The JET Alfvén Eigenmode Local Manager for the real-time detection and tracking of MHD instabilities

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Abstract

In this work we report the successful application of an innovative method, based on the Sparse Representation of signals, to perform a real-time, unsupervised detection of the individual components in a frequency degenerate, multi-harmonic spectrum, using a small number of data un-evenly sampled in the spatial domain. This method has been developed from its original applications in astronomy, and is now routinely used in the JET thermonuclear fusion experiment to obtain the decomposition of a spectrum of high-frequency (~10-500kHz range) magnetic instabilities with a sub-ms time resolution, allowing the real-time tracking of its individual components as the plasma background evolves. This work opens a path towards developing real-time control tools for electro-magnetic instabilities in future fusion devices aimed at achieving a net energy gain. More generally, the speed and accuracy of this algorithm is recommended for instances of physics measurements and control engineering where an unsupervised, real-time decomposition of a degenerate signal is required from a small number of data.

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1) Introduction

The problem of unsupervised detection of different components in a multi-harmonic spectrum using un-even sampling is common to many areas of physics and engineering [1], and particularly in the field of Astronomy and Astrophysics (A&A), where the very long measurement series are often interrupted due to different environmental constraints (such as the weather and the Earth's rotation). Much work has been done over the past 20 years to address this mathematical issue and improve on the limitations of the original methods, which were essentially based on the Lomb-Scargle periodograms [2-5].

In the field of plasma physics, analysis of electro-magnetic fluctuations is important for understanding and controlling the magneto-hydrodynamic (MHD) stability of the plasma. These instabilities can occur when a plasma is trapped and heated in a magnetic configuration and responds by generating its own oscillating magnetic field, i.e. an MHD instability. Considering the case of magnetically confined thermonuclear fusion plasmas in a tokamak device [6], the MHD analysis is based on magnetic and turbulence measurements, and involves an initial time/frequency Fourier decomposition of the data to obtain the individual frequency components $\psi(\omega)$. Since a tokamak has, to a first approximation, 2D boundary conditions along the longitudinal (toroidal) axis and on the plane perpendicular to it (the poloidal direction), the spatial structure of the MHD instabilities is determined by further decomposing each frequency component in its toroidal (n) and poloidal (m) harmonics: $\psi(\omega)=e^{-i\omega t}\Sigma_{n,m}A_{mn}e^{in\phi}e^{im\theta}$. Here ϕ and θ are the toroidal and poloidal angle coordinates, respectively, and we have used the fact that in tokamak geometry one single toroidal component with a given n is usually made up of multiple poloidal components m*'s* due to toroidicity and various other geometrical effects. Note also that the data are actually acquired only at some specific angle positions ϕ_p and θ_p, $p=\{1, \ldots, P\}$, with a small number of unevenly spaced sensors, i.e. there is not a continuous measurement coverage of the toroidal and poloidal angle coordinates. The sensors are pick-up coils mounted on the vessel wall (as those used in this work), and 2D imaging systems such as reflectometry, X-rays and electron cyclotron emission.

The number of mathematical methods in use to analyse MHD instabilities in tokamaks is rather high. With evenly spaced sensors, a simple discrete Fourier transform reveals the mode amplitude for each individual (n, m) component up to the Nyquist number [7]. However, such measurement arrangement is in general not available because of installation constraints. Hence, other methods have been developed for fusion plasmas [8-14]. All these methods work well for a spectrum of modes where the different components are well separated in frequency, but are unsuitable in case of modes superposition because of intrinsic mathematical difficulties or the too long computational time needed for the analysis. Hence these methods cannot be used for real-time detection of MHD instabilities in tokamaks for protection of the machine or optimization of the fusion performance [15], a functionality very much required for future thermonuclear fusion devices aimed at achieving a net energy gain, such as ITER [6, p.711; 16].

The problem of unsupervised decomposition of a multi component, frequency-degenerate spectrum, is a typical case where scientific cross-fertilization between two different fields, thermonuclear fusion in tokamaks and A&A, yields excellent results. Considering the similarities between these two areas, it is clear that an un-evenly distributed measurement time series of A&A data corresponds to an un-evenly spaced measurement array in the toroidal (poloidal) direction in a tokamak. Hence, the canonical time-conjugate in A&A (a temporal frequency) corresponds to the spatial toroidal (poloidal) mode number in a tokamak. There are, however, minor differences between these two fields: real-valued data and real-valued temporal frequencies in A&A, complex electro-magnetic data and integer-valued (positive and negative) mode numbers in thermonuclear fusion plasmas in tokamaks. Moreover, the role of the mean value of the data is of no scientific interest in A&A, and even perturbs the data analysis, while for tokamak plasmas it represents the $n=0$ ($m=0$) mode, which can causes a major magnetic instability leading to an abrupt vertical displacement of the entire plasma column. Finally, the long measurement series of A&A data do not call for real-time analysis, whereas in tokamak plasma, sub-millisecond calculations are needed in order to protect the machine from the dangerous effect of MHD instabilities.

To tackle the problem of an unsupervised signal decomposition of irregularly sampled A&A data, a new method has been recently proposed for fitting complex sinusoids to such data [17]. This algorithm is based on the *Sparse Representation of Signals*, as implemented in the *SparSpec* code. This method has also been initially adapted for post-pulse data analysis of MHD instabilities in the Joint European Torus (JET) tokamak [6, p.617], where it has been fully benchmarked with simulated and real data [18]. For JET data, the *SparSpec* method has proven to be extremely robust, and is especially useful for resolving the amplitudes and phases of multiple Alfvén Eigenmodes (AEs) (see chapter 7.15 in [6]) in the ~100-300kHz frequency range, which are ringing with the same or nearly the same frequency.

This paper reports on the use of *Sparse Representation* methods for real-time application, and on their specific use for the sub-millisecond detection, discrimination and tracking of the individual toroidal mode numbers in the multi-components, frequency-degenerate spectrum of AEs excited in JET by an array of antennas used for MHD diagnostic purposes. Section2 reviews the mathematical foundation of *Sparse Representations* and the numerical approach used in the *SparSpec* code. Section3 gives a brief overview of the active MHD diagnostic system used in JET. Section4 shows the first examples of the application of the real-time version of the *SparSpec* code to the detection and discrimination of the different toroidal components in the multi-harmonics spectrum driven by the active MHD diagnostic system. Finally, in Section5 we briefly summarize our results and give an outlook towards future work.

2) Sparse Representations and the SparSpec principle

Our aim is to detect the toroidal (poloidal) mode numbers n (m) of the MHD instabilities present in the plasma and to estimate their amplitude from data y_p acquired with detectors unevenly positioned at angles ϕ_p (θ_p) in radians, $p=\{1, \ldots, P\}$. Each y_p is modelled as $y_p = \sum_{l=1}^{L} \alpha_l \exp\left(i n_l \phi_p\right) + \varepsilon_p$, where n_l and α_l are the unknown mode numbers and amplitudes, respectively, L the unknown number of modes and ε_p corresponds to the noise on the data for the given p-th sensor. The data y_p, the mode amplitudes α_l and noise ε_p are all complex values variables. This problem amounts to fitting multiple complex sinusoids to the data and is a general signal processing problem in many fields of physics.

From an estimation viewpoint, evaluating the amplitudes α_l and the modes number n_l is a very difficult problem. Consider the best lest-square (LS) fitting: even if the number of modes is known, minimizing the LS criterion requires a combinatorial exploration for integer-valued mode numbers n_l. A way to circumvent the problem is to estimate the amplitudes of all mode numbers in the range $\{-K, \ldots, K\}$ (where $|K|$ is the maximum mode number), but to enforce the fact that most of these modes have a null amplitude. This amounts to approximating the data with the best linear combination of a small number of elementary known signals, which is called a *Sparse Approximation* [19]). Theoretically, the *Sparse Approximation* problem consists of minimizing the LS criterion penalized by the number of non-zero elements (the L0 norm). To minimize such criterion, one must sift through all possible combinations of elementary signals, which becomes intractable when multiple unknown modes could be present in the input spectrum. Hence, two methods have been proposed to solve this problem. The first one, based on *greedy pursuit* algorithms, iteratively adds elements to the approximation of the signal to improve it [20]. The second one, called *convex relaxation*, replaces the L0 norm with another penalization term, such that the criterion may be minimized more easily. Here we follow this *convex relaxation* approach, and use the *L1*-norm [19], i.e. the sum of the amplitudes of all the non-zero elements. Even if both penalizations may not lead to the same solution, such approximation has given excellent results in the performed simulations and real data processing [18]. This approach leads to the criterion:

$$J(x) = \left\|\mathbf{y} - W\mathbf{x}\right\|^2 + \lambda \left\|\mathbf{x}\right\|_1 = \left\|\mathbf{y} - W\mathbf{x}\right\|^2 + \lambda \sum_{k=-K}^{K} \left|x_k\right|. \quad (1)$$

Here $\boldsymbol{y}=[y_1, y_2, \ldots, y_P]^T$ is the vector of data; $\boldsymbol{W}=\{exp(i n_k \phi_p)\}_{p,k}$, for $p=\{1, \ldots, P\}$ and $k=\{1, \ldots, M\}$, with $n_k=k-K+1$ and $M=2K+1$ and $\boldsymbol{x}=[x_1, x_2, \ldots, x_M]^T$ is the vector of complex amplitudes of modes n_k. For applications where the variance on the noise ε_p, or covariance matrix of the noise vector $[\varepsilon^1, \varepsilon^2, \ldots, \varepsilon_p]^T$, is known, it may be taken into account in the criterion through a weighted LS-term. It can then be easily shown that this criterion is convex, hence has no local minima. Many numerical methods are available to minimize such criterion, and we use an algorithm based on an *Iterative Block Coordinate Descent* procedure [17]. This algorithm is very efficient: a correct solution can be typically found in less than 1ms using the modest computational resources available to process real-time JET data. Note that it has been shown that minimizing eq.(1) leads to an under-estimation of the amplitudes of the detected mode numbers due to the λ penalization term, but this is not an issue for real-time analysis, as the main objective is to detect the existence of the modes, their mode numbers and frequency width.

A real-time implementation of the proposed modes detection method requires an efficient tuning of the penalization parameter λ, which is related to the noise level. A rigorous interpretation of λ in a statistical framework can be found in [17]. An intuitive description of λ is that, since it increases the penalty for those solutions which invoke a larger number of modes, it influences the *SparSpe*c detection ability, hence its role when analysing frequency-degenerated spectra. For spectral analysis, this parameter has interesting physical meanings. It can be shown that for $\lambda>\lambda_{\mathrm{MAX}}=\max|\boldsymbol{W}^H\boldsymbol{y}|$ the minimizer $\boldsymbol{x}_{MIN}$ of eq.(1) is identically zero, i.e. the unique solution is the zero solution (no detected modes); and that for a given λ, the minimizer $\boldsymbol{x}_{MIN}$ of eq.(1) satisfies $\max|\boldsymbol{W}^H\boldsymbol{r}|<=\lambda$ where $\boldsymbol{r}=\boldsymbol{y}-\boldsymbol{W}\boldsymbol{x}_{\boldsymbol{MIN}}$ is called the residual (data minus the model corresponding to the estimated modes). Moreover, as $|\boldsymbol{W}^H\boldsymbol{r}|$ corresponds to the periodogram of the residual, λ can be interpreted as the maximum peak amplitude allowed in the periodogram of the residual. Finally, since $|\boldsymbol{W}^H\boldsymbol{y}|$ corresponds to the periodogram of the data, it can be shown that choosing λ to be a fraction $\lambda_{NORM}\in[0, 1]$ of the maximum of the periodogram of the data $\lambda=\lambda_{NORM}\times\max(|\boldsymbol{W}^H\boldsymbol{y}|)$, ensures the periodogram of the residual $\boldsymbol{r}$ to be lower up to this fraction relatively to the maximum of the data periodogram. The typical values for λ_{NORM} range from $\lambda_{NORM}=0.1$ in A&A where the number of time samples is very large (typically $>10^3$), to $\lambda_{NORM}=0.95$ when the *SparSpec* algorithm is applied to the real-time detection of toroidal mode numbers in a JET experiment. Following detailed simulations using various models and direct estimates for the noise on the magnetic sensors used for real-time MHD analysis at JET, in our applications the most suitable value is $\lambda_{NORM}=0.85$. This value allows for a very rapid convergence of the optimization algorithm (typically within ~600μs) and is sufficient to detect and discriminate multiple modes whose amplitudes are of interest for MHD diagnostic purposes (physics, plant protection and control issues) at JET.

3) The active MHD diagnostic system in use at JET

A key physics issue for a usable fusion reactor is the understanding and control of the burning plasma regime, a situation in which the energy carried by the fusion produced alpha particles (αs) exceeds that externally injected [15]. One of the main elements of this regime is the interaction of the αs with waves that are naturally excited in the plasma. Such interaction can be resonant, lead to efficient energy and momentum exchange between waves and particles and drive instabilities, tapping the free energy contained in the αs pressure gradient. If a significant spatial redistribution of the αs occurs, then the overall plasma performance can be limited; moreover, if this redistribution goes as far as the machine boundaries, causing net losses of the αs, then damage to the first wall can also occur. Conversely, the knowledge of the mechanisms behind the mode stability, the interaction of the modes with the αs and their redistribution can be turned into tools for the control of their phase-space behaviour and of the plasma burn itself. An example of waves that can interact resonantly with the αs is Alfvén Eigenmodes (AEs): these modes are particularly important as they are a natural Eigenmode of any magnetically confined plasma, and also because the fusion-produced αs are born with a supra-thermal speed that is typically super-Alfvenic in the usual thermonuclear tokamak plasma conditions. Hence, AEs constitute a unique way to "communicate" with the plasma, allowing them to be used as a powerful diagnostic tool for the αs and the background plasma, the so-called *MHD spectroscopy* technique [21].

A simple active method to drive and detect low amplitude modes in the plasma was pioneered and used in JET, the so-called AE Active Diagnostic (AEAD) system [22-25]. One essential and worldwide unique element of the AEAD system is the AE Local Manager (AELM). The AELM is a digital VME plant control system, with a 1kHz clock-rate, used to control the AE excitation and track in real time the evolution of the detected modes. The AELM crate contains a VME Crate Service Module (to provide timer and trigger synchronisation with the plant systems), a Real-Time Processor (to execute software running under the Wind River VxWorks operating system), a Communications Processor (to setup pre-pulse information, synchronise the real-time processor with important time points within the pulse and communicate data recorded during the pulse for archiving), and four analogue/digital input/output cards [26]. One of two algorithms can be used to derive from the available input data a single amplitude and phase pair that will be for mode detection and tracking used in real-time: the first uses the "*SparSpec*" method, and the second performs a simpler signed sum of the selected input data. A Lorentzian-model fit of the complex antenna/plasma transfer function is used to obtain the mode frequency and quality factor in real-time [27]. When using *SparSpec*, the AELM has two methods, "*highest*" and "*any*", for selecting the pair to use for tracking. The *highest* method picks at any time point t_j the pair with the greatest amplitude. The *any* method looks for a pair where the amplitude is above a given threshold at the time point t_1; if a resonance is detected, the same pair will be selected for all time points t_j, $j>1$, until tracking is lost at t_{j+1} when the search for a new pair will commence.

Figure1 illustrates the AEAD plant in the tracking mode of operation. In the full-frequency spectrum (shown in the left frame, which was only measured from *t=13sec* to *t=17sec*) we see a very narrow triangular waveform in an otherwise completely clean portion of the fluctuation spectrum: this is the system driving frequency, set to look for resonances around the frequency of an *n=1* Toroidal AE (TAE) as evaluated in real-time for that shot. In the right frame, we have the synchronously detected signal ($|\delta B_{MEAS}|$) from one magnetic pick-up coil, showing the real-time TAE frequency (f_{TAE}) and the driving frequency. Narrow sweeps of the antenna frequency occur when the complex-valued δB_{MEAS} is sufficiently close to the pre-set resonant shape: it describes a circle in the complex plan representation with a corresponding bell-shape in the $|\delta B_{MEAS}(\omega)|$ representation (as shown in the two inserts). In these instances we are in the *tracking* mode of operation: the frequency (f_{MEAS}) and damping (γ_{DAMP}) of the detected mode are measured in real-time. Such narrow sweeps occur between *t=12→sec* and *t=18sec*. After *t=21sec*, a much larger frequency sweep is seen in the right frame: no resonances close to the pre-set value have been detected in real-time. We are then in the *scanning* mode of operation, and the AELM looks for suitable antenna-driven plasma resonances in a different frequency range.

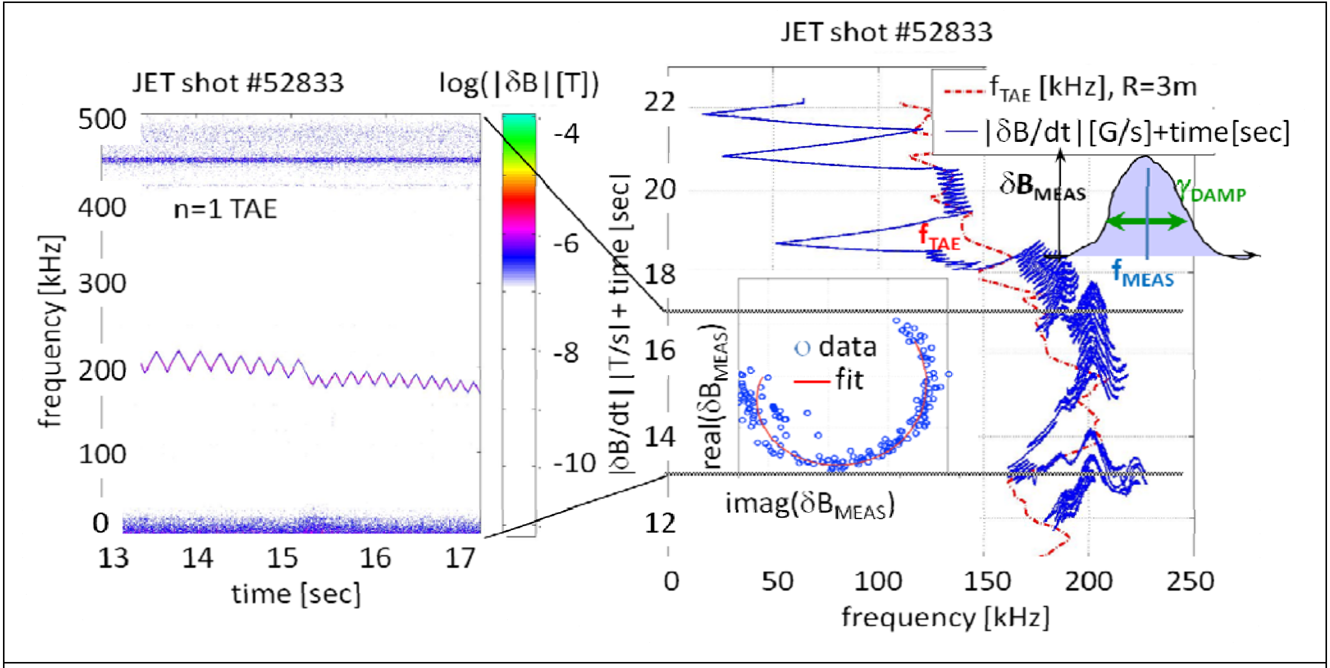

***Figure1**. An example of real-time tracking of a resonant n=1 Toroidal Alfvén Eigenmode (TAE).*

4) Real-time, unsupervised detection and discrimination of different toroidal components in a frequency-degenerated spectrum of MHD instabilities

One of the first results obtained with the AEAD diagnostic system in the most recent JET experimental campaigns has been that many modes with $|n|\sim 0\text{-}12$ and very low-damping rate $\gamma/\omega<0.2\%$ were found to be simultaneously present in plasmas without populations of resonant fast ions. Correct real-time detection and n-number discrimination of these modes is particularly important as their low γ/ω makes them very prone to become unstable if resonant fast ions were present in the plasma, and real-time control tools depend on their n's. This has been achieved imbedding *SparSpec* in the AELM tracking algorithm, allowing the detection and tracking of hundreds of individual resonances during one single discharge, which are guaranteed to have the same n-number.

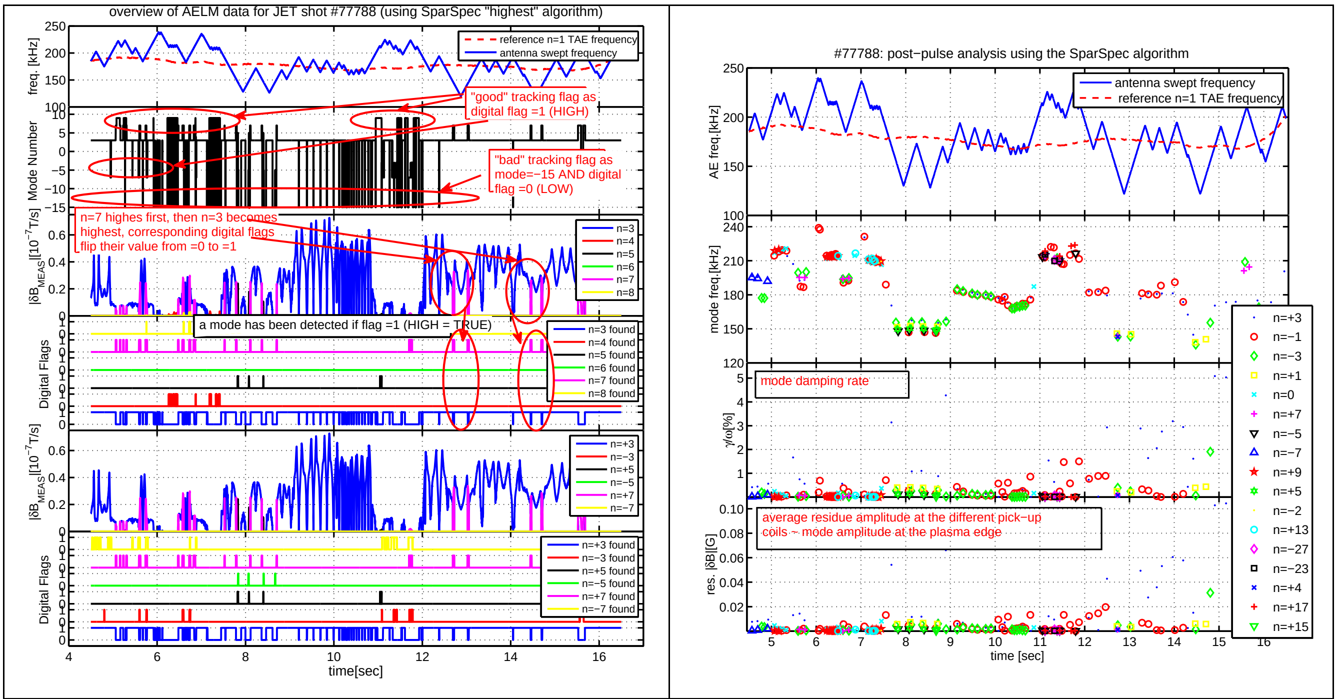

***Figure2**. An example of real-time tracking of individual TAEs for the shot #77788; the analogue signal $|\delta B_{MEAS}|$ shows the amplitude of each individual mode as detected in real-time using SparSpec.*

An example of unsupervised real-time detection and tracking of the individual n-number components in the antenna-driven spectrum is shown in fig2(a,b) for the JET shot #77788, where the AEAD system was configured to predominantly drive an odd-n spectrum, peaked towards the lower mode numbers $|n|=3,5,7$, with a negligible drive for components with $|n|>10$. In this shot the *highest* mode was selected in the real-time *SparSpec* algorithm: this explains the "switching" between detection of the different modes (and particularly between $n=3$ and $n=7$). In fig2a (left frame), we show separately the detection of all modes with $3\leq n\leq 8$, and the detection of odd-n modes rotating in the co-current ($n>0$, the preferential direction in the usual JET magnetic equilibrium) and counter-current ($n<0$) direction, with $|n|=[3,5,7]$. For comparison, in fig2b (right frame) we show the results of the post-pulse analysis using the full implementation of the *SparSpec* algorithm. All the digital signals shown at the bottom of fig2a indicate whether detection and tracking of a certain mode has been successful: if the flag is set to

high (=1), then the corresponding mode has been correctly detected and tracking is occurring, otherwise the digital signal is set to low (=0). At the start of each real-time scan *t*, the global "mode" value is initialised to a "bad" tracking value of $n=-15$ and its associated digital validity flag is set to 0. When a mode is successfully detected, the "mode" value is set to *n* and the validity flag is set to 1.
From fig2a, we note first that only very few even-*n* resonances ($n=[4,6,8]$) are detected in real-time (and confirmed by post-pulse analysis, see fig2b), compared to the number of $n=3$ resonance; second, the $n=3$ mode dominates the detected spectrum, as this is the one for which the AEAD system produces the maximum drive; third, not only co-and counter-rotating modes with the same *n* can be distinguished in real-time, but also we find that the plasma preferentially supports co-rotating modes (which are driven with the same amplitude as the corresponding counter-rotating modes). This result allows to discriminate whether the pressure profile of any resonant fast ion population is peaked on axis (reducing the damping rate of co-rotating modes) or off-axis (reducing the damping rate of counter-rotating modes), with important consequences for plasma control and burn optimization. A statistical analysis of the real-time vs. the post-pulse data (performed over many different discharges) indicates that the toroidal mode number and the mode frequency are detected in real-time with a confidence level >90%, the confidence level in the damping rate data being ~70%; this confirms the overall accuracy of the real-time calculations. The mode amplitude cannot be directly compared as, due to computational reasons, in real-time we do not perform the renormalization needed because of the λ-penalization term.

<u>5) Summary and Conclusions</u>

In this work we have reported on the application to tokamak plasmas of a new algorithm for the unsupervised and real-time detection and decomposition of a degenerate frequency spectrum, where the frequency separation between the various components is less than their full-width at half-maximum. This algorithm is based on the sparse representation of signals, as derived from its original applications to astronomical data using the *SparSpec* code. The real-time and post-pulse implementation of this algorithm has allowed an accurate and numerically efficient analysis of the measurements made with the AEAD system, which would have not been possible otherwise. Using the modest computational resources available with the AELM (a 1GHz PowerPC with a 512MB RAM running on a 1kHz clock-rate), the multi-components $|\delta B_{MEAS}(n)|$ spectrum can be fully resolved within ~650μs. The capability to perform an unsupervised and real-time detection and tracking of the individual n-components in the antenna driven spectrum constitutes an invaluable tool, which is unique to the JET tokamak. This provides accurate testing for the code prediction for the damping rate of Alfvén Eigenmodes [28], as it is paramount that the same mode be measured throughout the parameter scan. The efficiency with which the *SparSpec* method detects multiple modes in large datasets has suggested that it may be used for the concurrent real-time detection of different MHD instabilities, of interest for plasma control and machine protection in JET and ITER, as they can be accurately discriminated in real-time. This allows specifically tailored control schemes to be put in place for each individual instability, hence improving the overall control of plasma operation. This will be particularly important for future experiments approaching the burning plasma conditions, such as ITER, where real-time control of the stability of the fusion born alphas in the background "sea" of MHD modes that are expected to occur in such conditions, represents one of the key ingredients required to achieve a net energy gain.

Furthermore, and while specifically applied for the analysis of astronomical data and mode numbers in thermonuclear fusion plasmas in a tokamak device, the use of sparse approximations methods are ideally suited for applications to all domains of data analysis and control engineering where an efficient decomposition of a multi-harmonics degenerate spectrum is required from irregularly sampled data. Moreover, the computational speed and accuracy of algorithms such as SparSpec makes such a method ideally suited for real-time applications with a small number of data. These domains range from the analysis/optimization of measurement devices (an example of this being the work done for the ITER high-frequency magnetic diagnostic system [29]), to the numerous, sophisticated developments on the Sparse Representations of signals and digital images for pattern recognition, possibly in real-time.

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