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The Light-Cone Expansion and the Parton Model in Polarized Muon-Production

G. Altarelli and B. Lampe
CERN - Geneva

ABSTRACT

The relation between the light-cone expansion and the QCD improved parton model formalism for polarized deep inelastic muon-production is discussed in detail. The regulator dependence of the relevant quantities is explicitly studied at the two-loop level for the first moment of g_1^P. It is shown that the anomalous gluon component is well defined if the quark term is specified as a conserved quantity that does not evolve in Q^2. The behaviour of the first moment of g_1^P at the threshold for heavy quark production is also discussed.

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1. Introduction

Recently the EMC collaboration at CERN measured [1] the polarized proton structure function g_1^P in deep inelastic muon scattering, confirming and considerably extending previous measurements performed at SLAC [2]. These results have produced great surprise because, when interpreted in the naive parton model, they seem to imply that the total fraction of the proton helicity carried by all quarks and antiquarks nearly vanishes [3]. Even if one bears in mind the difference between constituent and parton quarks this result still remains surprising [4-7]. This is because in the unpolarized case the difference between constituent and parton quarks can be well understood, at least at the semi-quantitative level, by extrapolating the perturbative QCD evolution down to small values (of order Λ_{QCD}) of the energy scale Q [8]. The same approach applied to the polarized case appears to lead to a contradiction with the EMC results, because the spin carried by each flavour of quarks is separately conserved by the leading order QCD evolution [9,10].

A solution to this problem has been proposed in Refs. [11-12] (although in Ref. [11] the quantitative consequences were not correctly derived) and further developed in Refs. [13,14]. It was shown there that, due to the axial anomaly [15], the naive parton model relation with the quark helicity is in general not valid for the first moment M_1^S of the flavour singlet part of g_1^P:

$$M_1^S(Q) = \frac{2}{< e^2 >} \int_0^1 dx\, g_1^P(x, Q)\,|_{singlet} \tag{1}$$

where $< e^2 > \equiv \Sigma_i\, e_i^2/N_f = \frac{2}{9}$ for $N_f = 3$.

In fact, the gluon contribution to M_1^S is not in general suppressed by a power of $\alpha_s(Q)$. As a consequence the EMC result can after all be consistent with a large quark spin component. What the experiments show is that the matrix element between polarized protons of the flavour singlet axial current is nearly zero. But, due to the axial anomaly, an *a priori* non negligible gluon contribution at large Q is present in M_1^S, so that the singlet axial current matrix element is not so direclty related to the total spin of the quarks (the reasons why experimentally the value of M_1^S is nearly zero would, however, remain to be found [6,16]).

The explanation in terms of an anomalous gluon component has been criticized on different points. In the light cone formalism [17-19] only one operator couples to M_1^S: the singlet axial current. Thus the separation of this single contribution into a quark and a gluon part has been questioned [20]. Moreover, while the anomalous gluon term is obtained consistently in the case of massless quarks by several different methods [12-14] (e.g., from the known operator form of the anomaly, or from a direct diagrammatic evaluation), it was shown in Ref. [13] that the contribution to M_1^S of the diagrams in Fig. 1 depends on the regulator. It is zero for $m^2 \neq 0, p^2 = 0$ (where m is the mass of the produced quarks, and p^2 is the off-shell mass of the gluon), while it gives a finite result $c_g = -N_f \frac{\alpha_S}{2\pi}$ for the coefficient of the gluon moment Δg in the opposite limit $m^2 = 0, p^2 \neq 0$. It has been claimed that this regulator dependence implies that the anomalous gluon component cannot be properly defined [20,21]. Other authors [22-25] pointed out that, if care is not taken in properly separating a quark and a gluon term, instabilities might be produced when the values of the light quark masses are varied and large isospin violations can appear in each term (their sum M_1^S being stable and isospin invariant).

The present article is devoted to a discussion of some of these issues and of related ones. We discuss the relation between the light-cone formalism and the more general approach to hard processes based on the QCD improved parton model. We explicitly show that the reason why the gluon diagrams of Fig. 1 have a different value in the massless ($m^2 = 0$) and in the massive ($m^2 \neq 0$) cases is because the corresponding definition of the singlet quark moment $\Delta\Sigma$ is being changed at the same time. The anomalous gluon contribution actually remains unchanged if a fixed, objective definition of $\Delta\Sigma$ and Δg is kept when the regulators are varied. In particular, if the quark moments are defined as conserved quantities, i.e., the definition that should best correspond to the intuition based on constituent quarks, then the anomalous gluon component is specified by the coefficient $c_g = -N_f \frac{\alpha_S}{2\pi}$ as derived very simply in the theory with massless quarks [12-14]. The gluon component so specified is also directly related to large k_T jets produced in lepto-production [13] , which can then be used, as well as other hard processes sensitive to polarized gluons [26], for an experimental test of the reality of a large polarized gluon density. In Ref. [14] the technical reason for the variation with m of the quark definition was already pointed out. When the quark mass m is switched on, a new breaking term of chiral invariance is introduced which produces a non-vanishing two-loop anomalous dimension for the quark moment $\Delta\Sigma$ which otherwise would be conserved for $m = 0$. Here we explicitly demonstrate this mechanism by a calculation with and without masses of the two-loop diagrams shown in Fig. 2. We use the insight gathered from this study to discuss various aspects connected with the physical interpretation of the anomalous gluon term which have been brought up in the recent literature. In particular we discuss the effects associated to the opening of the charm threshold at sufficiently large Q.

2. The light-cone expansion and the parton approach to hard processes

We start by some remarks on the theoretical framework on which our discussion is based. As is well known, the light-cone operator expansion provides a method of general validity for the study of the structure functions of deep inelatic scattering and their scaling violations in QCD. But not all questions can be answered by the light-cone approach. For example, the magnitude of $\Delta\Sigma$ or of Δs (the strange quark contribution to $\Delta\Sigma$) is not restricted by the light-cone method. Moreover this method cannot be extended to other hard processes where polarized parton densities could also be measured.

The QCD improved parton model [10], based on the factorization theorem [27] derived by diagrammatic techniques, provides a generalization of the light-cone results that has been successfully applied and tested in all kinds of hard processes [28]. It is interesting that the application of the standard techniques of the QCD improved parton model to polarized leptoproduction, as done in Refs. [11-14], leads to a deeper understanding of the problem and provides testable predictions for other hard processes.

In the parton approach one assumes that all quark and gluon densities can be defined starting from a sufficient number of physical hard processes. The QCD evolution equations for quark and gluon densities can be written down (for both polarized and unpolarized densities) with kernels that at leading order are directly obtained from the QCD vertices without reference to the particular process used to define the densities. Of course, beyond the leading order, the two-loop evolution kernels start depending to some extent on the exact definition of the parton densities. In the parton approach the primary quantities are the parton densities: $q(x,Q)$ and $g(x,Q)$ for unpolarized targets, $\delta q(x,Q)$ and $\delta g(x,Q)$ in the polarized case. The

moments, which are the basic quantities for the light cone expansion, are derived entities in the parton picture. Provided that the corresponding x-integration is convergent, any moment (even non-integer ones) can be constructed from the densities. In the singlet sector of polarized leptoproduction there are two sets of local operators in the light-cone expansion [17,18] (for general n values): one set is constructed out of quark fields and their covariant derivatives and one set is made of gluon fields. However, for $n = 1$ (which corresponds to M_1^S) there is no gauge invariant gluon operator of dimension three: in the gluon set one element is missing. This fact does not necessarily imply that the first moment of Δg cannot be defined and measured in any hard process. In the parton picture, one sees no reason why the first moment of the polarized gluon density should not be considered. While it is true that the only operator which appears in the light-cone expansion for M_1^S is j_μ^5, the axial current, the problem remains of the relation between the operator j_μ^5, its matrix elements and coefficient functions and the first moments of $\Delta\Sigma(Q)$ and $\Delta g(Q)$. Usually, in similar cases, only a minor ambiguity can be expected: a quark operator corresponds to a moment of the quark density apart from a possible small correction of order $\alpha_s(Q)$ from the gluon density. The peculiarity of the present case is that $\Delta g(Q)$, the first moment of the gluon density, as computed with no ambiguity in leading order QCD evolves as $(\alpha_s(Q))^{-1}$ so that the product $\alpha_s(Q)\Delta g(Q)$ is not necessarily small. Then either Δg identically decouples from M_1^S for whatever objective definition one takes to define quark and gluon densities or the result obtained in the light-cone method from j_μ^5 must correspond to some combination of the quark and gluon moments which have been independently defined. One finds that the latter case is true in terms of a simple definition of $\Delta\Sigma$.

We now discuss in some detail the QCD parton model formalism in this somewhat special case. In lowest order the QCD evolution equations for the first moment of polarized densities lead [9,10] with no ambiguity to the results:

$$\frac{d}{dt}\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix}_t = \frac{\alpha_t}{2\pi}\begin{pmatrix}\gamma_{qq}^{(1)} & \gamma_{qg}^{(1)}\\ \gamma_{gq}^{(1)} & \gamma_{gg}^{(1)}\end{pmatrix}\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix}_t + 0(\alpha_t^2)$$
$$= \frac{\alpha_t}{2\pi}\begin{pmatrix}0 & 0\\ \frac{3}{2}C_F & \beta_0\end{pmatrix}\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix}_t + 0(\alpha_t^2) \qquad (2)$$

where $t = \ell n Q^2/\mu^2$ (with μ being a reference scale), $\alpha_t \equiv \alpha_s(t), \Delta\Sigma_t \equiv \Delta\Sigma(t), \Delta g_t \equiv$

$\Delta g(t)$, $C_F = \frac{4}{3}$ and β_0 is defined by

$$\frac{d}{dt}\frac{\alpha_t}{2\pi} = -\beta_0 \left(\frac{\alpha_t}{2\pi}\right)^2 - \beta_1 \left(\frac{\alpha_t}{2\pi}\right)^3 + 0(\alpha_t^4) \tag{3}$$

Note that $\gamma_{gq}^{(1)}$, $\gamma_{qg}^{(1)}$ and $\gamma_{gg}^{(1)}$ are all found to coincide with the $n = 1$ extrapolation of the light-cone results obtained [17] for $n > 1$. This means that one can think of the partonic gluon density as being derived (at leading accuracy) by inverse Mellin transforming the light-cone gluon moments with $n > 1$. Once the gluon density $\delta g(x, Q)$ has been constructed in this way, one can then obtain the $n = 1$ moment $\Delta g(Q)$ by integration of δg. Note that the same method leads to the correct Q dependence for non-integer moments. This procedure could in principle be wrong by $\delta(x)$ terms which indeed only contribute to the first moment. Such terms can only be of non-perturbative origin, because $\delta(x)$ terms clearly cannot be generated from any finite sum of Feynman diagrams for the physical cross-section. Note that these terms would in any case be totally irrelevant to the interpretation of the EMC/SLAC data (which only exist for $x \gtrsim 0.01$) and also to all data yet to come on other hard processes involving the same densities. However, the possible presence of such non-perturbative terms should be kept in mind when discussing spin sum rules, generalized Goldberger-Treiman relations [22-25] and instanton effects (like those related to the operator k_μ, discussed in Refs. [20,29,30], whose forward matrix elements are gauge invariant except for topologically non-trivial transformations that change the winding number).

The form of Eq. (2) and in particular the fact that $\gamma_{gg}^{(1)} = \beta_0$ suggest the change of variables $(\Delta\Sigma, \Delta g) \to (\Delta\Sigma, \Delta\Gamma = \frac{\alpha}{2\pi}\Delta g)$ or

$$D' \equiv \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{\alpha}{2\pi} \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} \equiv M(\alpha) D \tag{4}$$

The general solution of the evolution equations for moments has the form:

$$D(\alpha_t) = E(\alpha_t, \alpha) D(\alpha) \tag{5}$$

with

$$E(\alpha_t, \alpha) = T \exp \int_0^t \gamma(t')dt' \tag{6}$$

and $\alpha \equiv \alpha_s(t = 0)$. From Eqs. (4) and (5) it follows that:

$$D'(\alpha_t) = M(\alpha_t) E(\alpha_t, \alpha) M^{-1}(\alpha) D'(\alpha) \tag{7}$$

and

$$\frac{d}{dt} D'(\alpha_t) = \left[\frac{dM}{dt}(\alpha_t) M^{-1}(\alpha_t) + M(\alpha_t)\gamma(\alpha_t) M^{-1}(\alpha_t)\right] D'(\alpha_t) \tag{8}$$

where

$$\gamma(\alpha) = \frac{\alpha}{2\pi}\gamma^{(1)} + (\frac{\alpha}{2\pi})^2 \gamma^{(2)} + \dots \tag{9}$$

is the "anomalous dimension" matrix for $D = \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}$ expanded to one and two loops. With a little algebra, from Eqs. (8) and (9), we obtain

$$\frac{d}{dt} \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix}_t = (\frac{\alpha_t}{2\pi})^2 \begin{pmatrix} \gamma_{qq}^{(2)} & \gamma_{qg}^{(3)} \\ \gamma_{gq}^{(1)} & \gamma_{gg}^{(2)} - \beta_1 \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix}_t + 0(\alpha_t^3) \tag{10}$$

Here the explicit form of the matrix $\gamma^{(1)}$ given in Eq. (2) was taken into account and also the fact that $\gamma_{qg}^{(2)}$ must be zero for consistency. Equation (10) has been already introduced and discussed in Ref. [12] where the vanishing of $\gamma_{qg}^{(2)}$ was proved in the massless theory (in Ref. [12] the entry $\gamma_{gg}^{(2)} - \beta_1$ was simply denoted by $\gamma_{gg}^{(2)}$ which is not really precise).

An approximate solution of the evolution equation, Eq. (10), is given by:

$$D'(\alpha_t) \equiv \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix}_t = \left\{1 + \frac{\alpha - \alpha_t}{2\pi\,\beta_0} \begin{pmatrix} \gamma_{qq}^{(2)} & \gamma_{qg}^{(3)} \\ \gamma_{gq}^{(1)} & \gamma_{gg}^{(2)} - \beta_1 \end{pmatrix}\right\} \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix}_{t=0} \tag{11}$$

For the physical quantity of interest $M_1^S(Q)$ defined in Eq. (1), one has:

$$M_1^S(Q) = c(\alpha_t) D(\alpha_t) = c'(\alpha_t) D'(\alpha_t) \tag{12}$$

with

$$c'(\alpha) = c(\alpha)M^{-1}(\alpha)$$
$$= (1 + \frac{\alpha}{2\pi}c_\Sigma + \ldots, \frac{\alpha}{2\pi}c + (\frac{\alpha}{2\pi})^2 c_\Gamma + \ldots). \begin{pmatrix} 1 & 0 \\ 0 & (\frac{\alpha}{2\pi})^{-1} \end{pmatrix} \quad (13)$$
$$= (1 + \frac{\alpha}{2\pi}c_\Sigma + \ldots, \; c + \frac{\alpha}{2\pi}c_\Gamma + \ldots)$$

By combining Eqs. (11) - (13), on the one hand, one obtains the result:

$$M_1^S(Q) = [1 + \frac{\alpha}{2\pi}c_\Sigma + \frac{\alpha - \alpha_t}{2\pi\, \beta_0}(\gamma_{qq}^{(2)} + c\gamma_{gq}^{(1)} - \beta_0\, c_\Sigma)]\, \Delta\Sigma_{t=0}$$
$$+ [c + \frac{\alpha}{2\pi}\, c_\Gamma + \frac{\alpha - \alpha_t}{2\pi\, \beta_0}(\gamma_{qg}^{(3)} + c(\gamma_{gg}^{(2)} - \beta_1) - \beta_0\, c_\Gamma)]\, \Delta\Gamma_{t=0} \quad (14)$$

On the other hand, the light-cone formalism, in terms of the single operator j_μ^5, leads [18,19] to the corresponding expression:

$$M_1^S(Q) = [1 + \frac{\alpha}{2\pi}c_j + \frac{\alpha - \alpha_t}{2\pi\, \beta_0}\, (\gamma_j^{(2)} - \beta_0 c_j)] \; < j^5 >_{t=0} \quad (15)$$

By comparison one obtains:

$$c_j = c_\Sigma = c_\Gamma / c$$
$$\gamma_j^{(2)} = \gamma_{qq}^{(2)} + c\gamma_{gq}^{(1)} = \gamma_{qg}^{(3)}/c + \gamma_{gg}^{(2)} - \beta_1 \quad (16)$$
$$< j^5 >_t = (\Delta\Sigma + c\; \Delta\Gamma)_t$$

Finally one can write:

$$M_1^S(Q) = [1 + \frac{\alpha}{2\pi}\, c_\Sigma + \frac{\alpha - \alpha_t}{2\pi\, \beta_0}(\gamma_{qq}^{(2)} + c\gamma_{gq}^{(1)} - \beta_0\, c_\Sigma)]\, (\Delta\Sigma + c\Delta\Gamma)_{t=0} \quad (17)$$

This is an important result which makes the relation explicit between the operator formalism and the parton method. Due to the reshuffling of the perturbative expansion induced by the replacement of Δg by $\Delta\Gamma$, $\gamma_j^{(2)}$ is a combination of the two-loop anomalous dimension of the quark moment $\Delta\Sigma$ and of a gluon term. In general it is the coefficient of α_t which is independent of the regularization. However, in the present case, due to the different form of the colour and

flavour Casimir factors, c_j and $\gamma_j^{(2)}$ or equivalently c_Σ and $\gamma_{qq}^{(2)} + c\gamma_{gq}^{(1)}$ are separately independent of the regularization (see Section 3):

$$c_j = c_\Sigma = -\frac{3}{2}C_F$$
$$\gamma_j^{(2)} = \gamma_{qq}^{(2)} + c\gamma_{gq}^{(1)} = -\frac{3}{2}C_F N_F \quad (18)$$

As $\gamma_{gq}^{(1)}$ is unambiguously determined, one concludes that $\gamma_{qq}^{(2)}$ and c change in a related way and that there is a unique value of c that corresponds to $\gamma_{qq}^{(2)} = 0$, a necessary condition for conserved quarks.

We now consider the dependence on m, the mass of produced quarks. Here m is considered just as a regulator, because we assume that the massless theory is the relevant framework for light quarks u, d and s ($N_f = 3$). The case of heavy quarks, e.g., charmed quarks, will be considered at the end of this section.

For $m = 0$, as discussed in detail in Refs. [12-14], one has

$$[c]_{m=0} = -N_f \;\; or \;\; < j^5 >_t = (\Delta\Sigma - N_f \Delta\Gamma(t))_{m=0}$$
$$[\gamma_{qq}^{(2)}]_{m=0} = 0 \quad (19)$$

We have a different situation when the quark mass m is used as a regulator. First, as shown in Ref. [13]:

$$[c]_{m\neq 0} = 0 \;\; or \;\; < j^5 >_t = [\Delta\Sigma(t)]_{m\neq 0} \quad (20)$$

Also, as explicitly computed in the next section,

$$[\gamma_{qq}^{(2)}]_{m\neq 0} = -\frac{3}{2}C_F N_f \quad (21)$$

We see that in this case $\Delta\Sigma(t)$ is not conserved. When the mass m is introduced not only c is changed, but also $\gamma_{qq}^{(2)}$ (or in other words, the definition of $\Delta\Sigma$ as is evident from the relation $[\Delta\Sigma]_{m\neq 0} = [\Delta\Sigma]_{m=0} - N_f \Delta\Gamma$), so that the physics is unaltered.

Going back to Eqs. (16), we can show that actually the following relations hold:

$$\gamma_{qg}^{(3)} = c\gamma_{qq}^{(2)} = 0$$
$$\gamma_{gg}^{(2)} - \beta_1 = c\gamma_{gq}^{(1)} \tag{22}$$

These equations are valid both in the $m = 0$ and the $m \neq 0$ cases, with the appropriate values of c and $\gamma_{qq}^{(2)}$. For $m \neq 0$ one has $c = 0$, the Kodaira operator $< j^5 >_t$ coincides with $\Delta\Sigma(t)$ (see the last of Eqs. (16) or Eq. (20)) and $\gamma_{qg}^{(3)}$ vanishes because $< j^5 >_t$ is multiplicatively renormalizable. In the case $m = 0$, $\gamma_{qq}^{(2)} = 0$ and $c = -N_f$. The results in Eqs. (22) follow from the anomalous dimension matrix for the operators j_μ^5 and k_μ [12,14] (with $\partial^\mu k_\mu = \frac{\alpha}{2\pi} F_{\mu\nu}\tilde{F}^{\mu\nu}$):

$$\frac{d}{dt}\begin{pmatrix} j_\mu^5 \\ k_\mu \end{pmatrix}_t = (\frac{\alpha_t}{2\pi})^2 \begin{pmatrix} \gamma_j^{(2)} & 0 \\ \gamma_{gq}^{(1)} & 0 \end{pmatrix} \begin{pmatrix} j_\mu^5 \\ k_\mu \end{pmatrix}_t + 0(\alpha^3) \tag{23}$$

For $m = 0$, j_μ^5 and k_μ correspond to $\Delta\Sigma - N_f\Delta\Gamma$ (see the last of Eqs. (16)) and $\Delta\Gamma$ respectively. By comparing Eqs. (10) and (23), the relations in Eqs. (22) follow. We stress that the invoked correspondence between k_μ and $\Delta\Gamma$ is limited to the statement that they have the same anomalous dimensions. The actual relation between k_μ and $\Delta\Gamma$ is discussed in Ref. [25]. Finally, note that in general from Eqs. (10) and (22) one finds:

$$\frac{d\Delta\Gamma}{dt} = (\frac{\alpha_t}{2\pi})^2 \gamma_{gq}^{(1)} (\Delta\Sigma + c\Delta\Gamma)_t \tag{24}$$

We shall make use of this equation in the following.

It has been argued in Ref. [21] that the massless limit as defined here is not really orthodox. The claim is that there is no satisfactory set of regulators in the computation of the diagrams of Fig. 1 that leads to the results given in Eqs. (19) valid in the massless limit. For example, using an off-shell mass p^2 for the gluon as a regulator is considered dangerous because allegedly gauge invariance is not guaranteed for Green functions. We observe that a regulator is only needed for $m = 0$ if one simultaneously considers the whole set of moments derived from the diagrams of Fig. 1. But the first moment in itself can be completely studied for $m = 0$ without introducing any regulator. One possibility is to trade the integration over x for an integration

over the quark transverse momentum k_T. In fact at fixed k_T, the angular integration is finite, as observed in Ref. [13] and further discussed in Ref. [14]. The resulting k_T distribution is integrable both near $k_T \simeq 0$ and at large k_T. The integral over k_T leads to the result of the massless limit. This procedure directly shows that the corresponding quark and gluon moments can be defined in terms of observable quantities. Alternatively, one can use operator methods by considering the forward matrix elements of the operators j_μ^5 and k_μ as described in Refs. [12,14]. The forward matrix elements of k_μ are gauge invariant for ordinary gauge transformations [30]. This is sufficient to legalize the use of the forward matrix elements of k_μ in perturbative QCD for purposes of understanding the parton results. However, it has been objected [20] that the forward matrix elements of k_μ are not invariant under topologically non-trivial gauge transformations that change the winding number. One can construct a non-local generalization of k_μ, discussed in ref. [29], which coincides with k_μ at the perturbative level, but its forward matrix elements are invariant under all possible gauge transformations. We stress that while the consideration of the operator k_μ can be useful it is in no way necessary. What is important is that $\Delta\Sigma$ and $\Delta\Gamma$ can be related to physical processes and are useful to make predictions for other hard processes.

It has been shown in Ref. [13] that $[c]_{m\neq0} = 0$ is obtained because of a cancellation between the contribution to the integral from the whole range of finite values of $\lambda = \frac{4k_T^2}{Q^2}$ and a large spike of opposite sign concentrated at $\lambda \lesssim \frac{4m^2}{Q^2}$. The physical hard gluon density can in principle be defined at each finite λ by measuring the rate of jet production at $k_T \to \infty, Q \to \infty$ with fixed $\lambda = \frac{4k_T^2}{Q^2}$. This procedure leads to a smooth distribution at small λ. The contribution of the spike at $\lambda \lesssim \frac{4m^2}{Q^2}$ has to be reabsorbed into the light quark definition in order to make both the quark and the gluon terms smooth in the limit $\lambda \to 0$. (The case of heavy quarks with $m >> \Lambda_{QCD}$ will be considered later.) In addition we have seen that including into the quark definition contributions from the soft k_T region is also necessary if we want our quarks to be conserved. More generally the inclusion of analogous infra-red sensitive terms into the quark definition can also be important to make $\Delta\Sigma$ and $\Delta\Gamma$ separately isospin conserved and stable under mass effects. If quarks and gluons are not appropriately defined in terms of physical quantities then isospin non-invariant terms appear both in $\Delta\Sigma$ and $\Delta\Gamma$, while they cancel in the combination corresponding to M_1^S. These pathologies clearly show that those badly defined quark and gluon moments are not those defined in terms of physical hard processes.

It is instructive to make the relation between the massive and the massless case completely explicit. We start from the relation between the operator j_μ^5 at the scale Q and at the scale μ:

$$j_\mu^5(t) = [\exp \int_0^t (\frac{\alpha_t}{2\pi})^2 \gamma_j^{(2)}\, dt] j_\mu^5(0) \tag{25}$$

where higher orders in α_t are neglected in the integral. In the massive theory $c = 0$, so that $j_\mu^5(0) \Rightarrow [\Delta\Sigma(0)]_{m\neq 0}$. According to Eqs. (2) and (18), $\gamma_j^{(2)} = -\frac{3}{2} C_F N_f$ can always be written as $\gamma_j^{(2)} = -N_f \gamma_{gq}^{(1)}$. Then, Eq. (25) is equivalent to:

$$[\Delta\Sigma(t)]_{m\neq 0} = [\exp \int_0^t (\frac{\alpha_t}{2\pi})^2 (-N_f \gamma_{gq}^{(1)}) dt]\, [\Delta\Sigma(0)]_{m\neq 0} \tag{26}$$

Clearly this quark is not conserved. But we note that from Eq. (24) it follows that

$$\frac{d\Delta\Gamma}{dt} \simeq (\frac{\alpha_t}{2\pi})^2\, \gamma_{gq}^{(1)}\, [\Delta\Sigma(t)]_{m\neq 0} \tag{27}$$

or

$$\Delta\Gamma(t) = [-1 + \exp \int_0^t (\frac{\alpha_t}{2\pi})^2 \gamma_{gq}^{(1)} dt]\, [\Delta\Sigma(0)]_{m\neq 0} + \Delta\Gamma(0) \tag{28}$$

As a consequence, Eq. (26) at two-loop accuracy can be rewritten in the form

$$\begin{aligned} [\Delta\Sigma(t)]_{m\neq 0} &= [\Delta\Sigma(0)]_{m\neq 0} + [-1 + \exp \int_0^t (\frac{\alpha_t}{2\pi})^2 (-N_f \gamma_{gq}^{(1)})]\, [\Delta\Sigma(0)]_{m\neq 0} \\ &= [\Delta\Sigma(0)]_{m\neq 0} + N_f \Delta\Gamma(0) - N_f \Delta\Gamma(t) \\ &= \Delta\Sigma - N_f \Delta\Gamma(t) \end{aligned} \tag{29}$$

where

$$\begin{aligned} \Delta\Sigma &= [\Delta\Sigma(0)]_{m\neq 0} + N_f \Delta\Gamma(0) \\ &= [\Delta\Sigma(t)]_{m\neq 0} + N_f \Delta\Gamma(t) \\ &\equiv \Delta\Sigma_{m=0} \end{aligned} \tag{30}$$

is the quark moment defined in such a way that it is evidently conserved.

We can now consider what happens when the threshold for producing a heavy quark pair is passed. The most relevant example is the opening of the charm threshold.

We start from an indicative model where we consider the perturbative evolution with $N_f = 3$ to be valid up to $Q = Q_c$ while $N_f = 4$ is used for $Q > Q_c$ with Q_c being an appropriate scale of order m_c. For $t > t_c$ (with $t_c = \ell n Q_c^2/\mu^2$), Eq. (29) with $N_f = 4$ can be applied to the evolution from t_c up to t and it gives:

$$\Delta\Sigma(t)_{m\neq 0} = \Delta\Sigma(t_c)_{m\neq 0} + 4\Delta\Gamma(t_c) - 4\Delta\Gamma(t) \tag{31}$$

In general for $\Delta\Sigma(t_c)_{m\neq 0}$ one can write the expression:

$$\Delta\Sigma(t_c)_{m\neq 0} = \Delta c_{m\neq 0} + [\Delta\Sigma - 3\Delta\Gamma(t_c)] \tag{32}$$

The second term in bracket is the result that would be obtained by assuming that $\Delta\Sigma(t)_{m\neq 0}$ is continuous at $t = t_c$. $\Delta c_{m\neq 0}$ is a non-perturbative term arising from the lowest order diagram where the photon interacts with a charm quark inside the proton. By combining Eqs. (31) and (32), one obtains:

$$\Delta\Sigma(t)_{m\neq 0} = [\Delta\Sigma - 3\Delta\Gamma(t)] + [\Delta c_{m\neq 0} + \Delta\Gamma(t_c) - \Delta\Gamma(t)] \tag{33}$$

The first bracket is what would be obtained in absence of the threshold, i.e., if the smooth evolution with $N_f = 3$ was followed up to t. Consequently the second term is the contribution of charm:

$$\int g_1^\Gamma dx|_{above} - \int g_1^\Gamma dx|_{below} \simeq \frac{4}{18}[\Delta c_{m\neq 0} + \Delta\Gamma(t_c) - \Delta\Gamma(t)] \tag{34}$$

$\Delta\Gamma(t_c) - \Delta\Gamma(t)$ is the contribution to the charm cross-section obtained from the diagrams of Fig. 2. An approximate expression for this term is given by:

$$\Delta\Gamma(t_c) - \Delta\Gamma(t) = \frac{\alpha_{t_c} - \alpha_t}{2\pi\, \beta_0} \gamma_{gq}^{(1)} (\Delta\Sigma - 3\Delta\Gamma(t_c)) \tag{35}$$

where β_0 is computed with $N_f = 4$ and $\Delta\Sigma - 3\Delta\Gamma(t_c)$ is the nearly vanishing value measured by EMC. When $t \to \infty$ the corresponding result is of order $\alpha(m_c)$ (and not of order $\frac{\Lambda_{QCD}^2}{m_c^2}$ as the contribution evaluated in Ref. [20]). The coefficient of $\alpha(m_c)$ is, however, very small because of the EMC result. In Eq. (34), $\Delta\,\Gamma(t)$ is the hard component arising from large k_T, i.e., $k_T \sim 0(Q)$, while $\Delta\Gamma(t_c)$ is from $k_T \sim 0(m_c)$. Finally, $\Delta c_{m\neq 0}$ is a possible non-perturbative contribution from small $k_T \sim 0(\Lambda_{QCD})$. In Ref. [14] it was suggested that the sum $\Delta c_{m\neq 0} + \Delta\Gamma(m_c)$ should nearly vanish, so that a large variation of M_1^S, proportional to $\Delta\Gamma(t)$, would be observed at the opening of the charm threshold. We now think that a purely perturbative treatment of the heavy quark threshold should be adequate. Therefore we expect $\Delta c_{m\neq 0}$ to be of order $\frac{\Lambda_{QCD}^2}{m_c^2}$ and that the total effect of the charm threshold on M_1^S is expected to be of order $\alpha(m_c)$. The different rôle played by the heavy quark mass $m_c >> \Lambda_{QCD}$ with respect to a light quark mass $m << \Lambda_{QCD}$ is well demonstrated by the interesting study of Ref. [31]. This clear difference, which can be seen as a manifestation of the decoupling theorem, cannot be taken (as claimed in Ref. [20]) as evidence against the point of view that the conserved $\Delta\Sigma$ for $N_f = 3$ should more directly correspond to constituent quarks than $\Delta\Sigma(t)_{m\neq 0}$.

3. The two-loop anomalous dimensions $\gamma_j^{(2)}$ and $\gamma_{qq}^{(2)}$

In this section we present the explicit calculation of the diagrams in Fig. 2 for the process $\gamma(q) + q'(p) \to q(p_3) + \bar{q}(p_4) + q'(p_5)$. We denote by m the mass of the produced quark q and by m' the mass of the incoming quark q'. The value of m' can even coincide with m, but it is essential that the quarks q and q' are different. This eliminates additional exchange diagrams and correctly projects over the singlet channel [10]. The anomalous dimension $\gamma_j^{(2)}$ is proportional to the leading term of order $\alpha^2 t$ of the amplitude from the diagrams in Fig. 2. In fact:

$$\frac{\alpha(\mu) - \alpha(Q)}{2\pi\,\beta_0}\gamma_j^{(2)} < j^5 > \simeq \gamma_j^{(2)}\ \alpha^2 t\ < j^5 > + 0(\alpha^3) \qquad (36)$$

where, on the right-hand side, α is the fixed coupling appearing in the QCD Lagrangian. We want to explicitly show that the coefficient of $\ell n\ Q^2$ is independent of the regularization, i.e., from m and $\rho = \frac{m'^2}{m^2}$. This is essentially a derivation of Kodaira's result (originally obtained in the massless theory) with a different set of regulators.

So now we want to calculate the single logarithms of the diagrams in Fig. 2 in the massive theory. The problem is somewhat ugly, because there are three different on- and off shell masses

involved, namely the Q^2 of the photon, the mass m' of the incoming quark q' and the mass m of the quark q which is produced. We shall calculate our anomalous dimension for any ratio of masses $\rho = m'^2/m^2$, even $\rho = 0$ and $\rho = \infty$. For dimensional reasons the logarithms of Q^2 which we are looking for are always associated with logarithms of the masses. But the mass logarithms correspond to mass singularities, i.e., divergences of the propagators in the massless case. We have a divergence of the gluon propagator which corresponds to logarithms of m' and we have a divergence of the quark propagator which essentially corresponds to logarithms of m. In particular we expect squares of logarithms to arise in intermediate steps of the calculation and we shall see that only in the final result for the first moment these squares of logarithms drop out and only single logarithms remain.

The contribution of Fig. 2 to the first moment of g_1 can be written as

$$M_1^S(\gamma q' \to q\bar{q}q') = \int_0^1 dz\ PS_3\ |M_{\gamma\,q'\to q\bar{q}q'}|^2 = -\frac{1}{2}C_F N_f (\frac{\alpha_s}{2\pi})^2 \int_0^1 dz \mathcal{D}_p \mathcal{E} \qquad (37)$$

We have included a factor N_f from the projection over the singlet channel [10]. z is defined by $s = (p+q)^2 = \frac{Q^2}{z}(1-z)$ and can be interpreted as the fraction of momentum of the incoming quark carried by the struck quark. PS_3 is the phase space, $PS_3 = \frac{s}{2^8\pi^4}\mathcal{D}_p$, $|M_{\gamma q'\to q\bar{q}q'}|^2$ the matrix element and

$$\mathcal{E} = \frac{\epsilon(\mu\nu pq)}{2pq}\frac{\epsilon(\alpha\beta pp_5)}{(p-p_5)^2}\frac{s}{(p-p_5)^2}\Omega(\mu\nu\alpha\beta) \qquad (38)$$

where Ω is the Dirac trace for the subprocess $\gamma g \to q\bar{q}$ for general polarizations of photon and gluon, with quark mass m and gluon off-shellness $(p-p_5)^2$. The first ϵ tensor in Eq. (38) is the projector onto the structure function g_1. The second ϵ tensor comes from the Dirac trace of the light quark which contains a factor γ_5, because one has to calculate the difference of the contributions from a left-handed and a right-handed quark. What we have to show is

$$\int_0^1 dz \mathcal{D}_p \mathcal{E} = 3\ell n Q^2 + const(Q^2) \qquad (39)$$

This then implies $[\gamma_{qq}^{(2)}]_{m\neq 0} = -\frac{3}{2}C_F N_f$ and that $\gamma_j^{(2)}$ is independent of the regularization.

To prove Eq. (39), the first thing to know is $\mathcal{P}_p$. We shall work in the system where the incoming quark defines the z direction and which is the centre-of-mass system of the outgoing particles 4 and 5. In this system, the phase space can be factorized into a Lorentz invariant volume element $\frac{d^3\vec{p}_3}{2p_3^0}$ of the produced quark 3 and into an angle element $d\Omega_5$ of the quark 5. (The labels 1,2,3,4,5 of the particles are as shown in Fig. 2.) The invariant volume element can be evaluated in any system. We have found it convenient to use for it the centre-of-mass system of the incoming particles. In that system we can describe the invariant volume element for instance with the fraction x_3 of the total energy carried by the produced quark 3 and the scattering angle θ_{13} between the photon and the quark 3, or, alternatively, with the variables

$$
\begin{aligned}
v &= 1 - x_3(1+\cos\theta_{13})/2 \\
w &= 1 - (1-x_3)(1-z)/v
\end{aligned}
\tag{40}
$$

Here $\mu_c < v < 1$ and $z < w < 1$ with

$$\mu_c = \mu\frac{1-z}{1-w}(\rho + 2\sqrt{\rho}),\ \ \mu = \frac{m^2}{s}$$

The variables v and w have the advantage that the singularity of the quark propagator comes in a unique way from the region $v \to 0$. w has also a physical interpretation as we shall see later. Then we can write

$$\mathcal{P}_p = \frac{\sqrt{A}}{y_{15}}\frac{v}{1-z}dvdw\frac{1}{4\pi}d\cos\theta_{25}d\varphi \tag{41}$$

where $y_{45} = (p_4+p_5)^2/s = 1 - x_3 + \mu$ and

$$A = y_{15}^2 - 2y_{45}\frac{m^2+m'^2}{s} + (\frac{m^2-m'^2}{s})^2 \tag{42}$$

θ_{25} is the angle between the incoming and the outgoing quarks 2 and 5. The singularity from the gluon propagator in fact comes from the region $\theta_{25} \to 0$. φ is an azimuthal angle, which is defined by the following parametrization of the momentum p_5

$$p_5 = \frac{\sqrt{s}}{2\sqrt{y_{15}}}(y_{15} + \frac{m'^2-m^2}{s}, \sqrt{A}\sin\theta_{25}\sin\varphi, \sqrt{A}\sin\theta_{25}\cos\varphi, \sqrt{A}\cos\theta_{25}) \tag{43}$$

in the centre-of-mass system of particles 4 and 5.

Instead of working with the five variables z, v, w, θ_{25} and φ, we started to write down the matrix element in terms of the six variables $z, t_N, u_N, y_{25}, v_{35}$ and v_{45} where

$$
\begin{aligned}
t_N &= \frac{2qp_3 + Q^2}{s} \qquad u_N = \frac{2qp_1 + Q^2}{s} \\
y_{25} &= \frac{(p-p_5)^2}{s} \\
v_{35} &= \frac{2p_3p_5}{s} \qquad v_{45} = \frac{2p_4p_5}{s}
\end{aligned}
\tag{44}
$$

which are connected by a linear relation

$$v_{35} + v_{45} + u_N + t_N + \frac{m'^2}{s} = y_{25} + \frac{1}{1-z} \tag{45}$$

(t_N, u_N and y_{25} are the denominators of the quark and gluon propagator.) However, we did not make use of this linear relation, because in terms of those six variables the matrix element has an explicit symmetry under the interchange of the two produced quarks 3 and 4. We can use this symmetry to reduce the number of integrals to be calculated by almost a factor of two.

The part of the matrix element which leads to logarithms of Q^2 is

$$
\begin{aligned}
\mathcal{E} \approx{}& \frac{z}{1-z}(\frac{1}{t_N^2} + \frac{1}{u_N^2}) + \frac{1}{y_{25}}(\frac{2}{t_N} + \frac{2}{u_N} - 2 - \frac{u_N}{t_N} - \frac{t_N}{u_N}) \\
&+ \frac{2}{y_{25}}\{2z(\frac{v_{45}^2}{t_N^2} + \frac{v_{35}^2}{u_N^2}) + (3z-1)(\frac{v_{45}}{t_N} + \frac{v_{35}}{u_N}) + 2z - 1\} \\
&- \frac{2z}{(1-z)y_{25}}(\frac{v_{35}+v_{45}}{u_N t_N} + \frac{v_{45}}{t_N^2} + \frac{v_{35}}{u_N^2})
\end{aligned}
\tag{46}
$$

As an example we shall integrate the term $\frac{1}{y_{25}t_N}$. The first step is to express y_{25} and t_N in terms of phase space variables. There is no azimuthal dependence so that the φ integral is trivial. The only dependence on θ_{25} is a term linear in $\cos\theta_{25}$ in y_{25}. Therefore one can do the θ_{25} integral

$$\frac{\sqrt{A}}{y_{45}}\frac{1}{2}\int_{-1}^{1}\frac{d\cos\theta_{25}}{y_{25}} \approx -\frac{1-z}{\sqrt{v^2 + 2v\mu_g + \mu_j^2}}\ell n(\frac{m'^2}{Q^2}\frac{zw^2}{v(1-w)}) \tag{47}$$

with

$$\mu_g = \mu[1+\rho(1-z)(1-z-2(1-w))]$$
$$\mu_j = \mu[1-(1-z)^2\rho]$$

In Eq. (47), only those terms have been kept which finally contribute to the logarithms. One can see explicitly how the region $\theta_{25} \to 0$ yields logarithms of m'.

Now we want to do the v integral which will give logarithms of m. Instead of v we use the variable $r = \mu/v$ and find

$$\int_{\mu_c}^{1} \frac{v\,dv}{(1-z)t_N}(\text{Eq. } (47)\,) = -(1-z)\int_{\mu}^{\mu/\mu_c} \frac{dr}{r}\frac{\ell n[\rho(1-z)w^2 r/(1-w)]}{(w+r\mu_t/\mu)(1+2r\mu_g/\mu+r^2\mu_j^2/\mu^2)^{\frac{1}{2}}} \tag{48}$$

where for t_N we have used the approximation

$$t_N = \frac{vw+\mu_t}{1-z}, \quad \mu_t = (1-\rho z(1-z)).\mu \tag{49}$$

We see that in this particular case the actual form of μ_c, μ_t, μ_g and μ_j is irrelevant, because only the pole in r produces logarithms of Q^2. We can now also do the w and z integrals which are finite and give

$$\int_0^1 dz\mathcal{D}_p\frac{1}{y_{25}t_N} = \frac{3}{8}\ell n^2 Q^2 - \frac{3}{4}\ell n\ m'^2\ \ell n Q^2 + \frac{23}{8}\ell n Q^2 + const.(Q^2) \tag{50}$$

As expected we find squares of logarithms. The reason why these leading logarithms will drop out in the total result is the following: the coefficient of the leading logarithm is the product of the evolution kernels at the vertices, in this case $\Delta P_{qg}(z')\Delta P_{gq}(w)$, where w is our phase space variable and in this connection has the interpretation that it is the fraction of momentum of the incoming quark which is carried by the gluon. Similarly z' is the fraction of momentum of the gluon which is carried by the intermediate quark. Obviously we have $z' \cdot w = z$. Since we are interested in the first moment contribution we have to take the first moment of ΔP_{qg}, which is zero. This is the reason why the squares of logarithms will drop out.

The most complicated of all terms in Eq. (46) is $\frac{v_{35}}{y_{25}t_N \pi_N}$. The singularity structure of this term is similar to $\frac{1}{y_{25}t_N}$, because $lim_{\pi_N\to 0}v_{35} = 0$.

The most elegant procedure is to first calculate $\frac{1}{y_{25}t_N}\ [\frac{v_{35}}{\pi_N}]_{\theta_{25}=0}$ and then $\frac{1}{y_{25}t_N}\ \{\frac{v_{35}}{\pi_N} - [\frac{v_{35}}{\pi_N}]_{\theta_{25}=0}\}$, because the difference contains only a single logarithm.

The final result for all $\ell n\ Q^2$ terms in the matrix element is

$$\begin{aligned}\mathcal{D}_p\mathcal{E} = &(2\ell n\ m'^2 - \ell n\ Q^2)\ell n\ Q^2[5(1-z)+2(1+z)\ell n\ z] \\ &+\{22(1-z)+(28-20z)\ell n\ z - 4(1+z)\ell n\ z\ell n(1-z) \\ &+8(1+z)\ell n^2 z - 10(1-z)\ell n(1-z) - 4(1+z)L_2(1-z)\}\ell n\ Q^2 \\ &+ const.(Q^2)\end{aligned} \tag{51}$$

before the z integration. L_2 is the dilogarithm. After the z integration we find Eq. (39) as needed to get $[\gamma_{qq}^{(2)}]_{m\neq 0} = -\frac{3}{2}C_F N_f$.

Note that the corresponding massless calculation in which the incoming quark is taken off-shell also gives

$$\int_0^1 dz(\mathcal{D}_p\mathcal{E})_{m=0} = 3\ell n\ Q^2 + const.(Q^2) \tag{52}$$

but that this corresponds to $[\gamma_{qq}^{(2)}]_{m=0} = 0$!

We have checked that the limits $\rho \to 0$ and $\rho \to \infty$ are smooth. To see that, we did an independent calculation in which we kept all logarithms (not just logarithms of Q^2). We found that in addition to Eq. (46) the matrix element contains terms which give logarithms of the mass ratio ρ. These are of such a form that

$$\int_0^1 dz\ \mathcal{D}_p\mathcal{E} = \begin{cases} 3\ell n\frac{Q^2}{m^2} & (+\text{ non logarithmic terms}) \ \text{ for } \rho\to 0 \\ 3\ell n\frac{Q^2}{m'^2} & (+\text{ non logarithmic terms}) \ \text{ for } \rho\to\infty \end{cases} \tag{53}$$

A typical term which gives a logarithm of ρ but not of Q^2 is of the form $\frac{\mu}{y_{25}t_N^2}$. At first sight it seems to give no contribution at all. However, the integration of the double pole in t_N gives a factor $\frac{1}{\mu}$. We can see this if we do the θ_{25} integral according to Eq. (47) and then introduce our variable $r = \mu/v$ again. We get

$$\int_{\mu_c}^{1} \frac{vdv}{1-z}\frac{\mu}{t_N^2}\frac{1}{2}\int_{-1}^{1}\frac{d\cos\theta_{25}}{y_{25}} = -(1-z)^2 \int_{\mu}^{\mu/\mu_c} \frac{dr\ \ell n\,[\rho(1-z)w^2 r/(1-w)]}{(w+\mu_t r/\mu)^2(1+2r\mu_g/\mu + r^2\mu_j^2/\mu^2)^{\frac{1}{2}}} \qquad (54)$$

which is regular for $Q^2 \to \infty$, but contains logarithms of ρ. This time the actual form of μ_t, μ_g, μ_j and μ_c is important.

In summary we have found that in the massive theory we have a non-vanishing entry in the upper-left corner of the anomalous dimension matrix

$$[\gamma^{(2)}]_{m\neq 0} = \begin{pmatrix} -\frac{3}{2}C_F N_f & 0 \\ \frac{3}{2}C_F & 0 \end{pmatrix} \qquad (55)$$

As compared to the massless case

$$[\gamma^{(2)}]_{m=0} = \begin{pmatrix} 0 & 0 \\ \frac{3}{2}C_F & -\frac{3}{2}C_F N_f \end{pmatrix} \qquad (56)$$

there is a shift along the diagonal which destroys the conservation of $\Delta\Sigma$. Therefore we see that the ambiguity in the matrix element is associated with an ambiguity in the anomalous dimensions in such a way that the physical quantity which is the product of the coefficient function, anomalous dimension and matrix element is scheme independent.

4. Conclusion

In our opinion it is clear enough that the difference between the measured value of the singlet axial current for polarized protons and the naive expectations based on the constituent quark model is due to the presence in this channel of the axial anomaly. In the absence of the anomaly, the helicity carried by each kind of quarks would be conserved in the massless theory. We would then expect the constituent and the parton quarks to carry approximately the same amount of the proton helicity. The most conservative point of view is just to say [19] that in presence of the anomaly the conservation of the singlet quark helicity is broken at the two-loop level so that in principle the helicity of parton and constituent quark can be different. In spite of the smallness of the corresponding effect in the perturbative region, one can still attribute the large difference which is observed to the effect of the anomaly in the non-perturbative region above and around the confinement scale. In the approach of the anomalous gluon component, one accepts the starting point that the difference is due to the anomaly and goes beyond this statement by establishing testable connections with other hard processes. The additional input that leads to the new information is provided by the QCD improved parton model. In this model the polarized gluon density and its moments can in principle be defined from observable hard processes (e.g., the production of jets at large k_T in deep inelastic scattering). Due to the anomaly the evolution of the first moment of the polarized gluon density is such that the quantity $\Delta\Gamma \simeq \frac{g_s}{2\pi}\Delta g$ is a constant in leading order. The difference between the helicity of parton and constituent quarks is attributed to this anomalous gluon component. It is a challenge for future experiments to measure the polarized gluon density and to check whether its size and x dependence are suitable for an explanation of the EMC result in terms of the anomalous gluon component.

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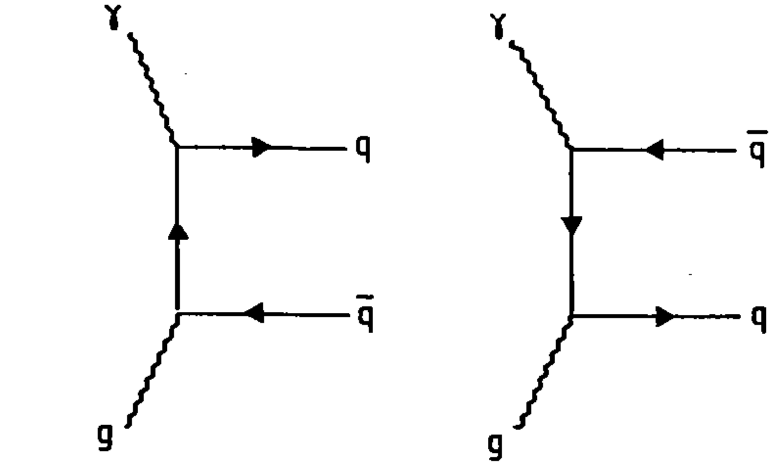
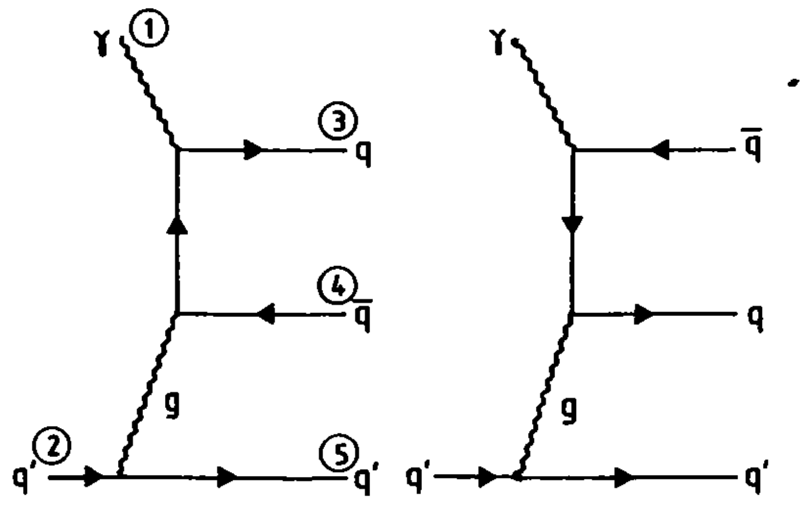


- Figure 1 -



- Figure 2 -