

# The Josephson plasmon as a Bogoliubov quasiparticle

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## Abstract

We study the Josephson effect in alkali atomic gases within the two-mode approximation and show that there is a correspondence between the Bogoliubov description and the harmonic limit of the phase representation. We demonstrate that the quanta of the Josephson plasmon can be identified with the Bogoliubov excitations of the two-site Bose fluid. We thus establish a mapping between the Bogoliubov approximation for the many-body theory and the linearized pendulum Hamiltonian.

## 1. Introduction

The Josephson effect and vortices are at the heart of superfluid phenomena studied for decades in superconductors and liquid helium. The achievement of Bose–Einstein condensation in trapped alkali atomic gases [1] opened the way to further extend our understanding of these macroscopic quantum phenomena in a new system: cold dilute bosonic gases. Predicted 40 years ago [2] for Cooper pair tunnelling from one superconductor to another through an insulating junction, the Josephson effect is, mathematically, a result of restricting the number of states available in the one-particle Hilbert space to two orbitals into which  $N$  bosons are distributed. In real life this is usually done by weakly connecting two superfluids. This generates a wealth of physical phenomena that are essentially due to collective oscillations between the two superfluids. For trapped alkali gases the two superfluids can either be two condensates of the same species spatially separated by a potential barrier (external Josephson effect) [3,4] or correspond to spatially overlapping condensates made of atoms in two different hyperfine states (internal Josephson effect) [5].

The standard Josephson Hamiltonian is the two-mode version of the Bose–Hubbard model,

$$H = -\frac{E_J}{N}(a^\dagger b + b^\dagger a) + \frac{E_c}{4}[(a^\dagger a)^2 + (b^\dagger b)^2], \quad (1)$$

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with the Josephson coupling energy  $E_J$  and the charging energy  $E_c$ . Intrinsic to the two-mode approximation is the assumption that the parameters  $E_J$  and  $E_c$  are constant, i.e. independent of the number of particles in a given state. Thus we assume implicitly that fluctuations in the relative particle number do not change the shape of the condensate wavefunctions significantly.

In the case of identical wells,  $a$  and  $b$  are the annihilation operators corresponding to the condensate wavefunctions of the left and right wells; an evaluation of the parameters  $E_J$  and  $E_c$  has been given in [4]. In the case of the internal Josephson effect, atoms in two different hyperfine states  $|1\rangle$  and  $|2\rangle$  are confined in an optical or magnetic trap; to implement the Josephson coupling term a Raman transition is driven between the two levels [6]. The many-body Hamiltonian describing the mixture of the two quantum fluids with scattering lengths  $a_{11} = a_{22}$  can be reduced to the effective form (1) with parameters  $E_c \propto (a_{11} - a_{12})$  [6] and  $E_J = N\hbar\Omega/2$  [5], where  $\Omega$  is the Rabi frequency and the detuning is assumed to be zero.

Coherent particle exchange between two different hyperfine states has been observed in the Rabi limit [5] of the internal Josephson effect. Then the two modes  $a$  and  $b$  can be for example the hyperfine states  $|F = 2, m_F = 1\rangle$  and  $|F = 1, m_F = -1\rangle$  of  $^{87}\text{Rb}$ , for which the robustness of phase coherence has already been experimentally checked out in magnetic traps [7] using a combination of microwave and rf fields to drive the two-photon transition, or  $F = 1, m_F = \pm 1$  states of  $^{23}\text{Na}$ , which are miscible and can be trapped in optical traps [8].

A vertical array of cold atoms trapped in the anti-nodes of an optical standing wave was used to create an analogue of the ac Josephson effect [9] and a lattice of quasi-one-dimensional confining tubes formed by a pair of laser fields was recently employed to investigate the phase coherence between neighbouring sites [10]. The physics of these experiments is based on the macroscopic coherent tunnelling of atoms between the wells, and is very similar to that of the Josephson effect between two superconductors connected by an insulating junction [11].

The model Hamiltonian (1) has been studied in the classical and the quantum regime by using different mathematical techniques. Most popular are the phase representation [12], the angular momentum representation [13] and the time-dependent Gross–Pitaevskii (GP) equation in either its reduced phase-number formulation [3, 14] or its hydrodynamic version [15]. As expected on very general grounds [16, 17] and also checked numerically [18], both GP approaches yield the same excitation spectrum as that obtained by solving the time-independent Bogoliubov–deGennes equations directly. Up to now, only the relationship between the phase representation and the angular momentum representations has been pointed out [13, 17, 19]. In this paper we focus instead on the equivalence between the harmonic limit of the phase representation and the Bogoliubov approximation [20] of the Hamiltonian (1). In particular, we establish the relationship between the creation and annihilation operators in the phase representation and the Bogoliubov operators, and show that the quanta of small oscillations of the Josephson pendulum are Bogoliubov excitations in disguise.

## 2. Phase representation and Josephson plasmon

Let us introduce the relative number operator

$$n = \frac{1}{2}(a^\dagger a - b^\dagger b), \quad (2)$$

and let  $N = a^\dagger a + b^\dagger b$  be the total number of atoms, which will be treated as a  $c$ -number since it is conserved. Then a simple way to obtain the phase representation of the Josephson Hamiltonian (1) is to insert a polar decomposition of the operators  $a$  and  $b$ ,

$$a = \sqrt{N/2 + n} e^{-i\varphi/2}, \quad b = \sqrt{N/2 - n} e^{i\varphi/2}. \quad (3)$$

This yields the momentum-shortened pendulum Hamiltonian [3, 14] whose classical limit reads

$$H = \frac{1}{8}E_c N^2 - E_J \sqrt{1 - \frac{4n^2}{N^2}} \cos \varphi + \frac{1}{2}E_c n^2. \quad (4)$$

This procedure, initiated by Dirac for the one-mode field operators in his original description of the electromagnetic field, is not without problems when it comes to finding explicitly a Hermitian operator  $\varphi$  [21]. A similar idea is often invoked in condensed matter physics: it is argued that, in certain conditions and generally for systems with large numbers of particles, the  $U(1)$  gauge symmetry is spontaneously broken so that the mode operators  $a$  and  $b$  can be replaced with well defined complex numbers in an amplitude–phase representation, yielding the form (4).

In the following we shall adopt a definition of the relative phase operator inspired by [22]. We consider the states

$$|\varphi_p\rangle = \frac{1}{\sqrt{N+1}} \sum_{n=-N/2}^{N/2} e^{-in\varphi_p} |n\rangle, \quad (5)$$

where  $|n\rangle$  is shorthand for  $|N/2 + n, N/2 - n\rangle$  and the phase  $\varphi_p = 2\pi p/(N+1)$ ,  $p = -N/2, \dots, N/2$  has a discrete structure. Then we can define a phase operator by

$$e^{i\varphi} \equiv \sum_{p=-N/2}^{N/2} e^{i\varphi_p} |\varphi_p\rangle \langle \varphi_p| = \sum_{n=-N/2}^{N/2-1} |n+1\rangle \langle n| + |-N/2\rangle \langle N/2|. \quad (6)$$

It is easy to check that the Lerner criterion  $[n, e^{i\varphi}] = e^{i\varphi}$  is satisfied so one can also write  $[\varphi, n] = i$ . The eigenstates  $|\varphi_p\rangle$  satisfy the orthogonality relation  $\langle \varphi_p | \varphi_{p'} \rangle = \delta_{pp'}$  and form a complete set,  $\sum_{p=-N/2}^{N/2} |\varphi_p\rangle \langle \varphi_p| = 1$ , which makes them suitable for the definition of a phase representation. This construction solves the problem of defining a (relative) phase operator by imposing a discrete spectrum for it. In the limit of large  $N$  the phase spectrum becomes quasi-continuous and one may define a derivative  $d/d\varphi$  to obtain for the number operator the phase representation  $n = -i\partial/\partial\varphi$ .

The representation given by the phase states  $|\varphi_p\rangle$  allows us to write the Hamiltonian (1) in the form of the momentum-shortened pendulum Hamiltonian (4). In the limit of small oscillations, this effective Hamiltonian becomes

$$H \approx \frac{1}{8}E_c N^2 - E_J + \frac{1}{2}E_J \varphi^2 + \frac{1}{2}\tilde{E}_c n^2, \quad (7)$$

with the effective charging energy

$$\tilde{E}_c \equiv E_c + \frac{4E_J}{N^2}. \quad (8)$$

Equation (7) describes a harmonic oscillator with the frequency

$$\omega_0 = \sqrt{E_J \tilde{E}_c / \hbar} \quad (9)$$

and the root mean square of the number and phase fluctuations of its ground state

$$\Delta n = \frac{1}{\sqrt{2}} \left( \frac{E_J}{\tilde{E}_c} \right)^{1/4}, \quad \Delta \varphi = \frac{1}{\sqrt{2}} \left( \frac{\tilde{E}_c}{E_J} \right)^{1/4}, \quad (10)$$

fulfil the minimum uncertainty relation  $\Delta n \Delta \varphi = 1/2$ . Writing  $n$  and  $\varphi$  in terms of creation and annihilation operators,

$$n = (\alpha + \alpha^\dagger) \left( \frac{E_J}{4\tilde{E}_c} \right)^{1/4}, \quad (11)$$

$$\varphi = i(\alpha - \alpha^\dagger) \left( \frac{\tilde{E}_c}{4E_J} \right)^{1/4}, \quad (12)$$

where  $\alpha$  and  $\alpha^\dagger$  fulfil the bosonic commutation relation  $[\alpha, \alpha^\dagger] = 1$ , diagonalizes the Hamiltonian, i.e. brings it to the form

$$H = \frac{1}{8}E_c N^2 - E_J + \hbar\omega_0(\alpha^\dagger\alpha + \frac{1}{2}). \quad (13)$$

It is useful to define three regimes [5, 17] for the Josephson two-mode Hamiltonian according to the interaction strength  $E_c$ , namely the Rabi regime  $E_c \ll E_J/N^2$ , the Josephson regime  $E_c/N^2 \ll E_c \ll E_J$  and the Fock regime  $E_J \ll E_c$ . In the Rabi regime, the atoms are all in the bonding state, but behave independently. The phase is well defined and the excitation is the promotion of a single atom to the anti-bonding state. In the Josephson regime, the ground state still has a well defined phase, but the excitation forms a collective motion, the Josephson plasmon with the plasma frequency  $\sqrt{E_J E_c}/\hbar$ . In the Fock regime, the Josephson link is dominated by the interaction energy and  $n$  is a good quantum number. Therefore the ground state has a well defined atom number on each side, the phase is completely undefined and the harmonic approximation (7) is no longer appropriate.

We close this section with a comparison between this and similar phase representations that can be found in the literature. In [5] the phase is allowed to take continuous values from the beginning. In [23] the problems of defining a Hermitian phase operator are overcome by introducing an over-complete phase representation, in a basis of phase-coherent states. Far from the Rabi regime, this representation becomes equivalent to ours (cf appendix A). Finally, in [24] a number representation is used, which obeys the same commutator relations but with ‘position’ and ‘momentum’ interchanged (cf appendix B).

### 3. The Bogoliubov approximation

To implement the Bogoliubov approximation [20] for the Hamiltonian (1), we first determine the condensate wavefunction. As an ansatz we use the most general many-body state for  $N$  atoms all occupying the same mode,

$$|\Psi\rangle = \frac{1}{\sqrt{N!}} [\cos\theta e^{i\phi/2} a^\dagger + \sin\theta e^{-i\phi/2} b^\dagger]^N |\text{vac}\rangle. \quad (14)$$

Its energy expectation value is

$$\langle\Psi|H|\Psi\rangle = -E_J \sin 2\theta \cos\phi + \frac{1}{8}E_c N^2 (2 - \sin^2 2\theta), \quad (15)$$

which becomes minimal for  $\theta = \pi/4$  and  $\phi = 0$ . Thus, the condensation occurs in the bonding state, so that the operator

$$c_0 = \frac{a+b}{\sqrt{2}} \quad [c_0, c_0^\dagger] = 1, \quad (16)$$

destroys a condensate atom. The remaining mode

$$c_1 = \frac{a-b}{\sqrt{2}} \quad (17)$$

is orthogonal to  $c_0$  and consequently the commutation relations  $[c_i, c_j^\dagger] = \delta_{ij}$  hold true.

The central assumption of the Bogoliubov approximation is that one can replace the operator  $c_0$  by  $(N - c_1^\dagger c_1)^{1/2} \approx \sqrt{N} - \frac{1}{2}c_1^\dagger c_1/\sqrt{N}$  keeping only terms up to second order in  $c_1$ . This results in the Hamiltonian

$$H = \frac{1}{8}E_c N^2 - E_J + \left(\frac{1}{4}E_c N + 2\frac{E_J}{N}\right) c_1^\dagger c_1 + \frac{1}{8}E_c N (c_1 c_1 + c_1^\dagger c_1^\dagger). \quad (18)$$

We now employ the symplectic transformation

$$c_1 = u\gamma - v\gamma^\dagger, \quad (19)$$

$$c_1^\dagger = u^*\gamma^\dagger - v^*\gamma, \quad (20)$$

where the ansatz  $u = \cosh \chi$ ,  $v = \sinh \chi$  ensures  $|u|^2 - |v|^2 = 1$  and, thus, the canonical commutation relation  $[\gamma, \gamma^\dagger] = 1$ . The choice

$$\tanh 2\chi = \frac{E_c}{E_c + 8E_J/N^2} \quad (21)$$

brings the Hamiltonian (18) to the form

$$H = \frac{1}{8}E_c N(N-1) - E_J \left(1 + \frac{1}{N}\right) + \hbar\omega_0 \left(\gamma^\dagger\gamma + \frac{1}{2}\right). \quad (22)$$

The ground-state energy of this Hamiltonian is slightly lower than the mean-field energy  $E_c N^2/8 - E_J$ ; this reflects the role of interactions, which in general distribute particles on modes other than the condensation state. In the limit  $N \gg 1$  this ground-state energy becomes the same as that predicted by the phase representation. Also, the diagonal form of the Hamiltonian indicates that the energy of the Bogoliubov quasiparticles is the same as that of the Josephson–Rabi oscillator in equation (9).

The corresponding Bogoliubov ground state is defined by  $\gamma|\text{BdG}\rangle = 0$ . Its depletion number, i.e. the number of atoms that do not reside in the one-particle ground state,  $N'_1 = \langle c_1^\dagger c_1 \rangle$ , is easily evaluated to read

$$N'_1 = \sinh^2 \chi = \frac{N}{8} \sqrt{\frac{\tilde{E}_c}{E_J}} + \frac{1}{2N} \sqrt{\frac{E_J}{\tilde{E}_c}} - \frac{1}{2} = \frac{1}{4} \left( \sqrt{N} \Delta\varphi - \frac{1}{\sqrt{N} \Delta\varphi} \right)^2. \quad (23)$$

The condition for the applicability of the Bogoliubov approximation is  $N'_1 \ll N$ , which means that it is not valid in the regime  $E_J < E_c$ . Comparing with equation (10) yields that the Bogoliubov approximation breaks down when the phase is not well defined. In this case the depletion is so large that the Penrose–Onsager criterion is no longer satisfied; the notion of a single condensate is no longer appropriate and the ground state of the system will be fragmented. Indeed, we find that the  $2 \times 2$  one-particle density matrix has elements  $\langle a^\dagger a \rangle = \langle b^\dagger b \rangle = N/2$  and  $\langle a^\dagger b \rangle = \langle b^\dagger a \rangle = N/2 - N'_1$  so to have condensation on the state  $(a+b)/\sqrt{2}$  one needs to make sure that the off-diagonal elements in the one-particle density matrix are of the order  $N$ , i.e. the depletion number  $N'_1$  is negligible with respect to  $N$ .

The structure of the Bogoliubov ground state  $|\text{BdG}\rangle$  can be obtained by noticing that the transformation (19), (20) is a squeezing transformation [25]

$$\gamma = c_1 \cosh \chi + c_1^\dagger \sinh \chi = S(\chi) c_1 S^\dagger(\chi) \quad (24)$$

with the squeezing operator

$$S(\chi) = \exp \left[ \frac{1}{2} \chi (c_1^2 - c_1^{\dagger 2}) \right]. \quad (25)$$

The ground-state structure is then

$$|\text{BdG}\rangle = S(\chi)|\text{GP}\rangle, \quad (26)$$

where  $|\text{GP}\rangle$  denotes the GP ground state.

Let us establish a relation between the corresponding creation and annihilation operators by writing the number difference operator  $n$  in terms of the Bogoliubov operators,

$$n = \frac{1}{2}(a^\dagger a - b^\dagger b) = \frac{1}{2}(c_1^\dagger c_0 + c_0^\dagger c_1) \approx \frac{1}{\sqrt{2}} \left( \frac{E_J}{\tilde{E}_c} \right)^{1/4} (\gamma + \gamma^\dagger). \quad (27)$$

To obtain the final expression, we have again used  $\langle c_0 \rangle \approx \sqrt{N}$  and  $e^{-4x} = 4E_J/N^2 \tilde{E}_c$ . The operator conjugate to  $n$  is uniquely defined by the commutation relation  $[\varphi, n] = i$  and must therefore read

$$\varphi \approx \frac{i}{\sqrt{2}} \left( \frac{\tilde{E}_c}{E_J} \right)^{1/4} (\gamma - \gamma^\dagger). \quad (28)$$

This demonstrates that the operator sets  $\alpha, \alpha^\dagger$  and  $\gamma, \gamma^\dagger$  are identical. Therefore the Hamiltonian (13) of the linearized pendulum is identical to the Bogoliubov Hamiltonian (22) and, consequently, the Josephson plasmon can be viewed as a Bogoliubov quasiparticle.

#### 4. Concluding remarks

In the Rabi and in the Josephson regime, a split condensate has a well defined relative phase and that is why Bogoliubov theory works. With increasing effective interaction  $E_c$ , the uncertainty in the relative particle number decreases from  $\sqrt{N}/2$  in the Rabi regime via  $(E_J/4E_c)^{1/4}$  in the Josephson regime to a value much smaller than unity in the Fock regime. At the same time, the fluctuations of the relative phase keep growing until the phase becomes completely undefined. In the Bogoliubov approach, this corresponds to an increasingly larger depletion that finally becomes of order  $N$  and, thus, violates the condition that most of the atoms have to reside in the same one-particle state.

In conclusion, we have established the equivalence between the harmonic limit of the phase representation and the Bogoliubov approximation in both the Rabi and the Josephson regimes. The quanta of the Josephson–Rabi oscillator are the quasiparticles of the Bogoliubov theory. The vacuum of the Bogoliubov theory is the ground state of the phase-number harmonic oscillator. Thus, despite their different mathematical appearance, both approaches can be mapped onto each other and describe the same physics.

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#### Appendix A. The phase-coherent state representation

The states (5) are not the only possible meaningful phase states. Another widely used choice is the over-complete set

$$|\theta\rangle = \frac{1}{\sqrt{2^N N!}} [a^\dagger e^{-i\theta/2} + b^\dagger e^{i\theta/2}]^N |\text{vac}\rangle, \quad (A.1)$$

where  $\theta$  is a continuous variable. In this representation and for  $E_J \ll N^2 E_c$  the Hamiltonian reads

$$H = \frac{1}{8} E_c N^2 - E_J \cos \theta - \frac{1}{2} E_c \frac{d^2}{d\theta^2}, \quad (A.2)$$

which agrees with the pendulum Hamiltonian (4) in the Josephson regime and  $n \ll N$ . The reason for this agreement is that for  $n \ll N$  the coefficients of the expansion of (A.1) in the basis  $|n\rangle$  can be approximated by

$$|\langle n|\theta\rangle| \propto e^{-n^2/N}. \quad (A.3)$$

Since in the Josephson regime the number fluctuations are much smaller than  $\sqrt{N}$ , the Hilbert space is explored for  $n$  only in the range between  $\pm\sqrt{N}$ , and the coefficients (A.3) become flat. But this is precisely what characterizes the phase states (5), so this argument proves that in this regime the two descriptions indeed become identical.

### Appendix B. The number representation

Instead of working in a basis of phase states one can also decompose  $|\Psi\rangle$  into the number states  $|n\rangle$ ,

$$|\Psi\rangle = \sum_{n=-N/2}^{N/2} \Psi(n)|n\rangle, \quad (\text{B.1})$$

and assume that  $\Psi(n)$  changes smoothly between consecutive values of  $n$ . Going from one representation to another is achieved by the Fourier sum

$$\Psi(n) = \frac{1}{\sqrt{N+1}} \sum_{p=-N/2}^{N/2} e^{-in\varphi_p} \Psi(\varphi_p), \quad \varphi_p = \frac{2\pi p}{N+1} \quad (\text{B.2})$$

for wavefunctions, while for the phase operator it acts as a derivative,  $\varphi = i\partial/\partial n$ . This brings the Hamiltonian (1) to the form (4), but now with  $\varphi$  being a derivative. After a linearization, we obtain a harmonic oscillator where ‘position’ and ‘momentum’ are interchanged.

Another route, followed in [24], is to start directly from the two-mode Hamiltonian (1) and to decompose it into the number states  $|n\rangle$  to obtain

$$\langle n|H|\Psi\rangle = \frac{1}{8}E_c N^2 - E_J \sqrt{1 - \frac{4n^2}{N^2}} \frac{\Psi(n+1) + \Psi(n-1)}{2} + \frac{1}{2}E_c n^2. \quad (\text{B.3})$$

With the operator identity

$$\cos\left(i\frac{d}{dn}\right)\Psi(n) = \frac{1}{2}[\Psi(n+1) + \Psi(n-1)] \quad (\text{B.4})$$

there follows the desired expression.

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