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Research Article

The Interval-Valued Intuitionistic Fuzzy MULTIMOORA Method for Group Decision Making in Engineering

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Multiple criteria decision making methods have received different extensions under the uncertain environment in recent years. The aim of the current research is to extend the application of the MULTIMOORA method (Multiobjective Optimization by Ratio Analysis plus Full Multiplicative Form) for group decision making in the uncertain environment. Taking into account the advantages of IVIFS (interval-valued intuitionistic fuzzy sets) in handling the problem of uncertainty, the development of the interval-valued intuitionistic fuzzy MULTIMOORA (IVIF-MULTIMOORA) method for group decision making is considered in the paper. Two numerical examples of real-world civil engineering problems are presented, and ranking of the alternatives based on the suggested method is described. The results are then compared to the rankings yielded by some other methods of decision making with IVIF information. The comparison has shown the conformity of the proposed IVIF-MULTIMOORA method with other approaches. The proposed algorithm is favorable because of the abilities of IVIFS to be used for imagination of uncertainty and the MULTIMOORA method to consider three different viewpoints in analyzing engineering decision alternatives.

1. Introduction

Multicriteria decision making (MCDM) is a growing field of operations research both from theoretical and implementation perspectives. The importance of MCDM can be drawn from Zeleny [1]: "it has become more and more difficult to see the world around us in a uni-dimensional way and to use only a single criterion when judging what we see. We always compare, rank, and order the objects of our choice with respect to multiple criteria of choice." The MCDM field is further divided into two classes, including multiobjective decision making (MODM) and multiattribute decision making (MADM). These classes are, respectively, associated with planning and selection classification of Simon [2] for decision making problems. The main topic of this paper covers MADM.

A MADM problem can be formally characterized as the task to evaluate, compare, and rank a set of finite alternatives, options, or choices with regard to a set of finite attributes. According to Yu [3], a MADM problem is composed of the set of substitutive alternatives (1), the set of evaluation criteria (2), the outcome (or decision) matrix with regard to the alternatives scored based on the evaluation criteria (3), and the preference structure of decision making about the criterion significances or weights (4). The information about the third and fourth parts of a decision making problem is determined exactly. However, a decision maker always deals with approximate or partial information [4]. Therefore,

exactness is an unrealistic assumption. Different frameworks have been proposed to handle uncertainty in practice. Liu and Lin [5] classified these frameworks into probability and statistics (1) and grey system theory and fuzzy set theory (2). Fuzzy set theory introduced by Zadeh [6] is widely applied to decision making problems. In other words, the fuzzy set theory is a generalization of ordinal sets, where a membership degree is assigned to each element of a set. Each type of uncertainty has its own characteristics and is appropriate for special cases. While probability is concerned with the occurrence of well-defined events, fuzzy sets deal with gradual concepts and describe their boundaries [7]. In fact, the fuzzy set theory is appropriate for recognition-based uncertainty that is common in decision making.

Determining a single membership degree is a difficult task for decision makers and, therefore, Grattan-Guinness [8] believes that presentation of a linguistic expression in the form of a fuzzy set is not sufficient. In 1986, Atanassov [9] introduced the notion of intuitionistic fuzzy sets (IFSs) as an extension of ordinal fuzzy sets. In addition to a membership degree of each element, IFS assigns a degree of nonmembership to each element. Later, Atanassov and Gargov [10] extended the interval-valued intuitionistic fuzzy sets (IVIFSs), where membership and nonmembership degrees are stated as closed intervals.

When it became obvious that the type 1 fuzzy sets were not always sufficient for MADM under uncertain environment, type 2 fuzzy sets, involving interval-valued as well as intuitionistic fuzzy sets, were proposed. As a result, the MULTIMOORA method was updated by using generalized interval-valued trapezoidal fuzzy numbers [11] or intuitionistic fuzzy numbers [12]. The method has been successfully applied to making economic, technological, or management decisions. The applications of various approaches in 2006–2013, including crisp and extended methods, were summarized by T. Baležentis and A. Baležentis [13]. The most recent applications cover the extended versions of the method, in particular, the fuzzy MULTIMOORA [14, 15] or MULTIMOORA based on the interval 2-tuple linguistic variables [16]. Also, Li [17] extended MULTIMOORA with hesitant fuzzy numbers, where membership degree of elements is defined over a set of different values.

The current research aims to extend the MULTIMOORA method, using the interval-valued intuitionistic fuzzy sets. Taking into account the advantages of IVIFS, the interval-valued intuitionistic fuzzy MULTIMOORA (IVIF-MULTIMOORA) method for group decision making is presented in the paper. For this reason, three parts of MULTIMOORA were extended under IVIF information and the appropriate algorithm was proposed.

The paper is organized as follows. In Section 2, the literature review is presented and then the crisp version of MULTIMOORA is briefly overviewed in Section 3. Section 4 includes a brief description of the interval-valued intuitionistic fuzzy sets. The proposed IVIF-MULTIMOORA algorithm is presented in Section 5. The solution of two numerical examples of civil engineering problems is given in Section 6, while the results obtained are compared with the data yielded by some other methods of IVIF decision making. Finally, the discussion and the conclusions are presented in Sections 7 and 8.

2. Literature Review

Taking into account the flexibility and general character of IVIFS, researchers developed decision making methods under the IVIFS environment. Some researchers developed the well-known MADM methods in the IVIF form. Some authors [18–20] proposed different frameworks of TOPSIS method under the IVIF environment. Li [21, 22] developed some mathematical programming-based method to solve the MADM problems with both ratings of the alternatives on attributes and weights of attributes expressed with the help of IVIFS. Park et al. [23] presented an IVIF version of the VIKOR method. Li [24] proposed a closeness coefficient based on the nonlinear programming method for solving IVIF MADM problems. Chen et al. [25] solved the MADM problems based on the interval-valued intuitionistic fuzzy weighted average operator and newly defined fuzzy ranking method for intuitionistic fuzzy values. Yu et al. [26] introduced some IVIF aggregation operators and applied them to solving various decision making problems. Zhang and Yu [27] presented an optimization model to determine the attribute weights in an IVIF decision making problem and, then, used these weights in an extended TOPSIS to rank the alternatives. Ye [28] introduced a cosine similarity measure and a weighted cosine similarity measure to MADM problems. Meng et al. [29] also proposed two new aggregation operators, that is, the arithmetical interval-valued intuitionistic fuzzy generalized λ-Shapley Choquet operator and the geometric interval-valued intuitionistic fuzzy generalized λ-Shapley Choquet operator, and investigated their application to solving MADM problems. Meng et al. [30] used Shapley function in extending a generalized IVIF hybrid Shapley averaging operator and proposed its application to solving MADM problems. Razavi Hajiagha et al. [31] developed Complex Proportional Assessment (COPRAS) method with IVIF data. Chai et al. [32] proposed a rule-based decision model when decision information in a group decision making problem is provided as IVIF values. Wan and Dong [33] defined the possibility degree of comparison between two IVIF numbers and introduced two ordered averaging operators based on the Karnik-Mendel algorithm to solve MADM problems with IVIF information. Chen [34] developed a method based on the traditional linear assignment method for solving decision making problems in the IVIF context. Xu and Shen [35] presented the IVIF outranking choice method to solve MADM problems, while Zavadskas et al. [36] extended the IVIF weighted aggregated sum product assessment (WASPAS) method. Geetha et al. [37] proposed a new ranking method of IVIF numbers and extended its application in decision making problems. Zhang et al. [38] proposed a new definition of IVIF entropy an entropy-based MADM method. Later, Wei, and Zhang [39] proposed applying an entropy measure for IFSs and IVIFSs to assess the experts' weights for multicriteria fuzzy group decision making. Tong and Yu [40] introduced a novel approach for

ranking the alternatives based on the IVIF cross entropy and TOPSIS in the IVIF environment. Wu and Chiclana [41, 42] introduced a risk attitudinal ranking method for IVIF numbers and applied this ranking method in a MADM problem. They imposed a risk attitude parameter over IVIF numbers ordinal score and accuracy functions [43] and used this method for solving MADM problems.

As mentioned before the current paper has focused on the extension of the MULTIMOORA (Multiobjective Optimization by Ratio Analysis plus Full Multiplicative Form) method. This method belongs to the group of complete aggregation methods, based on the reference point technique. The crisp MOORA method (Multiobjective Optimization by Ratio Analysis) was presented by Brauers and Zavadskas in 2006 [44]. This method was further supplemented by the full multiplicative form. The MULTIMOORA was suggested by Brauers and Zavadskas in 2010 [45]. The method is based on the theory of dominance [46] and enables us to summarize MOORA, consisting of the ratio system and the reference point approaches, as well as the full multiplicative form. The robustness of the method has been proved [47] and it has been successfully applied to evaluate decisions by using a single approach or its combinations with other MADM methods, such as TOPSIS [48] and WASPAS [49, 50].

Several developments of MULTIMOORA for the uncertain environment have been presented. The fuzzy MULTIMOORA was suggested by Brauers et al. [52]. To overcome some drawbacks of the fuzzy set theory, the two-tuple linguistic representation method for computing with words can be applied [53]. Accordingly, the considered method has been extended for group decision making based on two tuples (MULTIMOORA-2T-G) [54, 55]. The fuzzy MULTIMOORA, based on triangular fuzzy numbers and designed for group decision making (MULTIMOORA-FG), was introduced [56, 57]. As a further modification of the method, MOORA with grey numbers was developed [58, 59].

3. MULTIMOORA

The method summarizes three approaches, that is, MOORA, consisting of the ratio system and the reference point, and the full multiplicative form.

Regardless of the approach used, initial decision criteria are transformed by applying vector normalization:

$$\overline{x}_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \tag{1}$$

where x_{ij} is the initial criterion value, that is, the response of the alternative i to objective j; $i = 1, 2, \ldots, m$; m is the number of alternatives; $j = 1, 2, \ldots, n$; n is the number of decision criteria (objectives); $\overline{x}_{ij}$ is a dimensionless value of a decision criterion.

The first part of the approach is based on the ratio system [44]. To calculate the relative significance, $\overline{y}_i$, of each alternative i with respect to all objectives j, the weighted normalized criteria values should be added in the case of maximization, while, in the case of minimization, the weighted normalized criteria values should be subtracted as follows:

$$\overline{y}_i = \sum_{j=1}^{j=g} \overline{x}_{ij} w_j - \sum_{j=g+1}^{j=n} \overline{x}_{ij} w_j, \tag{2}$$

where $j = 1, 2, \ldots, g$ are maximized decision criteria; $j = g+1, g+2, \ldots, n$ are minimized decision criteria; w_j is the weight (or relative significance) of a criterion.

The values of variables $\overline{y}_i$ show the preference of the alternatives according to the ratio system approach.

The second part of the approach is based on the maximal objective reference point [44]. First, the desirable ideal alternative should to be established. The virtual ideal alternative consists of the best values of the considered criteria r_j. It is formed by selecting the best criteria values from every decision alternative based on their optimization direction, that is, the maximal values from the criteria set $j = 1, 2, \ldots, g$ and the minimal values from the criteria set $j = g+1, g+2, \ldots, n$.

All criteria values were transformed by applying vector normalization (1). Having the dimensionless values of the criteria $\overline{x}_{ij}$ and relative significances (weights) of the criteria w_j, decision alternatives are ranked based on the Min-Max metric of Tchebycheff [60]:

$$\min_{(i)} \left\{ \max_{(j)} \left| r_j - \overline{x}_{ij} w_j \right| \right\}. \tag{3}$$

The second part of the approach is based on the full multiplicative form, as presented by Brauers and Zavadskas [44]. The full multiplicative form for calculating the utility of the alternatives U_i is applied as follows:

$$U_i = \frac{A_i}{B_i}, \tag{4}$$

where A_i and B_i are calculated separately for maximized decision criteria $j = 1, 2, \ldots, g$ and minimized decision criteria $j = g+1, g+2, \ldots, n$, respectively. A_i and B_i are calculated as follows:

$$A_i = \prod_{j=1}^{g} w_j \overline{x}_{ij},$$
$$B_i = \prod_{j=g+1}^{n} w_j \overline{x}_{ij}. \tag{5}$$

The MULTIMOORA is based on the theory of dominance [46] and summarizes the MOORA, involving the above described ratio system and the reference point approaches, as well as the full multiplicative form [45].

Robustness of the method has been verified proving that accuracy of aggregated approaches is larger comparing to accuracy of single ones [47]. The method has been widely applied to evaluate alternatives and to select the best decisions in engineering technology or management problems by using a single approach or its combinations with other MADM methods [13, 48–50]. However, the farther the more complex

decisions should be made and decision makers usually face a larger amount of approximate or partial information. Accordingly, extensions of conventional MADM methods under uncertain environment for group decision making are essential. The further developed IVIF-MULTIMOORA method could be applied in innovation themes in engineering, such as sustainable building life-cycle modelling and evaluating and selecting new business models to finance, build, and manage public buildings and infrastructures.

4. Interval-Valued Intuitionistic Fuzzy Sets

Zadeh [6] generalized the characteristic function of classic sets into membership function and introduced fuzzy sets, where a membership degree was assigned to each element of a fuzzy set. Later, Atanassov [9] proposed the idea of intuitionistic fuzzy sets (IFSs), when a nonmembership degree is assigned to each element of the set alongside its membership. The interval-valued intuitionistic fuzzy sets (IVIFSs) present an extended form of IFSs.

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$ and let $X(\neq \Phi)$ be a given set. An IVIFS in X was defined as $\widetilde{A} = \{\langle x, \mu_{\widetilde{A}}(x), \nu_{\widetilde{A}}(x)\rangle \mid x \in X\}$, where $\mu_{\widetilde{A}} : X \to D[0,1]$ and $\nu_{\widetilde{A}} : X \to D[0,1]$ with condition $0 \prec \sup_x \mu_{\widetilde{A}}(x) + \sup_x \nu_{\widetilde{A}}(x) \leq 1$. The intervals $\mu_{\widetilde{A}}(x)$ and $\nu_{\widetilde{A}}(x)$ denote the degree of membership and nonmembership of the element x of the set A. Thus, for each $x \in X$, $\mu_{\widetilde{A}}(x)$ and $\nu_{\widetilde{A}}(x)$ are closed intervals whose lower and upper end points are denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$, $\nu_{AL}(x)$, and $\nu_{AU}(x)$.

The IVIFS was denoted by

$$A = \{\langle x, [\mu_{AL}(x), \mu_{AU}(x)], [\nu_{AL}(x), \nu_{AU}(x)]\rangle \mid x \in X\}, \tag{6}$$

where $0 \prec \mu_{AU}(x) + \nu_{AU}(x) \leq 1, \mu_{AL}(x), \nu_{AL}(x) \geq 0$. For convenience, the IVIFS value was denoted by $\widetilde{A} = ([a,b],[c,d])$ and referred to as an interval-valued intuitionistic fuzzy number (IVIFN).

The algebraic operations were extended over IVIFSs. Let $\widetilde{A}_1 = ([a_1,b_1],[c_1,d_1])$ and $\widetilde{A}_2 = ([a_2,b_2],[c_2,d_2])$ be any two IVIFNs; then [43]

$$\widetilde{A}_1 + \widetilde{A}_2 = \left([a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2], [c_1c_2, d_1d_2]\right), \tag{7}$$

$$\widetilde{A}_1 \cdot \widetilde{A}_2 = \left([a_1a_2, b_1b_2], [c_1 + c_2 - c_1c_2, d_1 + d_2 - d_1d_2]\right), \tag{8}$$

$$\lambda\widetilde{A}_1 = \left(\left[1 - (1-a_1)^\lambda, 1 - (1-b_1)^\lambda\right], \left[c_1^\lambda, d_1^\lambda\right]\right), \quad \lambda \geq 0. \tag{9}$$

The division and subtraction operations were defined as follows, using the extension principle [61]:

$$\widetilde{A}_1 \div \widetilde{A}_2 = \{[\min(a_1,a_2), \min(b_1,b_2)], [\max(c_1,c_2), \max(d_1,d_2)]\}, \tag{10}$$

$$\widetilde{A}_1 - \widetilde{A}_2 = \{[\min(a_1,a_2), \min(b_1,b_2)], [\max(c_1,c_2), \max(d_1,d_2)]\}. \tag{11}$$

Since the final rating of the alternatives is usually determined as IVIFNs, it requires their comparison. This comparison was made, using the score and accuracy functions [43]. Let $\widetilde{A} = ([a,b],[c,d])$ be an IVIFN. Then,

$$s\left(\widetilde{A}\right) = \frac{1}{2}(a - c + b - d) \tag{12}$$

was called the score function of $\widetilde{A}$, where $s(\widetilde{A}) \in [-1,1]$, while

$$h\left(\widetilde{A}\right) = \frac{1}{2}(a + c + b + d) \tag{13}$$

was referred to as the accuracy function of A, where $h(\widetilde{A}) \in [0,1]$.

For $\widetilde{A}_1$ and $\widetilde{A}_2$ as two IVIFNs, it follows that [43]

(1) if $s(\widetilde{A}_1) \prec s(\widetilde{A}_2)$, then $\widetilde{A}_1$ is smaller than $\widetilde{A}_2$, $\widetilde{A}_1 \prec \widetilde{A}_2$;

(2) if $s(\widetilde{A}_1) = s(\widetilde{A}_2)$, then

(a) if $h(\widetilde{A}_1) = h(\widetilde{A}_2)$, then $\widetilde{A}_1 = \widetilde{A}_2$,

(b) if $h(\widetilde{A}_1) \prec h(\widetilde{A}_2)$, then $\widetilde{A}_1$ is smaller than $\widetilde{A}_2$, $\widetilde{A}_1 \prec \widetilde{A}_2$.

Another requirement of group decision making is associated with the aggregation of different judgments of decision makers into a single estimate. In this case, aggregation operators on IVIFNs can be used. Let $\widetilde{A}_j = ([a_j,b_j],[c_j,d_j])$, $j = 1,2,\ldots,n$, be a collection of IVIFNs. Then, the generalized interval intuitionistic fuzzy weighted average $\mathrm{GIIFWA}_w(\widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_n)$ was defined as

$$\mathrm{GIIFWA}_w(A_1, A_2, \ldots, A_n) = \left(w_1\widetilde{A}_1^\lambda + w_2\widetilde{A}_2^\lambda + \cdots + w_n\widetilde{A}_n^\lambda\right)^{1/\lambda}, \tag{14}$$

where $\lambda \succ 0$, and $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector with $w_j \geq 0$, $j = 1,2,\ldots,n$, and $\sum_{j=1}^n w_j = 1$. It can be shown

that GIIFWA is also an IVIFN and can be calculated as follows [62]:

$$\begin{aligned}
&\mathrm{GIIFWA}_w\left(\widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_n\right) \\
&= \left(\left[\left(1 - \prod_{j=1}^{n}\left(1 - a_j^{\lambda}\right)^{w_j}\right)^{1/\lambda}, \right.\right. \\
&\left.\left(1 - \prod_{j=1}^{n}\left(1 - b_j^{\lambda}\right)^{w_j}\right)^{1/\lambda}\right], \left[1 \right. \\
&- \left(1 - \prod_{j=1}^{n}\left(1 - \left(1 - c_j\right)^{\lambda}\right)^{w_j}\right)^{1/\lambda}, 1 \\
&\left.\left. - \left(1 - \prod_{j=1}^{n}\left(1 - \left(1 - d_j\right)^{\lambda}\right)^{w_j}\right)^{1/\lambda}\right]\right).
\end{aligned} \tag{15}$$

If $\lambda = 1$, then GIIFWA is turned into the interval intuitionistic fuzzy weighted average (IIFWA).

5. IVIF-MULTIMOORA for Group Decision Making

5.1. IVIF-MULTIMOORA. Assume that a group of K decision makers (experts) wants to appraise a set of m alternatives $A_1, A_2, \ldots, A_m$, based on a set of n criteria $C_1, C_2, \ldots, C_n$. Also, the weight vector $w_1, w_2, \ldots, w_n$, with $w_j \geq 0$, $j = 1, 2, \ldots, n$, and $\sum_{j=1}^{n} w_j = 1$ is determined that shows the relative importance of different criteria on final decision. Due to incomplete and ill-defined data, the required information in the considered problem is expressed by IVIFNs.

At the first step, each decision maker expresses his/her decision matrix $\widetilde{X}^k$, $k = 1, 2, \ldots, K$, as follows:

$$\widetilde{X}^k = \begin{bmatrix} \widetilde{x}_{11}^k & \widetilde{x}_{12}^k & \cdots & \widetilde{x}_{1n}^k \\ \widetilde{x}_{21}^k & \widetilde{x}_{22}^k & \cdots & \widetilde{x}_{2n}^k \\ \vdots & \vdots & \cdots & \vdots \\ \widetilde{x}_{m1}^k & \widetilde{x}_{m2}^k & \cdots & \widetilde{x}_{mn}^k \end{bmatrix}, \tag{16}$$

where $\widetilde{x}_{ij}^k$ represents the IVIF performance of the alternative A_i over the criterion C_j from the viewpoint of kth decision maker. The aim of a decision making group is to rank the alternatives.

To solve this MAGDM problem, an IVIF version of the MULTIMOORA method, called IVIF-MULTIMOORA, was extended and described in this section. Initially, the aggregation was required to transform the set of K decision matrices to an aggregated decision matrix. This aggregated decision matrix had the form of $\widetilde{X} = [\widetilde{x}_{ij}]$. If an equal weight was assumed for decision makers, then

$$\widetilde{x}_{ij} = \left(\left[\mu_{ij}^l, \mu_{ij}^u\right], \left[\nu_{ij}^l, \nu_{ij}^u\right]\right) = \mathrm{IIFWA}\left(\widetilde{x}_{ij}^1, \widetilde{x}_{ij}^2, \ldots, \widetilde{x}_{ij}^K\right). \tag{17}$$

The aggregation operation was performed using the GIIFWA operator and considering the weight vector $V = (v_1, v_2, \ldots, v_K)$ for different decision makers.

Since the aggregated decision matrix $\widetilde{X} = [\widetilde{x}_{ij}]$ was available, the IVIF-MULTIMOORA method was proposed to solve the MAGDM problem. As mentioned in Section 3, the MULTIMOORA method involves three parts: the ratio system (1), the reference point approach (2), and the full multiplicative form (3).

5.2. The Part of MOORA Based on the IVIF-Ratio System. The first step in the ratio system is the normalization of the decision matrix. However, since IVIFNs are commensurable numbers, normalization is not required. The set of criteria was decomposed into two subsets of the benefit criteria (where more is better) and cost criteria (where less is better). Then, the $\widetilde{y}_i$, $i = 1, 2, \ldots, m$, score of alternatives was computed by adding its weighted benefit criteria and subtracting the weighted cost criteria. If the benefit criteria were ordered as $C_1, C_2, \ldots, C_g$ and cost criteria as $C_{g+1}, C_{g+2}, \ldots, C_n$, then

$$\widetilde{y}_i = \sum_{j=1}^{g} w_j \widetilde{x}_{ij} - \sum_{j=g+1}^{n} w_j \widetilde{x}_{ij}. \tag{18}$$

This score could be computed by applying the IVIFNs algebraic operators and the extension principle given in Section 4. However, a slight modification could simplify the computations. Let $W^+ = \sum_{j=1}^{g} w_j$ and $W^- = \sum_{j=g+1}^{n} w_j$. Then, $w_j' = w_j / W^+$ for each $j \in \{1, 2, \ldots, g\}$, and $w_j'' = w_j / W^-$ for each $j \in \{g+1, g+2, \ldots, n\}$. It was clear that $w_j' \geq 0$, $j = 1, 2, \ldots, g$, and $\sum_{j=1}^{g} w_j' = 1$. Similarly, $w_j'' \geq 0$, $j = 1, 2, \ldots, g$, and $\sum_{j=1}^{g} w_j'' = 1$. Then,

$$\widetilde{y}_i' = \frac{\widetilde{y}_i}{W^+ W^-}. \tag{19}$$

Multiplying both sides of (18) by positive value of $1/(W^+W^-)$, (18) was transformed as follows:

$$\widetilde{y}_i' = \frac{1}{W^-}\sum_{j=1}^{g} w_j' \widetilde{x}_{ij} - \frac{1}{W^+}\sum_{j=g+1}^{n} w_j'' \widetilde{x}_{ij}. \tag{20}$$

Based on the IIFWA operator definition, (18) was represented as

$$\begin{aligned}
\widetilde{y}_i' = &\frac{1}{W^-}\mathrm{IIFWA}_{W^+}\left(\widetilde{x}_{i1}, \widetilde{x}_{i2}, \ldots, \widetilde{x}_{ig}\right) \\
&- \frac{1}{W^+}\mathrm{IIFWA}_{W^-}\left(\widetilde{x}_{ig+1}, \widetilde{x}_{ig+2}, \ldots, \widetilde{x}_{in}\right).
\end{aligned} \tag{21}$$

The multiplication of $1/(W^-)$ and $1/(W^+)$ was performed using (9). In addition, the subtraction of two IVIFNs in (21) was performed by applying (16). $\widetilde{y}_i'$, $i = 1, 2, \ldots, m$, is relative significance of the alternative i. The comparison of $\widetilde{y}_i'$, $i = 1, 2, \ldots, m$, values based on the score and the accuracy function resulted in the ranking of the alternatives based on the ratio system.

5.3. The IVIF-Reference Point Portion of the MOORA Method. According to the second part of the MOORA, the maximal objective reference point approach was used. The desirable ideal alternative in the IVIF environment was the IVIF vector with the coordinates $\widetilde{r}_j$, $j = 1, 2, \ldots, n$, which was formed by selecting the data from every considered decision alternative and taking into account the optimization direction of every particular criterion. To find the reference point based on the idea of the TOPSIS method, the weighted decision matrix $\widetilde{N} = [\widetilde{n}_{ij}] = W \cdot \widetilde{X}$ was constructed first, where

$$\widetilde{n}_{ij} = \left(\left[\eta_{ij}^{l}, \eta_{ij}^{u}\right], \left[\delta_{ij}^{l}, \delta_{ij}^{u}\right]\right) = w_j \widetilde{x}_{ij}. \tag{22}$$

The scalar multiplication was performed by applying (9). Then, for benefit criteria, $C_1, C_2, \ldots, C_g$, the reference point was constructed as follows:

$$\begin{aligned}\widetilde{r}_j &= \left(\left[\eta_{ij}^{*l}, \eta_{ij}^{*u}\right], \left[\delta_{ij}^{*l}, \delta_{ij}^{*u}\right]\right) \\ &= \left(\left[\max_{1\le i\le m}\left(\eta_{ij}^{l}\right), \max_{1\le i\le m}\left(\eta_{ij}^{u}\right)\right],\right. \\ &\quad \left.\left[\min_{1\le i\le m}\left(\delta_{ij}^{l}\right), \min_{1\le i\le m}\left(\delta_{ij}^{u}\right)\right]\right),\end{aligned} \tag{23}$$

and for cost criteria, $C_{g+1}, C_{g+2}, \ldots, C_n$,

$$\begin{aligned}\widetilde{r}_j &= \left(\left[\eta_{ij}^{*l}, \eta_{ij}^{*u}\right], \left[\delta_{ij}^{*l}, \delta_{ij}^{*u}\right]\right) \\ &= \left(\left[\min_{1\le i\le m}\left(\mu_{ij}^{l}\right), \min_{1\le i\le m}\left(\mu_{ij}^{u}\right)\right],\right. \\ &\quad \left.\left[\max_{1\le i\le m}\left(\nu_{ij}^{l}\right), \max_{1\le i\le m}\left(\nu_{ij}^{u}\right)\right]\right).\end{aligned} \tag{24}$$

Then, the Min-Max metric of Tchebycheff [26] was used for ranking the alternatives:

$$\operatorname*{Min}_{1\le i\le m}\left\{\max_{1\le j\le n}\left|\widetilde{r}_j - \widetilde{x}_{ij} w_j\right|\right\}. \tag{25}$$

Based on (22)–(24), (25) could be simplified. The following steps were taken to determine the rankings of the alternatives, using the reference point approach.

Step 1. Compute for each alternative A_i, $i = 1, 2, \ldots, m$, the score function in the weighted decision matrix:

$$S\left(\widetilde{n}_{ij}\right) = \eta_{ij}^{l} - \delta_{ij}^{l} + \eta_{ij}^{u} - \delta_{ij}^{u}. \tag{26}$$

Step 2. Compute the score function of the reference points:

$$S\left(\widetilde{r}_j\right) = \eta_{ij}^{*l} - \delta_{ij}^{*l} + \eta_{ij}^{*u} - \delta_{ij}^{*u}. \tag{27}$$

Step 3. Find the maximum deviation from the reference points:

$$d_i = \max_{1\le j\le n}\left|\widetilde{r}_j - \widetilde{x}_{ij} w_j\right| = \max_{1\le j\le n}\left|S\left(\widetilde{r}_j\right) - S\left(\widetilde{n}_{ij}\right)\right|. \tag{28}$$

Step 4. Rank the alternatives in the ascending order of d_i, $i = 1, 2, \ldots, m$.

5.4. The IVIF-Full Multiplicative Form. Finally, the full multiplicative form was applied as follows:

$$\widetilde{U}_i = \frac{\widetilde{A}_i}{\widetilde{B}_i}, \tag{29}$$

where $\widetilde{U}_i$ denotes the overall utility of the alternative i, and

$$\begin{aligned}\widetilde{A}_i &= \prod_{j=1}^{g} w_j \widetilde{x}_{ij}, \\ \widetilde{B}_i &= \prod_{j=g+1}^{n} w_j \widetilde{x}_{ij}.\end{aligned} \tag{30}$$

Multiplication of a set of IVIFNs was performed by sequentially applying (8). To give a compact form of this multiplication, it was supposed that $\widetilde{x}_{ij}$, $j = 1, 2, \ldots, g$, was determined as an IVIFN as indicated in (17). First, $\widetilde{n}_{ij} = w_j \widetilde{x}_{ij}$, $j = 1, 2, \ldots, g$, was formed by applying (9) in the same manner as previously performed in (22). Then,

$$\begin{aligned}\widetilde{A}_i &= \prod_{j=1}^{g} \widetilde{n}_{ij} = \left(\left[\underline{\eta}_{ij}^{l}, \overline{\eta}_{ij}^{l}\right], \left[\underline{\delta}_{ij}^{l}, \overline{\delta}_{ij}^{u}\right]\right) \\ &= \left(\left[\prod_{j=1}^{g}\eta_{ij}^{l}, \prod_{j=1}^{g}\eta_{ij}^{u}\right], \left[\sum_{j=1}^{g}\delta_{ij}^{l} - \sum_{\substack{j_1<j_2 \\ j_1,j_2\in\{1,2,\ldots,g\}}} \delta_{ij_1}^{l}\cdot\delta_{ij_2}^{l}\right.\right. \\ &\quad + \cdots + (-1)^{k+1} \sum_{\substack{j_1<j_2<\cdots<j_k \\ j_1,j_2,\ldots,j_k\in\{1,2,\ldots,g\}}} \left(\delta_{ij_1}^{l}\cdot\delta_{ij_2}^{l}\cdots\delta_{ij_k}^{l}\right) + \cdots \\ &\quad + (-1)^{g+1}\left(\delta_{i1}^{l}\cdot\delta_{i2}^{l}\cdots\delta_{ig}^{l}\right), \sum_{j=1}^{g}\delta_{ij}^{u} \\ &\quad - \sum_{\substack{j_1<j_2 \\ j_1,j_2\in\{1,2,\ldots,g\}}} \delta_{ij_1}^{u}\cdot\delta_{ij_2}^{u} + \cdots \\ &\quad + (-1)^{k+1} \sum_{\substack{j_1<j_2<\cdots<j_k \\ j_1,j_2,\ldots,j_k\in\{1,2,\ldots,g\}}} \left(\delta_{ij_1}^{u}\cdot\delta_{ij_2}^{u}\cdots\delta_{ij_k}^{u}\right) + \cdots \\ &\quad \left.\left. + (-1)^{g+1}\left(\delta_{i1}^{u}\cdot\delta_{i2}^{u}\cdots\delta_{ig}^{u}\right)\right]\right).\end{aligned} \tag{31}$$

Then $\widetilde{B}_i$, $i = 1, 2, \ldots, m$, was also calculated in a similar way. Then, $\widetilde{U}_i$, $i = 1, 2, \ldots, m$, was determined by (10) for division. The ranking of the alternatives was specified by calculating the score functions $S(\widetilde{U}_i)$, $i = 1, 2, \ldots, m$, and ranking them in a descending order.

Applying three distinct parts of MULTIMOORA, three sets of rankings of the alternatives were determined. The obtained rankings formed a ranking pool, based on which the final ranking of the alternatives could be determined. To find

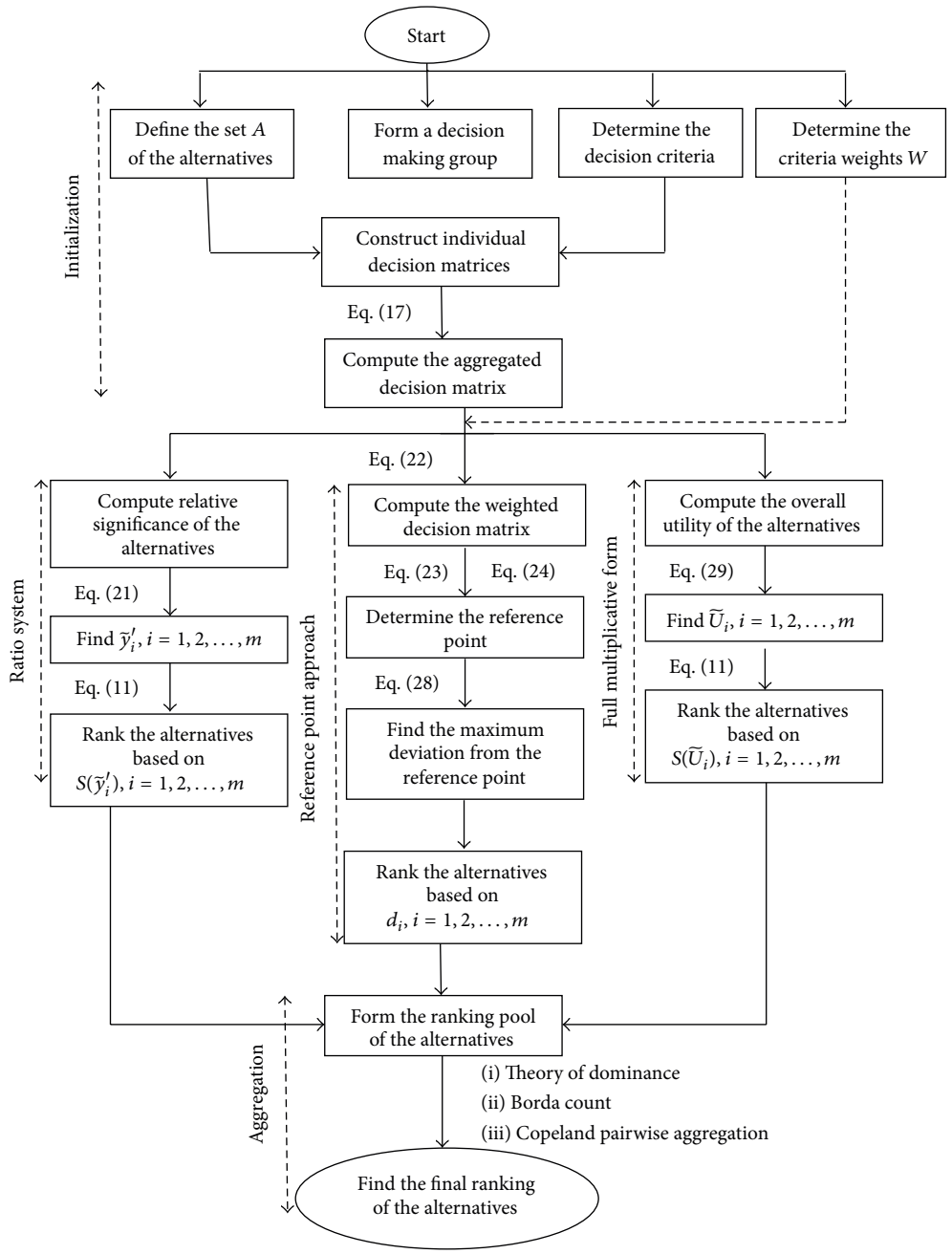

Figure 1: The IVIF-MULTIMOORA algorithm.

the final ranking of the alternatives, the theory of dominance [46] could be used. Also, the techniques, such as the Borda count method [63] or Copeland pairwise aggregation method [64], could be applied to determine the final ranking of the alternatives.

5.5. The IVIF-MULTIMOORA Algorithm. MULTIMOORA is the extension of the MOORA method and the full multiplicative form of multiple-objective. The distinct parts of the MULTIMOORA method are presented in Sections 5.1 to 5.3. The algorithmic scheme of MULTIMOORA is presented in the current section. The IVIF-MULTIMOORA algorithm includes five different stages: (1) initialization, (2) IVIF-ratio system, (3) IVIF-reference point approach, (4) IVIF-full multiplicative form, and (5) final ranking. These stages are presented in Figure 1.

6. A Numerical Example

In this section, two applications of the proposed method are presented and the results are compared to the rankings yielded by other multiple criteria decision making methods.

Table 1: Decision matrix of rational revitalization of derelict and mismanaged buildings.

Criteria	Optimization direction	Weight	A_1	A_2	A_3
$\widetilde{x}_1$	Max	0.0600	([0.70, 0.80], [0.10, 0.15])	([0.45, 0.60], [0.20, 0.30])	([0.30, 0.40], [0.45, 0.50])
$\widetilde{x}_2$	Min	0.0727	([0.60, 0.70], [0.15, 0.20])	([0.45, 0.55], [0.20, 0.30])	([0.25, 0.40], [0.45, 0.55])
$\widetilde{x}_3$	Max	0.0747	([0.75, 0.80], [0.10, 0.15])	([0.45, 0.55], [0.25, 0.35])	([0.30, 0.40], [0.40, 0.50])
$\widetilde{x}_4$	Max	0.0627	([0.80, 0.90], [0, 0.05])	([0.50, 0.65], [0.15, 0.25])	([0.35, 0.50], [0.30, 0.40])
$\widetilde{x}_5$	Max	0.0673	([0.70, 0.85], [0.05, 0.10])	([0.55, 0.65], [0.20, 0.25])	([0.35, 0.45], [0.30, 0.40])
$\widetilde{x}_6$	Max	0.0627	([0.75, 0.85], [0.10, 0.15])	([0.55, 0.65], [0.20, 0.25])	([0.35, 0.45], [0.30, 0.40])
$\widetilde{x}_7$	Min	0.0667	([0.80, 0.85], [0.10, 0.15])	([0.45, 0.55], [0.25, 0.35])	([0.15, 0.35], [0.40, 0.50])
$\widetilde{x}_8$	Max	0.0667	([0.85, 0.90], [0, 0.05])	([0.20, 0.35], [0.35, 0.50])	([0.10, 0.15], [0.75, 0.80])
$\widetilde{x}_9$	Max	0.0667	([0.80, 0.90], [0, 0.05])	([0.10, 0.25], [0.55, 0.65])	([0.05, 0.10], [0.80, 0.85])
$\widetilde{x}_{10}$	Max	0.0667	([0.45, 0.60], [0.25, 0.35])	([0.10, 0.25], [0.55, 0.65])	([0.05, 0.10], [0.80, 0.85])
$\widetilde{x}_{11}$	Max	0.0667	([0.75, 0.80], [0.10, 0.15])	([0.15, 0.30], [0.30, 0.50])	([0.05, 0.10], [0.75, 0.85])
$\widetilde{x}_{12}$	Max	0.0667	([0.75, 0.85], [0.05, 0.10])	([0.25, 0.40], [0.20, 0.40])	([0.70, 0.80], [0.10, 0.15])
$\widetilde{x}_{13}$	Min	0.0667	([0.70, 0.85], [0.05, 0.10])	([0.10, 0.15], [0.75, 0.80])	([0.50, 0.55], [0.35, 0.40])
$\widetilde{x}_{14}$	Min	0.0667	([0.70, 0.80], [0.10, 0.15])	([0.50, 0.55], [0.30, 0.40])	([0.10, 0.15], [0.70, 0.80])
$\widetilde{x}_{15}$	Max	0.0667	([0.60, 0.75], [0.15, 0.20])	([0.45, 0.60], [0.20, 0.30])	([0.25, 0.45], [0.25, 0.40])

6.1. The First Example. Zavadskas et al. [36] solved a problem of rational revitalization of derelict and mismanaged buildings in Lithuanian rural areas. Three alternatives included the reconstruction of rural buildings and their adaptation to production (or commercial) activities (A_1), as well as their improvement and use for farming (A_2) or dismantling and recycling the demolition waste materials (A_3). The following criteria were taken into consideration: the average soil fertility in the area x_1 (points), quality of life of the local population x_2 (points), population activity index x_3 (%), GDP proportion with respect to the average GDP of the country x_4 (%), material investment in the area x_5 (LTL per resident), foreign investments in the area x_6 (LTL $\times 10^3$ per resident), building redevelopment costs x_7 (LTL $\times 10^6$), increase in the local population's income x_8 (LTL $\times 10^6$ per year), increase in sales in the area x_9 (%), increase in employment in the area x_{10} (%), state income from business and property taxes x_{11} (LTL $\times 10^6$ per year), business outlook x_{12}, difficulties in changing the original purpose of the site x_{13}, the degree of contamination x_{14}, and the attractiveness of the countryside (i.e., image and landscape) x_{15}.

A decision matrix is presented in Table 1.

This problem was solved with the proposed IVIF-MULTIMOORA method as follows.

6.1.1. Ratio System. In the ratio system ranking, the relative importance (significance) of the alternatives was determined by (21). For this purpose, initially, the benefit criteria, including $x_1, x_2, x_3, x_4, x_5, x_6, x_8, x_9, x_{10}, x_{11}, x_{12}$, and x_{15}, were separated from the cost criteria, including x_7, x_{13}, and x_{14}. Then, the aggregations of the benefit criteria weights, W^+, and cost criteria weights, W^-, were computed. These weights were then used in (21), and the values of $\widetilde{y}_i'$, $i = 1, 2, 3$, were calculated as follows:

$$\widetilde{y}_1' = ([0.285, 0.363], [0.531, 0.602]),$$
$$\widetilde{y}_2' = ([0.110, 0.136], [0.787, 0.833]),$$
$$\widetilde{y}_3' = ([0.077, 0.110], [0.824, 0.858]). \quad (32)$$

Ranking these values according to their score functions led to the ranking of the alternatives as $A_1 \succ A_2 \succ A_3$.

6.1.2. Reference Point Approach. Ranking of the alternatives in this approach was initialized by determining a reference point from the weighted decision matrix. Then, the deviation of each alternative from the reference point was computed by (28). These deviations were as follows: $d_1 = 0.468$, $d_2 = 1.332$, and $d_3 = 1.391$. By ranking the alternatives in the descending order of deviations, the final ranking was determined as $A_1 \succ A_2 \succ A_3$.

6.1.3. Full Multiplicative Form. In applying the third part of the method, the full multiplicative approach was used to rank the alternatives. Using (30), the multiplicative forms of benefit and cost criteria were obtained. These multiplicative forms were constructed by sequential multiplication of a set of IVIFNs. The overall utility of the alternatives was determined as follows:

$$\widetilde{U}_1 = ([6.010E-14, 2.028E-12], [9.999999825E-1, 9.9999999998E-1]),$$
$$\widetilde{U}_2 = ([4.127E-20, 2.310E-17], [9.99999999999909E-1, 9.99999999999998E-1]), \quad (33)$$
$$\widetilde{U}_3 = ([0, 0], [1, 1]).$$

Ranking these values according to their score functions led to ranking of the alternatives as $A_1 \succ A_2 \succ A_3$.

Table 2: IVIF-MULTIMOORA ranking of the alternatives.

Alternatives	IVIF-MOORA ratio system	IVIF-MOORA reference point	IVIF-full multiplicative form	IVIF-MULTIMOORA
A_1	1	1	1	1
A_2	2	2	2	2
A_3	3	3	3	3

Table 3: Ranking of the alternatives by applying different methods.

Alternatives	IFOWA	TOPSIS-IVIF	COPRAS-IVIF	WASPAS-IVIF	IVIF-MULTIMOORA
A_1	1	1	1	1	1
A_2	2	2	3	2	2
A_3	3	3	2	3	3

Table 4: IVIF decision matrix for investment alternatives.

	C_1	C_2	C_3	C_4
A_1	([0.42, 0.48], [0.4, 0.5])	([0.6, 0.7], [0.05, 0.025])	([0.4, 0.5], [0.2, 0.5])	([0.55, 0.75], [0.15, 0.25])
A_2	([0.4, 0.5], [0.4, 0.5])	([0.5, 0.8], [0.1, 0.2])	([0.3, 0.6], [0.3, 0.4])	([0.6, 0.7], [0.1, 0.3])
A_3	([0.3, 0.5], [0.4, 0.5])	([0.1, 0.3], [0.2, 0.4])	([0.7, 0.8], [0.1, 0.2])	([0.5, 0.7], [0.1, 0.2])
A_4	([0.2, 0.4], [0.4, 0.5])	([0.6, 0.7], [0.2, 0.3])	([0.5, 0.6], [0.2, 0.3])	([0.7, 0.8], [0.1, 0.2])

Table 5: IVIF-MULTIMOORA ranking of the investment alternatives.

Alternatives	IVIF-MOORA ratio system	IVIF-MOORA reference point	IVIF-full multiplicative form	IVIF-MULTIMOORA
A_1	3	2	3	3
A_2	4	1	4	4
A_3	1	4	1	1
A_4	2	3	2	2

Table 6: Ranking of the investment alternatives by applying different methods.

Alternatives	Wang et al.	IFOWA	TOPSIS-IVIF	COPRAS-IVIF	WASPAS-IVIF	IVIF-MULTIMOORA
A_1	3	4	2	3	3	3
A_2	4	3	4	4	4	4
A_3	2	1	1	2	1	1
A_4	1	2	3	1	2	2

The three sets of rankings are summarized in Table 2. The final ranking was obtained by using the dominance theory in the last column of the table.

Table 3 also presents the rankings obtained by some different methods, including IFOWA [62], TOPSIS-IVIF [19], COPRAS-IVIF [31], and WASPAS-IVIF [36].

Kendall's coefficient of concordance in Table 3 is 0.840, which shows a high degree of concordance. Moreover, there is complete coincidence among the IVIF-MULTIMOORA method and the other three methods. These facts imply that there is great homogeneity between IVIF-MULTIMOORA and other methods.

6.2. The Second Example. Consider the problem of appraising four different investment alternatives $\{A_1, A_2, A_3, A_4\}$ based on four criteria of risk (C_1), growth (C_2), sociopolitical issues (C_3), and environmental impacts (C_4). The vector of criteria significance is (0.13, 0.17, 0.39, 0.31). Also, C_1 and C_3 were considered as cost criteria, while C_2 and C_4 were benefit criteria. The problem was initially formulated by Wang et al. [51] as the decision matrix which is given below (Table 4).

The above problem was solved by using the proposed IVIF-MULTIMOORA method. The results obtained by using the ratio system, the reference point approach, and the full multiplicative form are presented in Table 5. Moreover, the final ranking obtained based on the dominance theory is presented in the last column of the table.

The results obtained by solving the above problem by different methods, including the method of Wang et al. [51], IFOWA [62], TOPSIS-IVIF [19], COPRAS-IVIF [31], and WASPAS-IVIF [36], are presented in Table 6.

In this case, Kendall's coefficient of concordance is 0.766, which shows a high degree of concordance, compared to the critical value of 0.421 [65]. Spearman's rank correlation is used to demonstrate the similarity between different rankings. Table 7 shows Spearman's rank correlation between the

Table 7: Ranking similarity of the investment alternatives by applying different methods.

Methods	Wang et al.	IFOWA	TOPSIS-IVIF	CORAS-IVIF	WASPAS-IVIF	IVIF-MULTIMOORA
Wang et al. [51]	—	0.6 (0%)	0.4 (25%)	1 (100%)	0.8 (50%)	0.8 (50%)
IFOWA		—	0.4 (25%)	0.6 (0%)	0.8 (50%)	0.8 (50%)
TOPSIS-IVIF			—	0.4 (25%)	0.8 (50%)	0.8 (50%)
CORAS-IVIF				—	0.8 (50%)	0.8 (50%)
WASPAS-IVIF					—	1 (100%)
IVIF-MULTIMOORA						—

considered methods. The percentages specified in parenthesis in Table 7 show the matched ranks obtained by using the considered five methods with ranks of Wang et al. [51] method as the percentages of the matched alternatives.

Based on the above findings, it can be concluded that, except for COPRAS-IVIF, which completely matches the method of Wang et al. [51], the IVIF-MULTIMOORA, along with WASPAS-IVIF, has the highest similarity (reaching 80%) to all other methods. Moreover, the percentage of the matched rankings with those of Wang, Li, and Wang is 50%, which is the second high value of congruence. These results have shown great similarity of IVIF-MULTIMOORA to other methods, which can be considered to be one of its advantages.

7. Discussion

Vincke [66] believes that the main difficulty in solving multiple criteria decision making problems lies in the fact that, usually, there is no optimal solution to such a problem, which could dominate other solutions in all the criteria.

As a response to this dilemma, various methods have been proposed for solving MCDM problems. The selection of the most appropriate and the most robustly and effectively useable method is a continuous object of scientific discussions for decades [67, 68].

This means that the experts usually look for Pareto optimal solution. Therefore, while an optimal solution is obtained by several methods, this emphasizes that proposed method will achieve the near to optimum answer. In this regard, MULTIMOORA method has a strong advantage over the other methods as it summarizes three approaches. The robustness of the aggregated approach has been substantiated in the previous research [47].

On the other hand, the above-mentioned difficulty may be considered from two additional perspectives. First, the uncertainty of decision maker in his judgments should be taken into consideration. To deal with this challenge, several frameworks have been introduced, starting from application of Zadeh fuzzy numbers [6] and followed by numerous extensions of decision making methods using type 1 and type 2 fuzzy sets, involving interval-valued as well as intuitionistic fuzzy sets [69]. The extensions of the considered MULTIMOORA method till 2013 were summarized in a review paper [13]. The most recent extensions not covered by [13] should also be mentioned [14–17]. Seeking the best imagination of uncertainty, another up-to-date extension, namely, the interval-valued intuitionistic fuzzy MULTIMOORA method is presented in the current research.

Second, decisions are usually made by several decision makers. This makes it necessary to consider a suitable procedure to aggregate the opinions. Therefore, this problem is handled as a group decision making problem.

As it is not easy to prove that a newly proposed method is applicable, it is useful to test it in solving several multiple criteria problems. To make sure that the method is better or at least it is not worse than the other existing methods, it is appropriate to apply several related approaches to compare their ranking results for the same problem [70]. Accordingly, two illustrative examples have been presented to fulfill the task. It is encouraging that the results have shown great similarity of IVIF-MULTIMOORA to other methods. The fact can be considered to be one of advantages of the novel approach.

8. Conclusions and Future Work

In the current paper, the interval-valued intuitionistic fuzzy MULTIMOORA method is proposed for solving a group decision making problem under uncertainty. The IVIF is a generalized form of fuzzy sets which considers the nonmembership of objects in a set beside their membership and with respect to hesitancy of decision makers.

The MULTIMOORA method is a decision making procedure composed of three parts, including the ratio system, the reference point approach, and full multiplicative form. The rankings yielded by this method are finally aggregated, and a unique ranking is determined.

The proposed algorithm benefits from the abilities of IVIFSs to be used in imagination of uncertainty and MULTIMOORA method to consider three different viewpoints in analyzing decision alternatives. The application of the proposed method is illustrated by two numerical examples and the obtained results are compared to the results yielded by other methods of decision making with IVIF information. The comparison, using Kendall coefficient of concordance and Spearman's rank correlation, has shown a high consistency between the proposed method and the other approaches that verifies the proposed method can be applied in case of similar problems in ambiguous and uncertain situation. This advantage strengthens the applicability of IVIF-MULTIMOORA as a method for group decision making

under uncertainty when solving engineering technology and management complex problems.

Future directions of the research will be focused on searching modifications of the method, the best reflecting uncertain environment, and producing the most reliable results. At the moment two novel and up-to-date possible extensions of the MULTIMOORA method could be suggested. One of the approaches could be based on the neutrosophic sets [71, 72] that are a generalization of the fuzzy and intuitionistic fuzzy sets. The other possible approach could be to apply D numbers that were recently proposed to handle uncertain information [73, 74].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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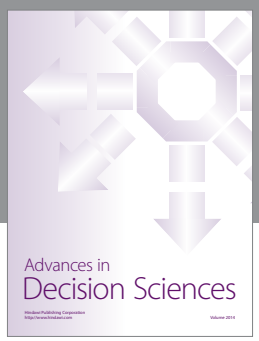

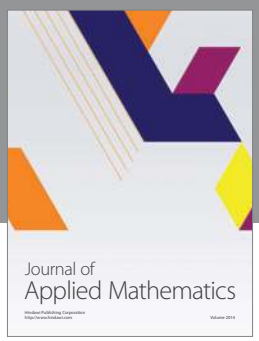

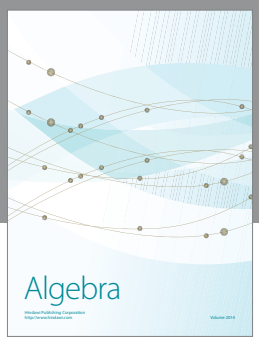

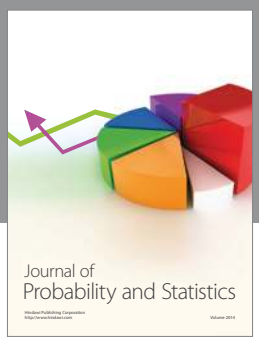

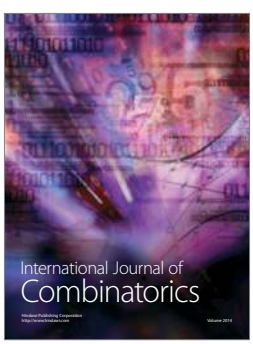

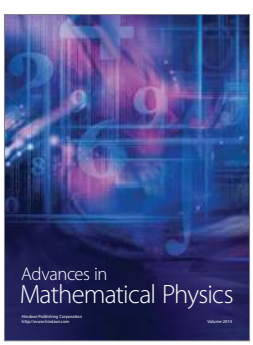

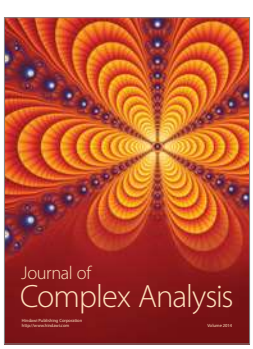

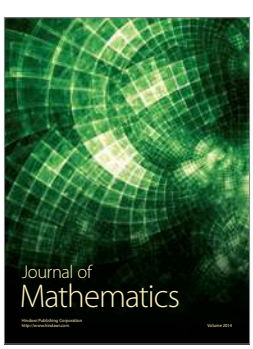

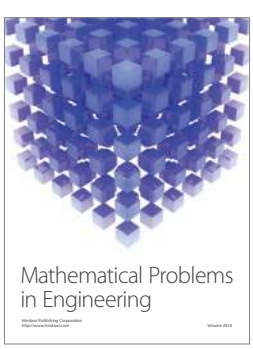

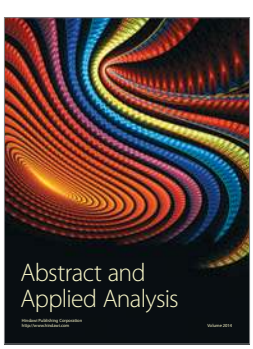

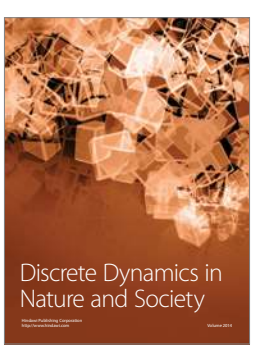

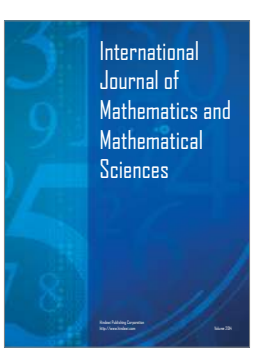

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