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ORIGINAL ARTICLE

The Liège Oscillation code

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Abstract The Liège Oscillation code can be used as a stand-alone program or as a library of subroutines that the user calls from a Fortran main program of his own to compute radial and nonradial adiabatic oscillations of stellar models. We describe the variables and the equations used by the program and the methods used to solve them. A brief account is given of the use and the output of the program.

Keywords Stars · Adiabatic oscillations · Stellar pulsations · Asteroseismology

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1 Introduction

The Liège Oscillation code (OSC) has been developed in the early 70 s for computing adiabatic pulsations of spherically symmetric stars (no rotation nor magnetic field). It has gone through minor updates and is still presently in use in the asteroseismology group of the Liège Institute of Astrophysics and Geophysics. Besides the frequencies and eigenfunctions, it produces also the coefficients needed to compute the first order rotational frequency splitting for a rigid rotation and the kernels needed to compute the splitting when a non rigid rotation is considered.

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2 Stellar models

A stellar model is input to OSC as a table describing a few physical quantities at discrete points of the star, ordered from the centre to the surface. In theory, only two functions are necessary to compute stellar oscillations, for instance $\rho(r)$ and $\Gamma_1(r)$, the density and the first adiabatic exponent in terms of the radius. However, for OSC, the model file must give at each point the values of the radius r, the mass m in the sphere of radius r, the total pressure P, the density ρ and the first adiabatic exponent Γ_1. It is clear that m and P could have been computed from the other quantities. The program does not require the Brunt–Väisälä frequency, often poorly computed by evolution codes.

If the first point is not at the centre, OSC computes the oscillations of the given envelope with a rigid boundary condition at the bottom. Of course, when neglecting the oscillatory behaviour of the core, great attention must be paid to the physical meaning of the output of the program.

The outer boundary conditions will be applied at the last point, considered as the surface of the star.

Inside the code, the model is described by the following five dimensionless quantities: $x = r/R$, q/x^3 (with $q = m/M$), $RP/GM\rho$, $4\pi R^3\rho/M$ and Γ_1, where R and M denote the radius and the total mass of the star.

3 Stellar oscillations

3.1 Oscillation modes

The small perturbations of a spherical star without rotation or magnetic field may be described as a superposition of normal modes of oscillation which are the solutions of a linear boundary eigenvalue problem. These normal modes may be

indexed by three integers k, ℓ and m. Index k is loosely related to the number of nodes of the radial displacement. Indices ℓ and m are the usual indices of the spherical function $Y_{\ell m}(\theta,\phi)$ describing the angular dependence. Index ℓ may take any null or positive integer value and m may take $2\ell+1$ values between $-\ell$ and $+\ell$.

In the following description we use the notation δX and X' for the Lagrangian and Eulerian perturbation of any quantity X and σ for the angular frequency. We often use the dimensionless angular frequency $\omega=\sigma\tau_{dyn}$, where the dynamical time τ_{dyn} is defined as $\sqrt{R^3/GM}$.

3.2 Oscillation equations

The theory of stellar oscillation has been developed in a number of textbooks. We are the most familiar with the paper of Ledoux and Walraven (1958) and the books of Unno et al. (1979) and Cox (1980). We will just write the needed equations in the form they are implemented in our code.

3.2.1 Radial oscillations

In the case of radial oscillations ($\ell=m=0$), the equation of Poisson can be integrated and the perturbation of the gravitational potential eliminated. The differential system is then reduced to order two. We describe a normal mode with two functions $Y(x)$ and $Z(x)$. Disregarding an arbitrary phase, the displacement $\boldsymbol{\delta}\mathbf{r}$ and the Lagrangian perturbation of the pressure δP are written in terms of $Y(x)$ and $Z(x)$ in the following way,

$$\boldsymbol{\delta}\mathbf{r}=\Re\{a(r)e^{-i\sigma t}\mathbf{e}_r\}=\sqrt{4\pi}\Re\{a(r)Y_{00}(\theta,\phi)e^{-i\sigma t}\mathbf{e}_r\},\quad(1)$$

where $\mathbf{e}_r$ is a unit vector in the radial direction. Near the centre, $a(r)\propto r$ and may be written

$$a(r)/r=Y(x)\quad\text{or}\quad a(r)/R=xY(x).\quad(2)$$

In a similar way,

$$\delta P/P=\Re\{Z(x)e^{-i\sigma t}\}=\sqrt{4\pi}\Re\{Z(x)Y_{00}(\theta,\phi)e^{-i\sigma t}\}.\quad(3)$$

Now, the differential equations read

$$\frac{dY}{dx}=-\frac{3}{x}Y-\frac{1}{\Gamma_1 x}Z,\quad(4)$$

$$\frac{dZ}{dx}=\left(4\frac{q}{x^3}+\omega^2\right)\frac{GM\rho}{RP}xY+\frac{GM\rho}{RP}\frac{q}{x^3}xZ.\quad(5)$$

These equations must be completed by the boundary conditions. At the centre, the regularity of the solution is ensured by the condition

$$3\Gamma_1 Y+Z=0\quad\text{at } x=0.\quad(6)$$

When an envelope model is given, the condition at the centre is replaced by a condition at the bottom of the envelope,

$$Y=0.\quad(7)$$

At the surface, we generally apply a condition deduced from the vanishing of the pressure. In this case, the coefficient $GM\rho/RP$ in (5) tends to infinity and the regularity of the solution requires that

$$\left(4\frac{q}{x^3}+\omega^2\right)Y+\frac{q}{x^3}Z=0.\quad(8)$$

Note that if R is the value of r at the last point, $x=q/x^3=1$ at the surface but this is not mandatory. The user can choose to apply the more usual condition

$$\delta P=0\quad\text{or}\quad Z=0.\quad(9)$$

3.2.2 Nonradial oscillations

In the case of nonradial oscillations ($\ell\neq 0$), the differential system is of order four. We describe a normal mode with four functions $Y(x)$, $Z(x)$, $U(x)$ and $V(x)$. These functions as well as the frequency do not depend upon index m ($2\ell+1$-fold degeneracy). The displacement reads

$$\overrightarrow{\delta r}=\sqrt{4\pi}\Re\left\{\left[a(r)Y_{\ell m}(\theta,\phi)\mathbf{e}_r+b(r)\left(\frac{\partial Y_{\ell m}(\theta,\phi)}{\partial\theta}\mathbf{e}_\theta+\frac{1}{\sin\theta}\frac{\partial Y_{\ell m}(\theta,\phi)}{\partial\phi}\mathbf{e}_\phi\right)\right]e^{-i\sigma t}\right\},\quad(10)$$

where $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ form the usual local cartesian basis of spherical coordinates. Near the centre, $a(r)$ and $b(r)\propto r^{\ell-1}$ and are written

$$a(r)/R=x^{\ell-1}Y(x),\quad(11)$$

$$b(r)/R=\frac{x^{\ell-1}}{\omega^2}\left[U(x)+\frac{RP}{GM\rho}Z(x)+\frac{q}{x^3}Y(x)\right].\quad(12)$$

The Lagrangian perturbation of pressure δP and the Eulerian perturbation of the gravitational potential Φ' are given by

$$\frac{\delta P}{P}=\sqrt{4\pi}\Re\{x^\ell Z(x)Y_{\ell m}(\theta,\phi)e^{-i\sigma t}\},\quad(13)$$

$$\frac{R\Phi'}{GM}=\sqrt{4\pi}\Re\{x^\ell U(x)Y_{\ell m}(\theta,\phi)e^{-i\sigma t}\},\quad(14)$$

$$\frac{R^2}{GM}\frac{\partial\Phi'}{\partial r}=\sqrt{4\pi}\Re\left\{x^{\ell-1}\left[V(x)-\frac{4\pi R^3\rho}{M}Y(x)\right]\times Y_{\ell m}(\theta,\phi)e^{-i\sigma t}\right\}.\quad(15)$$

Of course, the solution of a linear problem may be multiplied by an arbitrary factor and the $\sqrt{4\pi}$ factor in the above

expressions may be dropped. We have put it there for aesthetic reasons because the spherical functions are normalized in such a way that

$$\int_{4\pi} |Y_{\ell m}|^2 \, d\Omega = 1. \tag{16}$$

With this $\sqrt{4\pi}$ factor, the time-average kinetic energy of a mode is given by

$$\begin{aligned}\overline{E}_{kin} &= \int \frac{1}{2}\rho \overline{v^2}\, dV \\ &= \frac{\sigma^2}{4}\int [a^2 + \ell(\ell+1)b^2] 4\pi r^2 \rho \, dr \\ &= \frac{\sigma^2}{4}\int [a^2 + \ell(\ell+1)b^2]\, dm, \end{aligned} \tag{17}$$

without any π factor.

With these definitions, the oscillation equations read

$$\frac{dY}{dx} = \frac{\ell+1}{x}\left\{-Y + \frac{\ell}{\omega^2}\left(\frac{q}{x^3}Y + \frac{RP}{GM\rho}Z + U\right)\right\} - \frac{x}{\Gamma_1}Z, \tag{18}$$

$$\begin{aligned}\frac{dZ}{dx} = \frac{GM\rho}{RP}\left\{\left(\omega^2 + 4\frac{q}{x^3}\right)\frac{Y}{x} + x\frac{q}{x^3}Z - \frac{V}{x}\right. \\ \left. - \frac{\ell(\ell+1)}{x\omega^2}\frac{q}{x^3}\left(\frac{q}{x^3}Y + \frac{RP}{GM\rho}Z + U\right)\right\} - \frac{\ell}{x}Z, \end{aligned} \tag{19}$$

$$\frac{dU}{dx} = \frac{1}{x}\left(V - \frac{4\pi R^3\rho}{M}Y - \ell U\right), \tag{20}$$

$$\begin{aligned}\frac{dV}{dx} = \frac{\ell+1}{x}(\ell U - V) \\ + \frac{\ell(\ell+1)}{x\omega^2}\frac{4\pi R^3\rho}{M}\left(\frac{q}{x^3}Y + \frac{RP}{GM\rho}Z + U\right). \end{aligned} \tag{21}$$

These equations have been published by Boury et al. (1975), but their equation (9) has been affected by a typo (an extra factor ℓ).

The regularity of the solution at the centre imposes two conditions at $x = 0$,

$$Y = \frac{\ell}{\omega^2}\left[\frac{q}{x^3}Y + \frac{RP}{GM\rho}Z + U\right], \tag{22}$$

$$V = \frac{4\pi R^3\rho}{M}Y + \ell U. \tag{23}$$

In the case of an envelope, the bottom of the envelope is supposed to behave as a rigid boundary,

$$Y = 0. \tag{24}$$

There are no movement nor density perturbations in the core, where the perturbation of the gravitational potential obeys a Laplace equation and has a simple analytical expression. We thus require a continuous match between Φ' and its gradient at the bottom of the envelope. This is expressed as

$$V = \ell U. \tag{25}$$

Two boundary conditions must be imposed at the surface. The first one involves the Lagrangian perturbation of the pressure δP. As for the radial case, the default choice is deduced from the requirement of regularity of the solution when P vanishes at the surface. In our variables, this condition reads

$$\begin{aligned}\left[\omega^2 + 4\frac{q}{x^3} - \frac{\ell(\ell+1)}{\omega^2}\left(\frac{q}{x^3}\right)^2\right]\frac{Y}{x} + x\frac{q}{x^3}Z \\ - \frac{\ell(\ell+1)}{\omega^2 x}\frac{q}{x^3}U - \frac{V}{x} = 0. \end{aligned} \tag{26}$$

The user can however choose to impose the more usual condition

$$\delta P = 0 \quad \text{or} \quad Z = 0. \tag{27}$$

The second boundary condition ensures the matching of Φ' and its gradient with the regular solution of the Laplace equation outside the star,

$$V + (\ell+1)U = 0. \tag{28}$$

Our choice of variables and the way the differential equations are written call for three remarks:

(1) The inclusion of a term in Y in the definition of V (15) ensures that the boundary condition (28) is still valid in the (unphysical) case of a non vanishing density at the surface of the model.
(2) The use of the Lagrangian perturbation of the pression results in a better precision in the external layers.
(3) We do not use the Brunt–Väisälä frequency n in the coefficients of the equations, often badly computed by stellar evolution codes.

However, we use non independent functions $\rho(r)$, $m(r)$ and $P(r)$. Troubles may stem from their possible inconsistencies. Maybe it would have been wiser to let OSC compute m and P from ρ.

The solutions computed by OSC are normalized in such a way that

$$\int [a^2 + \ell(\ell+1)b^2]\, dm = MR^2.$$

3.3 Mode classification

The radial modes owe their existence to the compressibility of the stellar material (acoustic or pressure modes). The mode with the lowest frequency is called the fundamental mode, it has no node in the displacement (except at the origin) and is called p_1. By order of increasing frequency and number of nodes, we have then the first harmonics (one node, p_2), the second harmonics (two nodes, p_3), ...

For nonradial modes, the situation is more complicated. For each ℓ, we have a spectrum of p-modes. They are of the same nature as the radial modes, owing their existence to the compressibility of the stellar material. They are numbered p_1, p_2, ... by order of increasing frequency. If the stellar model has a radiative zone, it has a spectrum of g^+-modes. Their frequencies are lower than those of the p-modes and have an accumulation point at zero. They are numbered g_1^+, g_2^+, ... by order of decreasing frequency. They owe their existence to the buoyancy force. For each value of $\ell > 1$, there exists one f-mode, with its frequency between those of the p-modes and the g-modes. This mode does not disappear when the stellar material is incompressible nor when the buoyancy force is zero. When the star has a convective zone, another spectrum appears, the g^--modes. They have an exponential temporal behaviour (their frequencies are imaginary) and are associated with convection. We neglect them in the following discussion.

In OSC, we use an integer to denote the type and order of a computed mode: n for p_n, $-n$ for g_n^+ and 0 for f. In the Cowling's approximation (the Eulerian perturbation of the gravitational potential is neglected), the order of a given mode can be easily deduced from the behaviour of the vertical and horizontal displacements (Scuflaire 1974b; Gabriel and Scuflaire 1979, 1980). In our case, the mode number obtained in the same way is generally correct for models which are not too evolved. But as the condensation (measured by the ratio $\rho_c/\bar{\rho}$) increases with the age of the model, it can just be considered as a clue and finally looses any meaning. The algorithm described by Lee (1985) gives a clue to the mode number (also computed by OSC) which keeps its utility a bit longer. But when the condensation of the model is really too high, the only reliable identification method consists in the computation of a large number of contiguous modes, up to the asymptotic domain, where the implemented algorithms continue to give reliable mode numbers.

Though the order of the mode cannot be obtained safely, its parity can and is provided by OSC.

3.4 Influence of rotation

The rotation of the star removes the degeneracy of the nonradial oscillation frequencies. If $\sigma_{k\ell}^0$ denotes the frequency in the absence of rotation, a slow solid rotation with angular velocity Ω slightly alters the frequencies in the following way

$$\sigma_{k\ell m} = \sigma_{k\ell}^0 + m\beta_{k\ell}\Omega, \tag{29}$$

with

$$\beta_{k\ell} = 1 - \frac{\int (b^2 + 2ab)\, dm}{\int [a^2 + \ell(\ell+1)b^2]\, dm}. \tag{30}$$

When the angular velocity depends on the radius, the altered frequencies may be written

$$\sigma_{k\ell m} = \sigma_{k\ell}^0 + m \int K_{k\ell}(x)\Omega(x)\, dx, \tag{31}$$

where the kernel $K_{k\ell}(x)$, computed by OSC, is given by

$$K_{k\ell}(r) = \frac{\rho r^2 [a^2 + \ell(\ell+1)b^2 - 2ab - b^2]}{\int \rho r^2 [a^2 + \ell(\ell+1)b^2]\, dr}. \tag{32}$$

3.5 Physical description of the modes

For low order modes, specially for evolved models, the physical characteristics of a mode is not tightly linked to its g or p label (Scuflaire 1974a, 1980). OSC outputs different indexes allowing the user a quick analysis of the physical behaviour of a computed mode (gravity or pressure wave, trapped mode, ...).

4 Technique of solution

4.1 Interpolation of the model

The grid of points used for the computation of the model is rarely appropriate for the computation of oscillations, as the eigenfunctions can exhibit rapid spatial oscillations in regions where the variables describing the model are well-behaved. As the oscillatory behaviour of eigenfunctions is easy to foresee, we interpolate the model before any oscillation computation, increasing the number of points where they will prove necessary. The interpolation method we use preserves the continuity of the first derivatives.

4.2 Difference equations

We have adopted a difference equation scheme of the fourth order. That is why we do not need to use Richardson extrapolation method to increase the precision of the eigenfrequency.

Our difference scheme rests on the following identity satisfied by any vector function $\mathbf{y}(x)$ with continuous derivatives up to the fifth order.

$$\mathbf{y}_i + \frac{h}{2}\mathbf{y}'_i + \frac{h^2}{12}\mathbf{y}''_i = \mathbf{y}_{i+1} - \frac{h}{2}\mathbf{y}'_{i+1} + \frac{h^2}{12}\mathbf{y}''_{i+1} + O(h^5), \tag{33}$$

where $\mathbf{y}_i$ and $\mathbf{y}_{i+1}$ are the values of $\mathbf{y}(x)$ at points x_i and x_{i+1} and $h = x_{i+1} - x_i$. If $\mathbf{y}$ is a solution of the linear differential system

$$\frac{d\mathbf{y}}{dx} = A(x)\mathbf{y}, \tag{34}$$

identity (33) may be written

$$\left\{1 + \frac{h}{2}\alpha_i + \frac{h^2}{12}\beta_i\right\}\mathbf{y}_i = \left\{1 - \frac{h}{2}\alpha_{i+1} + \frac{h^2}{12}\beta_{i+1}\right\}\mathbf{y}_{i+1} + O(h^5), \tag{35}$$

with

$$\alpha = A, \tag{36}$$

$$\beta = A^2 + \frac{dA}{dx}. \tag{37}$$

The difference equations are easily obtained from the above equations, neglecting the term $O(h^5)$. At the centre, certain coefficients of the matrix A are singular and a slightly different treatment is needed. The matrix can be written as

$$A(x) = \frac{1}{x}B(x), \tag{38}$$

with all the odd order derivatives of matrix B vanishing at $x = 0$. It is clear that the regularity of the solution requires, at $x = 0$,

$$B\mathbf{y} = 0. \tag{39}$$

It is worth noticing that the rank of $B(0)$ is lower than its dimension and that equation (39) gives the right number of boundary conditions (1 for the radial case and 2 for the non-radial one). The matrices α and β assume the following different forms at the centre (index 0),

$$\alpha = 0, \tag{40}$$

$$\beta = (2 - B_0)^{-1}\left(\frac{d^2B}{dx^2}\right)_0. \tag{41}$$

4.3 Inverse iteration method

After the discretization of the differential equations we are left with an algebraic eigenvalue problem where the eigenvalue is $\lambda = \omega^2$. OSC uses the inverse iteration method. It is a powerful tool in linear eigenvalue problems. It is described in the book of Wilkinson (1965, Chap. 9, Sect. 47). It has been used by Keeley (1977) to compute stellar radial nonadiabatic oscillations. The principle of the method is easy to describe. Consider the following eigenvalue problem

$$(A - \lambda B)\mathbf{y} = 0. \tag{42}$$

The method is generally exposed with a unit matrix in place of B. Suppose that we know an approximation λ_0 of an eigenvalue λ. Starting with an arbitrary vector $\mathbf{y}_0$, we build the sequence

$$\mathbf{y}_{n+1} = -(A - \lambda_0 B)^{-1}B\mathbf{y}_n, \quad n = 0, 1, 2, \ldots. \tag{43}$$

Then,

$$\frac{\mathbf{y}_n \cdot \mathbf{y}_{n+1}}{\mathbf{y}_{n+1} \cdot \mathbf{y}_{n+1}} \to \lambda - \lambda_0. \tag{44}$$

The convergence is quite fast. Practically, the $\mathbf{y}_n$ must be normalized at each step of the computation to avoid overflow. Moreover, these normalized $\mathbf{y}_n$ tend to the eigenvector associated with λ.

It is true that, in the case of nonradial oscillations, the problem to solve is not exactly in the form of (42). But it is put in the right form if we write

$$\lambda = \lambda_0 + \Delta\lambda \tag{45}$$

and linearize with respect to the correction $\Delta\lambda$. The solution is then obtained in a few iterations of this process.

5 Use of the program

OSC is written in Fortran and can be used either as a stand-alone program or as a library of subroutines that the user calls from a main program of his own. The stand-alone program is in fact just a user interface to the library. It accepts instructions from the standard input (or from a command file) and prints all kind of information to the standard output. At the request of the user, the eigenfunctions can be saved to a file. For heavy work, the user had better write his own Fortran main program and call the routines of the library. He gains a better control on the computation and access to results not available otherwise (the rotation kernels for a r-dependent angular velocity Ω, for instance).

6 Applications

OSC is routinely used in our group in Liège and by members of the Belgian Asteroseismology Group (BAG) for seismic studies of solar-like pulsators such as, e.g., α Cen A+B (Thoul et al. 2003a; Miglio and Montalbán 2005) and of classical β Cephei variables (Aerts et al. 2003; Thoul et al.

2003b; Dupret et al. 2004; Ausseloos et al. 2004; Thirion and Thoul 2006; Briquet et al. 2007). It is worth mentioning that in these studies, we obtained indications on the internal rotation of the β Cephei stars HD 129929 and θ Ophiuchi.

The adiabatic frequencies computed by OSC are also used as first approximations by the program MAD which computes the non adiabatic oscillations of stellar models. See Dupret (2001) for a description of the code.

The Liège oscillation code has also taken part in the work and code comparisons realized within the Corot/ESTA group (Montalbán and Scuflaire 2007; Montalbán et al. 2007).

7 Discussion

There is not any general agreement on what the outer boundary condition on the perturbation of pressure should be and different boundary conditions are implemented in the existing codes. In our opinion, the precise choice of the outer boundary condition does not matter so much, as, in any way, the oscillation is far from adiabatic in the very external layers of the star. Moreover, Dzhalilov et al. (2000) have shown that in the Sun, waves in the frequency range $\nu \approx 2$–10 mHz may reach the chromosphere-corona transition regions by means of a tunneling through the atmospheric barrier. When comparing with observations, a reasonable strategy is thus to use expressions of frequencies which are insensitive to the very external stellar layers (Roxburgh and Vorontsov 2003; Roxburgh 2005).

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