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The Lie Theory of Connected Pro-Lie Groups

A Structure Theory for Pro-Lie Algebras, Pro-Lie Groups and Connected Locally Compact Groups

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Lie groups were introduced in 1870 by the Norwegian mathematician Sophus Lie. A century later Jean Dieudonné quipped that Lie groups had moved to the center of mathematics and that one cannot undertake anything without them.

If a complete topological group G can be approximated by Lie groups in the sense that every identity neighborhood U of G contains a normal subgroup N such that G/N is a Lie group, then it is called a *pro-Lie group*. Every locally compact connected topological group and every compact group is a pro-Lie group. While the class of locally compact groups is not closed under the formation of arbitrary products, the class of pro-Lie groups is.

For half a century, locally compact pro-Lie groups have drifted through the literature, yet this is the first book which systematically treats the Lie and structure theory of pro-Lie groups irrespective of local compactness. This study fits very well into that current trend which addresses infinite dimensional Lie groups. The results of this text are based on a theory of pro-Lie algebras which parallels the structure theory of finite dimensional real Lie algebras to an astonishing degree even though it has to overcome greater technical obstacles.

This book exposes a Lie theory of connected pro-Lie groups (and hence of connected locally compact groups) and illuminates the manifold ways in which their structure theory reduces to that of compact groups on the one hand and of finite dimensional Lie groups on the other. It is a continuation of the authors' fundamental monograph on the structure of compact groups (1998, 2006), and is an invaluable tool for researchers in topological groups, Lie theory, harmonic analysis and representation theory. It is written to be accessible to advanced graduate students wishing to study this fascinating and important area of current research, which has so many fruitful interactions with other fields of mathematics.

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