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The light curves of soft X-ray transients

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ABSTRACT

We show that the light curves of soft X-ray transients (SXTs) follow naturally from the disc instability picture, adapted to take account of irradiation by the central X-ray source during the outburst. Irradiation prevents the disc from returning to the cool state until central accretion is greatly reduced. This happens only after most of the disc mass has been accreted by the central object, on a viscous time-scale, accounting naturally for the exponential decay of the outburst on a far longer time-scale ($\tau \sim 20$–40 d) than seen in dwarf novae, without any need to manipulate the viscosity parameter α. The accretion of most of the disc mass in outburst explains the much longer recurrence time of SXTs compared with dwarf novae. This picture also suggests an explanation of the secondary maximum seen in SXT light curves about 50–75 d after the start of each outburst, since central irradiation triggers the thermal instability of the outer disc, adding to the central accretion rate one viscous time later. The X-ray outburst decay constant τ should on average increase with orbital period, but saturate at a roughly constant value ~ 40 d for orbital periods longer than about a day. The bolometric light curve should show a linear rather than an exponential decay at late times (a few times τ). Outbursts of long-period systems should be entirely in the linear decay regime, as is observed in GRO J1744 − 28. UV and optical light curves should resemble the X-rays but have decay time-scales up to 2–4 times longer.

Key words: accretion, accretion discs – instabilities – X-rays: stars.

1 INTRODUCTION

The disc instability picture is generally accepted as the most successful explanation for dwarf nova outbursts (see Cannizzo 1993 for a review). It is natural to try to extend this picture to soft X-ray transients, which are after all close, low-mass binaries like dwarf novae, but with black hole or neutron star primaries in place of the white dwarf. The main difficulty in doing this is the striking difference in time-scales: dwarf nova outbursts last a few days and recur in a few weeks, whereas the corresponding time-scales for soft X-ray transients (SXTs) are months and years. One can formally account for this to some extent simply by reducing the viscosity parameter α by factors ~ 10–100. However, such a step lacks all plausibility, and in any case further difficulties appear at a slightly more detailed level. Typical SXT outburst light curves (see Tanaka & Lewin 1996 and Tanaka & Shibazaki 1996 for reviews) show a quasi-exponential decay quite unlike the fairly abrupt decline seen in dwarf novae. Many SXTs have a second local X-ray maximum about 50 d after the start of the outburst, after which the X-rays continue their quasi-exponential decline as before.

There have been several attempts to modify the standard disc instability picture to account for these features, often by invoking particular functional forms for α (Cannizzo, Chen & Livio 1995). However, we shall show here that all of these differences from the dwarf nova paradigm are natural consequences of irradiation of the disc by the central source. Observationally, this has long been known to dominate the disc emission of all low-mass X-ray binaries (LMXBs), and SXTs in particular; the optical/X-ray flux ratios are systematically far too large for the optical emission to result from intrinsic dissipation alone (van Paradijs & McClintock 1994; Shahbaz & Kuulkers 1998). van Paradijs & McClintock also show that the optical luminosity of the disc in SXT outbursts correlates with the quantity $L_X P^{2/3}$, where L_X is the X-ray luminosity and P the orbital period, as expected if irradiation dominates the disc brightness (cf. equation 1 below). Shahbaz & Kuulkers show by an independent method that the optical surface brightness of SXT discs in outburst is considerably larger than that of dwarf nova discs in the same state. While some of this is a result of the higher outburst accretion rates in SXTs, it is again clear that the discs are brighter than would be expected from the intrinsic dissipation alone. Attempts to model observed outbursts using un-irradiated discs have had mixed results. Huang & Wheeler (1989) used a disc of the size (2×10^{11} cm) inferred from observations of A0620−00 in order to model its outbursts, finding optical/X-ray ratios more than an order of magnitude below those observed (the X-rays are too bright and the optical too faint). They pointed out that X-ray reprocessing probably affects the optical flux. Mineshige & Wheeler (1989) used (for reasons of numerical stability) a disc radius $R_{\rm out}=3.16\times10^{10}$ cm, smaller even than the circularization radius $R_{\rm circ}=5\times10^{10}$ cm, and found reasonable agreement with the observed optical/X-ray ratio in the initial part of the outburst, but not later. Even the apparent early agreement is unclear, as matter has to be added to the edge of the unphysically small disc with the specific angular momentum of the circularization radius, artificially altering the local dissipation rate. Mineshige & Wheeler indeed suggested that the neglect of irradiation must be important in computing the optical light curve in particular.

We note that Cannizzo et al. (1995; cf. Cannizzo 1994) claim that numerical modelling of irradiated discs actually produces a disc surface which is convex rather than concave, and would therefore shield the outer disc from irradiation. Since the observational evidence for irradiation is extremely strong, we regard this result as highlighting a limitation of current theoretical disc models. In the following we therefore assume that irradiation is able to heat the disc faces, and limit ourselves to simple analytic considerations using the most basic properties of the disc instability picture, rather than detailed numerical calculations.

2 DISC IRRADIATION

The disc instability picture relies on the possibility of partial ionization of hydrogen. The instability is suppressed in any disc region the effective temperature of which exceeds a value $T_{\rm H}\sim6500$ K characteristic of this state. A disc will thus be steady if the local viscous energy release achieves this, as in nova-like variables. Irradiation of the disc from outside can also produce a surface temperature $T_{\rm irr}>T_{\rm H}$ and stabilize it (Tuchman, Mineshige & Wheeler 1990). van Paradijs (1996) demonstrated the importance of this effect in LMXBs, where the compact central X-ray source is able to heat the concave disc faces, producing a surface temperature given by

$$T_{\rm irr}(R)^4=\frac{\eta\dot M_{\rm c}c^2(1-\beta)}{4\pi\sigma R^2}\left(\frac{H}{R}\right)^n\left[\frac{{\rm d}\,\ln\,H}{{\rm d}\,\ln\,R}-1\right], \qquad (1)$$

where η is the efficiency of rest-mass energy conversion into X-ray heating, $\dot M_{\rm c}$ is the central accretion rate, H is the disc scaleheight at disc radius R, β is the albedo of the disc faces, and the factor in square brackets lies between 1/8 and 2/7. The index $n=1$ or 2 depending on whether there is a central irradiating point source; this is discussed further below. The ratio H/R is roughly constant in a disc, so $T_{\rm irr}$ falls off as $R^{-1/2}$. Thus for a large enough disc, $T_{\rm irr}$ dominates the effective temperature of the disc, which goes as $R^{-3/4}$. In agreement with this, LMXBs have a much higher ratio of optical-to-X-ray flux than one would expect for an un-irradiated disc. Noting this, van Paradijs (1996) showed that observed LMXBs are correctly divided into persistent or transient systems according to whether $T_{\rm irr}$ (computed from the observed X-ray fluxes) exceeds $T_{\rm H}$ throughout the disc or not. By potentially stabilizing the disc, irradiation sharply decreases the lower limit $\dot M_{\rm crit}$ for the mass transfer rate permitting steady accretion. This is defined by $T_{\rm irr}(R_0)=T_{\rm H}$, where R_0 is the outer disc radius. Thus SXTs must have very low mass transfer rates $-\dot M_2$. The significance of this for their evolution, and that of LMXBs in general, is explored in a series of papers by King, Kolb & Burderi (1996), King & Kolb (1997), King et al. (1997a) and King, Kolb & Szuszkiewicz (1997b). For neutron star systems, the central star acts as a point source of irradiating flux, so $n=1$ is appropriate. For black hole systems, the correct criterion for deciding their long-term stability evidently has $n=2$ (King et al. 1997b): this is very reasonable, as black hole systems have no hard stellar surface, so that the source irradiating the outer disc is the foreshortened inner disc (Fukue 1992; King et al. 1997b), at least at central accretion rates $\dot M_{\rm c}$ equal to the mean mass transfer rate $-\dot M_2$ (but see the end of this section).

All of these studies are concerned with the *mean* mass transfer rate $-\dot M_2$; if this is high enough for stable disc accretion, allowing for irradiation, i.e. $-\dot M_2>\dot M_{\rm crit}$, it is assumed that the disc will find this steady state and the system will appear as persistent with $\dot M_{\rm c}=-\dot M_2$. Here we are concerned with SXTs, where by definition the mean transfer rate is too low for this to happen, i.e. $-\dot M_2<\dot M_{\rm crit}$. Nevertheless irradiation can obviously in principle affect the *outburst* state of these systems, when $\dot M_{\rm c}>-\dot M_2$.

In particular, the standard (un-irradiated) disc instability is ended by a cooling wave which moves inwards from the outer edge of the disc, returning it to the cool state and initiating quiescence, but for most of an outburst of a sufficiently small irradiated disc we will have $\dot M_{\rm c}>\dot M_{\rm crit}$, implying that $T_{\rm irr}>T_{\rm H}$ at the outer edge. Clearly this cannot return to the cool state until $T_{\rm irr}<T_{\rm H}$. From (1) this is just the condition $\dot M_{\rm c}<\dot M_{\rm crit}$. In other words, the whole disc is forced to remain in the hot state until the central accretion rate drops to the value $\dot M_{\rm crit}$. Since in outburst the disc surface density $\Sigma(R)$ adopts a quasi-steady radial profile, $\dot M_{\rm c}$ only drops significantly as the total disc mass is reduced. Even when $\dot M_{\rm c}$ is below $\dot M_{\rm crit}$, the cooling front cannot sweep inwards to shut off the outburst in the usual way, but moves in only as the radius $R_{\rm h}$ of the hot region of the disc, defined by $T_{\rm irr}(R_{\rm h})=T_{\rm H}$, slowly retreats in response to still further decreases in the disc mass and thus $\dot M_{\rm c}$. This reasoning shows that we can expect an SXT outburst to consume most of the disc mass which is heated above $T_{\rm H}$ by irradiation. Except in very large discs this is almost all the disc mass. By contrast, in a normal dwarf nova outburst only a small fraction (∼3–5 per cent) of the disc mass is accreted by the white dwarf (Cannizzo 1993). Moreover, since $\dot M_{\rm c}$ decreases only as the disc mass drops, we can expect a long exponential decay of the X-rays in SXT outbursts. Mineshige, Tuchman & Wheeler (1990) considered this effect in numerical calculations, but limited themselves to fairly weak irradiating fluxes that did not fully suppress the tendency of the cooling front to shut off the outburst. They nevertheless already found that the X-ray outburst was longer than the comparable dwarf nova outburst, with an 'X-ray plateau'. We shall show in the next section that irradiation of the strength suggested by observation produces long outbursts characterized by an exponential decay.

In the rest of the paper we shall give results for both $n=1$ and $n=2$. From the discussion above we might expect these to refer to neutron star and black hole systems respectively. However, during outbursts, black hole sources are observed to have a stronger power-law continuum in addition to the thermal emission from the inner disc. The power-law continuum probably arises from a central corona which develops during the outburst, thereby mimicking the neutron star case $n=1$. This is compatible with the $n=2$ adopted for the long-term stability criterion (i.e. deciding if the system is transient or persistent), provided that the corona only appears at central accretion rates $\dot{M}_c$ exceeding the mean mass transfer rate $-\dot{M}_2$. Clearly, more work on the formation of the corona is required to decide if this is reasonable. In any case, the differences between outbursts with $n=1$ and $n=2$ are rather small, as we shall see.

3 OUTBURSTS OF IRRADIATED DISCS

A full treatment of the effects described above requires the use of a disc instability code, modified to take account of irradiation. We shall do this in a future paper. Here we give a simple analytic discussion which reveals the main qualitative features. We make use of the fact, confirmed by many numerical calculations, that in the outburst state the disc adopts a quasi-steady surface density profile. Specifically, we assume that the surface density in the hot stage goes as

$$\Sigma(R)\simeq\frac{\dot{M}_c}{3\pi\nu}. \quad (2)$$

Integrating this gives the total initial mass of the hot zone as

$$M_h=2\pi\int_0^{R_h}\Sigma R\,\mathrm{d}R\simeq\dot{M}_c\frac{R_h^2}{3\nu}, \quad (3)$$

since the inner disc radius is much smaller than the outer radius R_h reached by the heating front. In (3) we interpret ν as some average of the kinematic viscosity in the disc, in practice its value near R_h. Note that (3) is effectively a dimensional relation, and simply asserts the obvious fact that the mass of a steady disc is given by the product of the accretion rate and the viscous time at its outer edge: it does not for example assume that ν is constant through the disc.

The only way in which the mass of the hot zone can change is through central accretion, so we have $\dot{M}_c=-\dot{M}_h$, so that

$$-\dot{M}_h=\frac{3\nu}{R_h^2}M_h \quad (4)$$

or

$$M_h=M_0\exp(-3\nu t/R_h^2). \quad (5)$$

Here M_0 is the initial mass of the hot zone. Before the outburst the density must have been between the maximum and minimum values

$$\Sigma_{\rm min}\simeq3\times10^{-8}R\ \mathrm{g\ cm}<\Sigma(R)\le\Sigma_{\rm max}\simeq1\times10^{-8}R\ \mathrm{g\ cm}, \quad (6)$$

attained in the local limit cycle (cf. Cannizzo 1993), so we write

$$\Sigma_{\rm max}(R)\simeq\frac{\rho}{2\pi}R, \quad (7)$$

where $\rho\sim3\times10^{-8}$ g cm^{-3}. Then we can estimate M_0 as

$$M_0\simeq2\pi\int_0^{R_h}\Sigma_{\rm max}R\,\mathrm{d}R\simeq\frac{\rho R_h^3}{3}. \quad (8)$$

Inserting this in (5) gives

$$M_h=\frac{\rho R_h^3}{3}\exp(-3\nu t/R_h^2), \quad (9)$$

and

$$\dot{M}_c=-\dot{M}_h=R_h\nu\rho\exp(-3\nu t/R_h^2). \quad (10)$$

Thus the first part of the X-ray light curve is indeed an exponential decay, as observed. Of course this result is only approximate, as the ν appearing everywhere is an average value characteristic of the outer disc. Including a detailed radial dependence for ν (e.g. the alpha parametrization) will result in a slightly different analytic form, but with the same general shape. Note that the usual decay law ($\dot{M}_c\sim t^{-(1+\epsilon)}$) for a viscously accreting alpha disc (e.g. Cannizzo, Lee & Goodman 1990) is inapplicable to an irradiated disc, for which the temperature is determined by $\dot{M}_c$ and not locally. A future paper will present analytic solutions showing that the resulting decay follows a much larger power of t, closely resembling the simple form (10).

Once the X-rays irradiate the outer disc this too will get hot, effectively increasing R_h to some value R_0 (which will be the outer edge of the disc in short-period systems, see below). This will increase the normalization in (10) after one viscous time, when the increased hot mass becomes noticeable at the centre of the disc. This raises the normalization of the X-ray light curve by a constant factor fixed by the combination $R_h\nu$. Thus the light curve will be displaced upwards by a fixed factor, after a time of order 1–3τ, where

$$\tau=\frac{1}{3}t_{\rm visc}\sim\frac{R_0^2}{3\nu}\sim40\ \mathrm{d}, \quad (11)$$

and we have used $R_0\sim10^{11}$ cm and $\nu\sim10^{15}$ cm^2 s^{-1} as typical values for the outer disc (where most of the mass is). Since the disc light is dominated by X-ray reprocessing, the light curves at other wavelengths will move in parallel also (the small rise in local viscous dissipation as the extra mass moves inwards is negligible compared with the reprocessed flux, so the longer-wavelength light curves cannot rise until the increased mass flow reaches the centre). This is presumably the origin of the 'secondary maximum' always seen in SXT light curves about 50–75 d after the start of the outburst. Since the rise in normalization is typically no more than a factor of 2, and $R_h \nu \sim R_h^{7/4}$ (e.g. Frank, King & Raine 1992), we deduce that the initial heat front typically reaches to a radius $R_h \gtrsim 0.7 R_0$ at the beginning of the outburst. The precise shape of the displacement in the X-ray (and therefore optical) light curve is fixed by the way the central accretion rate $\dot{M}_c$ rises in response to the increased mass flow diffusing inwards, and cannot be determined without a full disc calculation. We take it as a step function in the remainder of the paper, bearing in mind that this is an oversimplification.

Our explanation of the secondary maximum differs from that proposed by Augusteijn, Kuulkers & Shaham (1993). They indeed invoked irradiation of a new mass source, which would add to the central accretion rate after a viscous time: however, their suggested mass source was the secondary star rather than the outer disc. We do not regard this latter suggestion as plausible. First, it is very likely that the L_1 point is shielded from irradiation by the disc during outbursts. Secondly, even if L_1 is irradiated, it is extremely difficult to get the mass transfer rate to rise instantaneously in response to X-ray heating: unless the spectrum is very hard the X-rays do not penetrate the stellar photosphere, and merely produce short time-scale oscillations in the mass transfer rate (King 1989). Models assuming that the X-ray flux does penetrate the photosphere do not produce a rapid response in terms of mass transfer (Hameury, King & Lasota 1986).

From (9) with $R_h \sim R_0 \sim 10^{11}$ cm we predict a total heated disc mass $M_h \sim 3 \times 10^{24}$ g at the start of the outburst. From (10) the central accretion rate is initially $\sim 10^{18}$ g s^{-1} ($\sim 10^{-8}$ M$_\odot$ yr^{-1}), in good agreement with observation. The X-rays should decline on the e-folding time-scale $\tau \sim 40$ d.

The X-rays decay according to this law until they are weak enough that the outer edge of the disc cannot be kept in the hot state. Then the hot-state edge radius R_h is defined by $T_{ir}(R_h) = T_H$, i.e. $\dot{M}_c = \dot{M}_{crit}$. From (11) we have

$$R_h^2 = \frac{\eta \dot{M}_c c^2 (1-\beta)}{4\pi\sigma T_H^4} \left(\frac{H}{R}\right)^n \left[\frac{\mathrm{d}\ln H}{\mathrm{d}\ln R} - 1\right] = B_n \dot{M}_c, \tag{12}$$

where

$$B_1 \simeq 4 \times 10^5, \qquad B_2 \simeq 5 \times 10^4 \text{ (cgs)} \tag{13}$$

with typical parameters $\eta = 0.15$, $\mathrm{d}\ln H/\mathrm{d}\ln R = 45/38$ and $\eta = 0.10$, $\mathrm{d}\ln H/\mathrm{d}\ln R = 43/36$ for neutron star and black hole systems respectively, and $\beta = 0.8$, $H/R = 0.2$ in both cases (King et al. 1997b; de Jong, van Paradijs & Augusteijn 1996). Thus R_h retreats inside R_0 once $\dot{M}_c < \dot{M}_{crit}$, with

$$\dot{M}_{crit}(\mathrm{NS}) = 4.1 \times 10^{-10} R_{11}^2 \,\mathrm{M}_\odot\,\mathrm{yr}^{-1}, \tag{14}$$

and

$$\dot{M}_{crit}(\mathrm{BH}) = 2.9 \times 10^{-9} R_{11}^2 \,\mathrm{M}_\odot\,\mathrm{yr}^{-1}. \tag{15}$$

(Here the labels 'NS' and 'BH' imply $n = 1$, 2 respectively, even though some black hole systems may have $n = 1$.) In short-period systems (small R_{11}) this occurs typically at a time $T \sim$ a few times τ after the outburst began (see below), while in longer-period systems $\dot{M}_{crit}$ may already exceed the Eddington limit. In this case the systems may never be in the exponential regime described above, but always have only the inner part of the disc irradiated.

Once this state is reached the X-rays are controlled by the heated part of the disc inside the decreasing R_h. If all of the disc was irradiated initially, the part now outside R_h (and therefore no longer irradiated) must have been severely depleted by the 'forced' high-state accretion. Its surface density is thus likely to be below the minimum value (sometimes called Σ_A) reached at the end of an un-irradiated outburst, so we neglect any contribution to $\dot{M}_c$ from this region. This argument loses some of its force when R_h is very close to the centre of the disc, where irradiation is relatively less important. Here the surface density might exceed the value Σ_B needed to initiate one or more small outbursts after the main one. Inserting (12) into (3) we get

$$\dot{M}_c = \left(\frac{3\nu}{B_n}\right)^{1/2} M_h^{1/2}. \tag{16}$$

Now the hot mass decreases not only by central accretion, but also because the outer radius R_h shrinks as $\dot{M}_c$ drops:

$$\dot{M}_h = -\dot{M}_c + 2\pi\Sigma R_h \dot{R}_h. \tag{17}$$

Using (3) and (12) we get

$$\dot{M}_h = -\dot{M}_c + \frac{B_n \dot{M}_c}{3\nu} \ddot{M}_c, \tag{18}$$

which using (16) gives

$$\frac{B_n \ddot{M}_c}{3\nu} = -1 \tag{19}$$

so that (18) gives

$$\dot{M}_h = -2\dot{M}_c. \tag{20}$$

Hence, using (16), we get

$$\dot{M}_h = -2\left(\frac{3\nu}{B_n}\right)^{1/2} M_h^{1/2}, \tag{21}$$

yielding

$$M_h = \left[M_h^{1/2}(T) - \left(\frac{3\nu}{B_n}\right)^{1/2} (t-T)\right]^2 \tag{22}$$

and

$$\dot{M}_c = \left(\frac{3\nu}{B_n}\right)^{1/2} \left[M_h^{1/2}(T) - \left(\frac{3\nu}{B_n}\right)^{1/2} (t-T)\right], \tag{23}$$

where $M_h(T)$ is the hot-state disc mass when this phase begins at time T ($T = 0$ for long-period systems). This linear decay finally turns off the central X-rays ($\dot{M}_c = 0$) after a further time

$$t_{end} - T = \left(\frac{B_n}{3\nu}\right)^{1/2} M_h^{1/2}(T) = \left[\frac{M_h(T)\,\tau}{\dot{M}_{crit}}\right]^{1/2}, \tag{24}$$

where we have used (11) and (12) in the second step. Since $M_h(T)$ and $\dot{M}_{crit}$ are related by equations (9) and (10) we have $M_{\rm h}(T)/\dot M_{\rm crit} = \tau$, and (24) gives

$$t_{\rm end} - T = \tau. \qquad (25)$$

The full X-ray outburst therefore lasts a total time $t_{\rm end} \sim$ a few times τ, although there is an obvious selection effect against detecting the very faint X-ray fluxes predicted in the later part of this episode.

Almost all the original heated disc mass M_0 is accreted in the outburst. SXT outbursts appear to be rather similar when they repeat: the recurrence time until the next outburst is therefore presumably given by the accumulation time

$$t_{\rm rec} = \frac{M_0}{-\dot M_2} \simeq \frac{\rho R_0^3}{-3\dot M_2} \sim 50 \text{ yr} \qquad (26)$$

for a typical initial disc mass $M_0 \sim 3 \times 10^{24}$ g and mass transfer rate $-\dot M_2 \sim 10^{-10}\ {\rm M_\odot\ yr^{-1}}$ (cf. King et al. 1996; King et al. 1997a,b). It is possible, however, that a shorter outburst could recur after a smaller interval. A full disc code is required to discover what actually occurs.

4 THE OPTICAL OUTBURST

In the previous section we discussed the evolution of the central accretion rate during an SXT outburst. This essentially determines the X-ray light curve of the transient, up to effects governed by any change of the X-ray spectrum. SXTs are also observed at longer wavelengths, in particular in the UV and the optical. Here the light curve no longer reflects the bolometric luminosity, but is dominated by the local radiation spectrum of the disc. In order to see the main features of the outburst at these wavelengths we approximate this spectrum as a blackbody. (This is extremely crude, and must be replaced by a more realistic treatment for detailed comparison with observations.) Then the continuum spectrum is given by

$$F_\nu \propto \int_{R_{\rm in}}^{R_{\rm out}} B_\nu[T(R)]R\ {\rm d}R, \qquad (27)$$

where $R_{\rm in}$, $R_{\rm out}$ are the inner and outer disc radii. For unirradiated discs it is well known (see e.g. Frank et al. 1992) that (27) gives a Wien tail at high frequencies $\nu \gg kT_{\rm max}/h$ and a Rayleigh–Jeans spectrum at low frquencies $\nu \ll kT_{\rm min}/h$. Between these extremes we have

$$F_\nu \sim \int_0^\infty \frac{\nu^3}{\exp[h\nu/kT(R)] - 1} R\ {\rm d}R. \qquad (28)$$

This approximation is poor for CV discs, where $R_{\rm out}/R_{\rm in} \sim 10$, but should work well for LMXB discs, where this quantity is $\sim 10^5$. As is well known, for an un-irradiated disc $[T(R) \propto R^{-3/4}]$, the substitution $x = h\nu/kT(R) \propto \nu R^{3/4}$ implies $F_\nu \propto \nu^{1/3}$. For an irradiated disc we have $T(R) \propto R^{1/2}$; now substituting $x \propto \nu R^{1/2}$ leads instead to $F_\nu \propto \nu^{-1}$. The outburst disc spectrum in the UV and optical should therefore lie between a ν^{-1} slope and whatever is produced by a cool disc (whose radiating area can be much larger than the irradiated one at late stages, or at all times in long-period systems with very large discs – see below).

Since irradiation has such an important effect on the disc light, we expect the optical light curve to follow the X-ray light curve. However, the decay time-scale $\tau_{\rm opt}$ of the optical and UV should be considerably longer, reflecting the spectral shift to longer wavelengths as the central accretion rate drops. Thus in the blackbody approximation we have $F_{\rm X} \propto \dot M_{\rm c} \sim {\rm e}^{-t/\tau}$. However, the optical part of the spectrum varies roughly as $F_{\rm opt} \propto T^2$ (cf. van Paradijs & McClintock 1994), giving $F_{\rm opt} \propto \dot M_{\rm c}^{1/2} \sim {\rm e}^{-t/2\tau}$, while the Rayleigh–Jeans (near-IR) part of the spectrum will vary as $F_{\rm IR} \propto T \propto \dot M_{\rm c}^{1/4} \sim {\rm e}^{-t/4\tau}$. The decay time-scale should therefore increase from τ in the X-rays to $\tau_{\rm opt} \sim 2$–4τ in the optical–IR. We note that Chevalier & Illovaisky (1995) find that the optical decay time-scale in GRO J0422+32 is about three times that in the X-rays.

5 DISCUSSION

We have shown that irradiation forces SXT light curves to have long quasi-exponential tails because the outburst can only shut off through the viscous decay of central accretion rather than via a cooling front. This was also a feature of the now-discarded mass-transfer burst model of dwarf novae, and explains the resemblance of SXT light curves to the slow decays predicted by that model. However, because the viscosity is controlled by the central accretion rate rather than locally, the light curves differ in detail from that model. We can crudely represent the central accretion rate as

$$\dot M_{\rm c} \simeq 4.8 \times 10^{-8} R_{11}^{7/4} {\rm e}^{-t/\tau}\ {\rm M_\odot\ yr^{-1}} \qquad (29)$$

for $t \lesssim \tau$,

$$\dot M_{\rm c} \simeq 9.5 \times 10^{-8} R_{11}^{7/4} {\rm e}^{-t/\tau}\ {\rm M_\odot\ yr^{-1}} \qquad (30)$$

for $\tau \lesssim t < T$, and

$$\dot M_{\rm c}({\rm NS}) \simeq 4.1 \times 10^{-10} R_{11}^2 \left[1 - \frac{t-T}{\tau}\right] {\rm M_\odot\ yr^{-1}} \qquad (31)$$

or

$$\dot M_{\rm c}({\rm BH}) \simeq 2.9 \times 10^{-9} R_{11}^2 \left[1 - \frac{t-T}{\tau}\right] {\rm M_\odot\ yr^{-1}} \qquad (32)$$

for $T < t < T + \tau$, where 'NS, BH' refer to neutron stars and (possibly) black holes (i.e. to $n = 1$, $n = 2$ respectively).

$$T \simeq (5.4 - 0.25 \ln R_{11})\,\tau, \qquad (3.5\text{–}0.25 \ln R_{11})\,\tau \qquad (33)$$

for $n = 1$, 2, and

$$\tau \simeq 40 R_{11}^{5/4}\ {\rm d}. \qquad (34)$$

Here R_{11} is the maximal hot-state disc radius R_0 in units of 10^{11} cm, and we have used the fact that $\nu \sim R_0^{3/4}$ (e.g. Frank et al. 1992). Equations (31) and (32) result from combining (23) with (14) and (15), and equation (33) follows by enforcing continuity between the exponential and linear regimes. Fig. 1 shows the X-ray light curve predicted by equations (29)–(31). This figure neglects any central accretion from matter originally outside the irradiated region ($R > R_{\rm h}$): the surface density might nevertheless eventually manage to reach the critical value there, triggering further small outbursts during the linear decay phase, which could mask its shape.

The exponential regime given by equations (29) and (30) may not exist at all for longer-period systems, where larger

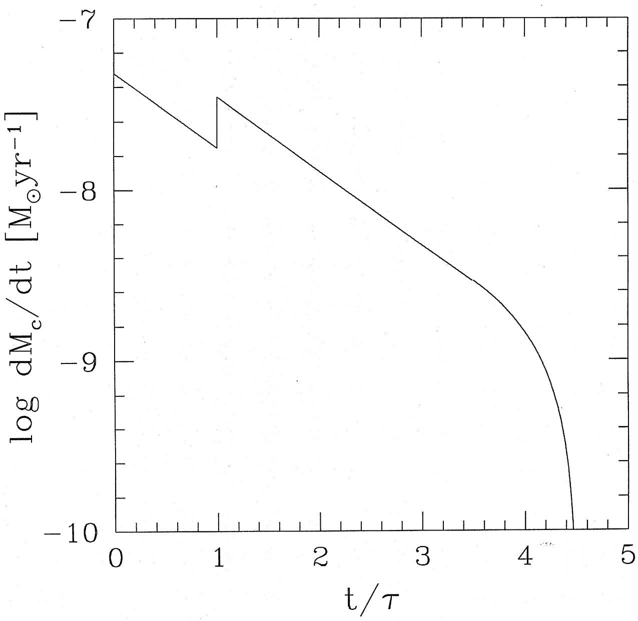

Figure 1. Central accretion rate ($\simeq$X-ray light curve) of a soft X-ray transient, as predicted by equations (29), (30) and (32), with $R_{11}=1$, $\tau=40$ d. A full disc calculation is required to resolve the secondary rise at $t=\tau$, shown as a step function. There could be additional small outbursts during the linear decay phase $t>T\simeq3.5\tau$.

values of R_{11} may cause the normalizations of (31) and (32) to exceed the Eddington limit. In this case the outburst will be entirely within the linear regime (31) and (32), with $T=0$, unless a heating wave propagating outwards from R_h triggers a further small outburst as envisaged above. The recurrence time can be expressed using (26) as

$$t_{rec}\sim 50\,R_{11}^3\left(\frac{-\dot{M}_2}{10^{-10}\ M_\odot\ yr^{-1}}\right)^{-1}\ yr. \tag{35}$$

If the central source is able to irradiate the entire disc, R_0 is given by Kepler's law and Roche geometry. We can then derive scaling laws describing SXT outbursts in terms of the binary period and evolutionary state of the system. It should be possible to check some of these scalings as a test of our picture, and we shall consider their implications for the detection of SXTs in a future paper. These scalings must, however, change for $R_{11}\gtrsim 1$–2, as then $\dot{M}_c$ formally exceeds the Eddington rate even for a black hole mass $m_1\sim10$. For such systems R_{11} stabilizes at the value given by the Eddington limit, so that τ should be constant at a value $\sim$40–80 d for the longest-period transients. This appears to agree, for example, with the decay time $\tau\sim40$ d seen in V404 Cygni ($P=6.47$ d) (cf. Tanaka & Shibazaki 1996). In such systems much of the disc mass is not heated directly. Unless this mass is reached by a heating wave propagating outwards from R_h it would not be accreted (the outburst would always be in the linear regime). In this case the meaning of the recurrence time t_{rec} becomes more complex: one might for example get a series of outbursts repeating on something like the cold-state viscous time (typically $\sim10\tau\sim400$ d) followed by a much longer quiescent interval characterized by the accumulation time of the whole disc.

It may be possible to detect the change from the exponential to linear decline (cf. equations 29–31) of the bolometric light curve in the brighter transients, although changes in the overall spectrum may obscure this. Short-period transients with $R_{11}\lesssim1$ will already be quite faint when this regime starts, particularly if $n=1$ (the 'neutron star' case). The best chance of seeing the linear decline is in long-period systems. Outbursts in systems with sufficiently long periods should be entirely in the linear regime, because their discs are so large that even irradiation by the Eddington luminosity cannot ionize them. We note that the X-ray decay of GRO J1744 − 28, which has the longest period of any SXT (12 d), is indeed entirely linear (Giles et al. 1996).

We have necessarily presented here a very simplified picture of SXT outbursts, using the assumption of a quasi-steady density profile to describe the global disc structure. To improve on this treatment we need to use a full disc outburst code. We stress, however, that it is hardly surprising that irradiation has a powerful effect on the outburst behaviour of SXTs. After all, it is already known directly from observation of the optical-to-X-ray flux ratio that the disc brightness is dominated by X-ray heating. Accordingly we expect the effects discussed here to form the basis of a fuller understanding of the details of SXT light curves. We note that both the general shape of the predicted outburst and the predicted accretion rates and duration are in excellent agreement with observation.

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