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THE KINEMATICS OF THE SCORPIO–CENTAURUS ASSOCIATION AND GOULD'S BELT

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SUMMARY

The convergent point of the proper motions of all stars in the FK4 catalogue, Gould's belt (Clube) and the Scorpio–Centaurus association (Bertiau) have been discussed by a maximum likelihood method. About 10 per cent of them seem to form an expanding group with vertex $l^{\mathrm{II}} = 240°$, $b^{\mathrm{II}} = -23°$ but the exact membership is vague and the group cannot be used for calibrating luminosity criteria.

Confining the discussion to the neighbourhood of Scorpio–Centaurus greatly improves the concentration of co-movers and a firm list of 47 stars can be isolated. Their vertex is $l^{\mathrm{II}} = 236°\cdot5$, $b^{\mathrm{II}} = -25°\cdot0$. The uncertainty is $\pm 6°$ in the most important co-ordinate which produces a negligible uncertainty in the luminosities. The K term is uncertain by a factor of two because of the uncertainty of the vertex. The discussion is in good accord with Bertiau except that the FK4 proper motions are about 20 per cent bigger than the N30 proper motions used by him and consequently all the luminosities are about $0^{\mathrm{m}}\cdot4$ fainter. The distances correlate well with galactic longitude; the residual scatter is 18 per cent at most. Luminosities derived from hydrogen line indices and spectral types are re-discussed. The changes in luminosity produced by rotation predicted by Hardorp & Strittmatter are compared with observation.

Of the double stars, one visual binary, γ Lup, provides a measure of mass and two others should provide orbits if carefully followed. The eclipsing binary μ^1 Sco is a member and provides a measure of mass, radius, luminosity and surface brightness.

1. INTRODUCTION

Kapteyn (1914) pointed out that the bright blue stars in the neighbourhood of the constellations Scorpius and Centaurus had proper motions converging to a point in the same manner as the Hyades stars. This discovery provoked a great deal of interest, especially among radial velocity observers who found that most of these stars had radial velocities several km s^{-1} too great to be accounted for by parallel motion. This work was reviewed and extended in an encyclopaedic paper by Blaauw (1946). In a later paper (1952) he argued that the excess radial velocity arose from an expansion of the association. The position of the vertex, the derived absolute magnitudes and any expansion are all sensitive to systematic errors in the proper motions. Bertiau (1958) subsequently redetermined these quantities using newly available N30 proper motions and other fresh data.

On the other hand, Smart (1939) has denied the separate existence of such a group, holding that it is merely a group of stars of small peculiar motion whose convergence reflects the solar motion. A similar conclusion has been reached by Petrie (1961). These objections implicitly raise the question of the overall frequency distribution of stellar space velocities. Is it an ellipsoidal or similarly smooth distribution with a preponderance of stars of small peculiar motion? Is it solely made up of clumps—moving groups—whose dimensions are limited by the accuracy of the observations? Or are the motions clumped in the direction of galactic rotation but not otherwise; or some combination of the two?

Some astronomers have linked the space motions of Scorpio–Centaurus with those of other nearby B-stars. Eggen (1961) presented evidence that they consisted of one system whose motions toward the galactic centre were a linear function of distance. He used the N30 system of proper motions but emphasized that all these conclusions were very sensitive to the proper motion system in the Southern Hemisphere. Similar motions were exhibited by the cepheids δ Cephei and α Ursa Minoris. Using the same data Clube (1967) linked Scorpio–Centaurus with the bright B-type stars in Gould's belt, a configuration of bright stars tilted with respect to the Milky Way. He showed that the radial velocities of stars in a limited spatial region were consistent with uniform velocity plus an expansion towards the galactic centre. Lesh (1968) discussed the space motions of B-stars North of declination $-20°$ using much new data, especially proper motions on the FK4 system and spectral types. She formulated the interaction of expansion with galactic rotation. There was evidently an expanding system among her stars but it could not be unambiguously separated.

In the upper Scorpius region the physical connexion of the stars is obvious from their entanglement in the bright nebulosity. This is well shown in Fig. 1 of Morgan, Stromgren & Johnson (1955) and others published by Garrison (1967). These nebulosities lie on the Northern edge of chart 4 of the *Mount Stromlo Atlas* (Rodgers *et al.* 1960a) where they border nebulosity No. 129 of Rodgers *et al.* (1960b). An anonymous nebulosity of low surface brightness extends from this region towards the Milky Way on chart 3 of the *Mount Stromlo Atlas*. However, association with nebulosity is at best a subjective criterion and there are many stars with closely similar proper motions and radial velocities which have no neighbouring nebulosity.

2. THE METHOD ADOPTED

2.1 *Maximum likelihood and the Blaauw Model*

The proper motions of a group of stars converge to a vertex either if their space motions are parallel or if they are expanding uniformly from a moving point (Blaauw 1964). In the following such a configuration is termed a ' Blaauw Model '. He demonstrates that, in addition to the convergence of the proper motions, the following relations are obeyed

$$\mu_{\parallel} = \pi S \sin \lambda$$
$$\rho = S \cos \lambda + rk + K$$

where $\mu_{\parallel}$ is the component of observed proper motion directed towards the vertex, π is the parallax and S is the stream motion. λ is the angular distance of the star from the vertex and ρ its radial velocity. r is the distance and rk is the ' expansion term ' in the radial velocities. K comprises any systematic effect in the radial velocities which arises from non-kinematic causes. The same notation is used throughout this paper.

In practice strict convergence always fails, firstly because of errors of observation and secondly because there is no *a priori* method of knowing whether all stars are truly co-moving. If a realistic estimate of the errors of the proper motions is available one can compute the expected departure from convergency; if the departure from convergency of any star rates a very low probability then it may be supposed not to co-move. But even if the frequency function of the errors is accurately known such a method will not produce a perfect membership list; for one can rank only probabilities of membership and these cannot be translated into certainties.

The method developed here is similar to the ' maximum likelihood ' method of Brown (1950). One chooses an array of vertices and for each vertex and each star one computes the component of its proper motion perpendicular to the great circle joining the stars to the vertex $\mu_\perp$. The corresponding component of the error in proper motion is also computed, assuming that the errors in Right Ascension and Declination are uncorrelated, $\sigma_\perp$. If the stars were truly moving towards the vertex then $\mu_\perp$ would be an error of observation sampled from a parent population with standard error $\sigma_\perp$. In practice $\mu_\perp$, $\mu_\parallel$ and $\sigma_\perp$ are evaluated from vector formulae which are readily adaptable to machine computation; it is not appropriate to enlarge on the details here.

Writing $t = \mu_\perp/\sigma_\perp$ and assuming the normal law of errors, the probability, P, of t occurring is given by

$$P = (1/\sqrt{2\pi}) \exp(-t^2/2)$$

and for n stars the probability of a given set of errors is

$$\bar{P} = \prod^{n} P.$$

Considering all the trial vertices, contours of $\bar{P}$ can be drawn and the most probable vertex quickly found. At the first attempt P may be absurdly low because of the inclusion of stars which are not co-moving. This is tested by forming

$$\sum^{n} t^2$$

which is distributed as χ^2 with $n-2$ degrees of freedom when t is normally distributed. If an unacceptably high value of χ^2 is obtained then the star with lowest P must be rejected and the whole solution repeated. Further stars can be rejected until a reasonably probable value of χ^2 is obtained. In practice one may eliminate more than one star at a time from the solution provided that the most probable vertex does not move so far between solutions that it changes the probability rankings. The assumption of the normal law is essential to the use of the χ^2 distribution and the contours of $\bar{P}$ depend critically upon it; but the rejection of individual stars depends on the much weaker assumption that P decreases monotonically as t increases.

The present investigation was carried out with proper motions from the FK4 catalogue (Fricke & Kopff 1963) because it was believed that this catalogue provided a substantial improvement over previous fundamental catalogues at the Southern declinations where most of the stars investigated lie. The corrections to the precession and motion of the equinox derived by Fricke (1967) were applied. The FK4 tabulates not only the standard error for each star but also (on p. 8) an estimate of the uncertainty of the system. They were combined by taking the square root of the sum of the squares. In a limited area of sky this exaggerates the errors because the errors of the system correlate between neighbouring stars and will be partially removed by a spurious shift in the vertex. This is compensated by the fact that no allowance has been made for the uncertainty in Fricke's corrections (his tabulated errors are larger than the random errors of the best observed stars). The normal law of errors was assumed. It is probably a good approximation for the random errors but has been adopted purely for convenience for the systematic errors. Especially in the South the FK4 system has been set up more by educated judgement than by uncritical weighting of the data.

2.2 *Parent Population*

In order to throw as much light as possible on the objections made by Smart and Petrie and on the different kinematical models proposed by Eggen and Clube the parent population of stars was extended to extremely wide limits to remove any suggestion of *a priori* bias. The parent list consisted of: (1) The whole FK4 catalogue; (2) All Clube's stars in Gould's belt; (3) All bright cepheids (Blaauw & Morgan 1954); (4) All stars in Bertiau's (1958) Table II. All stars under (2), (3) and (4) were corrected for double star motion where this was judged to be physical. The mass-ratio was inferred from the light ratio using the mass-luminosity relation of Harris, Strand & Worley (1963). For wide pairs the observed double star motion was combined with the mass-ratio to reduce the motion of the brighter star to that of the centre of gravity. It was assumed in close pairs the meridian observers had observed the centre of light and the motion adjusted accordingly. Where any star did not occur in the FK4 it was searched for in the GC (Boss 1937) and the N30 (Morgan 1952). The proper motions were reduced to the FK4 system following Brosche, Nowacki & Strobel (1964). The errors in the GC were multiplied by 1·3 following Schlesinger & Barney (1939). All the N30 stars appear in the GC as well and weighted means were taken of the proper motions from the two sources. The proper motions of stars in Bertiau's Table II but not in the FK4 were taken explicitly from that source and reduced from the N30 to the FK4 system. The Morgan–Oort precession corrections were removed before applying those of Fricke. Properly the errors in the proper motions should have been augmented to allow for the uncertainties in the tables reducing one meridian system to another. This was neglected because the accuracies of the stars not in the FK4 were not high enough to make the alterations significant.

No allowance was made for galactic rotation because the present investigation is searching for a group of stars whose galactic orbits intersect. The corrections for galactic rotation are based on circular motion which is incompatible with intersection. Lesh (1968) has now formulated the motion to be expected from expansion modified by the galactic force field but it is not clear how to make these formulae the the subject of an unambiguous statistical test.

2.3 *Cut-off at low accuracy*

If a star has a proper motion commensurate with its error then $\mu_\perp/\sigma_\perp$ will remain small whatever vertex is tested. Such stars would never be rejected from the solution. Accordingly all stars with $\mu/\sigma < 4$ were rejected at the outset. This implies the rejection of true members which are too distant in space or too close in angle to the vertex or its pole. Subsequent discussion will show that members of Scorpio–Centaurus have $\mu_\perp/\sigma_\perp < 2$ and median $\mu/\sigma = 8$. Such a star, if 90° from the vertex could be a field star whose true vertex is 2/8 radians from the cluster vertex. This is doubled for the adopted cut-off $\mu/\sigma = 4$. Because of the proximity of the Scorpio–Centaurus vertex to the antapex of Solar Motion any relaxation of the μ/σ restriction brings in a large number of stars of small peculiar motion.

This restriction removed nearly all the cepheids in the original list.

3. THE WHOLE SKY

3.1 *Application to whole sky*

In this paragraph we apply the method described in Section 2.1 to the parent population of stars listed in Section 2.2 covering the whole sky. Initially vertices were chosen over an area of 40° in galactic longitude and 30° in galactic latitude centred on Bertiau's (1958) preferred solution I (vertex $l^{\mathrm{II}} = 238$, $b^{\mathrm{II}} = -28$). The most probable vertex had an improbably high value of χ^2 so approximately a quarter of the stars were rejected—those least likely to be members. The process was continued with progressively less stars and over a progressively smaller area of sky until a value of χ^2 was arrived at which did not exceed the 0·05 level of significance. There were 233 stars remaining, converging towards $l^{\mathrm{II}} = 240$, $b^{\mathrm{II}} = -23$, about 7° from Bertiau's vertex. If these stars were truly co-moving then they should lie on a straight line when cos λ (λ the angular distance between the star and the vertex) is plotted against radial velocity ρ. Unfortunately, this diagram exhibits far more scatter than can be ascribed to errors of observation. Stars held by Thackeray (1967) to be members of Scorpio–Centaurus define a line

$$\rho = 6+21\cos\lambda$$

with deviations up to 9 km s^{-1}. The other stars seem to cluster loosely about this line and 125 of them lie within the range of Thackeray's stars.

3.2 *Comparison with Andrews*

If these 125 stars are truly co-moving then their astrometric distances should correlate with distances derived from astrophysical criteria. Andrews (1968) has estimated the distances of 43 of them from the strength of their Hα absorption. The distance moduli derived by Andrews are compared in Fig. 1 with the present study. The open circles are members of Sco–Cen as listed in Bertiau's Table 1. The broken lines are at 45° through the extreme members of Sco–Cen. Table I lists 12 stars for which there is strong evidence of co-motion with Sco–Cen and two for which there is not. Omitting these last two, the mean astrometric distance modulus of 41 stars is 5·52 where Andrews' mean is 6·34. It appears that Andrews' distance moduli are overestimated by $0^{\mathrm{m}}{\cdot}82 \pm 0^{\mathrm{m}}{\cdot}08$ s.e. of mean. Removing this difference there is a r.m.s. scatter of $\pm 0^{\mathrm{m}}{\cdot}52$ of which about $\pm 0^{\mathrm{m}}{\cdot}38$ is contributed by astrometric errors. The residual scatter, $\pm 0^{\mathrm{m}}{\cdot}36$ is rather smaller than Andrews own estimate of his error, $\pm 0^{\mathrm{m}}{\cdot}5$ on the main sequence and $\pm 0^{\mathrm{m}}{\cdot}3$ for supergiants. Apart from the rejected star HR 6714 there are no stars brighter than luminosity class III in question.

3.3 *Comparison with Eggen*

Eggen (1961) published distances of the apparently bright B-stars based on their MK luminosities calibrated by Johnson & Iriarte (1958) and freed from interstellar

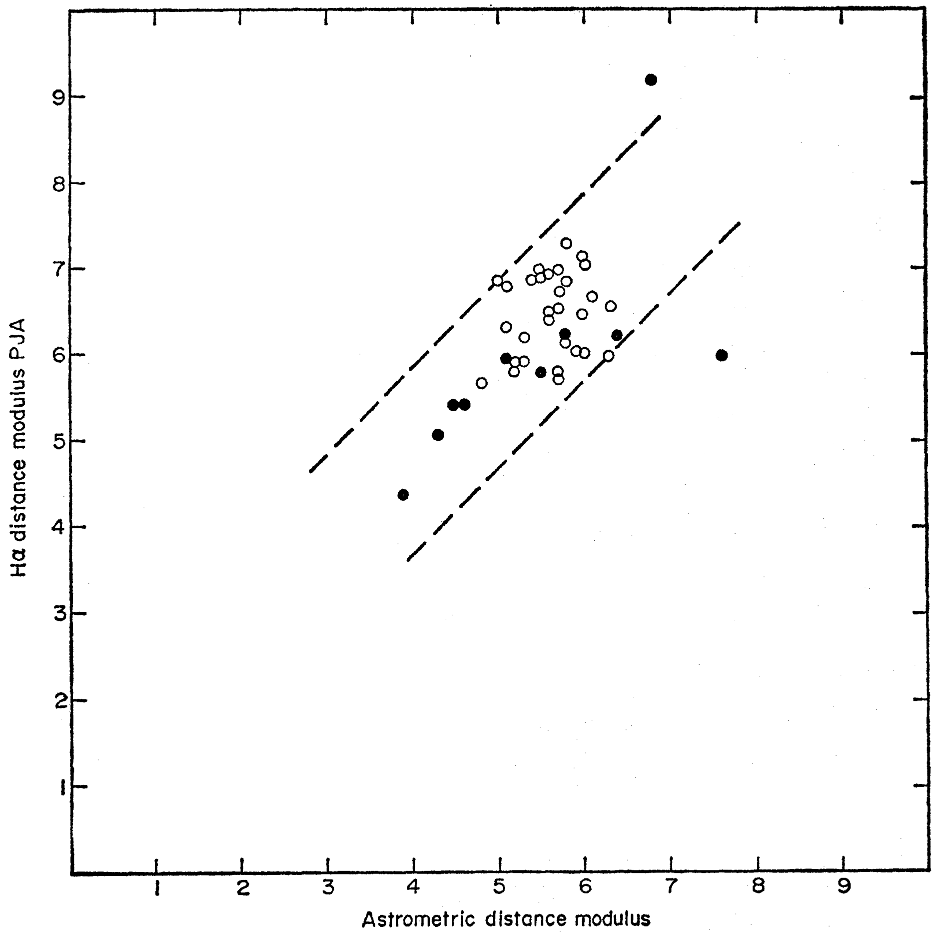

Fig. 1. Comparison between distance modulus derived by Andrews from Hα and the astrometric value. Open circles are members of Scorpio–Centaurus (Bertiau Table I) and filled circles other stars. The broken lines are drawn at 45° to the axes through the extreme members of Sco–Cen.

absorption. There are 72 stars in common with the 125 stars selected in Section 3.1. The distance moduli are compared in Fig. 2 with the same symbols as Fig. 1. Table I lists the stars in common which are not members of Sco–Cen (Bertiau Table I). Omitting the five stars which lie outside the scatter of the Sco–Cen stars the remainder show a difference in distance modulus (Astrometric—MK) of $0^m.41 \pm 0^m.08$ (s.e. of mean). The r.m.s. scatter is $\pm 0^m.61$ of which about $\pm 0^m.38$ is contributed by astrometric errors. The residual error $\pm 0^m.49$ is rather larger than that shown by Andrews' Hα photometry but is still only of the order of ± 1 luminosity class.

3.4 *The remaining stars*

Of the 125 stars selected in Section 3.1 there are 49 stars which do not appear in the lists of Andrews or Eggen. Eight are in Scorpio–Centaurus according to Bertiau and the remainder are mostly of later type. Distance moduli were derived for all these stars from either their spectroscopic or their trigonometric parallax, whichever was the more accurate and the two distance moduli are compared in Fig. 3. Little correlation is apparent. Of the 41 not in Sco–Cen eight have irreconcilable distances and a further five are irreconcilable with any modern theory of stellar evolution. They

TABLE I

Stars which share the vertex and radial velocity behaviour of Scorpio–Centaurus

HR	(*m–M*) Jones	(*m–M*) Eggen	(*m–M*) Andrews	Spectral Type	Remarks
		A. Astrometric and astrophysical distances compatible			
153	6·4	6·1	6·2	B2 V	Cass–Tau
1044	5·4	4·9	5·8	B5 V	α Per cluster
1087	5·4	4·9	—	B5e	α Per cluster
1145	4·6	4·9	5·4	B6 V	Pleiades Taygeta
1149	4·4	5·3	5·4	B7 III	Pleiades Maia
1156	4·4	5·5	—	B6 IVnn	Pleiades
1350	5·0	6·3	6·0	B6 III	
3849	4·8	6·0	—	B5 V	
4390	5·5	5·5	—	B5 Vn	
5056	4·6	5·3	—	B1 V	
5211	4·3	—	5·1	B8 V	3 Cen B
5358	6·5	7·4	—	B5 II	
5902	5·8	—	6·2	B3 V	II Sco
6161	3·9	—	4·4	B9 IV	
6588	6·0	5·4	6·0	B3 V	
6875	6·4	—	6·9	B3 Vn	Sco–Cen
7029	6·0	7·3	6·9	B2 V	Sco–Cen
7129	6·7	—	6·9	B9 III	Sco–Cen
7623	5·9	6·7	—	B3 IV	
		B. Astrometric and astrophysical distances incompatible			
801	7·6	6·2	6·0	B3 V	Cass–Tau
3663	4·2	6·5	—	B3 IV	
6527	5·8	4·9	—	B1 V	
6714	6·8	8·9	9·2	B5 Ib	
8773	7·1	5·4	—	B5 pe	

have spectral types around K0 III which no theory can make contemporary with the B2 V stars of which there are several in Sco–Cen.

3.5 *Comparison with Clube*

All 110 stars believed by Clube (1967) to belong to Gould's belt were put on the parent list in Section 2.2. Of the 125 selected in Section 3.1 there remained

In Scorpio–Centaurus	37	
In α Per cluster	2	but not α Per itself
In Pleiades	3	but not Alcyone
Others	4	one in Cass–Tau has an astrometric distance incompatible with its astrophysical distance
Total	46	

Thus the only part of Gould's belt compatible with a Blaauw model is Scorpio–Centaurus. This is not a direct criticism of Clube, because his model, expanding

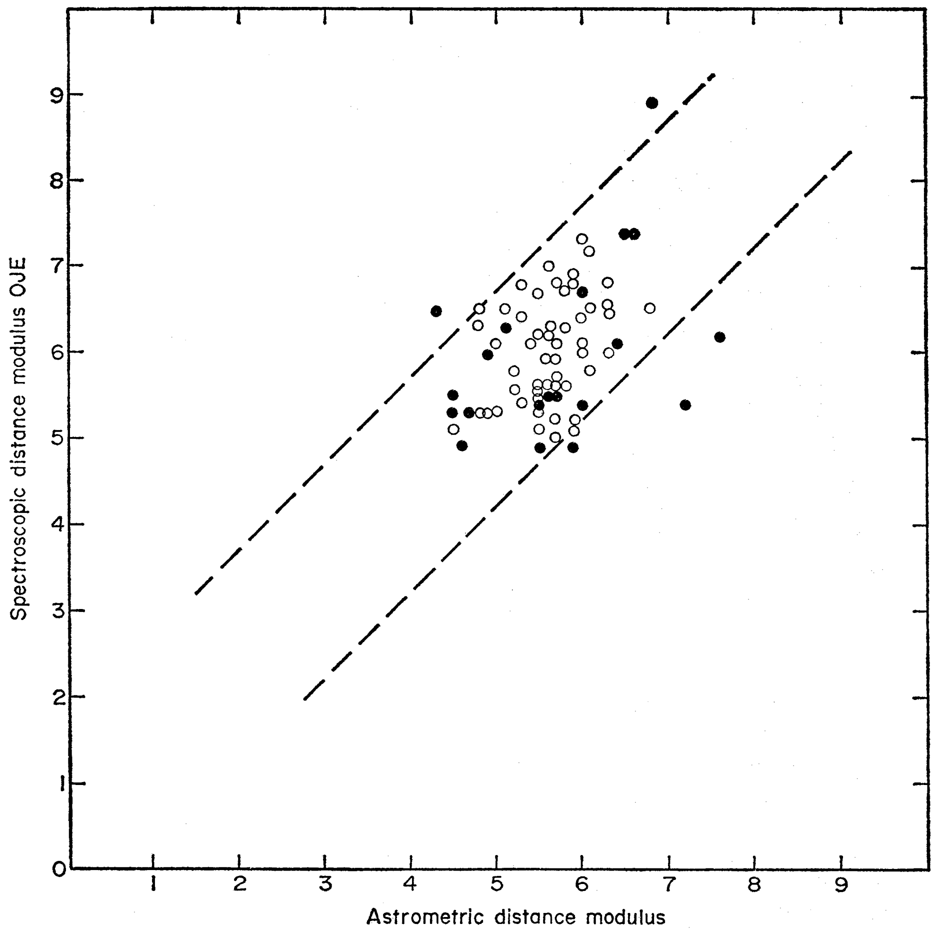

FIG. 2. *Comparison between Spectroscopic Distance Modulus after Eggen* (1961) *and the astrometric value. The symbols are the same as in Fig.* 1.

along one dimension only is not strictly comparable with Blaauw's. It was nevertheless thought worthwhile to confront the observations with Blaauw's model because Clube believes Gould's belt to be elongated along its direction of expansion and the two models are identical when the stars lie exactly on their expansion vector. The same arguments apply to the kinematic model of Eggen (1961) who argues that the bright early-type B-stars show a velocity gradient of 40 km s^{-1} kpc^{-1} towards the galactic centre.

3.6 *Weakness of the whole-sky approach*

The weakness of the present discussion lies in the segregation of stars in the (ρ, cos λ) plot. Although the Scorpio–Centaurus stars do seem to define a ridge line the exact boundaries are very ill-defined. Once a view has been taken over the region of the (ρ, cos λ) plot to include, then S the stream motion is automatically defined. This arbitrary value of S then affects all the distances and consequently all the luminosities. For this reason the discrepancies with the Hα and MK luminosities have not been emphasized; although S would have to be increased to 30 km s^{-1} to reconcile the Hα luminosities and this does seem to be more than the observations will allow.

For the same reason it is hopeless to look for the correlation between ρ and distance which should appear in an expanding association.

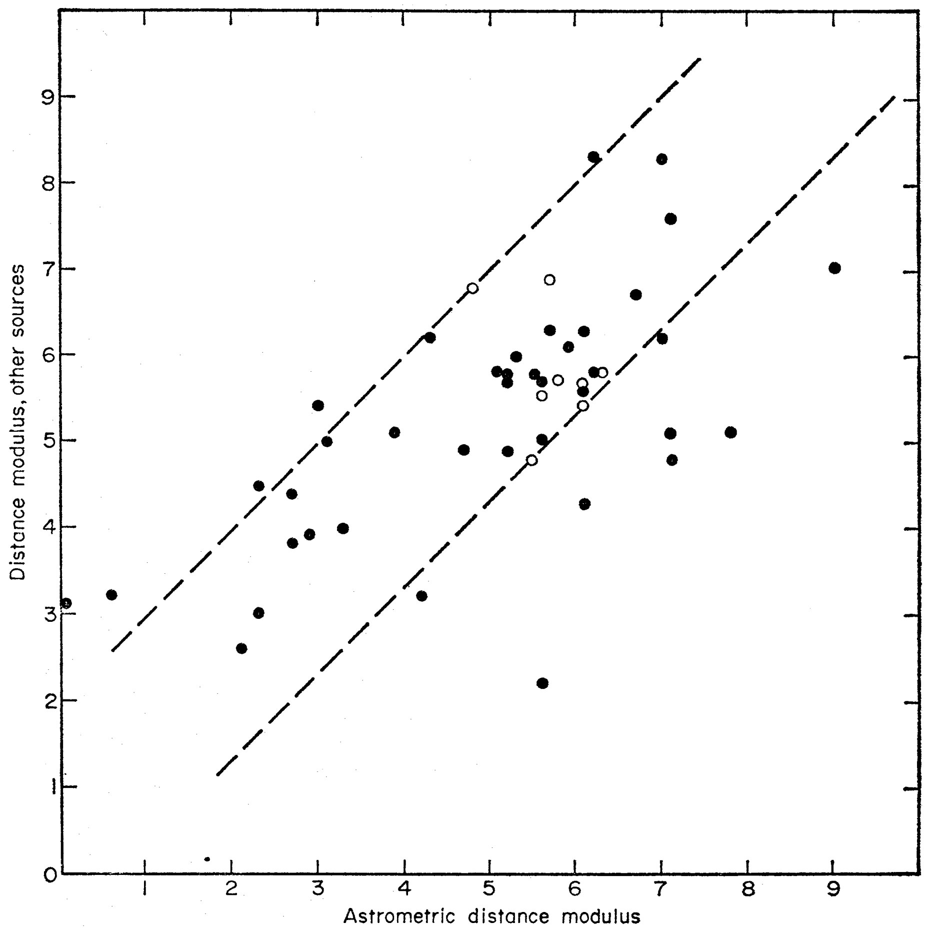

FIG. 3. *Comparison of best available distance modulus with astrometric distance modulus for stars which do not appear in Figs* 1 *or* 2. *The symbols have the same meaning as in Fig.* 1.

In this Section (3.1–3.5) we have investigated whether the motions can be reconciled with a Blaauw (1964a) model. This model was preferred because the most inaccurate quantity—the distance—was eliminated and the proper motions could be subjected to a straightfoward statistical test. More complicated kinematic models require a combination of the distances and proper motions to be tested. Disregarding the difficulty of segregating the stars in the (ρ, cos λ) diagram this approach gave only limited success. Even if we confine oureslves to OB stars there remain a few whose astrometric and astrophysical distances are incompatible. Any kinematic pattern which has been revealed in this paragraph is not consistent enough to provide either reliable new astrometric distances or a list of stars with common kinematic behaviour which might betray a common origin.

4. THE NEIGHBOURHOOD OF SCORPIUS AND CENTAURUS

4.1 *Application to a limited area of sky*

To make any further progress more information is needed. This should not be astrophysical for fear of becoming involved in a circular argument. The obvious approach is to appeal to the star positions. Fig. 4 shows the boundaries of Scorpio–Centaurus as drawn by different investigators.

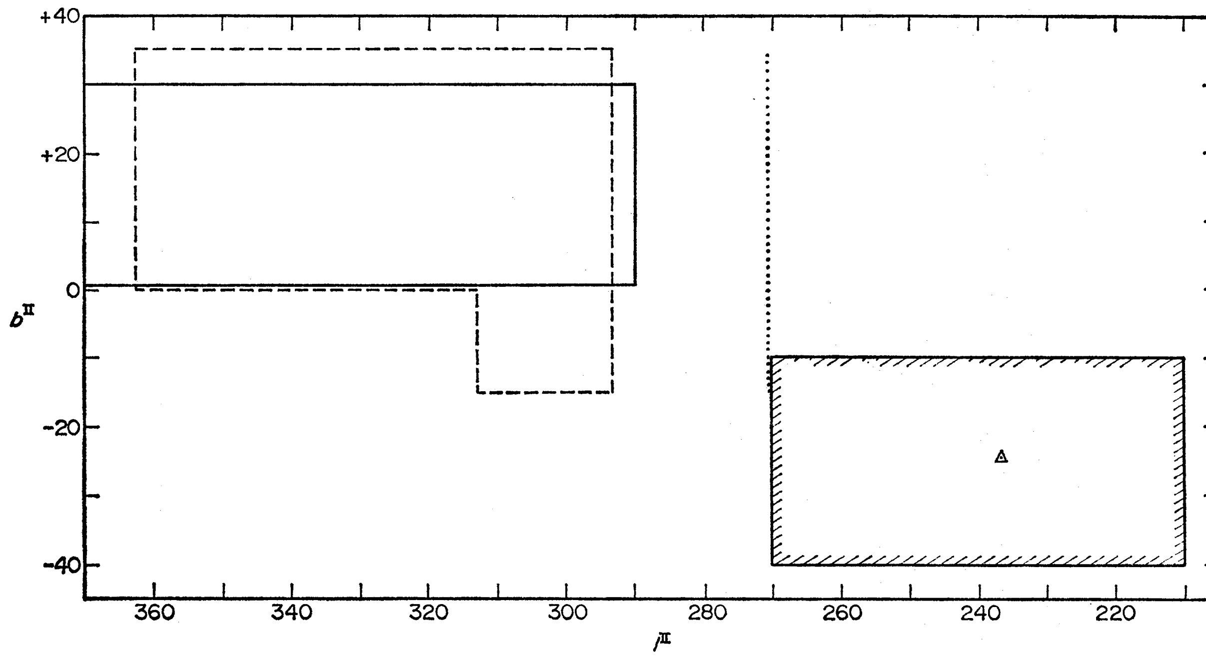

FIG. 4. *Galactic coordinate boundaries of (a) limited area discussed in this paper (Section 4.1* et seq.), *continuous line; (b) Bertiau's (1958) area, broken line; (c) lower limit of galactic longitude adopted by Blaauw (1946) dotted line; (d) neighbourhood of convergent, hatched line. This last area is expanded in Fig. 7. The triangle is the adopted convergent.*

Before drawing the boundaries for the present investigation it is interesting to compare the 125 stars selected in Section 3.1 with the discussion of Bertiau who also used only a limited area of sky. There are 61 stars in common which exhibit a systematic difference in proper motion. The mean total centennial proper motion is $2''\cdot75$ as given by Bertiau and $3''\cdot25$ here—the difference arises almost entirely from the systematic difference between the N30 and FK4 catalogues; the precession corrections have only a minor effect. The corresponding change in luminosity is $0^{m}\cdot36$ with the FK4 proper motions making the stars fainter.

4.2 *Comparison with Thackeray*

All Thackeray's (1967) radial velocity stars are in Bertiau's list and so were included in the parent list of Section 2.2. Of his 45 full weight stars six were rejected on proper motion grounds (HD 113791, 120908, 144217, 144218 and 148605); of his 26 half weight stars three were rejected (HD 103884, 106490 and 111123). After they were removed the most probable convergent for Thackeray's stars was also found to be $l^{II} = 240$, $b^{II} = -23$.

4.3 *Delineation of the area*

The boundaries of the limited area discussed in the present paragraph were selected to provide as high a concentration of co-moving stars as possible. It is not suggested that no members lie outside this area—it is merely suggested that in this area the number of co-moving stars is high enough for the true kinematic pattern to emerge without being obscured by too high a number of field stars. The boundaries adopted were

$$290^\circ \leqslant l^{II} \leqslant 10^\circ$$
$$1^\circ \leqslant b^{II} \leqslant 30^\circ$$

and the limiting value of μ/σ was raised to 5·0. There were 68 stars remaining. Their most probable vertex was again $l^{II} = 240$, $b^{II} = -23$.

4.4 *Radial velocities*

The radial velocities used in this discussion are identical with those used by Thackeray (1967) except for the inclusion of data presented by van Albada & Sher (1969). The author is very grateful to Dr Thackeray for communicating his valuable unpublished compilation and evaluation. In Fig. 5 the radial velocity ρ, of each star is plotted against cos λ. There is a broad correlation and a few wildly divergent stars. The quality of the radial velocity does not seem to affect the scatter. Clearly the most probable relation between the variables does not go through the origin. There is a 'K term' of about 6 km s^{-1}.

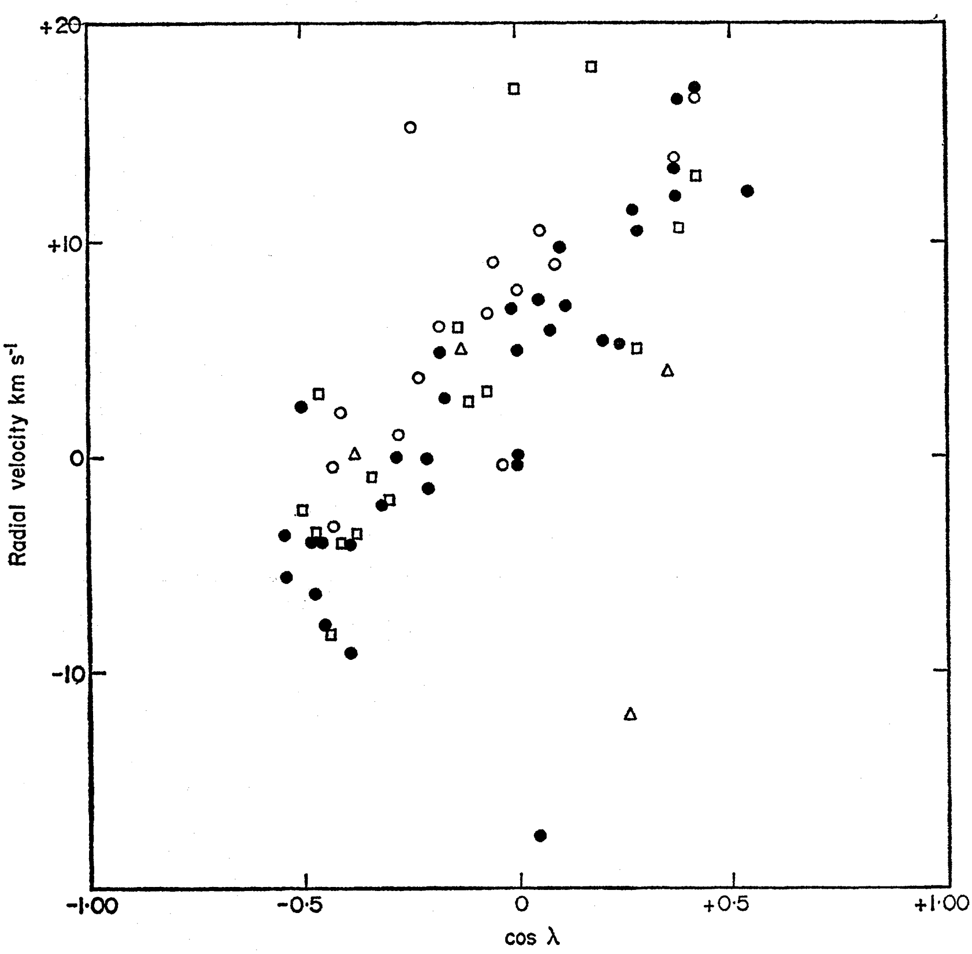

FIG. 5. *Comparison of radial velocity with cos λ in the field demarcated in Fig.* 4. *Symbols denote quality of radial velocity*: *open circles* a, *filled circles* b, *open squares* c *and triangles* d.

Three explanations have been put forward for this K term. Firstly, there is the relativistic red shift. Greenstein & Trimble (1967) give the formula

$$K\,(\text{relativistic}) = 0{\cdot}635 \text{ km s}^{-1}\,(\mathscr{M}/\mathscr{M}_{\odot})(R/R_{\odot})^{-1}$$

where the second factor is about two for main sequence stars of the temperatures considered here (Harris *et al.* 1963). Many of the stars considered here are evolved so that the mean correction must be about 1 km s^{-1} which was the value adopted. Bertiau (1958) adopted 1 km s^{-1} for B0 stars and 0·5 for B1 stars.

Secondly, it has been postulated that the excess radial velocity arises from some form of mass motions in the atmospheres of the stars themselves. This has been disproved by Thackeray (1967) who found from the observations of sixteen visual binaries in Scorpio–Centaurus a mean difference in radial velocity of $0{\cdot}45 \pm 0{\cdot}9$ km s^{-1}. This is in the sense bright–faint and the relativity shift has already been removed. The companions are all late-type main sequence stars and any systematic error in their radial velocities can scarcely amount to $0{\cdot}5$ km s^{-1}. Thackeray's result is also a confirmation of the correctness of the relativity corrections.

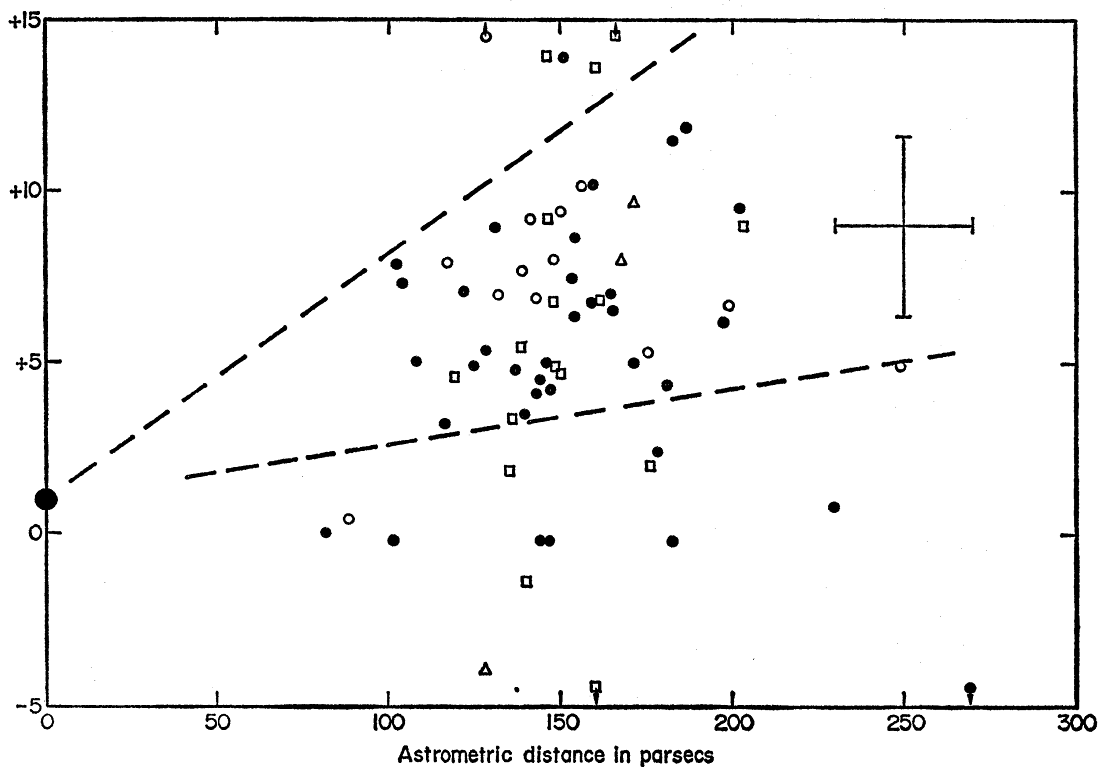

FIG. 6. *Comparison of radial velocity residual* $\rho - 23\cos\lambda$ *with astrometric distance in parsecs. The symbols have the same significance as in Fig.* 5. *The broken lines demarcate the supposed extent of Scorpio–Centaurus. The large circle on the left-hand axis represents the assumed relativity shift. The large cross indicates the estimated errors.*

The third explanation is that the stars form an expanding system. If this be true then the excess velocity of the star must correlate with its distance. The dependence of ρ on $\cos\lambda$ was judged to have a slope of 23 km s^{-1}. In Fig. 6 $\rho - 23\cos\lambda$ has been plotted against the astrometric distance based on $S = 23$ km s^{-1}. If the stars are members of an expanding association then there should be a correlation between $\rho - 23\cos\lambda$ and d whose intercept on the left-hand margin equals the relativistic velocity shift. No such correlation is manifest, partly because of the large observational errors whose estimated sizes are shown by the cross in the figure. The broken lines have been drawn in to include the majority of stars in the diagram; they intercept the radial velocity axis at an assumed relativistic shift of 1 km s^{-1}. All these 47 stars may be supposed to lie on an intrinsic linear relation and to be affected only by the acknowledged errors of observation.

Fig. 6 shows that the stars have a nearer limiting distance which is scarcely affected by the demarcation lines. It is not clear whether a distant limit exists or not. As explained in Section 2.3, all stars have been rejected whose proper motions are less than a given multiple of their errors—five in this case. If the proper motions all have the same accuracy the association would appear to have a spurious distant boundary.

4.5 *Convergent point*

The convergent point of these 47 stars is $l^{II} = 236{\cdot}5$, $b^{II} = -25{\cdot}0$. The associated probability contours for this vertex are shown in Fig. 7 together with the vertices found by other investigators. The present discussion has moved the vertex only 4° from Bertiau's preferred vertex I and in a direction nearly perpendicular to

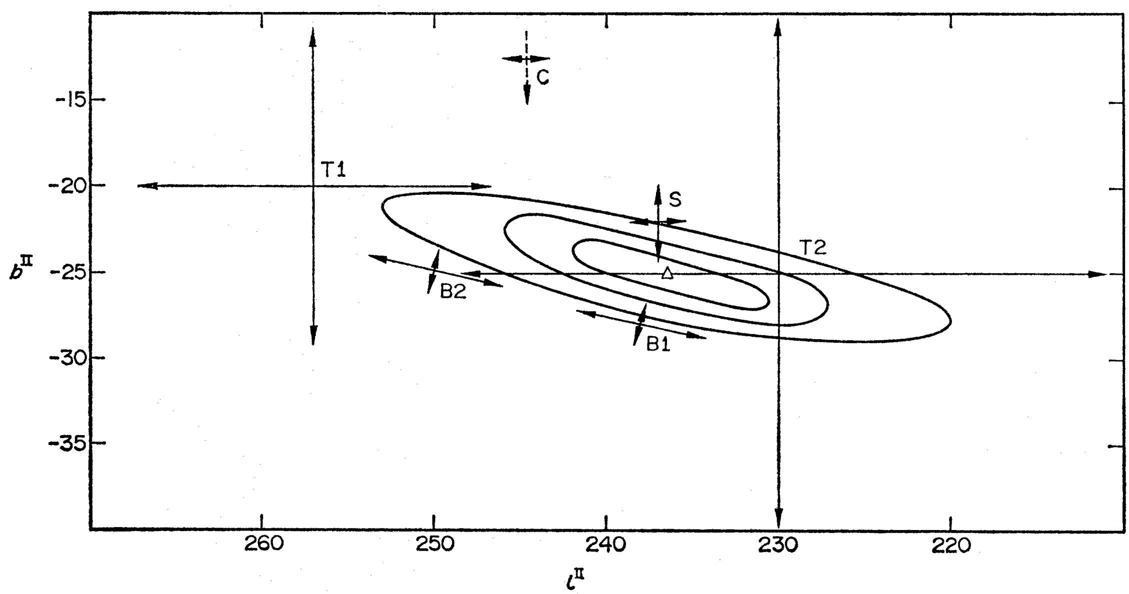

FIG. 7. *Galactic coordinates of Most Probable Convergent of the 47 stars delimited in Fig.* 6 *plotted as a triangle. The three surrounding contours have* $\Delta \log \bar{P}$ *of* $-0{\cdot}22$, $-0{\cdot}87$ *and* $-1{\cdot}95$ *respectively, corresponding to one, two and three standard errors when the normal law obtains. B1 and B2 are the two solutions derived by Bertiau, the first on the expansion hypothesis and the second with corrections applied for galactic rotation*; (*the plotted errors are probable errors*). *T1 and T2 are the two solutions by Thackeray* (1967) *based on radial velocities alone. The first solves for the solar motion alone but the second allows for a K term. C is the vertex found by Clube from the radial velocities of stars in his model of Gould's belt. The latitude is arbitrary because he was forced to adopt a value of W by the ill-conditioning of his equations. S is the antapex of the Solar Motion relative to nearby stars* (*Allen* 1963).

the great circle joining the vertex to the centroid of the stars; the associated changes in luminosity are correspondingly slight. The chief contributor to the decrease in distance found between the present investigation and Bertiau's lies in the size of the proper motions on the FK4 system (cf. Section 4.1) and not on the position of the convergent point. The convergent of the 47 stars is also close to the solar antapex; thus, as explained in 2.3 there will be a tendency to include in the association any stars of small peculiar motion; but these stars should lie on a line with $S \approx 20$ km s^{-1} $K \approx 0$ km s^{-1}, in Fig. 5. Stars near such a line do appear there and the lower broken

16§

line in Fig. 6 was drawn to eliminate them. The radial velocities of these 47 stars have the following regression equation on d and cos λ.

$\rho = 1$	$+0{\cdot}048\,d$	$+22{\cdot}64\cos\lambda$	(4.5.1)
	$\pm 0{\cdot}0023$ s.e.	$\pm 0{\cdot}99$ s.e.	
assumed relativity term	expansion term	stream motion	

The r.m.s. deviation from this relation is $\pm 2{\cdot}03$ km s^{-1} which compares favourably with the errors of observation. Its size depends critically on the boundaries drawn in Fig. 6 so this argument is not compelling. The interpretation of the radial velocities is more subjective than the treatment of the proper motions and this may have introduced some bias. Considering first the random errors, S was determined by regressing ρ on cos λ. If the regression were reversed then S would become $24{\cdot}64$ km s^{-1}. This is only achieved by allowing each star to converge to a different vertex and have its own peculiar motion—a violation of the kinematic model. The amount of bias introduced by rejecting the non-members from Fig. 6 is more difficult to assess. As $S = 23$ km s^{-1} was adopted in drawing up the limits, the final value of S must be biased towards this value. An upper limit was obtained by returning to Fig. 5 and determining the slope of the maximum likelihood relation between ρ and cos λ. All 68 stars were used and the ratio of standard errors was assumed to be 23 between ρ and cos λ. The solution was

$$S = 24{\cdot}86 \pm 2{\cdot}96 \text{ (s.e.) km s}^{-1}. \qquad (4.5.2)$$

The increase in standard error is caused by the inclusion of the non-members. It is because of the non-members that the maximum likelihood slope is preferred. A straightforward regression of ρ on cos λ gives the absurdly low value; $S = 17{\cdot}52$ km s^{-1}. This standard error for S is very much an upper limit because it is determined largely by the few deviant stars which are clearly non-members by any reasonable criterion. It is believed that (4.5.1) represents the best solution and that (4.5.2) represents the upper limit of any bias which may have arisen from a misinterpretation of Fig. 5 or Fig. 6. The variation of ρ with distance is much smaller than with cos λ and the manner in which they are related is not critical.

Turning to the systematic errors in the solution, the stars are found to lie roughly on a great circle passing through the vertex with λ evenly distributed between 60° and 120°. Under these conditions it may be verified that S is independent of the vertex position. The reverse is true for K where the standard error $\pm 6°$ in vertex position produces an error of $\pm 2{\cdot}1$ km s^{-1} in K. Thus K is only 2·5 times the sum of its different errors and a value of zero is not beyond the bounds of credibility. The value of (4.5.1) implies an expansion time of 2×10^7 years in good agreement with Bertiau (1958).

4.6 *Variation of convergent point with longitude*

The distance moduli based on this vertex and stream motion are plotted as a function of of galactic longitude l^{II} in Fig. 8. The vertical lines are two standard errors in length, based solely on the errors in proper motion. The centroid of longitude is $l^{\mathrm{II}} = 331$ and the regression of m–M on l^{II} is

$$m\text{–}M = \underset{\pm 0{\cdot}04\text{ s.e.}}{5{\cdot}79} \quad \underset{\pm 0{\cdot}0023\text{ s.e.}}{+0{\cdot}008147\,(l^{\mathrm{II}} - 331)} \qquad (4.6.1)$$

The dispersion about this relation is $0^{\mathrm{m}}\cdot 30$ where the mean of the standard deviations in m–M is $0^{\mathrm{m}}\cdot 29$. The immediate implication of these figures is that the true dispersion in distance about equation (4.6.1) is extremely small, about $\pm 0^{\mathrm{m}}\cdot 08$. It is difficult to accept that the distance should be such a closely defined function of l^{II} alone and the conclusion may be doubted for the following reasons: (i) the sampling error of the dispersion about equation (4.6.1) is $\pm 0^{\mathrm{m}}\cdot 14$ and the true dispersion

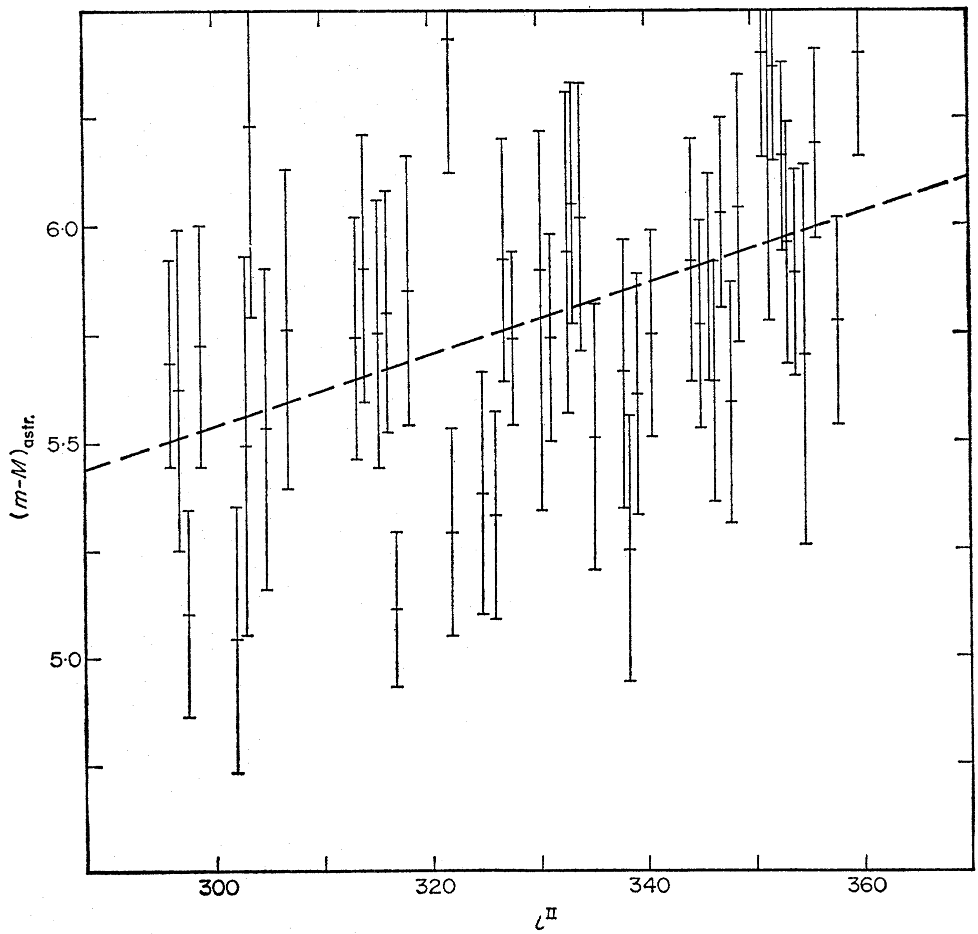

FIG. 8. *Astrometric distance modulus plotted against galactic longitude l^{II}. The vertical bars are two standrad errors in length. The broken line is equation* (4.6.1).

must be at least as uncertain; (ii) the uncertainties in the proper motions include the uncertainties in the proper motion system. As explained in Section 2.1 this procedure overestimates the scatter in a limited area of sky where the errors of the system correlate. The removal of the systematic errors reduces the mean error in (m–M) to $0^{\mathrm{m}}\cdot 23$ and increases the scatter about equation (4.6.1) to $0^{\mathrm{m}}\cdot 19$ with the same uncertainty. The adopted value was $0^{\mathrm{m}}\cdot 18$; this provided the weight with which the distance modulus from (4.6.1) was combined with the astrometric distance modulus whose weights were computed from the errors of the proper motions.

4.7 *Individual distances*

The weighted mean distance moduli have standard errors ranging from $0^{\mathrm{m}}\cdot 13$ to $0^{\mathrm{m}}\cdot 17$. These are internal errors. The uncertainty in the stream motion produces an

TABLE II

(1) HD	(2) Name	(3) $(m-M)$ ast	(4) s.e.	(5) $(m-M)$ l^{II}	(6) $(m-M)$ wt	(7) s.e.	(8) $(B-V)_o$	(9) $(U-B)_o$	(10) V_o	(11) M_v	(12) M_v Hα	(13) M_v Hβ	(14) Sp. type	(15) $v \sin i$ km s^{-1}	*Notes*
105382		5·62	0·37	5·50	5·52	0·16	−0·20	−0·72	4·38	−1·14		−1·85	B2 IIIne3		
105435	δ Cen	5·68	0·24	5·50	5·56	0·14	−0·22	−0·78	2·25	−3·31			B2 IVne2+		
105937	ρ Cen	5·10	0·24	5·51	5·36	0·14	−0·18	−0·66	3·90	−1·46		−1·07	B3 V		
108483	σ Cen	5·72	0·28	5·53	5·59	0·15	−0·22	−0·82	3·85	−1·74		−2·41	B2 V		
110956		5·04	0·31	5·55	5·42	0·16	−0·19	−0·68	4·58	−0·84		−1·15	B3 V		
112091	μ^2 Cru	5·49	0·44	5·56	5·55	0·17	−0·16	−0·56	4·68:	−0·87			B5 Vne1+		
112092	μ^1 Cru	6·23	0·44	5·56	5·65	0·17	−0·22	−0·80	3·99:	−1·66	−2·57	−2·02	B2 IV–V		
113703		5·53	0·37	5·57	5·56	0·16	−0·17	−0·62	4·61	−0·95	−1·28	−1·80	B5 V		1
115823		5·76	0·37	5·59	5·62	0·16	−0·15	−0·54	5·39	−0·23		−0·75	B6 V		
120307	ν Cen	5·74	0·28	5·65	5·68	0·15	−0·24	−0·90	3·67	−2·01		−2·75	B2 IV	90	
120324	μ Cen	5·90	0·31	5·65	5·71	0·16	−0·22	−0·78	3·38	−2·33			B2 IV–Ve	190	
120709	3 Cen	5·11	0·18	5·67	5·39	0·13	−0·18	−0·68	4·42	−0·97	−2·20		B5 III	⩽20	2
121743	ϕ Cen	5·80	0·28	5·66	5·70	0·15	−0·23	−0·86	3·78	−1·92	−2·67	−1·96	B2 IV		
121790	υ^1 Cen	5·75	0·31	5·66	5·68	0·16	−0·23	−0·84	3·80	−1·88		−2·50	B2 IV–V		
122980	χ Cen	5·85	0·31	5·68	5·72	0·16	−0·22	−0·81	4·29	−1·43	−2·43	−2·53	B2 V	⩽20	
125823	a Cen	5·29	0·24	5·71	5·56	0·14	−0·22	−0·79	4·35	−1·21	−2·43	−2·57	B7 IIIp	⩽20	3
126769	52 Hya	5·74	0·20	5·75	5·75	0·13	−0·12	−0·43	4·81	−0·94			B8 IV		
129056	α Lup	6·43	0·31	5·71	5·89	0·16	−0·25	−0·91	2·25	−3·64			B1·5 III		4
129116		5·33	0·24	5·75	5·60	0·14	−0·20	−0·74	3·92	−1·68	−1·96	−1·72	B3 V	200	
130807	ο Lup	5·38	0·28	5·74	5·63	0·15	−0·18	−0·66	4·25	−0·77	−1·49	−1·56	B5 IV		
132200	κ Cen	5·92	0·28	5·75	5·80	0·15	−0·22	−0·82	3·08	−2·72			B2 IV	⩽20	
132955		5·94	0·37	5·80	5·83	0·16	−0·18	−0·67	5·26	−0·68			B3 V	⩽20	
136298	δ Lup	5·74	0·24	5·79	5·77	0·14	−0·25	−0·93	2·97	−2·80			B1·5 IV	230	
136664	ϕ^2 Lup	6·02	0·31	5·81	5·86	0·16	−0·19	−0·69	4·42	−1·44			B4 V	210	
137432		5·51	0·31	5·82	5·74	0·16	−0·17	−0·63	5·35	−0·39	−1·41	−1·15	B4 Vp	130	
138690	γ Lup	6·05	0·28	5·80	5·88	0·15	−0·23	−0·85	3·42	−2·46		−3·00	B2 IV	270	
138769		5·78	0·44	5·79	5·79	0·17	−0·20	−0·74	4·57	−1·22	−1·93	−1·65	B3 IVp		
139365	τ Lib	5·75	0·24	5·87	5·83	0·14	−0·20	−0·73	3·91	−1·92	−2·17	−1·73	B2·5 V	130	5

140008	ψ^2 Lup	5.66	0.31	5.84	5.80	0.16	−0.16	−0.58	5.31	−0.49		−0.78	B5 V	50	
141637	1 Sco	5.77	0.24	5.91	5.86	0.14	−0.23	−0.85	4.15	−1.71	−2.92	−2.62	B1.5 Vn	300	
142184		5.59	0.28	5.93	5.82	0.15	−0.22	−0.78	4.87	−0.95		−2.37	B2.5 V	350	
142378	47 Lib	6.15	0.37	5.96	6.00	0.16	−0.18	−0.66	5.68	−0.32		−1.30	B5 V	240	
142669	ρ Sco	5.92	0.28	5.90	5.91	0.15	−0.23	−0.86	3.78	−2.13	−2.40	−2.60	B2 IV–V	140	
142983	48 Lib	6.19	0.22	5.99	6.07	0.14	−0.08:	−0.3:	4.8:	−1.3:			B5 IIIp	400	6
143018	π Sco	6.03	0.22	5.92	5.96	0.14	−0.26	−0.97	3.26	−2.70	−4.45	−3.35	B1 V+B2	100:	7
143118	η Lup	5.61	0.28	5.85	5.78	0.15	−0.24	−0.89	3.37	−2.41	−3.54	−3.25	B2.5 IV	240	
143699		5.25	0.31	5.85	5.70	0.16	−0.17	−0.62	4.82	−0.88	−1.43	−1.51	B6 IV	170	
144470	ω^1 Sco	5.89	0.24	5.97	5.94	0.14	−0.26	−0.98	3.28	−2.66	−3.58	−3.20	B1 V	140	
145482		6.04	0.31	5.93	5.96	0.16	−0.22	−0.79	4.42	−1.54	−2.60	−2.42	B2 V	240	
147165	σ Sco	6.40	0.24	5.95	6.11	0.14	−0.26	−0.96	1.7:	−4.4:	−4.37		B1 III	60	8
147888		5.70	0.44	5.97	5.93	0.17	−0.18	−0.67	5.68	−0.25	−1.31		B5 V	180	
148184	χ Oph	5.78	0.24	6.01	5.93	0.14	−0.21	−0.78	3.28	−2.65			B1.5 Ve4	130	9
148478	α Sco	6.37	0.22	5.96	6.12	0.14	1.83:	1.32:	1.0:	−5.1:			M1 Ib	⩽20	10
149438	τ Sco	6.16	0.22	5.96	6.04	0.14	−0.29	−1.06	2.72	−3.32	−3.99	−3.70	B0 V	20	
151890	μ^1 Sco	5.88	0.24	5.91	5.90	0.14	−0.24	−0.86	3.52	−2.38	−4.24	−3.12	B1.5 IV	220	11
151985	μ^2 Sco	5.64	0.28	5.91	5.83	0.15	−0.24	−0.89	3.49	−2.34		−3.26	B2 IV	40	
157056	θ Oph	6.40	0.24	6.02	6.16	0.14	−0.24	−0.89	3.17	−2.99	−3.38	−3.05	B2 IV	20	12

Notes to Table II:

1. Thackeray (1966) physical companion type K0 Ve.
2. A peculiar star with a high abundance of He^3 (Sargent & Jugaku 1961).
3. Spectrum variable (Jaschek *et al.* 1968, Norris 1968).
4. β Cephei variable.
5. Double-lined binary. An approximate orbit by van Albada & Sher (1969). The secondary has $v \sin i = 100$:
6. Variable range 0.19 in V, symbiotic star (Cousins 1967).
7. Double-lined binary; orbit Hetzler & Summers (1959). Both components have the same $v \sin i$ as given in the Table.
8. β Cephei variable.
9. Quasi-nova like variable, range about 0.24 in V (Cousins 1967).
10. 3″.4 from HD 148479, B2.5 V (Garrison 1967).
11. Eclipsing binary.
12. β Cephei variable.

additional standard error of $\pm 0^{m}{\cdot}09$. The uncertainty in the vertex causes an uncertainty in the stream motion producing a further uncertainty of $\pm 0^{m}{\cdot}01$ in the luminosities. The uncertainty in the position of the vertex produces no uncertainty in the distance of a star 90° from the vertex, which fortunately coincides with the centroid of these stars. The star most badly affected by a vertex change is HD 142983 for which $\lambda = 127°$ and the uncertainty of $\pm 6°$ in λ produces an additional standard error of $0^{m}{\cdot}17$ in distance modulus. In general the uncertainty in the vertex produces an uncertainty in the distance modulus of the remaining stars which is negligible compared with the other sources of uncertainty. It must not be forgotten that the present solution is based on these particular 47 stars. If they form a true kinematic grouping then the later addition of more true members should not affect the solution beyond its acknowledged uncertainty. On the other hand, if these 47 include a proportion of stars which are not truly co-moving then the solution will suffer from further errors for which no allowance has been made in the analysis. It is for this reason that the boundaries in Fig. 6 have been drawn so conservatively. A member erroneously rejected does far less harm than a non-member erroneously accepted.

Table II lists the 47 stars demarcated in Fig. 6 by HD number and name. Column (3) gives the distance modulus computed from the formula

$$-5 \log \left(\frac{4{\cdot}74\mu_{\parallel}}{S \sin \lambda}\right) - 5$$

where $\mu_{\parallel}$ is the proper motion component towards the vertex $l^{\mathrm{II}} = 236{\cdot}5$, $b^{\mathrm{II}} = -25{\cdot}0$ and λ is the angular distance of the star from the vertex. S is the stream motion of $22{\cdot}64$ km s^{-1}. Column (4) gives the standard error of (3) computed from the standard error of the proper motion augmented by the uncertainty of the catalogue system. Column (5) gives the distance modulus from equation (4.6.1). Column (6) is the weighted mean of (3) and (5) allowing (5) to have a standard error of $\pm 0^{m}{\cdot}18$ and (7) is the standard error of (6). Columns (8), (9) and (10) are the intrinsic colours and magnitudes of the stars from Gutierrez-Moreno & Moreno (1968). They have been freed from reddening following Johnson (1958). The stars HD 120709, 126769 and 147165 were taken from Cousins (1967) and similarly corrected for reddening. Column (11) is the absolute magnitude found by applying (6) to (10). Corrections for duplicity were applied following Bertiau except for HD 120709 where the standard colours of a B8 V star with $V = 6.17$ were used to subtract the light of the companion; HD 130807 with $\Delta m = 0{\cdot}61$, HD 142378, $\Delta m = 0{\cdot}24$ and HD 147999, $\Delta m = 0{\cdot}36$. A correction of $-0^{m}{\cdot}10$ was applied to HD 151985 to bring it out of eclipse. The mean duplicity correction for the 47 stars is $0^{m}{\cdot}11$. Column (12) gives M_v deduced from the Hα strength by Andrews (1968). Column (13) gives M_v deduced from the Hβ strength published by Gutierrez-Moreno & Moreno (1968), based on the calibration of Hardie & Crawford (1961). The spectral types in Column (14) are from Hiltner, Garrison & Schild (1969) for stars south of declination $-20°$ and Lesh (1968) for stars to the north. Exceptions are HD 126769 (Morris 1961) and HD 147888 and HD 148478 from Slettebak (1968) who also provides the rotational velocity $v \sin i$ in column (15).

4.8 *Colour-magnitude array*

$(B-V)_0$ has been plotted against M_v in Fig. 9 and $(U-B)_0$ against M_v in Fig. 10. The zero-age main sequence is plotted following Blaauw (1963) and Johnson (1963).

There are several stars fainter than the ZAMS similar to those found by Lloyd-Evans (1969) in his discussion of IC 2581. Also plotted are the lower envelopes from Lloyd-Evans' Figs 6 and 7. These figures include the Pleiades and α Per cluster as well as IC 2581 but it is the latter, because of its small age, which defines the lower envelope over the colour range plotted. There appears to be better agreement between IC 2581 and Sco–Cen than either with the ZAMS.

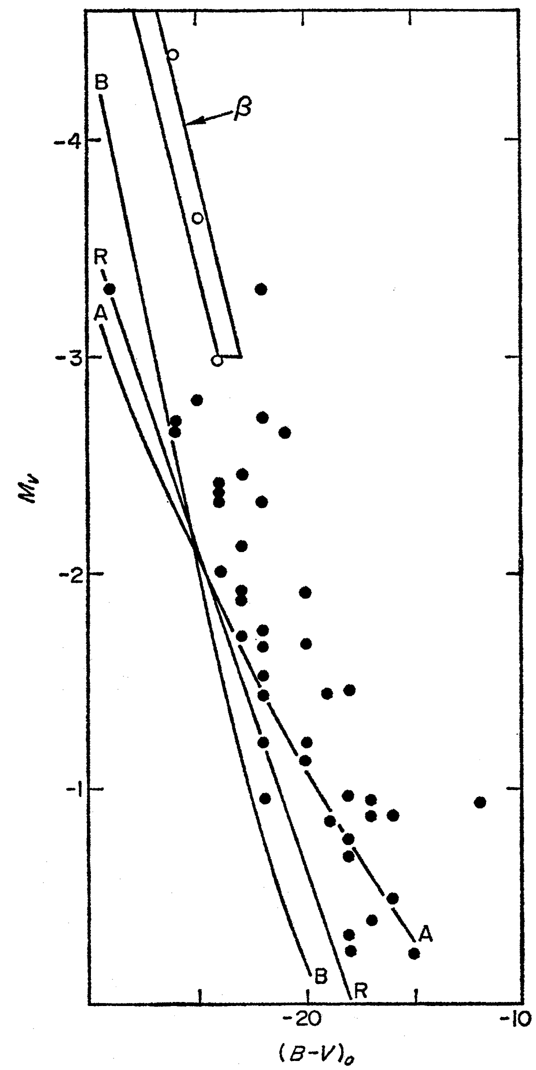

FIG. 9. *Plot of unreddened colour* $(B-V)_0$ *against luminosity* M_v. *Open circles are* β *Cephei variables, filled circles other stars.* β *marks the* β *Cephei strip after Percy* (1970). *AA is the zero age main sequence after Blaauw* (1963) *and Johnson* (1963). *BB is the lower envelope of IC* 2581 *following Lloyd Evans* (1969). *RR is the reference line used in discussing stellar rotation.*

The three β Cephei stars among the 47 are plotted as open circles in Fig. 9 and compared with their instability strip after Percy (1970). The agreement is very heartening; moreover no constant stars lie in or near the strip.

In Fig. 10, stars in the Upper Scorpius region, Blaauw's (1946) area 2, are shown as open circles. The *M*-type supergiant α Sco also lies in this region. The fact these stars occupy the bluest colours and the lowest luminosities is evidence of their relative youth. The two hottest stars not in Upper Scorpius are δ Lup and α Lup

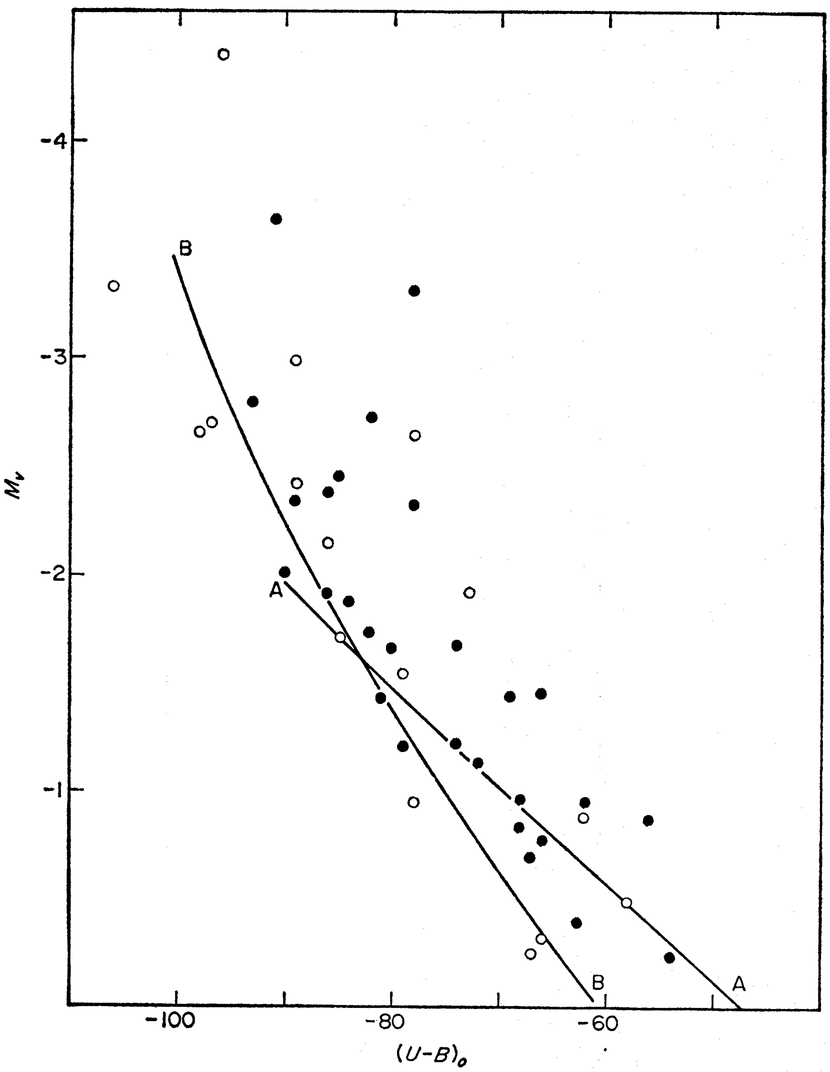

FIG. 10. *Plot of unreddened colour* $(U-B)_0$ *against luminosity* M_v. *Open circles, stars in Upper Scorpius (Blaauw's* (1946) *area* 2), *filled circles, others. AA is the zero age main sequence (Johnson* 1963). *BB is the lower envelope of IC* 2581 *following Lloyd Evans* (1969).

which are both in Blaauw's area 3. It appears that the youngest stars are at the highest longitudes where the nebulosity is strongest and that the older stars trail away towards lower longitudes. Fig. 10 is very similar to Blaauw's (1964b) Fig. 5 (viii).

4.9 *Balmer line luminosities*

The luminosities from Table II when compared with those of Andrews based on Hα are found to be fainter by $0^{m}{\cdot}85$ on the average with a standard deviation of $0^{m}{\cdot}49$ for 22 stars in common. Andrews predicts a standard deviation of $0^{m}{\cdot}5$ in excellent agreement. The residuals between the Hα and astrometric luminosities do not correlate with luminosity indicating that Andrews' calibration merely requires a translation, at least over the range of the present astrometric luminosities: $-0{\cdot}2$ to $-4{\cdot}4$.

The 31 stars in Table II with Hβ luminosities have astrometric luminosities fainter by $0^{m}{\cdot}58$ on the average with a standard deviation per star of $0^{m}{\cdot}41$. Hardie

& Crawford (1961) predict a scatter of $0^{m}.37$ from their 'single stars only' relation which is appropriate to the Table where the effects of duplicity have been corrected for as far as possible. The agreement is very satisfactory but largely reflects the fact that the Hardie–Crawford calibration was set up using stars in the Upper Scorpius region of Sco–Cen. Again the deviations do not correlate with luminosity indicating that the Hardie–Crawford relation merely requires a translation.

The deviations of neither the Hα luminosities nor the Hβ luminosities correlate significantly with the differences between the two luminosities derived here—the 'astrometric' and the longitude values. If the method of combining these two were erroneous—by overweighting one or the other—then such a correlation should appear. As no correlation occurs it is concluded that the weighting system is basically correct.

Gutierrez-Moreno & Moreno tabulated two values of V_0 for the stars in the Upper Scorpius region based on $R = 3$ and $R = 6$ respectively. The corresponding differences in M_v were examined for a correlation with either the Hα or Hβ residuals; no significant correlation was found indicating that the present material is incapable of distinguishing between the two values of R.

Omitting all peculiar and emission line stars the present luminosities were compared with the spectroscopic luminosities using the calibration of Lesh (1968). The mean residual was $+0^{m}.4$ with a standard deviation for one star of $0^{m}.5$, for 37 stars in common.

4.10 *Correlations among luminosity estimates*

It is instructive to compare the scatter between the different estimates of luminosity. Let J denote M_v derived in the present study, α that derived from the Hα index and β that from the Hβ index, all freed from the mean differences given above; let σ_J, σ_α, σ_β be their dispersions and $r_{J\alpha}$, $r_{\alpha\beta}$, $r_{\beta J}$ their mutual correlations, then

$$\sum^{n} \frac{(J-\alpha)^2}{n-1} = \sigma_J{}^2 - 2r_{J\alpha}\sigma_J\sigma_\alpha + \sigma_\alpha{}^2$$

$$\frac{\sum^{n} (\alpha-\beta)^2}{n-1} = \sigma^2 - 2r_{\alpha\beta}\sigma_\alpha\sigma_\beta + \sigma_\beta{}^2$$

$$\frac{\sum^{n} (\beta-J)^2}{n-1} = \sigma^2 - 2r_{\beta J}\sigma_\beta\sigma_J + \sigma_J{}^2.$$

There are 20 stars in Table II that have all three luminosity estimates. Setting the mutual correlations to zero we find

$$\sigma_\alpha = 0^{m}.36 \qquad \sigma_\beta = 0^{m}.22 \quad \text{and} \quad \sigma_J = 0^{m}.34.$$

It is difficult to believe that σ_J can be so large because the standard deviation about equation (4.6.1) is $0^{m}.30$ and most of this must arise from errors in the proper motions. But (4.6.1) provides the standard deviation in distance and before comparison with the standard deviation in luminosity must be augmented by: (i) errors in apparent magnitude; (ii) errors in the reddening corrections including the ratio R of total to selective absorption; (iii) errors in the duplicity corrections. None of these seem big enough to generate so large a scatter.

An alternative interpretation of the figures arises when we examine the correlation coefficients. There is no physical reason for $r_{J\alpha}$ and $r_{\beta J}$ to differ from zero and they have accordingly been neglected. Nevertheless there may well be physical effects which affect α and β similarly and produce a mutual correlation. Setting $\sigma_J = 0^{\mathrm{m}}{\cdot}15$ from the present work one finds

$$\sigma_\alpha = 0^{\mathrm{m}}{\cdot}47 \qquad \sigma_\beta = 0^{\mathrm{m}}{\cdot}38 \qquad r_{\alpha\beta} = +0{\cdot}55$$

which is significant at the $P < 0{\cdot}02$ level. The results are not greatly affected by the weighting system used to combine columns (3) and (5) in Table II. For using the astrometric distance moduli in column (3)

$$\sigma_\alpha = 0^{\mathrm{m}}{\cdot}31 \qquad \sigma_\beta = 0^{\mathrm{m}}{\cdot}29 \qquad \sigma_J = 0^{\mathrm{m}}{\cdot}40$$

or setting $\sigma_J = 0^{\mathrm{m}}{\cdot}29$ as before

$$\sigma_\alpha = 0^{\mathrm{m}}{\cdot}41 \qquad \sigma_\beta = 0^{\mathrm{m}}{\cdot}40 \qquad r_{\alpha\beta} = +0{\cdot}48 \text{ significant } P < 0{\cdot}05.$$

Alternatively, using the ' longitude distances ' in column (5)

$$\sigma_\alpha = 0^{\mathrm{m}}{\cdot}37 \qquad \sigma_\beta = 0^{\mathrm{m}}{\cdot}21 \qquad \sigma_J = 0^{\mathrm{m}}{\cdot}36$$

or setting $\sigma_J = 0^{\mathrm{m}}{\cdot}18$ as before

$$\sigma_\alpha = 0^{\mathrm{m}}{\cdot}49 \qquad \sigma_\beta = 0^{\mathrm{m}}{\cdot}38 \qquad r_{\alpha\beta} = +0{\cdot}54 \text{ significant } P < 0{\cdot}02.$$

Hardorp & Strittmatter (1968b) have predicted theoretically that intrinsic rotation should cause a total scatter of about $\pm 0^{\mathrm{m}}{\cdot}5$ in the plot of Hγ against M_v. It is very probable that Hα and Hβ are similarly affected and produce correlating errors in M_v. The r.m.s. scatter within the correlation here is about $\pm 0^{\mathrm{m}}{\cdot}2$ which is close to the amount expected. Hardorp & Strittmatter predict that this scatter is irreducible because it depends on v and i but not on $v \sin i$ which is the only parameter observable.

The Hα and Hβ luminosity residuals also correlate with the residuals from the spectroscopic luminosities. There are 17 stars in the 20 above which have spectroscopic luminosities. Giving the subscript s to the spectroscopic luminosity and $\alpha\beta$ to the mean of α and β we find either

$$\sigma_s = 0^{\mathrm{m}}{\cdot}48 \qquad \sigma_{\alpha\beta} = 0^{\mathrm{m}}{\cdot}17 \qquad \sigma_J = 0^{\mathrm{m}}{\cdot}36$$

or putting $\sigma_J = 0^{\mathrm{m}}{\cdot}15$

$$\sigma_s = 0^{\mathrm{m}}{\cdot}57 \qquad \sigma_{\alpha\beta} = 0^{\mathrm{m}}{\cdot}37 \qquad r_{\alpha\beta, s} = +0{\cdot}51 \text{ significant } \; P < 0{\cdot}05.$$

One of the criteria of spectroscopic luminosity is the strength of the hydrogen lines so the correlation with $\alpha\beta$ is not absurd. The values of the correlation coefficients can best be decided by further observation. The α and β observations might be made simultaneously in a cluster of small angular extent and, inferentially, small dispersion in distance. It would have to be substantially unreddened otherwise the solution would be vitiated by uncertainties in the reddening corrections, especially the ratio of total to selective absorption.

4.11 *Stellar rotation*

It has been suggested (Strittmatter 1966, Sargent & Strittmatter 1966, Roxburgh, Sargent & Strittmatter 1966) that fast rotating stars lie above slow rotators in the colour-luminosity array. Following the first of these treatments a reference line was

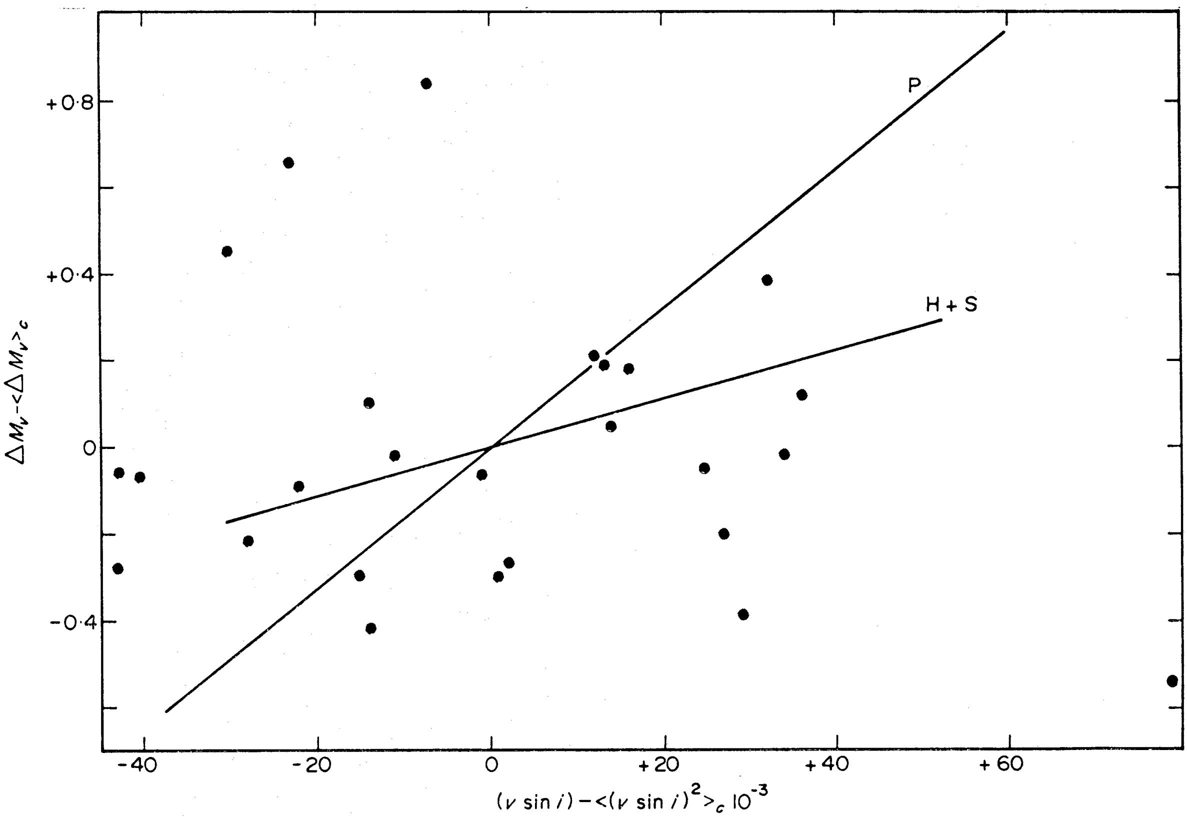

FIG. 11. *Deviation from mean main sequence* $\Delta M_v - \langle \Delta M_v \rangle_c$ *plotted against departure from mean apparent rotation* $(v \sin i)^2 - \langle (v \sin i)^2 \rangle_c$. *P is the empirical relation found by Strittmatter* (1966) *in Praesepe. H+S is the theoretical relation of Hardrop & Strittmatter* (1968*a*).

drawn in Fig. 9. Average values, $\langle \Delta M_v \rangle_c$ from this reference line, and $\langle (v \sin i)^2 \rangle_c$ were formed for each narrow colour range (0·01 to 0·03 depending on the number of stars). Following Strittmatter stars with $\Delta M_v > 1{\cdot}25$ were excluded because they were likely to be evolved. In Fig. 11, $\Delta M_v - \langle \Delta M_v \rangle_c$ for each star has been plotted against $(v \sin i)^2 - \langle (v \sin i)^2 \rangle_c$. The relation found by Strittmatter in Praesepe is also shown. It is reasonable to postulate that the three stars in the top left of the diagram are substantially evolved and need not be considered, but even so the remainder show no respect for the Praesepe relation. Hardorp & Strittmatter (1968a) have calculated the slope to be expected. Averaging over the range of $(B-V)$ in their Table 5 that covers the present observations and over all inclinations the expected shift at constant colour is $\Delta M_v = 0{\cdot}60$ between stars with no rotation and those at break-up velocity. This velocity is about 400 km s^{-1} on Slettebak's scale so that

$$\langle (v \sin i)^2 \rangle = 2/3 \langle v^2 \rangle = 107\,000 \text{ km}^2 \text{ s}^{-2}.$$

Discarding HD 142184 which lies in the extreme bottom right-hand corner of the diagram and the three stars in the top left-hand corner the remainder do seem to exhibit a slope of about this amount. The observed total spread in ΔM_v is about $0^{\mathrm{m}}{\cdot}7$ which agrees well with that predicted for rotational spread. It is also interesting to note that Hardorp & Strittmatter's zero rotation main sequence deviates from Johnson's ZAMS in much the same manner as the observations in Fig. 9.

The star most embarrassing to the theory is HD 142184 in the bottom right of the diagram. It is on the lower edge of the main sequence in both Figs 9 and 10 but has a high rotation so that no value of sin i will make it fit the theory. It may be significant that it is one of the most heavily reddened stars in the Upper Scorpius region with $E_{B-V} = 0{\cdot}18$. If the star were less reddened or if the ratio of total to selective absorption were greater then the star's position would not be so anomalous. It seems that the present material does not provide a critical test of the rotation theory. Much of the scatter in Fig. 11 is probably caused by uncertainties in the reddening and by the confusion arising from a range of ages in the association. Neither of these factors are significant in the clusters where the theory has been successfully applied.

5. DOUBLE AND OTHER INTERESTING STARS

5.1 *The visual doubles*

Table III summarizes the visual doubles from Table II. Columns (1) and (2) give the star names from the HD catalogue unless otherwise stated. Columns (3), (4) and (5) give m_{pg} for the two stars and their separation in seconds of arc. Column (6) gives log n where n is the number of pairs to be expected in the 2211 square degrees marked off in Fig. 4 with m_{pg} brighter than those given in (3) and (4) and separation closer than the value in (5). This is the total number expected if the stars were scattered randomly over the area; it has been calculated in the manner described by Jones (1970). For most pairs n is very small which indicates a small probability that

TABLE III

(1) Name	(2) (mostly HD)	(3) A m_{pg}	(4) B m_{pg}	(5) d''	(6) log n	(7) cpm?	(8) crv?	(9) π_d	(10) π_c	Notes
105382	105383	4·3	6·3	368	−1·4	yes	yes			
105435	105382	2·7	4·3	217	−4·0	yes	yes			
110956	comes	4·5	8·9	53	−1·6	yes	no			
112092	112091	3·8	5·0	38	−4·2	yes	yes			
113703	comes	4·6	11·7	12	−2·0	maybe	yes			
120324	comes	3·3	14·0	48	−0·6	yes	maybe			
120709	120710	4·6	6·1	9	−5·1	maybe	maybe	0·017	0·008	A
122980	123021	4·2	8·6	213	−1·1	yes	yes			
129056	comes	2·1	13·5	28	−1·7	maybe	maybe			
130807	comes	4·8	5·1	0·1	−8·8	yes	maybe			B
132200	comes	2·9	11·8	4	−3·9	yes	maybe			
137432	comes	5·3	14·5	20	+0·3	maybe	maybe			
138690	comes	3·1	3·4	0·1	−10·9	yes	maybe			C
138769	comes	4·4	7·7	3	−4·9	yes	yes	0·014	0·007	
140008	CD −34° 10497	4·6	8·8	231	−0·7	maybe	no			
142378	comes	5·9	7·2	0·4	−6·3	yes	maybe			D
142669	comes	3·7	14·8	38	−0·2	maybe	maybe			
143018	comes	2·7	9·0	51	−3·2	yes	maybe			
143118	GC 21479	3·2	8·0	15	−4·2	yes	yes			
143118	143099	3·2	9·9	139	−1·6	maybe	no			
147165	comes	3·1	10·6	21	−3·2	yes	maybe			
147888	comes	7·5	8·0	0·9	−4·6	yes	maybe	0·005	0·006	E
148478	148479	2·9	5·0	3	−7·3	yes	yes	0·004	0·006	F
151890	151868	2·9	9·4	71	−2·5	yes	yes			
151890	151985	2·9	3·3	346	−4·1	yes	yes			

the pairs occurred by chance and consequently a high probability of physical connection. In columns (7) and (8) a qualitative answer is given to the questions ' Do the two stars share a common proper motion? ' and ' Do the two stars share a common radial velocity? ' Column (8) is based very largely on Thackeray (1966). The smallest value of log n for a ' No ' is $-1\cdot6$ and the largest value of log n for both (7) and (8) to be ' Yes ' is $-1\cdot1$. The actual number of pairs has a Poisson distribution with mean n so some overlap is to be expected; but when $\log n \approx -1\cdot3$ the number of ' Noes ' should be zero not 2 out of 4 as observed. This indicates some small imperfection in the statistical model or the data. In particular most of the faint companions have values of m_{pg} based solely on the double star observers estimate of m_v and an assumed colour index. Nevertheless it is believed that log n provides a good ranking parameter for chance occurrences and that all pairs with $\log n \leqslant -2\cdot6$ can be unreservedly accepted as physical, and possibly physical for

$$-2\cdot6 < \log n < -0\cdot7.$$

Column (9) contains the dynamical parallax from Russell & Moore (1940) and column (10) the cluster parallax converted from column (6) of Table II. In all cases

Notes to Table III:

A. HD 120709 is of great astrophysical interest because of its high abundance of He^3 (Sargent & Jugaku 1961). The proper motion of the companion is discordant. The proper motions of both stars were corrected for the motion observed by the double star observers, assuming it to arise from a physical revolution of one about the other. However, this correction does not markedly improve the inter-agreement of the proper motions.

The usual rule adopted by transit circle observers when observing double stars is ' When the pair is resolved observe the brighter and when not resolved observe the centre of light ' (Yallop, private communication). One would suppose that a pair with a separation of 8″ would always have been resolved and the proper motion was corrected on this assumption. The most likely explanation is that the presence of the other star was recognized by the observers but still served to influence their bisection of the star under observation.

Following Thackeray (1967) the radial velocity of HD 120709 was corrected by 5 km s^{-1} to allow for the isotopic shift of the He^3 lines. There is still a difference of 5 km s^{-1} from the radial velocity of HD 120710 but this is probably not significant in view of Thackeray's (1966) comment ' very poor lines '. HD 120710 was part of the parent sample but was eliminated in Fig. 6 because its radial velocity was too low and its distance too small. In spite of these difficulties it is firmly believed that the two are physically connected because of the low value of n. Unfortunately the rotational broadening in HD 120710 is too high for it to be possible to analyse its chemical abundances.

B. The duplicity of this star ($=\phi 319$) was discovered by Finsen (1953). He found a small increase in position angle and decrease in separation up till 1964. For the next two years it was not separable and its motion must have accelerated. When it is resolved again it will be a good candidate for a reliable mass determination.

C. The orbit of γ Lupi was determined by Heintz (1956) who found $a = 0''.59$, $P = 147$ years. The cluster parallax is $0''\cdot0067$ so that the total mass is $31\cdot5\mathscr{M}_\odot$. The uncertainty in the cluster parallax produces an uncertainty of 50 per cent in the mass. It is 60 per cent more massive than would be expected from its luminosity, using the mass-luminosity relation of Harris *et al.* (1963).

D. This pair is ADS 9834. It was discovered by Hussey in 1904 when $\theta = 154$, $\rho = 0''\cdot4$. In 1958 $\theta = 129$ and $\rho = 0''\cdot5$. Nevertheless it is not certain that this is a smooth progression for it was $\approx 0''\cdot25$ in 1917 and 1920 and not separated at all by van Biesbroeck in 1923.5 and 1924.5. This may be a pair of short period which would yield an orbit if carefully followed.

E. This pair is ADS 10045 and also forms ADS 10049 DE ($\rho = 156''$). The double star measures seem to indicate common proper motion. ADS 10049 was not in the parent sample but was included in Table IV. It may well be a member but does not satisfy all the criteria necessary for inclusion in Table II.

F. ADS 10074. The magnitude of the companion is based on Garrison (1967).

the dynamical parallaxes are based on a short arc without the orbit being known. Because the inclination is unknown the agreement in the parallaxes is quite satisfactory.

5.2 μ^1 *Scorpii*

The eclipsing binary μ^1 Sco is a member of Table II and the radius determined by Stibbs (1948) can be combined with the distance to give an angular diameter of $0''{\cdot}354 \times 10^{-3}$. Angular diameters of several early-type stars have been measured by Hanbury Brown *et al.* (1967) and re-discussed by Wesselink (1969). He derived s_v the surface brightness of the stars on a magnitude scale. The data for μ^1 Sco imply

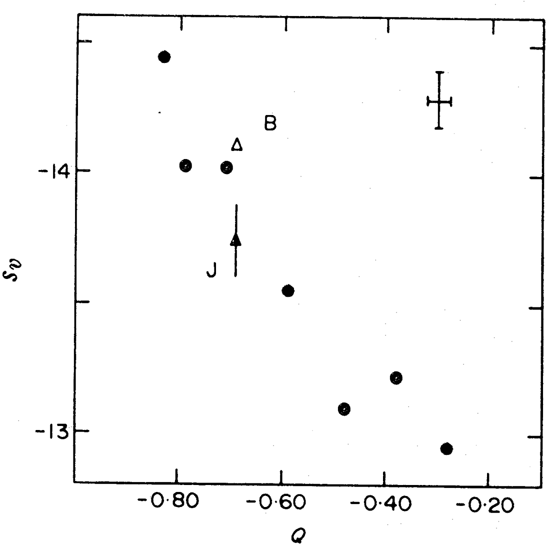

FIG. 12. *Surface brightness in the V-band on Wesselink's system,* s_v, *against*

$$Q = (U\text{–}B) - 0{\cdot}72(B\text{–}V).$$

Filled circles are stars observed by Hanbury Brown et al. (1967); *typical errors are shown by the cross. The open triangle are* μ^1 *Sco; B using Bertiau's distance and J using the present distance. The vertical bar is two standard errors in length, taken from Table II.*

$s_v = -13{\cdot}74$ where Wesselink's Table II gives $-14{\cdot}14$ for the same $(B\text{–}V)_0$. The discrepancy is almost completely removed if Bertiau's distance is used ($s_v = -14{\cdot}10$). However this is not a critical test, for the discrepancy could equally well be removed by changing the colour excess from $+0^{\mathrm{m}}{\cdot}04$ to $+0^{\mathrm{m}}{\cdot}01$, a not unreasonable amount. To remove the uncertainty in the reddening, in Fig. 12, $Q = (U\text{–}B) - 0{\cdot}72(B\text{–}V)$ has been plotted against s_v for all the B-stars. Q is independent of reddening but depends on both temperature and luminosity. For this reason stars of luminosity class I have been omitted from the plot. The hottest star, β Cru, has been corrected by 0·08 in s_v following Popper (1968). Fig. 12 shows that μ^1 Sco is consistent with the Narrabri brightness measurements but the brightness is not a critical test of the distance scale.

Stibb's mass when combined with the present luminosity is too great by about 25 per cent compared with the mass luminosity relation of Harris *et al.* (1963) and the discrepancy is insensitive to the distance. In Section 5.1 γ Lup was found to be 60 per cent over-massive but this is a visual binary and the mass is very sensitive to the distance.

The strange aspect of Stibb's solution (in common with earlier work) is that the more massive star has a smaller radius. This arises because the ratio of the surface brightnesses is greater than the ratio of the luminosities, determined from equivalent widths in the two spectra. At this colour the radius increases very slowly with luminosity on the main sequence; the pair appears to be slightly evolved which would tend to increase the radius difference. It would be interesting to repeat the solution using equal radii and an (assumed) realistic value of the limb darkening rather than the uniform discs assumed by Stibbs. It is doubtful if much can be learnt by re-observing μ^1 Sco. The difficulty lies in the unfavourable geometry of the system—primary eclipse is only partial and $0^m{\cdot}276$ deep.

5.3 *Interesting stars not in parent population*

There are several stars of great astrophysical interest in the region of the sky demarcated in Fig. 4 which were not included in 2.2. Several are listed in Table IV which is based on their proper motions on the FK4 system. The Table gives $-\log P$ where P is the probability of the perpendicular proper motion error occurring when the vertex is $l^{\mathrm{II}} = 236{\cdot}5$, $b^{\mathrm{II}} = -25{\cdot}0$, the same as used in Table II. The distance modulus depends solely on the proper motion and corresponds to $(m-M)$ astrometric given in column (3) of Table II. The observed radial velocity is given where known and its computed value from equation (4.5.1). The radial velocity of HD 136504 is from Thackeray (1970). The spectral types are from the same sources as Table II except for SEG 13 which is the author's estimate from spectra kindly made available by Dr M. Dixon.

The first four stars were taken from Eggen's (1969) study of Sco XR-1 as a member of the Upper Scorpius complex. Sco XR-1 could be a member of Sco–Cen but at 227 pc it would be more distant than the furthest member. This is not a critical objection because it is not clear whether there is a true distant boundary to the association or whether the more distant stars are rejected because their proper motions are so small that they become commensurate with their errors. On the other hand this distance is at the extreme lower limit of those proposed by Westphal, Sandage & Kristian (1968) or Wallerstein (1969). Of the companions SEG 37 has deviant proper motion and SEG 25 is apparently too far away; moreover its proper motion is too weak to include. Because its astrometric distance is greater than SEG 13 the expansion term produces a much greater discrepancy in radial velocity.

TABLE IV

Star	μ/ρ	$-\log P$	$(m-M)$ ast	radial velocity km s^{-1} O	C	Spectral type
Sco XR−1	6	0·46	6·78	—	—	
SEG 13	6	0·41	6·17	−7	−5	F2 V
SEG 25	4	0·96	7·77	−7	+4	
SEG 37	8	2·06	6·47	—	—	
HD 136504	2	0·80	6·74	+8	+9	B2 IV–V
HD 142301	7	1·97	6·30	−8	−1	B8 IIIp
HD 142445	7	4·76	5·08	—	—	
HD 147933	4	1·85	6·84	−10	0	B2 IV
HD 147934	4	0·42	6·51	−10	−1	B2 V
Limits of Table II	⩾5	<1·66	5·04–6·43			

SEG 13 has all the characteristics of a member of Sco–Cen. Eggen's *UBV* photometry implies an unreddened F0 star. The astrometric distance modulus implies that it lies a magnitude above the ZAMS, which is difficult to reconcile with its being coeval with the rest of Sco–Cen.

It is very surprising that so bright a star as HD 136504 has so weak a proper motion determination. Until it can be strengthened the star's membership cannot be confirmed or denied. Both HD 142301 and HD 142445 have proper motions which deviate significantly from the motion of Sco–Cen. HD 147933 and 4 form ADS 10049 and were included because of their proximity to HD 147888 (=ADS 10049 DE). Although they do not satisfy the criteria for inclusion in Table II their entangement with bright and dark nebulae (Garrison 1967, Fig. 2) strongly suggest a physical connexion. Strangely, while their astrometric distance is too great for membership, the dynamical parallax $0''{\cdot}013$ (Russell & Moore 1940) implies a distance which is too small.

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