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The interpretation of pulsar rotation measures and the magnetic field of the Galaxy

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Summary. A re-analysis of pulsar rotation measures suggests that the systematic component of the galactic magnetic field is confined to the gas-disc region of the Galaxy ($|Z| \lesssim 120$ pc), and exhibits a reversal in direction towards the inner spiral arm. Estimates are obtained for the direction, intensity and scale height of this systematic component, the distance of the reversal from the Sun, the vertical displacement of the Sun from the magnetoactive midplane, and an approximate value for the intensity of the random magnetic field component.

1 Introduction

Manchester (1974) analysed the rotation measures (*RM*) of 38 pulsars, and, by means of a least-squares fit to an assumed uniform component of the galactic magnetic field ($\mathbf{B}_u$), found $B_u = 2.2 \pm 0.4\,\mu$G in the direction $l_u = 94° \pm 11°$, with $b_u = 0°$. In order to consider only the local field, the set of sources used in the least-squares fit excluded pulsars with distance from the Sun $L > 2$ kpc (eliminating four pulsars), while six further sources in the direction of the North Polar Spur were excluded on the basis that they must have an anomalous rotation measure due to the Spur's magnetic field. The residuals of the remaining 28 pulsars were then weighted with $\cos b$, where b is the latitude of the source, in order to diminish the effect of the high-latitude sources which would be more influenced by the random component of the field ($\mathbf{B}_r$), and the weighted least-squares analysis led to the above result.

This choice of weighting is not the optimum for obtaining the most likely values of B_u and l_u. The best choice for the weighting factor (w) is one proportional to the reciprocal of the standard deviation of the random contributions (Bevington 1969), which corresponds to the well-known χ^2 fit. Assuming unknown random components to both the magnetic field ($\mathbf{B}_r$) and electron density (n_r) such that

$$RM \propto \int_0^L (n + n_r)\,(\mathbf{B}_u + \mathbf{B}_r) \cdot \mathbf{ds},$$

the best weighting-factor to use is one proportional to $1/nL^{1/2}$, since the standard deviation of the random walk in polarization position angle induced by the random components is proportional to $nL^{1/2}$ (see Section 3), where n is the average electron density along the line-of-sight and L is the path length to each pulsar. Having thus normalized the residuals to the same statistical population, their standard deviation can then be used to estimate the random component of the field, B_r.

In addition, the source sample used in this analysis is an extension of Manchester's original data, and consists of 48 pulsars listed by Manchester & Taylor (1977) which have measured *RM*s and a maximum allowed source distance of 3 kpc. The sources given zero weight by Manchester are retained in this analysis, under the assumption that these sources simply exhibit local aspects of the random magnetic field component (see Section 3).

2 The systematic magnetic field component $\mathbf{B}_u$

We first take $\mathbf{B}_u$ as uniform and the contribution of $\mathbf{B}_u$ to the *RM* of a source at galactic longitude l, latitude b, and distance L (pc) as given by

$$GRM = -0.81 B_u \cos b \cos (l - l_u) \int_0^L n_e \, ds, \tag{1}$$

where l_u is the direction of $\mathbf{B}_u$ (we assume $b_u = 0^\circ$), B_u is the magnitude of the field (μG), and the line integral over the electron density n_e (cm^{-3}) is given by the pulsar dispersion measure.

The residual for a source is defined by

$$\delta = RM - GRM.$$

These residuals are then normalized to the same statistical distribution, with average electron density along the line of sight of 0.03 cm^{-3} and path length of 500 pc, by multiplying them by the weighting factor $w = (0.03/n)(500/L)^{1/2}$. Minimizing $\Sigma(w\delta)^2$ with respect to the parameters B_u and l_u, we obtain the values $1.5 \pm 0.2\ \mu$G and $107^\circ \pm 7^\circ$ for B_u and l_u respectively.

This model has three unsatisfactory aspects:

(i) The scatter of the absolute value of the normalized residuals (NRES) is seen to increase with L (see Fig. 1). This is indicative of choosing incorrect values for the parameters B_u and/or l_u. We quantify this increase by comparing the standard deviation of the NRES for those sources nearer than 1000 pc (σ_1), which is the median distance of the sample, with that of the sources more distant than 1000 pc (σ_2), and obtain $\sigma_1 = 18.1 \pm 2.7$ and $\sigma_2 = 29.4 \pm 4.3$.

(ii) Plotting the source position projected onto the galactic plane ($X = L \cos b \cos l$, $Y = L \cos b \sin l$), with a symbol indicating whether the sign of the *RM* agrees, or disagrees, with that assigned to each source by the above model, we obtain Fig. 2. Neglecting those sources nearer than 1 kpc, where the random magnetic field component tends to dominate the observed *RM*s (see Section 3), there are six sources in the direction of the inner spiral arm ($X > 0$) which disagree with this uniform model, and only four which are in agreement. In the remaining region ($X < 0$), there are only two which disagree with the model and 12 which are in agreement. The probability that a source further than ~ 1 kpc will disagree with the model due to fluctuations of the random field component is $\lesssim 0.2$ (see Section 3). Thus, for $X < 0$, we expect to see $\lesssim 0.2 \times 14 \sim 3$ disagreements, which is consistent with the observed number. For $X > 0$, we expect to see $\lesssim 2$ disagreements, and actually observe six.

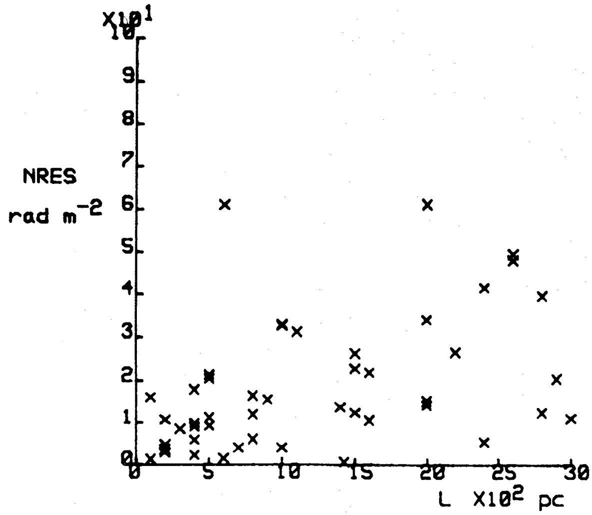

Figure 1. Absolute value of the normalized residuals, obtained using the best fitting uniform model of $\mathbf{B}_u$, plotted against pulsar distance.

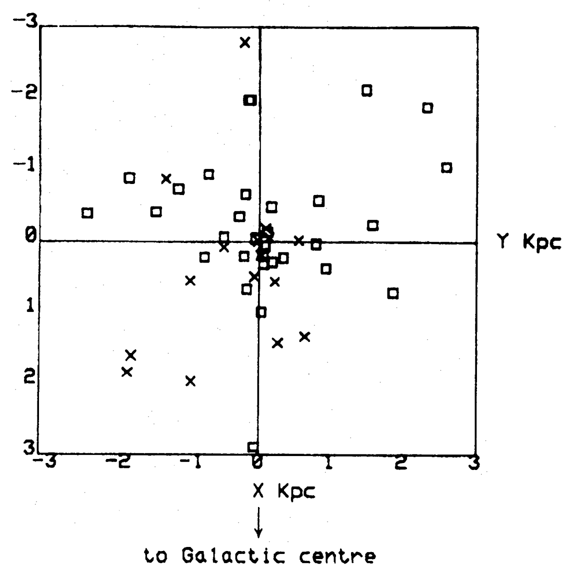

Figure 2. Source position projected onto the galactic plane, showing whether the sign of the source *RM* agrees (□), or disagrees (×) with that assigned to each source by the best fitting uniform model of $\mathbf{B}_u$.

This has a probability $\lesssim 0.01$ of being due to random fluctuations in $\mathbf{B}_r$, which implies that this model of the systematic component is inconsistent with the *RM*s observed in the direction of the inner spiral arm.

(iii) The field direction obtained ($l_u \sim 107°$) is at variance with results obtained from observations of the polarization of galactic synchrotron radiation (Mathewson & Milne 1964), and the polarization of starlight (Heiles 1976). These measurements imply that the systematic component of the galactic magnetic field is approximately aligned with the optical spiral arm of the Galaxy ($l \sim 70–80°$). The result obtained above is 4σ away from this direction.

2.1 THE Z DISTRIBUTION OF B_u

In the above model, B_u was assumed to be independent of Z ($= L \sin b$). This is not very realistic, since observations of galactic synchrotron and gamma radiation imply that the magnetic field intensity increases with the gas density in the disc region of the Galaxy (Paul, Cassé & Cesarsky 1976). We therefore allow B_u to have a Gaussian profile in Z, such that

$$B_u = B_0 \exp\,[-(Z + Z_\odot)^2/SH^2], \tag{2}$$

where SH is the scale height (pc), and $Z_\odot$ is the displacement of the Sun above the magneto-active midplane (pc). Using this model in equation (1) above (taking B_u inside the integral), the χ^2 fit gives values $B_0 = 2.4 \pm 0.4\ \mu$G, $l_u = 110° \pm 8°$, $SH = 75 \pm 40$ pc and $Z_\odot = 40 \pm 30$ pc.

This model of $\mathbf{B}_u$ reduces the standard deviation of the NRES by 1σ, and the increase in the scatter of the NRES with L has also been reduced, with $\sigma_1 = 18.0 \pm 2.6$ and $\sigma_2 = 25.2 \pm 3.7$ (see Fig. 3). However, there is still a tendency for the scatter of the NRES to increase with L, and the excessive number of disagreements between the model and the source RMs, noted above, is still present (see Fig. 4), together with the anomalous field direction.

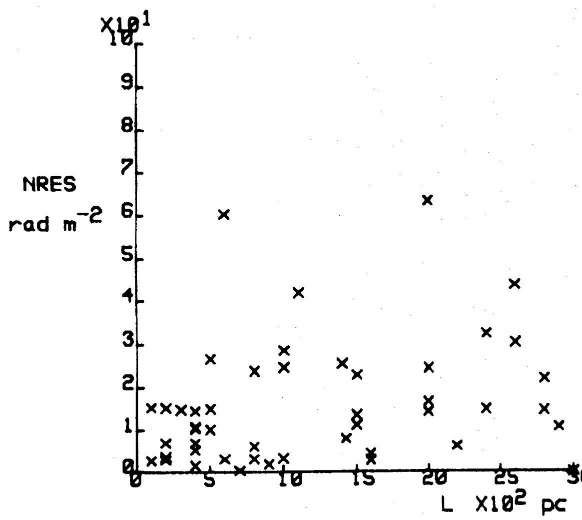

Figure 3. Absolute value of the normalized residuals, obtained using the best fitting model of $\mathbf{B}_u$, allowing a Gaussian profile in Z, plotted against pulsar distance.

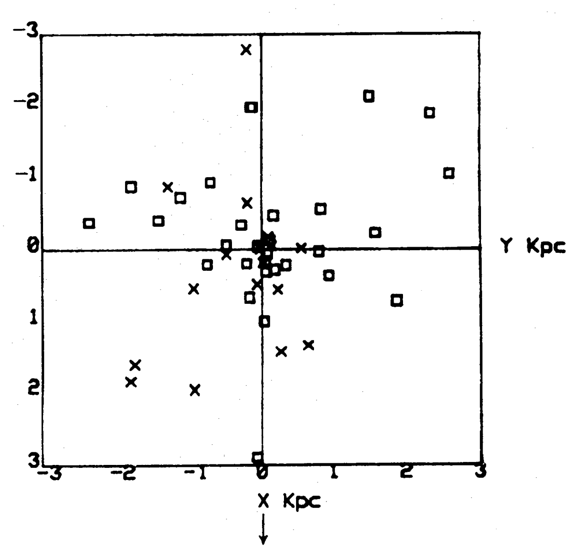

Figure 4. Source position projected onto the galactic plane, showing whether the sign of the source RM agrees (□), or disagrees (X) with that assigned to each source by the best fitting uniform model of $\mathbf{B}_u$, allowing a Gaussian profile in Z.

2.2 A REVERSAL IN DIRECTION TOWARD THE INNER SPIRAL ARM

In view of the excessive number of disagreements between the sign of the model and source *RMs*, a third model, including a field reversal, has been fitted to the data (see Fig. 5), such that

$$GRM = -0.81 \cos b \cos (l - l_u) \int_0^L Bn_e \, ds, \tag{3}$$

with

$$B = B_u \qquad X' < X_0$$

$$B = -B_u \quad X' > X_0,$$

where $X' = X \sin l_u - Y \cos l_u$, X_0 is the distance to the reversal (pc), and B_u has a Gaussian profile in Z, as given in equation (2) above. This model employs five parameters, B_0, l_u, X_0, SH, $Z_\odot$, and the χ^2 fit using this model gives values $B_0 = 3.5 \pm 0.3\ \mu$G, $l_u = 74° \pm 10°$, $X_0 = 170 \pm 90$ pc, $SH = 75 \pm 40$ pc and $Z_\odot = 25 \pm 30$ pc.

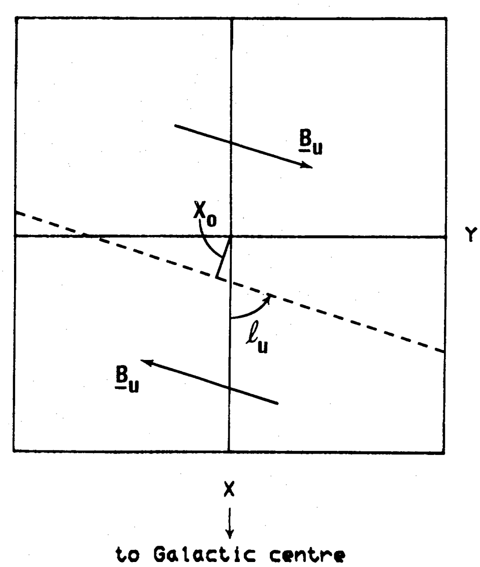

Figure 5. Diagram showing the field configuration employed in the reversal model.

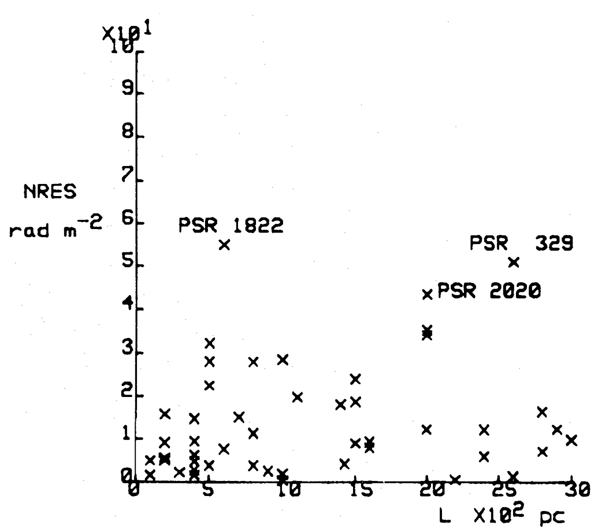

Figure 6. Absolute value of the normalized residuals, obtained using the best fitting reversal model of $\mathbf{B}_u$, allowing a Gaussian profile in Z, plotted against pulsar distance.

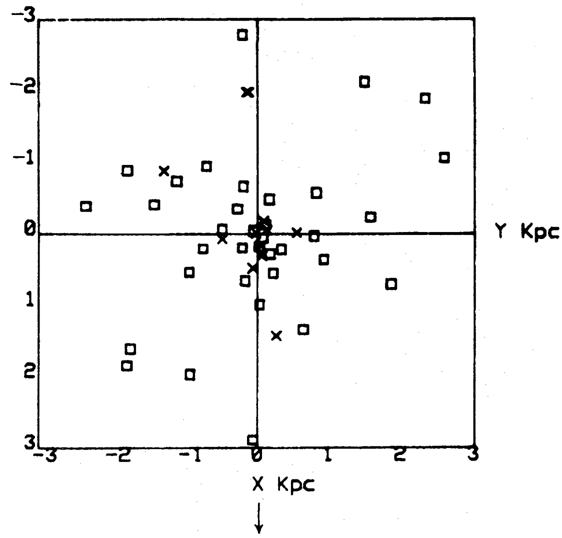

Figure 7. Source position projected onto the galactic plane, showing whether the sign of the source *RM* agrees (□), or disagrees (X) with that assigned to each source by the best fitting reversal model of $\mathbf{B}_u$, allowing a Gaussian profile in Z.

This model of $\mathbf{B}_u$ reduces the standard deviation of the NRES, compared to the original model (1) above, by 2.2σ, and the increase in the scatter of the NRES with L has now become insignificant, with $\sigma_1 = 17.4 \pm 2.5$ and $\sigma_2 = 21.5 \pm 3.2$ (see Fig. 6). Also, the excessive number of disagreements between the previous models and the source *RM*s has disappeared (see Fig. 7), and the direction of the magnetic field is now in better agreement with the other available data.

Noting that the scale height of the pulsar sample is ~ 250 pc, these results imply that the systematic field component is confined to the disc region of the Galaxy. In addition it should be noted that the existence of a large-scale field-reversal in the direction of the inner spiral arm has also been postulated from a study of extragalactic *RM*s (Simard-Normandin & Kronberg 1979). Further, Tosa & Fujimoto (1978) have analysed *RM*s evaluated at various positions across the disc region of the spiral galaxy M51, and proposed a bisymmetric spiral structure for the systematic component of the magnetic field. This configuration exhibits local field directions aligned with the spiral arms, and field reversals across lines running parallel to these arms.

The value of B_0 determined above may at first seem difficult to reconcile with the distribution of $B_\parallel = (RM/0.81DM)\mu$G discussed by Manchester (1974) since, for the pulsar sample used here, $\langle B_\parallel \rangle \simeq 1\ \mu$G. However, $B_\parallel = B_0 \cos\theta$, where θ is the angle between the line-of-sight to the source and $\mathbf{B}_u$, and the $\cos\theta$ factor will reduce $B_\parallel$ below B_0, while the fact that some pulsars lie above the B_u layer further reduces $B_\parallel$, so that this quantity can then have an average value $\simeq 1\ \mu$G with $B_0 \simeq 3.5\ \mu$G.

3 The random magnetic field component $\mathbf{B}_r$

For simplicity, we assume that the Z distribution of $\mathbf{B}_r$ has a scale height greater than that of the pulsar sample used (~ 250 pc), allowing $\mathbf{B}_r$ to be taken as independent of Z. This can be justified by considering the Z distribution of the galactic synchrotron radiation, since Paul *et al.* introduce a second Gaussian element into their magnetic field model having a scale height of approximately 400 pc in order to interpret the observed distribution. As we have seen, this cannot be due to the systematic field component, which has a scale height of only 75 pc. We thus conclude that the random field component, $\mathbf{B}_r$, extends ~ 400 pc from the galactic midplane.

We define n_R^2 to be the mean square along a line-of-sight of $\int_{\text{cell}} n_r \, ds$, with the integration taken over one fluctuation cell, divided by d^2, the mean square of the cell lengths. We also define B_R^2 in the same manner. Then, assuming a random-walk model for the residuals (Nelson 1973), induced by the random components, the standard deviation of the residuals, $\sigma(\delta)$, is given by

$$\sigma(\delta) = 0.81 \, (n^2 B_R^2 + n_R^2 B_R^2 + n_R^2 B_u^2)^{1/2} (dL/3)^{1/2}$$

where the cell correlation-length of both random components, B_r and n_r, is assumed to be the same, namely d pc. For the normalized residuals, $\delta' = \delta \, (0.03/n)(500/L)^{1/2}$, we obtain

$$\sigma(\delta') \simeq 0.3 \sqrt{d} \left[B_R^2 + \frac{n_R^2}{n^2} (B_R^2 + B_u^2) \right]^{1/2}.$$

For some sources L is determined independently from hydrogen absorption measurements, but for most sources L is determined using the pulsar dispersion measure ($L = DM/n$) with $n = 0.03 \text{ cm}^{-3}$, together with an estimate of the contribution to n from known H II regions within 1 kpc (Manchester & Taylor 1975). Since the positivity of n_e implies $n_R \lesssim n$, we take the extreme case of $n_R \sim n$, thus helping to compensate for the possible error involved in the estimation of n and obtaining a more conservative estimate for B_R. We thus have

$$\sigma(\delta') \sim 0.3 d^{1/2} (2B_R^2 + B_u^2)^{1/2}.$$

The best-fitting model employed above gives $\sigma(\delta') = 19.2 \pm 2.0 \text{ rad m}^{-2}$ and, since d probably lies between 10 and 100 pc (Heiles 1976), we estimate $4 < B_R < 14 \, \mu$G. We note here that a total galactic magnetic field intensity ~ 10 μG can satisfactorily account for the observed intensity of the galactic synchrotron radiation if the interstellar cosmic-ray electron density is the same as that near the Sun (Setti & Woltjer 1971). Also such field intensities will have important consequences for grain alignment mechanisms, and imply that the galactic magnetic field may be dynamically significant, since its energy density (~ 10^{-12} erg cm^{-3}) is of the same order as the thermal energy density in the Galaxy.

Polarization measurements of the non-thermal radiation from spiral galaxies yield fractional polarizations ~ 10–20 per cent (Segalovitz, Shane & de Bruyn 1976). We estimate the fractional polarization of non-thermal radiation from the Galaxy (averaged over regions $\gg d$ in extent) to be ~ 10 per cent, including an allowance for the variation of the cosmic ray electron density with Z (Paul *et al.* 1976).

In order to justify, *a posteriori,* the assumption made in Section 2, we consider the relative contributions of $\mathbf{B}_u$ and $\mathbf{B}_r$ to the *RM*s. Since the systematic field contribution averaged over longitude (RM_u) is proportional to L, and the random field contribution (RM_R) is proportional to $L^{1/2}$, we can use the values obtained above to estimate the probability, $P(L)$, that the random field contribution will alter the sign of the contribution from the systematic component over a path length L. It is straightforward to show that

$$P(L) = \tfrac{1}{2} \left[1 - \operatorname{erf} \left(\frac{RM_u(L)}{\sqrt{2} \sigma \, [RM_R(L)]} \right) \right]$$

where $\sigma \, [RM_R(L)]$ is the standard deviation of the random field contribution for sources at a distance L. In order to evaluate RM_u and $\sigma(RM_R)$ for sources 1 kpc distant, we assume

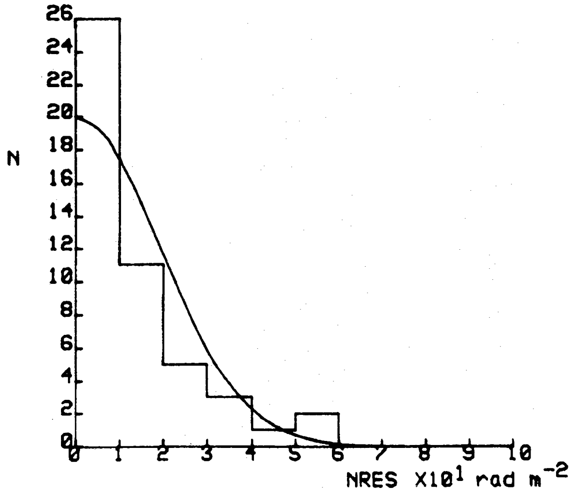

Figure 8. Frequency distribution of the absolute value of the normalized residuals, obtained using the best fitting reversal model of B_u, allowing a Gaussian profile in Z. The smooth curve represents the expected Gaussian frequency distribution with a standard deviation of 19.2 rad m^{-2}.

that the average path length of the systematic component is $\sim L/2 = 500$ pc, to take into account the limited scale height of this component, compared to that of the pulsar sample (see Section 2). We thus obtain the approximate value $P(L > 1\ \text{kpc}) \lesssim 0.2$ (again assuming $n_R \sim n$). It is worth noting here that, since $\sigma(RM_R) \propto L^{1/2}$ and $RM_u \propto L$, then, for $L \gg d$ (i.e. almost all sources), the dominant contribution to the *RM*s comes from B_u, even though B_R is the dominant field component.

Finally, in order to determine whether any sources exhibit anomalous *RM*s, the frequency distribution of the NRES is plotted in Fig. 8. As can be seen, this histogram is fairly well represented by the expected Gaussian distribution function, with a standard deviation of 19.2 rad m^{-2}, showing no obviously anomalous residuals. However, there is the possibility that some of the high NRES sources, named in Fig. 6, may be anomalously large, perhaps due to incorrect estimates of their distance. Even if this is so, the standard deviation of the NRES (and thus B_R) is reduced by only ~ 20 per cent if these sources are excluded from the sample.

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