taxes are levied on the middle-age generation, and the proceeds are used to finance, respectively, pensions for the old and education for the young. We assume a period-by-period balanced budget. Write

$$T_t^p = P_t \tag{3.1}$$

for the pension scheme, and

$$T_t^e = E_t \tag{3.2}$$

for the education plan. The budget constraints for the representative member of the generation born in period $t-1$ become

$$0 \leq d_{t-1} \leq E_{t-1} \tag{3.3a}$$

$$c_t^m + s_t \leq w_t h_t - T_t^p - T_t^e \tag{3.3b}$$

$$c_{t+1}^o \leq R_{t+1} s_t + P_{t+1}. \tag{3.3c}$$

Comparison of equations (3.3) with the budget restrictions of problem (2.1) shows that, if the lump-sum amounts satisfy

$$E_t = d_t^*, \qquad\qquad P_t = d_{t-1}^* R_t^* \tag{3.4}$$

the competitive equilibrium under the new policy achieves the CMA. A benevolent planner can restore efficiency, improve long-run growth rates, and preserve intergenerational fairness by establishing publicly financed education *and* pay-as-you-go pensions simultaneously, and by linking the two flows of payments via the market interest rate.[2]

Efficiency properties aside, a public education and public pension scheme (PEPP) satisfying restrictions (3.1), (3.2), and (3.4) would also be actuarially fair in the following sense. The pension payment (contribution) that a typical citizen receives (pays) during the third (second) period of life corresponds to the capitalized value of the education taxes (transfers) the citizen contributed (received) during the second (first) period of life. These quantities are capitalized at the appropriate market rate of interest:

$$E_t R_{t+1}^* = T_{t+1}^p \tag{3.5a}$$

$$T_t^e R_{t+1}^* = P_{t+1}. \tag{3.5b}$$

[2] Introducing individual heterogeneity and income uncertainty complicates but does not alter these conclusions. See Boldrin and Montes (2003a) for details.

In the applied literature on contribution-based social security systems, the issue of actuarial fairness between contributions paid and pensions received is an actively debated topic. Our model suggests that we should look for actuarial fairness somewhere else, that is, between contributions paid and the amount of public financing for education received on the one hand, and between taxes devoted to human capital accumulation and pension payments on the other.

3.2. Distortionary Taxation

We have assumed so far that the benevolent planner has access to lump-sum instruments of taxation. This is seldom the case. In this subsection, we take a brief look at the case of linear income taxes. Again, we assume that the period-by-period budget constraint must be satisfied and ask if the CMA can be implemented as a competitive equilibrium with linear income taxes. The novel result presented here is that, in fairly general circumstances, taxing the purchases of physical capital to finance education while also subsidizing the return from physical capital is the way to support the CMA.

The case in which only labor income can be taxed is easy. While, in the absence of credit markets for education, it may still be beneficial to introduce a PEPP system, there is no reason to expect that the CMA will be supported as a competitive equilibrium in such circumstances. The tax on labor income reduces the rate of return on human capital investment and distorts the borrowing/lending decisions of both young and middle-age individuals.

The fact that the CMA cannot be achieved by taxing labor income suggests considering other forms of taxation. Notice that, in our model, the quantities $T_t^e = E_t$ are effectively lump-sum. Young agents cannot do anything but acquire human capital, and middle age people supply their work inelastically. In reality, the assumption about young people is true only until the age of mandatory schooling, and the presence of a tax, even if it were lump-sum, would affect the labor supply decisions of the middle age, beside creating redistributional issues. This is not the appropriate place to get into a detailed discussion of how, in practice, one should design a non-regressive education financing scheme minimizing distortions. Still, it does not seem impossible to replicate at least the lending side of the CMA scheme: a governmental agency can stand ready to lend, at the going market rate, to all those individuals that satisfy a set of standardized requisites (previous education, age, school performances) and intend to attend further schooling. The quantity $P_t = T_t^p$ is harder to treat as a lump-sum amount; pension payments do affect retirement decisions and, as mentioned, lump-sum income taxation brings about complicated redistributional issues. Still, we fail to see how the scheme considered here would be more distortionary than current ones.

Notice that the reason we need to collect T_t^e from middle-age individuals is that they are unable to invest in human capital. Once T_t^e is taxed away, we want to pay a pension to the old agents because otherwise their return from physical capital is too little. This suggests that the following, somewhat unusual, scheme may work in practice.

Proposition 2. *Under our assumptions, when markets for financing human capital accumulation are absent, the following tax-and-transfer scheme restores the Complete Market Allocation: The planner taxes purchases of physical capital in period t (to bring $s_t = k_{t+1}$ to the CMA level) and subsidizes the return from physical capital in period $t+1$ (to increase third-period income to the CMA level).*

We provide a simple algebraic proof of the proposition. To avoid confusion with notation, denote with $\hat{s}_t$ the saving (consisting only of purchases of physical capital) that obtains in a competitive equilibrium with the proposed tax and subsidy scheme. Recall that, if the CMA is achieved, $\hat{s}_t = k^*_{t+1}$, where starred symbols still denote CMA quantities and prices. The government budget constraint requires $\tau_t \geq 0$ to satisfy

$$\tau_t \hat{s}_t = \tau_t k^*_{t+1} = E^*_t = d^*_t.$$

The household budget constraints become

$$0 \leq d_{t-1} \leq E^*_{t-1} \tag{3.6a}$$

$$c^m_t + (1+\tau_t)\hat{s}_t \leq w_t h_t - T^p_t \tag{3.6b}$$

$$c^o_{t+1} \leq R_{t+1}(1+\tau_t)\hat{s}_t. \tag{3.6c}$$

The first-order condition determining d_{t-1} is identical to (2.2b). The condition determining $\hat{s}_t$ becomes

$$u'\left[w_t h_t - (1+\tau_t)\hat{s}_t - T^p_t\right](1+\tau_t) = \delta R_{t+1} u'\left[(1+\tau_t)\hat{s}_t R_{t+1}\right](1+\tau_t). \tag{3.7}$$

Cancelling $(1+\tau_t)$ on both sides, replacing the lump-sum value T^p_t with $R_t d_{t-1}$, and setting $\tau_t = d^*_t / k^*_{t+1}$ yields (2.2a), which has the unique solution $\hat{s}_t = k^*_{t+1}$, as desired. Further,

$$\tau_t \hat{s}_t = d^*_t \qquad\qquad R_{t+1}\tau_t \hat{s}_t = R_{t+1} d^*_t$$

which corresponds to the CMA's investment in, and return from, human capital assets.

For the simple economy considered in our Example, the choice of a constant $\tau_t = \gamma$ suffices to implement the CMA along any path. In general, a constant tax rate suffices along a balanced growth path when the production functions are linearly homogeneous. For production functions that are not linearly homogeneous or outside the balanced growth path, the tax rate cannot be constant because the composition d_t^*/k_{t+1}^* of the CMA investment portfolio is neither in those circumstances.

3.3. Using Debt

We note finally that a third, in our view more compelling, implementation of our transfer scheme is possible. In this interpretation, the government issues one-period debt in the amount d_t in each period. Given the demographic structure assumed, this debt will be purchased only by the middle-age individuals. The resources collected are used to finance education for the young. In the following period, the government pays back $R_{t+1}d_t$ to the now old debt holders. Such repayment is financed by an income tax on the middle-age individuals. This tax should be proportional to the past usage of public financing for education.

This scheme is not exempt from the distortionary effects of labor income taxation. We should stress, though, that (even in an environment with heterogeneous individuals and income uncertainty) the amount $R_{t+1}d_t = T_{t+1}^p$ can be usefully broken down into two parts. The first, and likely larger, portion should be proportional to previous individual borrowing for education, and therefore lump-sum. A second portion may have to be collected for intragenerational insurance or redistributive purposes. It is only this portion of $R_{t+1}d_t$ which, being proportional in nature, distorts labor supply. To the extent that current social security contributions achieve some degree of redistribution or intragenerational risk-sharing, the suggested scheme cannot do worse in terms of economic efficiency.

3.4. The Model and the Real World

We view our analysis as essentially normative. When private competitive markets for financing education are not available, a properly designed PEPP scheme may restore the efficient CMA as a competitive equilibrium. Further, our analysis also shows that the distance between actual and efficient allocations, at least along this specific dimension, can be measured by looking at the difference between some implicit rates of return, which can be measured in the data, and the market rate of return. More formally, our hurler analysis has proved that

Proposition 3. *If the set of intergenerational transfers induced by the public education and the public pension systems support the CMA, the following should be observed. For a given generation, the implicit rate of return i_t which, along the life cycle, equalizes the discounted values of education services received and social security contributions paid, is equal to the market rate of interest r_t. Similarly, the implicit rate of return π_t that, along the life cycle, equalizes the discounted values of education taxes paid and pension payments received, is also equal to the market rate of interest r_t.*

As reality is seldom, if ever, fully efficient, it becomes relevant to ask how much "off the mark" current intergenerational arrangements are. The pair of numbers $|\pi_t - r_t|$ and $|i_t - r_t|$ is a reasonable way of measuring such distance. Should reality turn out to be not far from what we have shown to be the efficient allocation, it would become an interesting topic of research to ask how existing political mechanisms implement allocations that satisfy the Pareto criterion. Should reality turn out to be far from the efficient allocation, then it becomes relevant to ask how one should proceed to bring it closer.

These considerations lead us to entertain, albeit briefly, a positive reading of our model. In the real world benevolent planners are probably harder to come across than credit instruments for financing education. A priori, there are very few reasons to expect that existing public education and pension systems should strive to replicate the CMA and achieve the efficiency gains we have outlined here.

As a matter of fact, in none of the countries we are aware of is the welfare state legislation explicitly organized around the principles advocated in this paper. In general, social security contributions are levied as a percentage of labor income and bear no clear relation to the previous use of public education. Pension benefits received are related, in one form or another, to past social security contributions but never to some measure of lifetime contributions to aggregate human capital accumulation. Still, there are intuitive reasons to believe that intergenerational transfers that are either grossly inefficient or openly unfair (in the sense that some generations collect rates of return systematically higher than those of other generations) would be subject to strong public pressure to be either dismantled or improved upon. This is the intuition set forth by Becker and Murphy (1988) and which is captured in our model by conditions (3.9). In particular, as those equations show, both fairness and replication of the CMA are summarized by a simple present-value calculation that uses the market rate of return as a yardstick.

Further, in a recursive environment in which the middle-age generation decides whether and how to implement a PEPP system, an equilibrium satisfying (3.5) may arise. In previous versions of this paper, we present a dynamic game of generational voting, along the lines of Boldrin and Rustichini (2000), which possesses a subgame perfect equilibrium implementing the CMA. We refer the interested reader to Boldrin and Montes (2003b) for this result, a discussion of the circumstances under which the political equilibrium implementing the CMA is the unique subgame perfect and, finally, for extensions to other notions of recursive equilibrium, and to more general OLG environments. Results along the same lines have been derived independently by Rangel (1999) and, to a smaller degree, by Bellettini and Berti Cerroni (1999). All of the above reasons converge to make an examination of the data worthy of our time. This we do, using Spanish data, in the next section.

4. The Spanish Case

In this section, we use Spanish data to compute the values of i and π faced by Spanish citizens under the rules in place and the taxes and transfers implemented in 1990–91. To carry out our computations, the stationarity assumptions made in the model are first taken verbatim and then relaxed as we move along. We proceed in three stages. In the first, we abstract from demographic change and economic growth. We will show that, as long as growth takes place at a constant rate, it makes only a quantitative but not a qualitative difference in the results. In the second stage, we incorporate the forecasted demographic evolution for the period 1990–2089 and consider a number of reasonable policy scenarios. In the third, we use the same demographic predictions to evaluate the quantitative impact that economic growth at a constant rate would have on the implicit rates of return faced by different generations.

More specifically, in our empirical exercise we assume that the rules of the Spanish public education and public pension systems will not be changed for the very long future and that all individuals currently alive have also lived under those same rules in the past. This is obviously false, because both education and pension systems underwent large and frequent changes in the period 1960–85. In 1985 the pension system was reformed once more, and since then, it has kept its basic rules. The same goes for the public education system, which achieved its current structure in the early 1980s and has not changed much since. Hence, while our assumption of stationarity is only an approximation to reality, it is a good approximation for the last 20 years, and it appears to be a reasonable one for the foreseeable future.

In the first stage, we assume that the aggregate burden of taxation and its age distribution have not varied and will not vary over the lifetime of the individuals alive in 1990–91. In the second and third stages, we let aggregate public expenditure change according to specific scenarios. As for income, in the first and second stages we assume it remains constant, for each age group, over the simulation horizon. In the third, we let age-specific per capita income grow at a constant rate and adjust aggregate taxation accordingly, under the assumption of a constant age distribution of taxes and transfers. Notice that, if it were not for the changing demographic structure, this would imply constant tax and transfer rates for each age group and function. Finally, in all of our simulations we make the assumption that, for each function, the yearly budget is balanced.

We have decided to ignore deficit financing and the generational burden of public debt for a variety of reasons. First, the Spanish public sector deficit has varied a lot during the last 15 years and was much higher in the early 1990s than it is now. In fact, partly because of the European Monetary Union implementation, the fiscal deficit has decreased steadily since 1994, reaching very low values in the last five years. The same applies to the social security administration budget, which is often manipulated by changing accounting criteria and has generated a surplus since 1997. Secondly, we do not have a reliable method to allocate the debt burden over different age groups either for the last 10 years or for the future. The intergenerational distribution of the debt burden remains, nevertheless, an important issue to be addressed. It requires an explicit model of stochastic demographic change and of optimal fiscal policy. A first step in this direction is taken in Boldrin and Montes (2003a).

Ours are, indeed, relatively strong assumptions. Stationarity and balanced growth assumptions are often made in most empirical applications of dynamic models, and our case makes no exception. Given the available micro data, we find our approach to be a reasonable starting point.

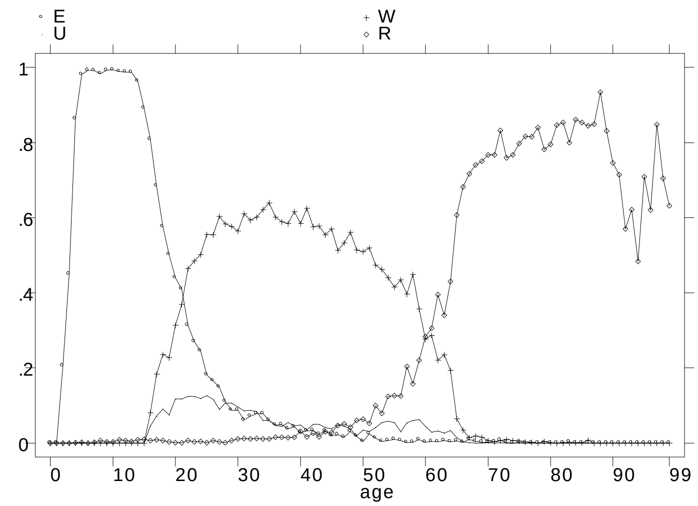
4.1. Data[3]

To compute the implicit rates i and π, we use several kinds of micro and macro data. The choice of the reference year is dictated by the availability of information about individual behavior along the life cycle. At present, there is only one reliable source of microeconomic observations of the allocation of personal time between school, work, and retirement at various stages of the life cycle. This is available only for 1980–81 and 1990–91 via the Spanish household budget survey (*Encuesta de Presupuestos Familiares*, or EPF). We have used the 1990–91 EPF because the Spanish public pension system underwent a major reform in 1985 and because the 1980–81 EPF contains only a severely limited subset of the information we need.

For each individual in the sample, conditional upon age and occupational status, the information in the EPF allows us to estimate (1) the amount and value of public educational services received, (2) the amount of direct and indirect taxes paid, (3) the amount of pension contributions paid, and (4) the amount of public contributive pensions received. The information in the EPF also affords the computation of the share of the population which, at each age, is studying, working, unemployed, or retired. Such lifetime distribution of activities is reported, in percentage terms and for each age group, in Figure 2. Together with quantities (1)–(4), it allows us to compute the implicit rates of return.

[3]Further details about the data sets we use are in the Appendix and in Montes (1998).

Figure 2: Life-time distribution among activities.



E=student, W=worker, U=unemployed, R=retired.

1-E-W-U-R=inactive, not reported.

4.1.1. The base case

Consider an individual living for a maximum of A periods, and let p_a denote the (conditional) probability of survival between age a and $a+1$. Denote with i the interest rate at which young people "borrow" through public education and with π the rate of return old people receive from their "investment" in public education. For a given sequence of taxes and transfers, the rates i and π (time invariant, because of the stationarity assumptions) are defined implicitly by

$$\sum_{a=1}^{A} \left(\Pi_{j=1}^{a} p_j \cdot \Pi_{j=a}^{A} \left(1+i_j\right)\right) \left[E_a - T_a^p\right] = 0 \tag{4.1a}$$

$$\sum_{a=1}^{A} \left(\Pi_{j=1}^{a} p_j \cdot \Pi_{j=a}^{A} \left(1+\pi_j\right)\right) \left[T_a^e - P_a\right] = 0. \tag{4.1b}$$

The representative agent for our base case is defined by the following assumptions:

(a) At age $a = 1, \ldots, 98$, the probability p_a of being alive at age $a + 1$ is the one reported by the *Instituto Nacional de Estadística* for that age group in 1990. The EPF does not contain any individual older than 99.

(b) At each age $a = 1, \ldots, 99$, the representative individual is working, studying, unemployed, or retired with a probability equal to the frequency of that activity in the EPF sample of people of age a.

(c) At each age $a = 1, \ldots, 99$, an individual receives or pays transfers and taxes equal to the average, in the EPF, for those individuals that at age a were in the same occupational status.

Assumptions (a)–(c) can be used to extract from the EPF the amounts E_a, P_a, T_a^e, and T_a^p that an individual of age a would pay or receive. Such estimation uses the age- and status-specific information contained in the EPF, according to assumptions (b) and (c). Let X be a stand-in for any of the four quantities. For each $a = 1, 2, \ldots, 99$, we use population data to compute the amounts X_a attributable to the representative individual of that age. Let L_a be the number of individuals of age a in the Spanish population in 1990 (INE (1991)). A four-tuple of weights x_a can be computed by setting

$$x_a = \frac{X_a}{\sum_{a=1}^{A} X_a L_a}.$$

Write this four-tuple of x_a as $[\alpha_a, \beta_a, \gamma_a, \delta_a]$. The terms denote, respectively, the share of total T^e and T^p paid and the share of total P and E received (according to the EPF) by the representative individual of age a.

Next, from the government and social security administration budgets for 1990, we compute the quantities X^{90} corresponding to the effective total tax or transfer relative to each function. We allocate these amounts over the life cycle of the representative agent by means of the weights x_a. The lifetime distribution of these four flows, in thousands of 1990 pesetas, is reported in Figure 3.

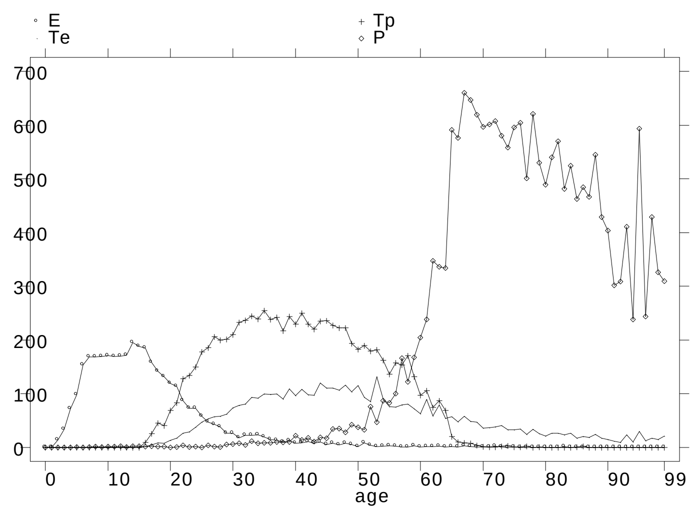
Equations (4.1) become

$$\sum_{a=1}^{99} \left(\Pi_{j=1}^{a} p_j\right) (1+i)^{99-a} \left[\delta_a \cdot E^{90} - \beta_a \cdot T^{90,p}\right] = 0 \tag{4.2a}$$

$$\sum_{a=1}^{99} \left(\Pi_{j=1}^{a} p_j\right) (1+\pi)^{99-a} \left[\alpha_a \cdot T^{90,e} - \gamma_a \cdot P^{90}\right] = 0. \tag{4.2b}$$

Notice in passing that, had we assumed a constant annual growth rate of $g > 0$ for both taxes and transfers, equations (4.2) would be modified by multiplying each annual entry by a factor of $(1+g)^a$. Dividing through by $(1+g)^{99}$ and replacing $(1+i)$ and $(1+\pi)$ by $(1+i)/(1+g)$ and $(1+\pi)/(1+g)$, respectively, leads back to (4.2). This implies that adding a constant growth rate changes the quantitative but not the qualitative conclusions of our exercise. We come back to this point at the end of the section, when we quantify the joint impact of economic growth and demographic change.

Figure 3: Life-time distribution of tax and transfer flows.



We solve expressions (4.2) numerically. Our point estimate of the implicit rate of return on education investment is

$$\pi = 4.238\%.$$

Our point estimate of the implicit rate of interest at which young people borrow is more ambiguous. It depends upon the convention with which we handle the yearly surpluses and deficits of the various Spanish social security administrations. In 1990, the social security administration for workers in the private sector (INSS) realized a surplus of pension contributions over pension outlays[4], while the social security administration for public employees (RCP) realized a deficit. The latter was covered by a transfer of funds from the general government budget. Our model assumes a year-by-year balanced budget. One possibility is to get rid of both the INSS surplus and the RCP deficit by assuming that the total amount of social security contributions was, in fact, equal to the public contributive pension payments made in that year (P^{90}). In this case, our point estimate is

$$i_1 = 3.6307\%.$$

A second possibility is to use the actual social security contributions paid to INSS and RCP in 1990 ($T^{90,p}$). In this case, we have

$$i_2 = 3.772\%.$$

Finally, a third alternative is to add to the total contributions paid in 1990 ($T^{90,p}$) the amount transferred from the general government budget to cover the

[4] The INSS is divided further into six different funds, some of which exhibited a deficit and others a surplus during the same year. Our micro data do not allow us to consider this finer partition.

RCP deficit. With adoption of this wider definition, the implicit rate of interest is computed to be

$$i_3 = 4.2601\%.$$

We believe that i_3 is a better estimate of the true implicit rate because the RCP "deficit" is a misnomer attributable to accounting practices. The government portion of the social security contributions for its employees is highly forecastable and, de facto, is always recorded as a transfer from the general budget to cover the RCP deficit. In other words, the transfers included in the computation of i_3 are functionally equivalent to the employers' contributions paid by private sector firms to the INSS. To us, this means they are part of the gross labor income of public employees and, for this reason, should be treated as part of the total social security contributions they pay.

4.2. Accounting for Changes in Mortality Rates

In our definition of the Spanish representative individual, we have used the mortality rates reported by the INE for 1990 (assumption (a) above). This is not very reasonable, because those rates were computed using observations prior to 1990; survival probabilities have changed greatly since then and are still changing. To correct for this, we run the base case simulation using the updated survival probabilities implicit in the Fernández Cordón (2000) demographic forecasts. The new estimates are $i = 4.4565\%$ (from now on, only i_3 will be reported) and $\pi = 4.7853\%$, reversing the order of our previous findings. As should be expected, public pensions are a good deal at lower mortality rates.

The population forecasts of Fernández Cordón (2000) are obtained by simultaneously using new mortality rates and expected immigration flows. Unfortunately, the two sources of change cannot be disentangled, so that for certain age groups the estimated probability of survival is slightly higher than one. Immigrants are concentrated in the 20 to 40 age groups so, if we force all probabilities of survival to be bounded above by one, we get $\pi = 4.859\%$ and $i = 4.388\%$, making pensions an even better deal.

4.2.1. Impact of nonstationarity

To provide a fuller account of the impact that demographic change and economic growth may have on i and π, we simulated a number of alternative scenarios, which are illustrated next.

We begin with the impact of expected demographic change. To do this, we replace the assumption of demographic stationarity with the projections of Fernández Cordón (2000), which take into account variations in both mortality and fertility rates. Removing demographic stationarity makes i and π dependent upon the date of birth. Each cohort faces different rates (i_a, π_a), depending on its age $a = 1, 2, \ldots, A$ in the first period. Here we report results for only two particular cohorts that were alive in 1990: those just born and those aged 16 (the latter corresponds to the minimum working age in Spain). Results for other cohorts are available upon request.

4.3. Impact of Demographic Change

Changing the demographic structure while keeping the balanced budget requirement satisfied in each period requires making assumptions on the policies for distributing taxes and transfers across individuals belonging to different generations. In particular, we need to make an assumption as to which features of the PEPP system that were observable in 1990 will be maintained in future periods. Many scenarios are conceivable. We selected four, which we consider most likely or, at least, most informative. In Scenario A, we assume that age-specific, per capita expenditure in education and pensions will remain at their 1990 level, in real terms. Symmetrically, in Scenario B, we assume that age-specific, per capita education taxes and social security contributions will remain at their 1990 values. In Scenario C, we take as fixed the borrowing rate realized in 1990, and apply it to the generation born in that year. Finally, in Scenario D we consider the case in which the lending rate is kept at its base case level for the generation born in 1990. In all scenarios, for the generation aged 16, we use 1990 age-specific quantities for the amounts that generation paid or received in the sixteen years between its year of birth and 1990.

The careful reader will notice that, for each scenario, the constant policy adopted plus year-by-year balanced budgets are not enough, for given demographics, to determine all remaining variables. Consider, for example, Scenario A. Using per capita expenditure in education and demographic data, we can compute total education expenditure E_t in each future year. The balanced budget constraint implies that $E_t = T_t^e$, for all t. This determines the total education tax to be levied in each year, but leaves its distribution, across generations, still open. The same is true for the distribution of T_t^p across generations in Scenario A, and for other quantities in the other scenarios. To address this problem, we proceed as follows.

For each of the four scenarios, let X_a be the amount paid or received by the representative individual of age a according to the 1990 data. For each $x = E, T^e, P, T^p$ and age $a, a' = 1, 2, \dots, A$, define the constants

$$k^x(a, a') = \frac{X_a}{X_{a'}}.$$

Then, for all future years and in all four scenarios we assume that, for each function x whose distribution over cohorts is to be determined endogenously, the payments from or transfers to the average individuals of age a and a' will yield the same $k^x(a, a')$ as in 1990. In other words, we assume that, while a certain policy may favor or hurt a given cohort over its entire lifetime, it will not do so by charging different taxes to individuals of different ages in any given year. The same is true for transfers.

Scenario A

We set real per capita expenditure E_a and P_a at the 1990 level, for all a. The demographic projections allow the computation of aggregate expenditures, E_t and P_t, for each year $t = 1990, \ldots, 2089$. We use a balanced budget in each year to compute T_t^e and T_t^p. We use the assumption of constant $k^{T^e}(a, a')$ and $k^{T^p}(a, a')$, together with demographic data, to compute the distribution of taxes across individuals in each year. Given this, we compute the rates of return. For the generation born in 1990, we obtain $i = 6.0643\%$ and $\pi = 6.8918\%$, while we have $i = 4.7620\%$ and $\pi = 6.3121\%$ for the generation aged 16 in 1990.

Scenario B

Here we fix real per capita taxation T_a^e and T_a^p at the 1990 level, for all a. Then we proceed as in Scenario A, using the assumption of constant $k^E(a, a')$ and $k^P(a, a')$ to compute the yearly distribution of E_t and P_t across individuals of different ages. For the generation born in 1990, this policy gives $i = 3.0018\%$ and $\pi = 2.2021\%$, while we have $i = 4.2322\%$ and $\pi = 2.1912\%$ for the generation aged 16 in 1990.

Scenario C

In this case, we take the borrowing rate $i = 4.4565\%$ for the generation born in 1990 as given. On the basis of the 1990 data, we fix the per capita expenditure in E for each age group. We use this per capita expenditure to project total lifetime transfers to each generation alive in 1990. This gives us the total education expenditure, E_t, in each fiscal year between 1990 and 2089. Next, we use 1990 per capita social security contributions (for each age group) to compute how much will be available to pay pensions, T_t^p, during each fiscal year between 1990 and 2089. Notice that, by doing this, we guarantee that the generation born in 1990 will pay $i = 4.4565\%$, as the representative individual in the base case. Finally, we use the yearly balanced budget restrictions together with the assumption of time-invariant $k^{T^e}(a, a')$ and $k^P(a, a')$ to determine endogenously the amount of taxes $T_t^{e,a}$ paid and pensions P_t^a received by an individual of age a in year t. For the generation born in 1990, this yields a lending rate of $\pi = 4.3843\%$, while the cohort aged 16 in 1990 faces implicit rates equal to 4.3843% and 3.9804%, respectively.

Scenario D

This case takes as given the lending rate $\pi = 4.7853\%$ for the generation born in 1990. Again, we start from the 1990 data for real per capita T^e and P^a and use demographic projections to compute future E_t and T_t^p. The yearly balanced budget restrictions together with the constants $k^E(a, a')$ and $k^{T^p}(a, a')$ determine the other two flows. For the generation born in 1990, this yields a borrowing rate of $i = 4.7497\%$, while the cohort aged 16 in 1990 faces implicit rates of $i = 4.6312\%$ and $\pi = 4.7323\%$ in this scenario.

Different policies, apparently, make a big difference when it comes to intergenerational distribution of resources, and economic efficiency as well. Current Spanish policies induce an allocation which is fairly close, in fact, surprisingly close, to the efficient and intergenerational fair one. Future policies will need to implement substantial adjustments to maintain efficiency and intergenerational fairness in the face of forthcoming demographic changes.

4.4. Impact of Economic Growth

After equations (4.2), we pointed out that the inclusion of a constant growth rate would not change the qualitative conclusions. In each of the four scenarios considered, introducing a constant growth rate g implies that $(1+g)(1+i)$ and $(1+g)(1+\pi)$ would be the new rates, where i and π are the numbers we just reported for each scenarios. Hence, for example, in the base case and accounting for recent mortality rates, a balanced growth rate of about 3% (which is pretty close to the historical experience of the last 15 years) would yield a borrowing rate of $i^* = 7.51\%$ and a lending rate of $\pi^* = 8.00\%$. Adjusting for growth does not change the qualitative conclusions, while making the comparison to historical rates of return on capital more meaningful.[5]

5. Extensions and Relations with Earlier Literature

In our model individuals are identical, live for three periods, face no uncertainty, are completely selfish, and supply their labor time inelastically. These are restrictive assumptions. Let us consider briefly how much they affect the central results.

Heterogeneity would introduce distributional considerations and, in the presence of uncertainty, insurance problems. The latter we consider when addressing uncertainty. Distributional considerations are certainly relevant to understanding actual education and pension systems but cannot alter our results. Aggregate efficiency conditions would not change but would have to be supplemented with individual efficiency conditions. The latter, though, would be of the same type as (2.2) and our conclusions would still apply in the aggregate.

[5] We thank Tim Kehoe for pointing this out to us.

Many periods would only complicate notation, but add no additional insight. As the previous section shows, our findings can be automatically applied to individuals living for one hundred periods without any relevant alteration.

Uncertainty in the rates of return of human and physical capital would enhance and strengthen the results. This is because, in general, one expects some degree of correlation between the two kinds of investment, in which case efficient portfolio considerations should also play a role in the analysis. This is, indeed, the point made originally by Merton (1983) in a static model, and which applies almost verbatim to our dynamic context. Uncertainty, in particular, strengthens our assertion (propositions 1 and 2) that it takes both a public education and a public pension system to restore efficiency, at least in the general case. The assertion is strengthened because the optimal retirement portfolio must include, in general, both physical and human capital. A pension system built along our guidelines satisfies exactly this requirement and, therefore, "completes the markets" when returns are uncertain.

Altruistic parents are a delicate issue, which is often the source of confusion. Naturally, if parents were "fully altruistic" in the sense of Barro and Becker (1989), lack of credit markets for education would not really matter for efficiency. In this case, it is well known that the OLG economy turns into one with infinitely lived agents, and optimality of competitive equilibrium is restored. More generally, any degree of parental altruism leading to a partial internalization of the utility of future generations would reduce, but not eliminate, the inefficiency of competitive equilibrium when credit markets for education are absent. Still, we insist, as long as this internalization is not complete our qualitative arguments apply, and a PEPP system like the one suggested here would move the economy closer to efficiency. This is particularly true for the kind of incomplete altruism which is often assumed in the literature, in which parents derive direct utility from the human capital of their children. In this case, the children's direct gains from human capital accumulation are not taken into consideration by parents when making their investment. More generally, any kind of altruism, other than full altruism as defined before, is likely to fail to satisfy the efficiency conditions (2.2), thereby leaving room for Pareto-improving transfers. Such transfers, obviously, would be quantitatively, but not qualitatively, different from those we present in the earlier sections.

Endogenous labor supply is also a complicated matter. On the one hand, when labor supply is endogenous, lack of credit markets for education leads to a suboptimal labor supply in every period, with either too much or too little labor being supplied, depending on the specific circumstances. On the other hand, when labor supply is endogenous, taxes and transfers that are not lump-sum introduce other, different, kinds of distortions. In this case, only a second best allocation is achievable and the whole analysis would have to be modified according to the specific context adopted. It seems to us, though, that our basic message should still be valid and that a properly designed PEPP system would still be part of the ingredients leading to a second best allocation.

We already discussed in the introduction that portion of the previous literature which we consider, either in spirit or in the actual findings, closest to ours. Among those not yet mentioned, the literature on Generational Accounting (GA, see, for example, Auerbach, Kotlikoff, and Leibfritz (1999) and references to earlier works therein) is the most relevant. Traditionally, works in the GA tradition used to treat education as government consumption, leaving aside its role as an investment/transfer favoring the young generations. More recent works have changed this assumption and started treating public education as a transfer toward the young. This modification is, in our view, quite appropriate and has led to empirical findings that go in a direction similar to ours, i.e. a lower burden of taxation on the young and future generations. A crucial distinction between our approach and the GA one is that we use a theoretical model to study the effect of a specific missing market, and restrict our analysis only to those policies that may alleviate the inefficiencies associated to the particular missing market, while the GA approach has the ambition of taking into account the whole set of public taxes and transfers without asking, though, which among those taxes/transfers are meant (at least in principle) to alleviate some market failure, and which are purely redistributive or a plain waste. In this sense our theoretical approach is complementary and not alternative to the GA one, and it may lead to a clearer theoretical justification of the empirical estimations obtained with the GA methodology. When looking at the whole collection of public policies and associated taxes, though, it remains a daunting task to model appropriately the "missing markets" these policies are supposed to take care of. The case for education and pensions is, in our view, much clearer and well defined than that for most other welfare policies.

Finally, we should compare our results to those of Cremer, Kessler, and Pestieau (1992). They also consider education and pensions as tools to alleviate inefficiency when altruism is absent in a world without production. In their cases, though, investment decisions are taken by parents on behalf of their children, which leads to conditions for efficiency which are different from ours. In particular, in their analysis efficiency fails due to a lack of coordination between contiguous generations and not because of the missing credit markets, hence public education alone is enough to restore efficiency. Because they use an exchange economy, the issue associated to capital accumulation, growth, efficient allocation of savings, and optimal retirement portfolio, which are central to our analysis, cannot be studied.

6. Conclusions

We have studied a three-period overlapping generations model with production and accumulation of physical and human capital. When the young generation cannot borrow to finance investment in human capital, the competitive equilibrium outcome does not satisfy either static or dynamic efficiency, and the aggregate growth rate of output and consumption is lower than under the complete market allocation. We have shown that a simple intergenerational transfer agreement could eliminate this problem and induce an efficient allocation.

The intergenerational transfer agreement we study is inspired by the argument advanced in Becker and Murphy (1988). Accordingly, we interpret public financing for education as a loan from the middle-age to the young generation. The latter uses this loan to finance its accumulation of human capital. Symmetrically, the pay-as-you-go public pension system can be seen as a way for the former borrowers to repay the capitalized value of their education debt to the previous generation. In this interpretation, the two institutions of the welfare state, public education and public pensions, support each other and achieve a more efficient allocation of resources over time.

There are important normative implications of this analysis. Our model suggests that utilization of either public or publicly financed education should be treated as accumulation of debt toward the older generations. Such debt, capitalized at the market rate of interest, should be paid back, during one's working life, by means of a tax levied upon labor income. Repayment of the education debt can be achieved by means of a voluntary mortgage plan or by means of a compulsory tax. Either choice has some obvious incentive and redistributive implications, which are, nevertheless, not dissimilar from those faced by current arrangements for financing public education. On the side of retirement pensions, the model requires earmarking some tax (paid by individuals) as a source of public financing of education and to capitalize at the market rate of interest the amounts paid by each single citizen. The capital so accumulated should then be paid out, in the form of annuities, to the same citizen once retirement age is reached. This is our main theoretical and normative finding. It suggests that public education financing and a properly redesigned public pension system could be useful tools to enhance economic efficiency and long run welfare.

While a benevolent planner could easily implement such a system of lump-sum taxes and transfers, it is not obvious that a benevolent planner is behind the design of modern welfare state institutions. Hence, it is worth investigating if existing systems are or are not far from the quantitative prescriptions of our normative model. We do so by computing the "borrowing" and "lending" rates implicit in the Spanish public education and public pension systems. We use both microeconomic and aggregate data for 1990–91. The model predicts that, at the CMA allocation, the borrowing and lending rates should equal each other and be equal, in turn, to the rate of return on capital. For the baseline case, our point estimates of borrowing and lending rates are relatively close to 4.0%, which corresponds to the risk-free real rate of return on Spanish Treasury bonds during the last 15 years or so. This optimistic finding, though, is based on the assumptions of demographic and policy stationarity.

Once the assumption of demographic stationarity is replaced by realistic projections of the future evolution of the Spanish population, results change dramatically. We carry out four simulations based on such projections, each scenario characterized by different assumptions about the form in which public policy may react to the demographic change. While the policies we consider are hypothetical, common sense suggests that they are a reasonable starting point for this kind of analysis. In each of the four cases considered, the implicit rates we estimate move apart from each other. In particular, unless they are held fixed by the assumptions underlying the policy scenario being considered, pensions tend to yield a rate of return (on the previous education investment) higher than the rate of interest the working cohorts are expected to pay (via social security contributions) on the education services they received.

A second finding is that the rates of interest paid by or accrued to generations born in different years move apart from each other when the demographic evolution is taken into account. Nevertheless, and contrary to a widespread presumption, such movements are not monotone; in particular, they do not seem to necessarily favor the older relative to the younger generations. In other words, rebus sic stantibus, the expected demographic evolution should not necessarily lead to a huge redistribution of resources away from the younger or not-yet-born generations and toward the older ones. Most previous findings, based on the generational accounting methodology pioneered by Auerbach and Kotlikoff (see, for example, Auerbach, Kotlikoff, and Leibfritz (1999)), have instead shown that the interaction between demographic change and current fiscal policies (in particular, current welfare policies) is likely to engender a large intergenerational redistribution in favor of the older cohorts. While our findings cannot rule out this conclusion and, in fact, lend support to it under certain policy scenarios, we believe our estimates have independent value and should shed some additional light on the intricacies of intergenerational public policy.

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Appendix: Data Sources and Treatment

A.1 Data sources

Our sources of data are the following.

We obtain the aggregate expenditure on public education from the *Estadística del Gasto Público en Educación* (EGPE 1995, in Ministerio de Educación y Ciencia (1995)) and the *Encuesta sobre Financiación y Gasto de la Enseñanza Privada* (EFGEP 1990–91, in INE (1992b)). The first database contains public expenditure for each schooling level; the second reports the amount of public funding going to private schools (*centros concertados*). Aggregate tax revenues are obtained from the *Cuentas de las Administraciones Públicas* (IGAE (1991b)). From this we extract the share of total tax revenues allocated to financing public expenditure on education, excluding the fraction covered with public debt. We assume that the fraction of public expenditure covered by debt financing is equal to the average share of public expenditure financed by debt during 1990–91.

Aggregate flows of public pension payments are obtained from the *Cuentas de las Administraciones Públicas* (IGAE (1991b)) and *Actuación Económica y Financiera de las Administraciones Públicas* (IGAE (1991a)).

The conditional survival probabilities at each age are equal to those obtained by the latest mortality tables published by the National Statistical Institute (INE (1991) with reference to the year 1990.

The aggregate data do not allow the study of individual life cycle behavior. To do this, we use a Spanish household budget survey (*Encuesta de Presupuestos Familiares*, or EPF) carried out by INE (1992a), in 1990–91. This survey contains data on individual income, expenditure, personal characteristics, and demographic composition for 21,155 households and 72,123 Spanish citizens. This survey is representative of the entire Spanish population and is calibrated on the Spanish Census data.

A.2 Treatment of the data

A.2.1 Lifetime distributions

We now detail how, using the data in the EPF, we calculated the lifetime distribution of the four flows associated to the two public systems.

The information in the EPF allows the estimation of the contributions and payments associated to the two public systems for each individual in the sample. These contributions and payments depend upon the labor market condition of the individual. Thus, we have considered five states in which each individual can be. For each state we compute contributions and payments the individual receives or makes. These five states are the following:

($\mathcal{E}$) *Student.* The individual is enrolled in a school or university receiving public funds. The individual is then receiving a transfer (E_a^i) of an amount equal to the average cost of a pupil of his/her age attending a school of the kind he/she specifies, during the fiscal year 1990–91. The same individual contributes toward financing of public education through a portion of his/her direct and indirect taxes, (T_a^i).

($\mathcal{W}$) *Worker.* This class includes all employed individuals. Such individuals pay direct or indirect taxes to support public education, (T_a^i), and also pay social security contributions, (T_a^{pi}).

($\mathcal{R}$) *Retired.* We consider as retired only those individuals receiving a contributive pension (P_a^i). Retired individuals are also financing the public education system with a portion of their taxes (T_a^i).

($\mathcal{U}$) *Unemployed.* If an individual receives unemployment benefits, he/she is financing the public pension system through the social security contributions paid, (T_a^{pi}). Again, the unemployed are also financing the public education system with a portion of their taxes (T_a^i).

$(\mathcal{I})$ *Inactive.* Here we include all the individuals that are not in any of the previous four states. If these individuals pay some income taxes, this is recorded in the EPF. Otherwise, we attribute to them a share of the indirect taxes based on their reported expenditure. The total gives (T_a^i).

These five states are mutually exclusive. For the very rare cases in which the same individual in the EPF reports to be in two or more of them, we create two or more "artificial" individuals and increase the sample size correspondingly . We define the universe of states to be $\mathcal{S} = \{\mathcal{E}, \mathcal{W}, \mathcal{P}, \mathcal{U}, \mathcal{I}\}$. The total population at each age $a = 1, \ldots, A$ is $\sum_{s \in \mathcal{S}} L_a(s)$, with $L_a(s)$ equal to the number of individuals of age a that are in state s. Denote the share of the population of age a in state s as $\mu_a(s) = L_a(s) / \sum_{s \in \mathcal{S}} L_a(s)$, with $\sum_{s \in \mathcal{S}} \mu_a(s) = 1$. For each a and $s \in \mathcal{S}$, $\mu_a(s)$ is the probability that an individual is in state s at age a.

A.2.2 Public education system

In Spain, public financing of education is allocated in part to public schools and in part to a special kind of private school, *centros concertados*, by means of school vouchers to students. At the compulsory school level (up to age 14 in 1990, 16 in the current legislation) schooling is completely free. After that, students attending public institutions pay only a small fraction of the total cost, the rest being born by general tax revenues. Students attending private institutions bear the full cost.

Cost of public schooling

For each educational level (primary, secondary, higher, and other), we have computed the real per-pupil public expenditure on education for various types of schools (public and *concertados*) and for the public universities. The EPF reports if an individual is enrolled in school, the type of school (public or private), and the level he/she is attending. This information is enough to compute the total number of students in each level, type of school, and age group. The criterion we followed to compute the cost of schooling for each "kind" of student (age a, level j, type k of school) is the following. From the EGPE and the EFGEP we obtain the actual total amount of public expenditures for each kind (kj) of school. We divide these amounts by the total number of pupils attending each. This gives us the effective per-student cost for each kind kj of school, E^{jk}. From the EPF we compute how many students of age a are attending a school of kind kj. Using this, we estimate public school expenditure on the representative individual at each age a as

$$E_a = \mu_a (\mathcal{E}) \sum_{k \in TC} \sum_{j \in NE} \mu_a \left(\mathcal{E}^{jk} \right) E_a^{jk} = \mu_a (\mathcal{E}) \overline{E}_a$$

where $\mu_a (\mathcal{E})$ denotes the fraction of the population of age a attending school, NE is the universe of educational levels, and TC is the universe of types of schools. Finally, $\mu_a \left(\mathcal{E}^{jk} \right)$ is the portion of students of age a enrolled in the educational level j in a school of type k.

The age distribution of public education "borrowing" is

$$\delta_a = \frac{E_a}{\sum_{a=1}^{A} E_a L_a}.$$

Hence, δ_a is the share of (lifetime total) education-related transfers the representative individual receives at age a.

Financing of the public education system

On the financing side, we need to compute the amount of education-related taxes paid by the representative individual at age a. The taxes we consider are the following: personal income tax (*Impuesto sobre la Renta de las Personas Físicas*, or IRPF), value added tax (VAT), special, and other local taxes.

The EPF provides detailed information about the income flow of each individual and the wealth and consumption baskets of each household. This allows a detailed reconstruction of the various taxes paid by an individual, which we then aggregate in a total burden of taxation (T_a^i) for individual i of age a. We calculate the average tax paid by a person of age a as

$$T_a = \sum_{s \in \mathcal{S}} \mu_a(s) \frac{\sum_{i \in s} T_a^i}{L_a(s)} = \sum_{s \in \mathcal{S}} \mu_a(s) \overline{T}_a^s$$

where $\overline{T}_a^s$ is the average tax paid by an individual in state s at age a.

Given the values T_a for $a = 1, \cdots, A$, the computation of the lifetime distribution of the total investment in public education is straightforward:

$$\alpha_a = \frac{T_a}{\sum_{a=1}^{A} T_a L_a}.$$

Hence, α_a represents the relative burden of taxation charged to the representative individual at age a, for $a = 1, \cdots, A$. Call this the age distribution of the total tax burden.

To impute the flow of real expenditures in education to the various years of one's life, we need to scale the coefficients α_a by the actual public expenditure on education. We retrieve this from IGAE (1991b); call it T_{90}^e. Then we compute $T_a^{e*} = \alpha_a \cdot T_{90}^e$ for $a = 1, \cdots, A$, the investment in public education for the representative agent.

A.2.3 Public pensions

Public contributory pensions are provided by the following programs. The General Social Security Regime (*Régimen General de la Seguridad Social*, or RGSS) is the main one and covers most private sector employees plus a (small but growing) number of public employees. The five plans included in the Special Social Security Regimes (*Regímenes Especiales de la Seguridad Social*, or RESS) are for the self-employed (*Régimen Especial de Trabajadores Autónomos*, or RETA), the agricultural workers and small farmers (*Régimen Especial Agrario*, or REA), the domestic employees (*Régimen Especial de Empleados de Hogar*, or REEH), the sailors (*Régimen Especial de Trabajadores de Mar*, or RETM), and the coal miners (*Régimen Especial de la Minería del Carbón*, or REMC). Finally, there exists a seventh, special pension system for the public employees (*Régimen de Clases Pasivas*, or RCP).

Financing the public contributive pension system

All seven pension regimes are of the pay-as-you-go-type and, presumably, are self-financing[6]. To estimate the lifetime distribution of social security payments, we identified all individuals in the EPF paying social security contributions and split them among the seven plans. For each individual we have enough information, either from the EPF or from current legislation (for example, for public employees) to compute the fictitious income (*bases de cotización* and *haberes reguladores*) upon which pension contributions are being charged. To each of the fictitious incomes we apply the social security contribution rate, as specified by the 1990–91 legislation, for the pension regime in which the individual was enrolled. Aggregating these amounts over all the individuals of age a, we obtain, for each $a = 1, \ldots, A$, the amount of social security contributions paid by individuals in state $\mathcal{W}$ ($T_a^{\mathcal{W}}$) and state $\mathcal{U}$ ($T_a^{\mathcal{U}}$). The social security contribution paid by the representative agent at age a is then

$$T_a^p = \mu_a(\mathcal{W}) \cdot T_a^{\mathcal{W}} + \mu_a(\mathcal{U}) \cdot T_a^{\mathcal{U}}.$$

[6] The RGSS shows a surplus. The five special regimes show small deficits.

Also in this case, we compute weights by setting

$$\beta_a = \frac{T_a^p}{\sum_{a=1}^{A} T_s^p L_a}.$$

Finally, from IGAE (1991a,b) we obtain the total amount of social security contributions paid to the seven plans during the year 1990, T_{90}^p. In our simulation, we use

$$T_a^{p*} = \beta_a \cdot T_{90}^p.$$

Benefits of the public pension system

The Spanish social security system provides five types of contributive pensions: old-age, disability, widowers, orphans, and other relatives. We have not considered payments of noncontributive pensions as part of our scheme, because they are not financed by means of social security contributions.

In the EPF, we are told if an individual is a pension recipient, what kind of pension he or she receives, and in what amount. The average contributive pension received at each age a is therefore easily computed as

$$P_a = \mu_a(\mathcal{P}) \cdot \sum_{k \in TP} \mu_a(\mathcal{P}^k) \cdot \frac{\sum_{i \in k} P_a^i}{L_a(\mathcal{P}^k)} = \mu_a(\mathcal{P}) \, \overline{P}_a$$

where $\mu_a(\mathcal{P})$ is the fraction of the population of age a receiving a contributive pension, TP is the universe of kinds of contributive public pensions, $\mu_a(\mathcal{P}^k)$ is the portion of pensioners at age a receiving a pension of type k, P_a^i is the actual pension received by individual i of age a, and $L_a\left(\mathcal{P}^k\right)$ is the number of individuals of age a receiving a pension of type k.

As in the previous cases, the lifetime weights are computed as

$$\gamma_a = \frac{P_a}{\sum_{a=1}^{A} P_a L_a}.$$

Finally, from IGAE (1991a,b) we obtain the total contributive pension payments effectively made, by the seven regimes, during the year 1990, P_{90}. The amounts used in our calculations are, therefore, $P_a^* = \gamma_a \cdot P_{90}$.