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THE LIMITING LUMINOSITY OF ACCRETING NEUTRON STARS WITH MAGNETIC FIELDS

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SUMMARY

Accretion on to a magnetized neutron star for high accretion rates, when one can no longer ignore the back-reaction of emergent light on the infalling material, is discussed in detail. The equations of hydrodynamics and radiative diffusion are solved in a one-dimensional approximation; the solution is expressed in an analytical form.

The luminosity L^* is evaluated beyond which one should allow for the dynamic effect of emergent light on the infalling material. The limiting X-ray luminosity L^{**} of accreting magnetized neutron stars is shown to depend crucially on the geometry of the accretion channel. Under certain conditions the value of L^{**} may appreciably exceed the critical Eddington value L_{Ed}, the main energy flux being carried away by neutrinos.

The effects connected with the gas flow along the magnetospheric surface are discussed in detail. The plasma layer on the Alfvén surface is shown to be optically thick with respect to Thomson scattering and to reradiate in soft X-rays $h\nu \lesssim 1$ keV a considerable fraction of the primary X-ray flux. A necessary condition for the X-ray luminosity to exceed the Eddington limit is a certain degree of asymmetry in the distribution of matter over the Alfvén surface. In the framework of the adopted model, regular pulsations of hard and soft X-rays are discussed.

1. INTRODUCTION

At least several tens of galactic X-ray sources and a number of X-ray sources in the nearest galaxies—the Magellanic Clouds—have a stellar origin and are, most likely, members of close binary systems. The X-ray emission of these objects can be naturally explained by the energy release, which occurs when matter supplied in excess by the normal star falls on to its compact companion—a neutron star or a black hole. In the case of spherically symmetric accretion the luminosity of a compact star has an upper bound,

$$L_{\mathrm{X}} \leqslant L_{\mathrm{Ed}} = 4\pi \frac{c}{\kappa} GM = 1\cdot 26 \times 10^{38} \left(\frac{\kappa_{\mathrm{T}}}{\kappa}\right)\left(\frac{M}{M_{\odot}}\right) \mathrm{erg\ s^{-1}},$$

which is called the Eddington limit (here κ is the opacity; $\kappa_{\mathrm{T}} = \sigma_{\mathrm{T}}/m_{\mathrm{p}}$ is the Thomson opacity; M is the mass of the accreting object). This limit originates from the fact that as $L_{\mathrm{X}} \to L_{\mathrm{Ed}}$ the pressure of the outgoing radiative flux inhibits the infall of matter towards the gravity centre, and at $L_{\mathrm{X}} = L_{\mathrm{Ed}}$ the force of gravitational attraction is precisely balanced by the opposing force of light pressure. One could expect that in the absence of spherical symmetry the luminosity of an accreting object might in principle exceed L_{Ed}. However, no self-consistent solution realizing this opportunity has been obtained up to now.

On the other hand, the observational data indicate that there is a number of

X-ray sources in the Galaxy and in the Large and the Small Magellanic Clouds with luminosity $L_X \sim 10^{38}$ erg s^{-1} (Margon & Ostriker 1973; Markert & Clark 1975). Of particular interest is the source SMC X-1. Its luminosity in the spectral range $2 \div 6$ keV amounts to $L_X \sim 10^{38}$ erg s^{-1} (Schreier *et al.* 1972), while the total X-ray luminosity $L_X \gtrsim 10^{39}$ erg s^{-1} (Price *et al.* 1971). The binary companion of SMC X-1 is a visible supergiant of an early spectral type—Sanduleak 160. The spectroscopic observations of Sanduleak 160 (Osmer & Hiltner 1974) indicate that SMC X-1 has a mass in the interval $1\ M_\odot < M < 4\ M_\odot$. Thus, there is a strong observational evidence that the luminosity of SMC X-1 exceeds the critical Eddington value L_{Ed}.

The main purpose of this study is to calculate the limiting luminosity of an accreting neutron star, the accretion channel of which is far from being spherical due to the presence of a strong magnetic field. It is rather clear that the key point in this problem is to allow properly for the back-reaction of the emergent radiation on the infalling gas. The effect of the emergent radiation on the accreting matter was taken into account by Davidson (1973) and Inoue (1975). However, the intention of both these authors was to construct models with a fixed accretion rate, in order to apply them to the X-ray pulsars Her X-1 and Cen X-3, rather than to obtain the limiting luminosity of an accreting magnetized neutron star. Moreover, in contrast to the previous papers mentioned above we have obtained an analytical solution to the problem and, among other things, have derived a simple expression (2) for the X-ray luminosity of the accretion column as a function of the accretion rate. The limiting X-ray luminosity L^{**} turns out to be very sensitive to the accretion geometry. When the shape of the accretion column is that of a thin wall (see Fig. 1(a)), the luminosity L^{**} may appreciably exceed L_{Ed}. And though one could not assert that the existence of such accretion geometry in real X-ray sources is firmly established, the possibility of surpassing the Eddington limit even in principle—is of great interest.

The other interesting result obtained in this paper is the value of the limiting luminosity L^* (see equation (1)) beyond which the mechanisms of X-ray beaming proposed earlier (Gnedin & Sunyaev 1973; Bisnovatyi-Kogan 1973; Basko & Sunyaev 1975) fail to work. And since the luminosity L^*, as contrasted to L^{**}, depends rather weakly on the accretion geometry, its value can be estimated accurately enough and the conclusion made is of greater generality. On the other hand, the observed pulse shape of the X-ray pulsars Her X-1 and Cen X-3 cannot be accounted for by the mere presence on the surface of the neutron star of isotropically emitting hot spots (Basko & Sunyaev 1976). It implies that, if only the luminosity of these X-ray pulsars exceeds L^* (the luminosity of Her X-1 seems to be approximately equal to L^*), the X-ray beam formation cannot be explained by the mechanisms proposed in the papers cited above (for a more exhaustive discussion see (Basko & Sunyaev 1976)).

This paper consists of four sections. In Section 2 we discuss in detail the geometric configuration of the accretion flow near the surface of a neutron star endowed with a dipole magnetic field. Three types of accretion geometry are considered; the analytical solution obtained below can be applied to any of them.

In Section 3 a qualitative description of accretion for different rates of mass inflow $\dot{M}$ is given. This qualitative description is based on the analytical solution of the system of hydrodynamic and radiative transfer equations; the derivation and analysis of this solution is presented in Section 4.

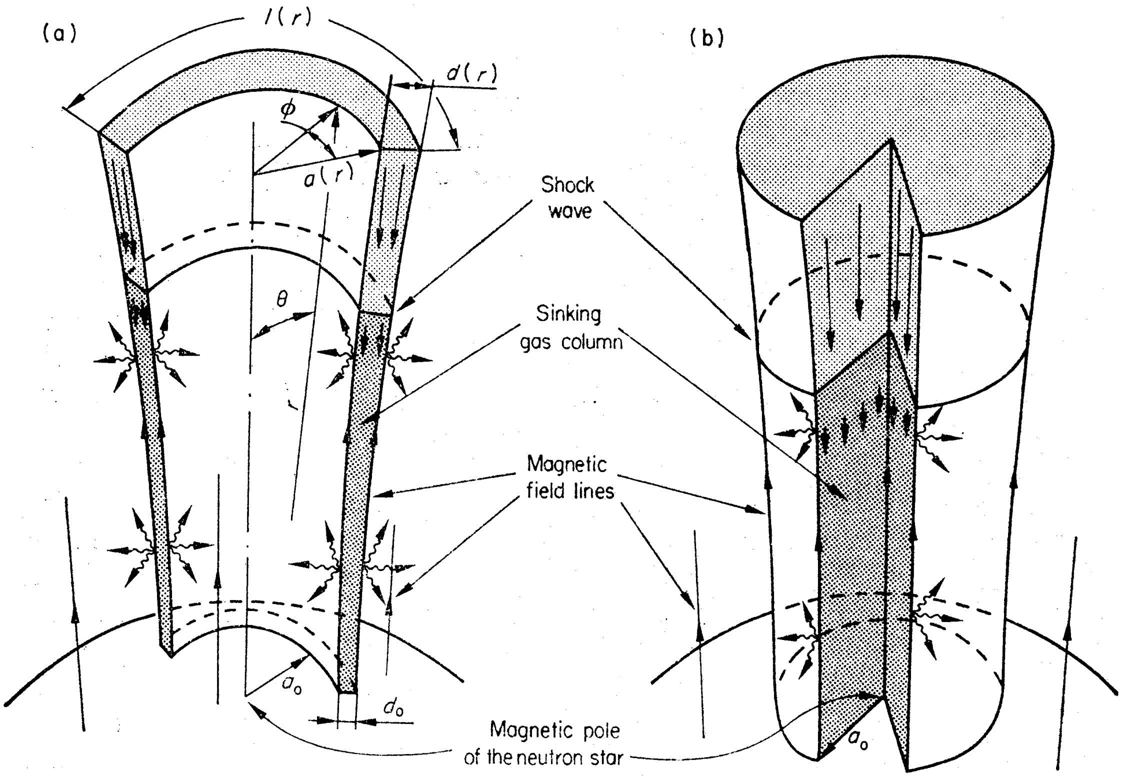

FIG. 1. *The configuration of accretion flow in two limiting cases: (a) material is confined to a narrow 'wall' of magnetic funnel, $d_0 \ll \alpha_0$; (b) material fills the whole of the funnel cavity. In the text the case (a) is adopted as the basic one.*

We show that at a low rate of accretion, when the luminosity $L_X < L^*$, the gas falls freely right down to the surface of the neutron star, where it gives up its kinetic energy by interaction with the particles of the stellar atmosphere. At a greater rate of accretion, when $L_X \simeq L^*$, the radiative shock stands above the surface of the neutron star; the infalling gas gives up its kinetic energy by interaction with the emergent photons. For this case the optical depth across the accretion channel for Thomson scattering becomes of the order of unity. As the accretion rate increases further ($L_X \gg L^*$), the radiative shock rises further above the surface of the neutron star, eventually reaching the Alfvén radius. At this stage the accretion funnel is filled with slowly sinking material immersed in a high-density radiation field. The X-ray luminosity L_X is the energy radiated per unit time by the side walls of the sinking gas column. At high accretion rates, when $L_X > L^*$, the dependence of the X-ray luminosity L_X on the accretion rate weakens, and it stabilizes at the level $L_X \sim L^{**}$ (see Fig. 2 and equation (2)). A considerable fraction of the energy released by accretion may escape in the form of neutrino emission (see Section 3.3).

The possibility of an accreting neutron star having a supercritical luminosity $L_X > L_{Ed}$ depends on the manner in which the accreting plasma flows along the Alfvén surface. In Section 3.5 we show that the gas layer on the Alfvén surface should be asymmetric and optically thick. Under these conditions it should reradiate an appreciable fraction of the primary X-ray flux in soft X-rays $h\nu \lesssim 1$ keV.

The rotation of the neutron star will result in periodic pulsations of the X-ray flux. These pulsations may be either due to the fact that the visible surface area of the sinking gas columns is less at one moment than at another, or that the gas on the Alfvén surface periodically shields the source of hard X-rays (see Section 3.6).

2. ACCRETION GEOMETRY

Outside the Alfvén surface, where the influence of the stellar magnetic field on the accreting matter can be neglected, it will be natural to restrict our consideration to two extreme cases of matter distribution—to the spherically-symmetric radial infall and to the disk accretion. The solution obtained below can be applied to both these patterns. However, in order to be specific, we shall everywhere assume the disk structure (for a more detailed discussion see Pringle & Rees 1972; Shakura & Sunyaev 1973), because the most favourable situation for large mass transfer rates in binary systems apparently occurs when the normal star fills its Roche lobe and the gas overflowing through the inner Lagrangian point forms a disk around the compact companion.

At the Alfvén radius R_A the pressure of the magnetic field (which is assumed to be that of a dipole) destroys the disk. We assume that at this radius the accreting plasma 'freezes' into the magnetic field of the neutron star. In general, the lines of constant magnetic pressure $H^2/8\pi$ in the disk plane, are not circles and that is why the matter freezes into magnetic field not along the whole of the inner disk edge but in two opposite regions, where the contour of constant magnetic pressure is remotest from the neutron star. The 'frozen-in' material falls along magnetic field lines towards the magnetic poles of the neutron star. Owing to the low resistivity, the depth to which the plasma penetrates into the dipole field is small compared to the Alfvén radius R_A. As a consequence, the accretion channel near the surface of the neutron star will look like a thin wall of a funnel (see Fig. 1(a)). At the stellar surface the cross-section of the accretion channel is a thin annular arc with radius $a_0 \sim 0{\cdot}1\ R$ (R is the radius of the neutron star) and length $l_0 \sim a_0$. The thickness of the accreting 'wall' $d_0 \ll a_0$. Below, in numerical estimates we shall use the following values for the parameters: $l_0 = 2a_0 \simeq 2\times 10^5$ cm, $d_0 = 5\times 10^3$ cm—which corresponds to a freezing depth $\simeq 0{\cdot}1\ R_A$ at the Alfvén radius.

If the flow pattern outside the Alfvén surface is a spherically-symmetric one, then the infalling matter will occupy the whole of the funnel cavity rather than a thin side wall (see Fig. 1(b)). In other words, we shall have a solid axisymmetric accretion funnel. However, this change in accretion geometry alters nothing in the qualitative picture described below. Moreover, this case of an axisymmetric geometry can be described by the same solution which corresponds to the plane-parallel geometry (thin accretion 'wall'), and only the values of some parameters should be changed (for details see Section 4.6).

In principle, the assumption that the accreting plasma is frozen into magnetic field may also be invalid, and one can imagine a situation when plasma falls in a narrow axisymmetric funnel and contains no external magnetic field. But this is precisely the case of a solid accretion funnel discussed in the previous paragraph. However, an external magnetic field must be always present in a *sinking* (rather than freely falling) gas column, because sinking gas liberates its gravitational energy locally. This energy is transported by radiative diffusion to the side walls of the accretion column. The resulting transverse gradient of radiative pressure (which greatly exceeds the pressure of matter) must be balanced by the opposite gradient of magnetic pressure $H^2/8\pi$. Thus, in view of the above argument it will be interesting to consider the other extreme case of the axisymmetric accretion channel—a hollow axisymmetric channel, in which material falls along the thin wall of a symmetric funnel (see Fig. 1(a)), the interior of the funnel containing only radiation with a constant (over the cross-section of the funnel) energy density, and no matter, nor magnetic field. However, one sees at once that this is but a special case of the geometry depicted in Fig. 1(a), which requires merely that the values of some constants in the general solution be changed (see Section 4.6).

3. QUALITATIVE PICTURE OF ACCRETION FOR DIFFERENT MASS FLOW RATES

Begin with a low accretion rate $\dot{M}$ (M is the mass of the neutron star), when the density of the emergent radiation is low and it has only a minor effect on the infalling matter. At the surface of the neutron star the total energy flux vanishes: the flux of the kinetic energy of the infalling material is exactly balanced by the opposite energy flux of the emergent radiation. We shall not discuss here the processes occurring in the atmosphere of the neutron star, which are responsible for the conversion of the kinetic energy of the accreting nucleons into radiation (they have been extensively discussed by Lamb, Pethick & Pines (1973), Gnedin & Sunyaev (1973), Basko & Sunyaev (1975)), and restrict our consideration to the structure of the accretion column above the stellar surface. By the term ' accretion rate ' we shall designate the quantity

$$L_t \equiv \dot{M}\frac{GM}{R}$$

which is equivalent to the above-mentioned $\dot{M}$.

3.1 *Low accretion rates*

If the accretion rate L_t is smaller than L^*, where

$$L^* = 2\frac{l_0}{R}\frac{c}{\kappa}\frac{GM}{R} = 4\times 10^{36}\left(\frac{\sigma_T}{\sigma_s}\right)\left(\frac{l_0}{2\times 10^5\ \mathrm{cm}}\right)\left(\frac{10^6\ \mathrm{cm}}{R}\right)\left(\frac{M}{M_\odot}\right)\ \mathrm{erg\ s^{-1}}, \qquad (1)$$

one can distinguish two distinct zones in the accretion column—the free-fall zone, and the shock wave zone. The free-fall zone extends almost down to the very surface of the neutron star; the radiation density in this zone $u = 0$, the gas velocity $v(r) = -(2\,GM/r)^{1/2}$.

The height of the shock zone is of the same order of magnitude as d_0 (in the case of a solid axisymmetric channel it is of the same order as a_0), i.e. it is much less than R; this region adjoins the surface of the neutron star and contains a radiation field of the density $u \sim \rho(R)v^2(R)$. The infalling material partly loses its kinetic energy while traversing the shock zone: the escaping photons slow down the infalling electrons when scattered by the latter; electrons, in their turn, slow down the protons through electrostatic interaction. The structure of the region being discussed resembles the structure of the extended shock that develops in gas where radiation pressure dominates.

The fraction of the kinetic energy lost by the accreting material in the shock rapidly decreases with decreasing accretion rate L_t: the velocity v_0 of the gas at a lower boundary of the accretion column vanishes at $L_t = L^*$, but $v_0 = -0{\cdot}8$ $(2\,GM/R)^{1/2}$ for the accretion rate as high as $L_t = \frac{1}{2}L^*$. Thus, we can say that *the neglect of the back influence of the emergent light on the accreting matter is justified so long as* $L_t < L^*$. Note that, allowing for the difference between σ_s and σ_T, the condition $L_t < L^*$ is in a good agreement with the analogous inequality obtained in the previous paper (Basko & Sunyaev 1975) from a more crude consideration.

The luminosity L^* has a clear physical meaning; for $L_t = L^*$ the distance, at which the infalling material is stopped by the emergent light, becomes equal to the thickness of the accretion 'wall' d_0 (or to the radius a_0 of a solid accretion funnel); or, in other words, the optical depth for scattering τ_t across the accretion channel becomes of the order of 1.

3.2 *High accretion rates*

If the accretion rate $L_t > L^*$, the infalling gas loses practically all its kinetic energy in the radiative shock—this energy transforms into radiation. At the same time, since at $L_t \gg L^*$ the transverse optical depth τ_t of the accretion channel is much greater than 1, only a small fraction of the accretion energy can be radiated by the side walls in the shock zone. From this we conclude that the shock rises above the surface of the neutron star as the accretion rate $L_t > L^*$ grows, and beneath the shock an extended sinking zone appears. In this latter zone—in contrast to the free-fall zone—the gas velocity is small and the radiative energy density is high; the gas slowly sinks, liberating its gravitational energy and emitting it sideways. Shakura (1974) has considered a similar accretion regime in a spherical geometry for $L_x \to L_{Ed}$. However, in his case there was no force (analogous to the magnetic pressure in our case) other than gravitation that could resist the radiative pressure gradient, and, as a consequence, the luminosity L_x could not exceed L_{Ed}.

To find the radius r_s at which the shock stands out, one should assign a boundary condition on the surface of the neutron star. However, whereas for $L_t < L^*$ this was a natural condition of vanishing of the total energy flux carried underneath the stellar surface, in a sinking regime, when $L_t > L^*$, one cannot satisfy this condition with a finite value of radiative energy density and non-zero flow velocity (for details see Sections 4.3(b) and 4.4). On the other hand, the approximation of a rigid magnetic channel breaks down when the radiative pressure becomes as high as $u/3 \sim H^2/8\pi$. And if the strength of magnetic field near the stellar surface exceeds the value $\sim 2 \times 10^{13}$ gauss, the neutrino emission resulting from electron–positron pair annihilations becomes of importance still earlier. For this reason, we shall define the lower boundary of the sinking gas column as the level with a fixed value of the radiative energy density u_0, which is stipulated by one of the two constraints mentioned above. We assume that the lower boundary introduced in this way coincides with the surface of the neutron star.

The analysis of the solution describing the sinking regime shows that the shock rises upwards with increasing accretion rate $L_t > L^*$ until it either (i) reaches the Alfvén surface, or (ii) ascends to some peak height and then drops downward (see Fig. 2) due to the fact that the ram pressure of the accreting flow approaches the magnetic pressure $H^2/8\pi$ at the stellar surface. The first of these two cases occurs in a more intensive magnetic field; the second, in a weaker one.

The total amount of energy L_x radiated per unit time by the side walls of two sinking gas columns also increases with L_t. However, at $L_t \sim L^{**}$ the rate of this increase diminishes (see Fig. 2) and some kind of a saturation regime sets up. As the rise of the shock terminates, the X-ray luminosity L_x attains its peak value equal to $\sim(2 \div 4)\, L^{**}$. In a wide range of parameter values the luminosity L_x can be represented accurately enough by the expression

$$L_x = L^{**} \exp\,(L^{**}/L_t)\, E_1(L^{**}/L_t), \tag{2}$$

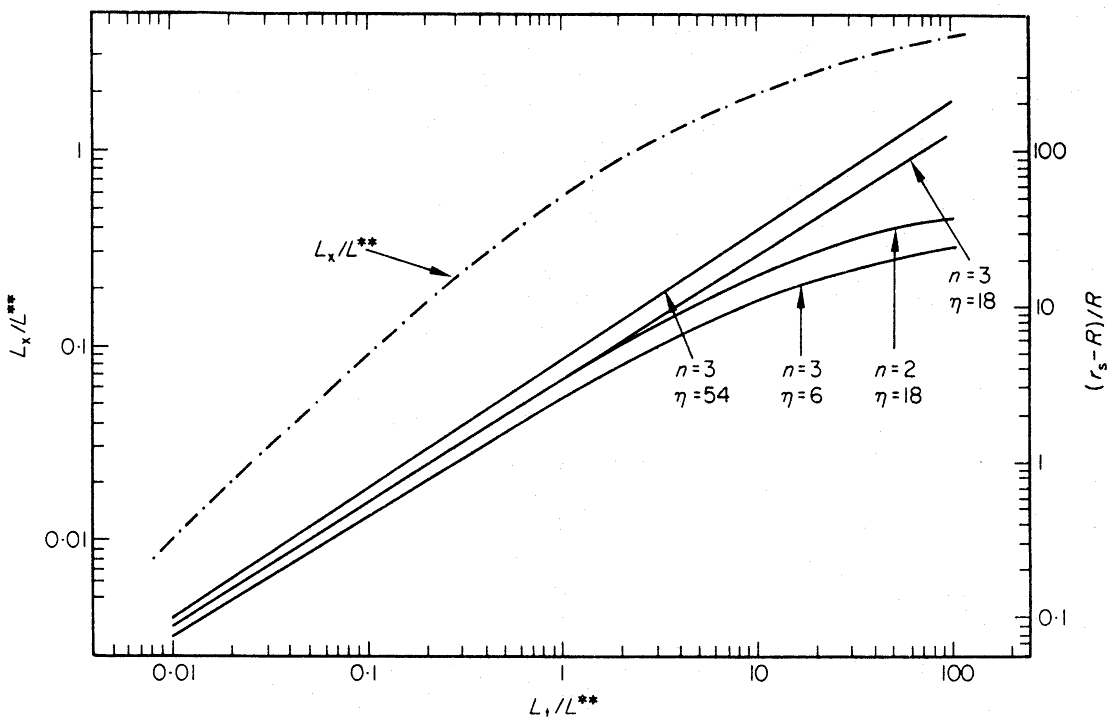

FIG. 2. *Two quantities are plotted in this figure: the dash-dotted line shows the run of the X-ray luminosity of two sinking gas columns L_x (in units of L^{**}, the left ordinate scale) with the rate of accretion; the solid curves represent the height of the shock r_s–R (in units of R, the right-ordinate scale) as a function of accretion rate for different values of model parameters n and η (see* (10) *and* (36)). *The abscissae are the values of $L_t/L^{**} = \gamma^{-1}$, where $L_t = GM\dot{M}/R$ is the total energy release by accretion.*

where

$$E_1(x) = \int_1^\infty t^{-1} \exp(-tx)\, dt$$

is the integral exponential, and the limiting luminosity L^{**} is given by

$$L^{**} = 2\frac{l_0}{d_0}\frac{c}{\kappa}GM = \frac{l_0}{2\pi d_0}L_{\mathrm{Ed}} = 8\times 10^{38}\left(\frac{l_0/d_0}{40}\right)\left(\frac{\sigma_{\mathrm{T}}}{\sigma_{\mathrm{s}}}\right)\left(\frac{M}{M_\odot}\right) \mathrm{erg\ s^{-1}}. \tag{3}$$

From (3) one sees that the luminosity L^{**} is very sensitive to the accretion geometry (for a solid axisymmetric funnel it is $L^{**} = \frac{1}{4}L_{\mathrm{Ed}}$, see Section 4.6) and under certain conditions may exceed the Eddington limit L_{Ed} by many times.

3.3 *Energy balance in a sinking regime*

The key feature of the sinking regime discussed above is the fact that the sinking gas column radiates sideways only a part $(1-\beta)\,L_t \equiv L_x$ of the total energy L_t released by accretion. The remaining part βL_t gets under the surface of the neutron star. To make the whole picture self-consistent, one should point out how this energy escapes from the interior of the neutron star—the coefficient β increases monotonically with L_t and amounts to $\beta \simeq 0.4$ as $L_t = L^{**}$. The excess energy βL_t cannot escape by ordinary radiative transfer since at $L_t \sim L^{**}$ the luminosity βL_t may exceed L_{Ed}.

(a) *Strong field, neutrino emission.* There is no problem at all if the magnetic field at the stellar surface is as strong as $H \gtrsim 2\times 10^{13}$ gauss. In this case magnetic walls can resist the pressure of such a strong radiation field that neutrinos, created by the annihilation of electron–positron pairs, carry away all the excess energy flux.†

The temperature corresponding to the radiative energy density $u \sim 3H^2/8\pi$ is then $T \gtrsim 10^{10}$ K. As was shown by Beaudet, Petrosian & Salpeter (1967), the basic contribution to the rate of neutrino losses at this temperature (and gas densities $\sim 10^5 \div 10^6$ g cm^{-3} that are of interest here) is due to the process $e^+ + e^- \rightarrow \nu_e + \tilde{\nu}_e$. The rate of cooling does not depend on the matter density and amounts to $\epsilon_\nu \gtrsim 10^{24}$ erg cm^{-3} s^{-1}. The emitting volume is $V \simeq 2l_0 d_0 \mathscr{H} \simeq 2 \times 10^{14}$ cm^3, where $\mathscr{H} \simeq 10^5$ cm is the scale height in the sinking gas column (*cf.* equation (33)). Thus, the neutrino luminosity is of the order of $L_\nu = \epsilon_\nu V \simeq 2 \times 10^{38}$ erg s^{-1}.

(b) *Weak field, non-steady accretion regime.* If the strength of magnetic field is 'small', $H < 2 \times 10^{13}$ gauss, the steady accretion regime considered hitherto, probably cannot exist. At a depth, where $u/3 \gtrsim H^2/8\pi$, the pressure of sinking gas pushes magnetic walls asunder. The 'magnetic bubbles' appearing are unstable. Their decay should result in X-ray flare-like phenomena accompanied with the interlocking of field lines, ejection of material etc. The characteristic time scale of irregular variations should be $\sim 10^{-4} \div 10^{-1}$ s.

3.4 *The influence of processes reducing the opacity*

From formula (3) one sees that the limiting luminosity of the neutron star L^{**} (just as the Eddington limit $L_{\rm Ed}$) is in inverse proportion to the opacity of material $\kappa = \sigma_s/m_p$. That means that any mechanism reducing the cross-section of scattering σ_s as compared to the Thomson value σ_T will increase the limiting luminosity of an accreting neutron star. Below we discuss two such mechanisms.

In a strong magnetic field the cross-section for photon scattering $\sigma_s \simeq (\nu/\nu_H)^2 \sigma_T$ drops appreciably at frequencies $\nu \ll \nu_H = eH/2\pi m_e c$ (Canuto, Lodenquai & Ruderman 1971). However, the effect of this mechanism on the limiting luminosity of accreting neutron stars becomes appreciable only in a magnetic field as strong as $H \gtrsim 10^{14}$ gauss. To prove this, we note that the main part of the X-ray energy flux is emitted from the bottom of the accretion column. The internal temperature of the accretion column for $L_X \sim L^{**}$ is $T_{\rm int} \sim (3H^2/8\pi a)^{1/4}$, where $a = 4\mathscr{C}/c = 7{\cdot}56 \times 10^{-15}$ erg cm^{-3} K^{-4}; so we have

$$\left(\frac{\nu}{\nu_H}\right)^2 \simeq \left(\frac{kT_{\rm int}}{h\nu_H}\right)^2 \simeq \left(\frac{2\pi m_e ck}{eHh}\right)^2 \left(\frac{3H^2}{8\pi a}\right)^{1/2} \simeq \frac{2 \times 10^{14}\ \text{gauss}}{H}.$$

Note that when $H > 2 \times 10^{13}$ gauss the temperature $T_{\rm int}$ ceases to increase because of neutrino losses (see Section 3.3). The mechanism being discussed here may, however, be of importance for the X-ray spectrum formation (see Section 4.5).

If the strength of magnetic field $H > 10^{13}$ gauss, then the temperature inside the accretion column becomes $kT_{\rm int} \gtrsim m_e c^2$, and quantum corrections to the cross-section of Compton scattering should be taken into account. The decrease of the scattering cross-section due to this effect (if not compensated by creation of electron–positron pairs) may result in an appreciable increase of the limiting X-ray luminosity L^{**}.

† The models with a supercritical accretion rate $L_t \gg L_{\rm Ed}$, in which all the excess energy $L_t - L_{\rm Ed}$ is carried away with neutrinos, have been proposed earlier too (Zeldovich, Ivanova & Nadezhin 1972; Ruffini & Wilson 1973). However, in none of these models has the photon luminosity of accreting object exceeded $L_{\rm Ed}$.

3.5 *Physical effects ensuing from the presence of the Alfvén surface*

(a) *Asymmetry of gas flow on the Alfvén surface.* Before getting into the polar magnetic funnels, accreting plasma flows along the Alfvén surface. Since the radius of this surface $R_A \gg R$, we may treat the material flowing along it as being irradiated by the point X-ray source of power L_X centred on the neutron star. If the matter were distributed uniformly along the whole of the Alfvén surface, the supercritical regime of accretion $L_X > L_{Ed}$ would be impossible, since the light pressure would overcome the gravity and have prevented material from falling into the magnetic funnels. Thus, the necessary condition for supercritical accretion to take place is a certain degree of asymmetry in the gas flow along the Alfvén surface; in other words, the fraction α of a solid angle subtended by the accretion flow must be less than 1. Below we show that the condition $\alpha \lesssim 0{\cdot}3 \div 0{\cdot}6$ is a sufficient one as well, i.e. we show that already for $\alpha \lesssim 0{\cdot}3 \div 0{\cdot}6$ the light pressure is unable to scatter the material flowing along the Alfvén surface, even if the accretion rate is arbitrarily large.

Let us first estimate the characteristic optical depth τ of the gas flowing along the Alfvén surface for $L_X > L_{Ed}$. For this we use the equation of energy balance

$$L_X = (1-\beta)\dot{M}\frac{GM}{R} = (1-\beta)\,\alpha 4\pi R_A \Delta r \rho v \frac{GM}{R}, \tag{4}$$

which is written under the assumption that gas of density ρ flows in a layer of thickness Δr along the sphere of radius R_A with a velocity v; α is the fraction of the sphere surface occupied by the gas flow. From equation (4) one readily obtains

$$\tau = \rho \Delta r \kappa = \frac{1}{\alpha\xi(1-\beta)}\frac{R}{(r_g R_A)^{1/2}}\frac{L_X}{L_{Ed}}, \tag{5}$$

where $r_g = 2GM/c^2$ is the gravitational radius of the neutron star, $\xi = v/(2GM/R_A)^{1/2} < 1$. Since $R_A \simeq (100 \div 300)\,R$ (Lamb *et al.* 1973), $R \simeq 3r_g$ and the quantity $\alpha\xi(1-\beta) \ll 1$, the optical depth $\tau \gtrsim 1$ so long as $L_X \gtrsim L_{Ed}$.

In order to find the restriction on α under which a supercritical accretion is possible, one should compare the force of light pressure with that of gravity at the Alfvén radius R_A. Strictly speaking, one should compare not the forces themselves but their projections on to the magnetic field lines; however, in our case this makes no difference because both these forces have one and the same radial direction. When estimating the light pressure, we assume that the impinging photons transfer the whole of their momentum to the gas layer (recall that $\tau \gtrsim 1$) and that the coefficient $\alpha \ll 1$. Then the condition for a supercritical accretion to be possible reads

$$\frac{L_X}{4\pi R_A^2 c} < \rho \Delta r \frac{GM}{R_A^2},$$

which is equivalent to

$$\tau > L_X/L_{Ed}. \tag{6}$$

Note that this inequality also implies $\tau \gtrsim 1$ at $L_X \gtrsim L_{Ed}$. Having substituted expression (5) into (6), we arrive at

$$\alpha \lesssim \alpha_0 = \frac{1}{\xi(1-\beta)}\frac{R}{(r_g R_A)^{1/2}}. \tag{7}$$

Here α_0 is the maximum value of α at which the X-ray flux from the neutron star still does not scatter the material on the Alfvén surface. Since the typical values of

β and ξ are $\beta \sim 0.5$ and $0.1 \lesssim \xi \lesssim 0.5$, we find that $\alpha_0 \sim 0.3 \div 0.6$ (recall that we have assumed $\alpha \ll 1$). That remarkable fact, that the X-ray luminosity L_X does not enter into (7), means that if the gas flow on the Alfvén surface has a high enough degree of asymmetry, then matter will fall into magnetic funnels at any accretion rate L_t.

This asymmetry requirement can be readily satisfied in the disk accretion picture (unless the magnetic axis is perpendicular to the disk plane), when accreting material 'freezes' into magnetic dipole at two diametrically opposite regions (see Section 2).

(b) *Emission of soft X-rays.* A considerable fraction ($\sim \alpha L_X$) of hard X-rays emerging from the neutron star is screened by the optically thick plasma layer on the Alfvén surface. If heavy elements in this layer are not appreciably ionized, then, as was shown by Basko, Sunyaev & Titarchuk (1974), about 70 per cent of the energy of the screened X-ray flux transforms into heat—soft X-ray photons $h\nu \lesssim 10$ keV are absorbed by the photoionization of heavy elements; those with $h\nu > 10$ keV 'redden' due to the recoil by Compton scattering and then are also absorbed. If heavy elements are highly ionized, an appreciable fraction of soft X-rays $h\nu \lesssim 10$ keV reflects diffusively from the screening plasma layer, while hard X-rays give up almost all of their energy by the recoil, as before. Thus, if primary X-rays are hard enough ($kT_X \gtrsim 30$ keV), about half of the screened energy flux is absorbed and heats the screening plasma layer.

To obtain a tentative estimate of the spectral range in which hard X-rays absorbed on the Alfvén surface are reradiated, we assume that the screening plasma layer radiates according to the blackbody law. Then its temperature T_A can be estimated from

$$L_X = \mathscr{C} T_A{}^4 4\pi R_A{}^2, \tag{8}$$

where $\mathscr{C}$ is the Stefan–Boltzmann constant. From (8) we have

$$T_A \simeq 2 \times 10^6 \,\mathrm{K} \left(\frac{10^8 \,\mathrm{cm}}{R_A}\right)^{1/2} \left(\frac{L_X}{L_{Ed}}\right)^{1/4} \left(\frac{M}{M_\odot}\right)^{1/4}. \tag{9}$$

Thus, the gas flowing along the Alfvén surface reradiates the energy flux $\sim \alpha L_X$ at frequencies $0.1 \text{ keV} \lesssim h\nu \lesssim 1$ keV. Since an overwhelming part of the primary X-ray flux is emitted in the spectral range $h\nu \gg 1$ keV, the soft X-ray flux (at $h\nu \lesssim 1$ keV) from the Alfvén surface may well exceed that from the neutron star itself.

We note that the soft X-ray source discussed here has nothing to do with that proposed by Avni *et al.* (1973), because its very existence is due to the presence of hard X-rays emerging from the neutron star. Remember that the soft X-ray source, proposed by Avni *et al.* (1973) to account for the optical light curve of HZ Her, must exist irrespective of the presence of hard X-rays from Her X-1. However, one cannot exclude the possibility that such phenomena as a 35-day cycle of Her X-1 and extended lows in Cen X-3 and other X-ray sources, revealed in hard X-rays, stem from the fact that the optically thick plasma layer on the Alfvén surface temporarily shields the neutron star from an observer. If this is the case, soft X-rays ($h\nu \lesssim 1$ keV) may be observed during the periods when the intensity of hard X-rays practically vanishes.

3.6 *Regular pulsations of the X-ray flux*

A characteristic feature of the model proposed here for X-ray sources with a supercritical luminosity is that the X-ray flux should pulsate with the period of rotation of the neutron star. These pulsations may originate in a two-fold way.

(i) The distant observer at one moment sees the larger surface area of the sinking gas columns than at the other, unless the magnetic axis is aligned with the spin axis or the latter coincides with the line of sight. If one assumes that the spin axis is perpendicular to the line of sight (which is a reasonable assumption for eclipsing X-ray binaries) and all the orientations of the magnetic axis have equal probabilities, then the probability of a pulsational amplitude $A = (f_{\max} - f_{\min})/2\langle f \rangle$ to be less than 0·1 can be estimated as $P(A \leqslant 0{\cdot}1) = 0{\cdot}19$ and, respectively, $P(A \leqslant 0{\cdot}2) = 0{\cdot}36$. In the other extreme case, when all the orientational probabilities for both these axes are equal, we have $P(A \leqslant 0{\cdot}1) = 0{\cdot}085$ and $P(A \leqslant 0{\cdot}2) = 0{\cdot}21$.

(ii) In the course of rotation of the neutron star the optically thick plasma layer on the Alfvén surface may periodically shield the X-ray source. In this latter case the amplitude of pulsations can be much more than that in the previous situation. We note that the characteristics of pulsations in soft X-rays $h\nu \lesssim 1$ keV emitted from the Alfvén surface may considerably differ from those in hard X-rays; the soft X-ray pulsations may, for example, be shifted in phase or appear at higher harmonics.

According to the UHURU data (Schreier *et al.* 1972) the X-ray source SMC X-1 reveals no regular pulsations with the amplitude $A > 10$ per cent in the range of periods $0{\cdot}3\ \mathrm{s} < p < 10\ \mathrm{s}$. This is the most serious obstacle for the interpretation of a supercritical luminosity of SMC X-1 in the framework of the model presented above (see Note added in proof).

One cannot exclude, however, that the rotational period of the neutron star greatly exceeds 10 s. The newly-found X-ray pulsar Ariel 1118–61 has a period 6·75 min (Ives, Sanford & Bell-Burnell 1975; Eyles *et al.* 1975). As a mechanism for slowing the magnetized neutron star up to such enormous rotational periods, the 'propeller' mechanism proposed by Illarionov & Sunyaev (1975) may be pointed out. Provided that the further accretion on to the slowly rotating neutron star proceeds from a stellar wind, where the gas has a negligible angular momentum, such a slow rotation can be maintained for an infinitely long time period.

4. SOLUTION OF THE EQUATIONS DESCRIBING THE ACCRETION IN A MAGNETIC CHANNEL

4.1 *Opacity*

Compton scattering is a dominant mechanism of opacity for practically all values of plasma parameters encountered in the present study. Below, for the sake of simplicity, we assume that the scattering cross-section σ_S is isotropic and independent of frequency, though it may in principle differ from the Thomson cross-section σ_T. The purpose, for which the scattering cross-section σ_S is distinguished from σ_T, is to retain the possibility of estimating the effects resulting from changes in the cross-section value (e.g. due to strong magnetic fields or to relativistic temperatures) in this or that accretion regime.

4.2 *Basic equations*

Let us consider a gas flow in the accretion channel. We adopt spherical polar coordinates (r, θ, ϕ) and make the following assumptions:

(1) The accretion channel extends in ϕ-direction—its width $d(r)$ is much less than its length $l(r)$ and its height $r-R$; all the quantities are independent of ϕ, the energy flux along ϕ can be neglected. The width and the length of the accretion column depend on its height as follows:

$$d(r) = d_0(r/R)^{n/2}, \quad l(r) = l_0(r/R)^{n/2};$$

here d_0 and l_0 are the width and the length of the accretion column at the surface $(r = R)$ of the neutron star. The case $n = 2$ corresponds to the spherically diverging accretion column; the case $n = 3$ is a good approximation to the flow pattern near the magnetic poles when matter falls along the field lines of a magnetic dipole.

(2) The radiative pressure $p = u/3$ (u is the radiative energy density) dominates over the gas pressure.

(3) The gas flow is in a steady state, $\partial/\partial t \equiv 0$. Under these assumptions the equations of mass, momentum and energy conservation take the form

$$\begin{cases} \rho v r^n = -sR^n = \text{const}, \\ v\dfrac{\partial v}{\partial r} + \dfrac{1}{3\rho}\dfrac{\partial u}{\partial r} + \dfrac{GM}{r^2} = 0, \\ \dfrac{1}{r^n}\dfrac{\partial}{\partial r}(r^n F_r) + \dfrac{1}{r}\dfrac{\partial F_\theta}{\partial \theta} = 0. \end{cases} \tag{10}$$

Here ρ is the gas density, v is the gas velocity, M and R are the mass and the radius of the neutron star, F_r and F_θ are the energy fluxes along r and θ respectively,

$$\begin{cases} F_r = -\dfrac{c}{3\kappa\rho}\dfrac{\partial u}{\partial r} + \dfrac{4}{3}uv + \rho v\dfrac{v^2}{2} - \rho v\dfrac{GM}{r}, \\ F_\theta = -\dfrac{c}{3\kappa\rho}\dfrac{1}{r}\dfrac{\partial u}{\partial \theta}; \end{cases} \tag{11}$$

$K = \sigma_s N_e/\rho = 0{\cdot}4(\sigma_s/\sigma_T)\ \text{cm}^2\ \text{g}^{-1}$ is the opacity. The factor $4/3$ by uv allows for the work of pressure $p = u/3$. In the third equation from (10) we used $d(r) \ll r\theta$.

A set of equations (10)–(11) essentially simplifies if one takes the values of ρ, v, u in the centre of accretion channel and averages the transverse radiative diffusion over θ. After the averaging we get

$$\begin{cases} \rho v r^n = -sR^n = \text{const}, \\ v\dfrac{dv}{dr} + \dfrac{1}{3\rho}\dfrac{du}{dr} + \dfrac{GM}{r^2} = 0, \\ \dfrac{1}{r^n}\dfrac{d}{dr}(r^n F_r) + \dfrac{2F_\theta}{d} = 0, \end{cases} \tag{12}$$

where

$$\begin{cases} F_r(r) = -\dfrac{c}{3\kappa\rho}\dfrac{du}{dr} + \dfrac{4}{3}uv + \rho v\dfrac{v^2}{2} - \rho v\dfrac{GM}{r}, \\ F_\theta(r) = \dfrac{2c}{3\kappa\rho}\dfrac{u}{d}. \end{cases} \tag{13}$$

Now all the functions depend on the argument r only; $F_\theta(r)$ is the energy flux radiated from the unit surface area by either of the two side walls of each accretion column (see Fig. 1(a)). Expression (13) for $F_\theta(r)$ used in the present study is an accurate one when the energy liberation is localized along the middle surface of accretion column. In the other extreme case, when energy sources $1/r^n\, \partial/\partial r\, (r^n F_r)$ are distributed uniformly over the whole of the channel width, the accurate expression will be $F_\theta = 4cu/3\kappa\rho d$. This case, however, reduces to the previous one as we increase the values of L^* and L^{**} (see equations (1) and (3)) by the factors of $\sqrt{2}$ and 2 respectively. The true values of L^* and L^{**} seem to lie somewhere between these bounds.

Below we consider the amount of matter s falling on to a unit surface area of the neutron star per unit time as a fixed quantity, determined by the properties of the normal star and by the parameters of the given binary system. To fix the value of s means to fix the value of L_t—the total amount of energy released by accretion; if accretion proceeds at both magnetic poles then

$$L_t = 2sd_0 l_0 \frac{GM}{R}. \tag{14}$$

In order to specify the only solution of (12)–(13) which is relevant here (for a fixed value of L_t), one should assign three boundary conditions. The choice of these boundary conditions is far from evident and depends on the value of L_t. For this reason we discuss the boundary conditions separately in each of the two accretion regimes.

4.3 *Shock regime*

(a) *Differential equation describing the shock regime.* At small accretion rates the region where the interaction of the emergent radiation with the infalling gas is of importance (the shock zone) localizes near the surface of the neutron star. Taking in (12)–(13) the limit $r-R \ll R$ and neglecting the gravitational force (validity of these assumptions will be checked after the solution is obtained), we get

$$\begin{cases} \rho v = -s, \\ v\dfrac{dv}{dr} + \dfrac{1}{3\rho}\dfrac{du}{dr} = 0, \\ \dfrac{dF_r}{dr} + \dfrac{4c}{3\kappa d_0^2}\dfrac{u}{\rho} = 0, \\ F_r = -\dfrac{c}{3\kappa\rho}\dfrac{du}{dr} + \dfrac{4}{3}uv - s\dfrac{v^2}{2}. \end{cases} \tag{15}$$

The boundary conditions for this set of equations are: (i) the condition $F_r(R) = 0$ at the bottom of the accretion column which means that the net energy flux carried in under the stellar surface vanishes, and (ii) two conditions $u(\infty) = 0$ and $v(\infty) = v_\infty = -(2GM/R)^{1/2}$ at the top of the shock. When we write in this section $r = \infty$, we mean that $r - R \gg d_0$.

The second equation in (15) can be immediately integrated, the result being

$$u = 3s(v - v_\infty) = -3sv_\infty(1-\tilde{v}). \tag{16}$$

Here and below in this section $\tilde{v} = v/v_\infty$ is the dimensionless velocity. Combining the third and the fourth equations from (15) with (16) and introducing a new independent variable

$$\tau = \kappa \int_R^r \rho \, dr,$$

we arrive at the second-order differential equation

$$\frac{d^2\tilde{v}}{d\tau^2} + \delta(7\tilde{v} - 4)\frac{d\tilde{v}}{d\tau} + 4\delta^2\epsilon^2\tilde{v}^2(1 - \tilde{v}) = 0, \tag{17}$$

which is equivalent to system (15). Here

$$\delta = -v_\infty/c \quad \text{and} \quad \epsilon \equiv L^*/L_t = c/\kappa s d_0 \tag{18}$$

are dimensionless constants. From (18) and (14) one easily derives the expression (1) for the luminosity L^*. In numerical calculations we use $\delta = \frac{1}{2}$ which is typical of neutron stars with $M \sim M_\odot$. The quantity ϵ is a measure of the accretion rate and has a clear physical meaning—$(\epsilon\delta)^{-1}$ is the optical depth across the accretion flow of a freely-falling material near the surface of the neutron star. The boundary conditions for (17) are as follows:

$$\begin{cases} F_r\Big|_{\tau=0} = 2sv_\infty^2\left(\dfrac{1}{2\delta}\dfrac{d\tilde{v}}{d\tau} - 2\tilde{v} + \dfrac{7}{4}\tilde{v}^2\right)\Big|_{\tau=0} = 0, \\ \tilde{v}(\infty) = 1. \end{cases} \tag{19}$$

Since τ does not appear in (17) explicitly, the latter can be reduced to the first-order differential equation. Introducing $w = 1/2\delta \, d\tilde{v}/d\tau$ as a function of $\tilde{v}$, we rewrite (17) in the form

$$\frac{dw}{d\tilde{v}} = 2 - \frac{7}{2}\tilde{v} - \epsilon^2\frac{\tilde{v}^2(1-\tilde{v})}{w}. \tag{20}$$

Thus, in order to solve equation (17) describing the accretion flow in the shock regime, one should proceed as follows:

(1) find the solution $w(\tilde{v})$ of (20) satisfying the boundary condition $w(1) = 0$ (which is equivalent to the second of the two conditions (19) since

$$\lim_{\tau\to\infty} \tilde{v}(\tau) = 1 \text{ implies } \lim_{\tau\to\infty} d\tilde{v}/d\tau = 0);$$

(2) calculate the velocity $\tilde{v}_0$ at $\tau = 0$ from the relationship

$$w(\tilde{v}_0) = 2\tilde{v}_0 - \tfrac{7}{4}\tilde{v}_0^2, \tag{21}$$

which is equivalent to the first of the two conditions (19);

(3) establish the relation $\tilde{v}(\tau)$ from the quadrature

$$\tau = \frac{1}{2\delta}\int_{\tilde{v}_0}^{\tilde{v}} \frac{d\tilde{v}}{w(\tilde{v})}; \tag{22}$$

the implicit form of the dependence $\tilde{v}(r)$ can be written in an analogous way

$$r = R + \frac{\epsilon d_0}{2}\int_{\tilde{v}_0}^{\tilde{v}} \frac{\tilde{v}\, d\tilde{v}}{w(\tilde{v})}; \tag{23}$$

(4) the relation between r and τ can be obtained from

$$r = R + d_0\delta\epsilon \int_0^\tau \tilde{v}(\tau')\, d\tau'. \tag{24}$$

(b) *Basic properties of the solutions of equation* (20). Since equation (20) plays a crucial role in elucidating the qualitative picture of accretion in magnetic channel, we shall discuss in detail the properties of its solutions for different values of ϵ. For this we introduce the phase plane $(\tilde{v}, w)$, see Fig. 3. The relevant solution must emerge from the point $(\tilde{v}, w) = (1, 0)$ which is a singular point of (20). One easily verifies that such a solution exists for any $0 \leqslant \epsilon < \infty$, and the first term of its expansion near $\tilde{v} = 1$ is

$$w = \left(\tfrac{3}{4} + \sqrt{\tfrac{9}{16} + \epsilon^2}\right)(1 - \tilde{v}) + o[(1 - \tilde{v})^2]. \tag{25}$$

Other essential properties of (20) and its solutions depend on whether $\epsilon > 1$ or vice versa.

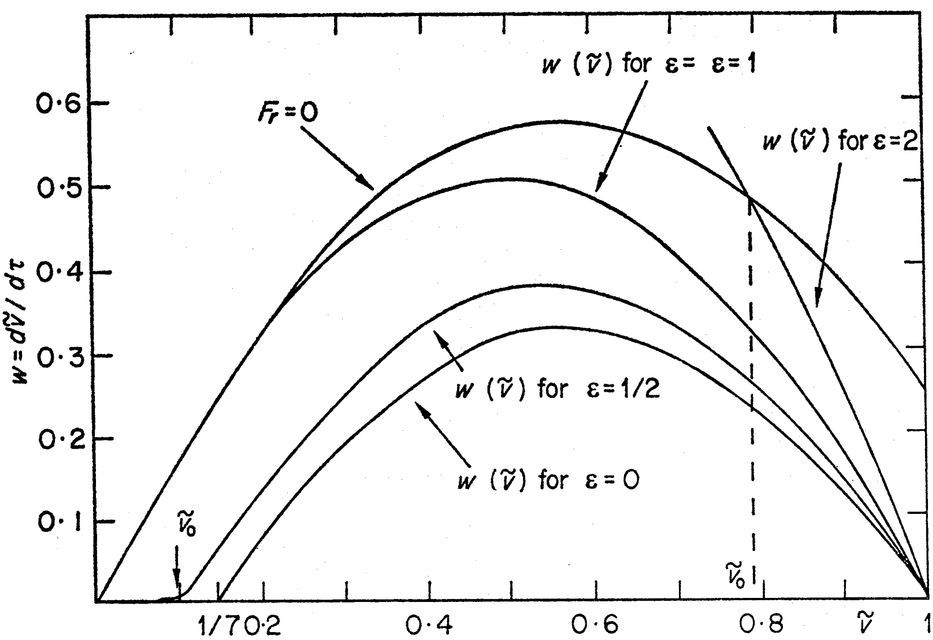

FIG. 3. *The phase plane of the differential equation* (17). *The integral curves are shown for different rates of accretion* ($\epsilon = L^*/L_t$ *where* $L_t = GM\dot{M}/R$ *is the energy release by accretion*). *The curve is also drawn along which the total energy flux* F_r *vanishes* (*the gravitational force is neglected*).

(1) *The case* $\epsilon > 1$ ($L_t < L^*$). From Fig. 3 one readily sees that the relevant solution $w(\tilde{v})$ necessarily intersects parabola (21) (along which $F_r = 0$) at a finite value of velocity $\tilde{v} = \tilde{v}_0 > 0$. In fact, this circumstance serves as a substantiation of the boundary condition $F_r = 0$ adopted above. It implies also a conclusion, stated in Section 3.1 somewhat groundlessly, that at $L_t < L^*$ the infalling material is stopped in the atmosphere of the neutron star rather than in the radiative shock and the influence of the emergent light can be neglected.

The height of the shock zone is given by the integral

$$r - R = \frac{\epsilon d_0}{2} \int_{\tilde{v}_0}^{1} \frac{\tilde{v}\, d\tilde{v}}{w}.$$

From (25) one sees that this integral diverges logarithmically on the upper limit. From a physical point of view this divergence stems from the fact that the radiative energy density does not vanish abruptly, as the distance from the shock front increases, but drops exponentially. Below we adopt the upper shock boundary as the level with $\tilde{v} = 0{\cdot}99$. Then for $\epsilon = 2$ ($L_t = L^*/2$) the height of the shock zone is $r-R = 1{\cdot}1\, d_0$, while for $\epsilon = 1$ it is $r-R = 1{\cdot}15\, d_0$.

Now we turn to the two basic simplifications under which equation (20) was derived. We immediately verify the first of them: $r-R \lesssim d_0 \ll R$. This implies also that one can neglect the gradient of the gravitational force. The neglect of gravity itself, which was the second simplification, is equivalent to the condition $v(dv/dr) \gg GM/R^2$, that in the above notation takes the form

$$w(\tilde{v}) \gg \frac{\epsilon}{4}\frac{d_0}{R}. \tag{26}$$

Fig. 3 shows that this requirement is always satisfied so long as $\epsilon > 1$. We note that all the above discussion becomes meaningless at $\epsilon \gg 1$, since the optical depth τ_t across the accretion channel is then much smaller than 1 and the diffusion approximation breaks down.

(2) *The case* $\epsilon = \epsilon^* = 1$ ($L_t = L^*$). In this particular case the relevant solution of (20) is a singular one. It is given by a very simple analytical expression

$$w^*(\tilde{v}) = 2\tilde{v}(1-\tilde{v}). \tag{27}$$

This solution satisfies the condition $F_r = 0$ at the point $(\tilde{v}, w) = (0, 0)$ which is also a singular one for (20). Solution (27) describes an intermediate accretion regime between low ($\epsilon > 1$) and high ($\epsilon < 1$) accretion rates. In this regime the accreting material gives up all of its kinetic energy ($\tilde{v}_0 = 0$) by interaction with the outgoing radiation. At the same time, the net energy flux penetrating under the surface of the neutron star vanishes. The flow velocity in the accretion column varies according to

$$v(r) = -\left(\frac{2GM}{r}\right)^{1/2}\left[1-\exp\left(-4\frac{r-R}{d_0}\right)\right].$$

Note that it is just this very case which corresponds to the formulation of the problem adopted by Davidson (1973).

(3) *The case* $0 \leqslant \epsilon < 1$ ($L_t > L^*$). Once $\epsilon \leqslant 1$, the relevant solution of (20) does not intersect parabola (21) within the interval $0 < \tilde{v} < 1$. In this case there exists such a velocity value $\tilde{v}_0 > 0$ that in the interval $\tilde{v}_0 < \tilde{v} < 1$ (the shock zone) the behaviour of $w(\tilde{v})$ is similar to that of (27), while at $0 < \tilde{v} < \tilde{v}_0$ (the sinking zone) the solution takes the form (see Fig. 3):

$$w(\tilde{v}) = \frac{\epsilon^2}{2}\tilde{v}^2 + o(\tilde{v}^3). \tag{28}$$

The numerical value of $\tilde{v}_0$ depends on the parameter ϵ—it increases monotonically with L_t from $\tilde{v}_0 = 0$ at $\epsilon = 1$ up to $\tilde{v}_0 = \frac{1}{7}$ at $\epsilon = 0$. Such a behaviour (28) of $w(\tilde{v})$ in the vicinity of $\tilde{v} = 0$ implies that (i) one can no more assign the condition $F_r = 0$ at the lower boundary of the accretion column, that (ii) one cannot use equation (20) to describe the accretion beneath the shock where $\tilde{v} < \tilde{v}_0$, and that (iii) in a sinking zone $\tilde{v} < \tilde{v}_0$ one can put $dw/d\tilde{v} = 0$. The first two statements result from the fact that for $w(\tilde{v})$ of the form (28) the integrals in (22) and (23) diverge; in other words, the zone $\tilde{v} < \tilde{v}_0$ may contain a considerable amount of matter and may be rather extended, as is really the case at $L_t \gg L^*$. To substantiate the third statement, it is sufficient to note that the solution of the algebraic equation, arising from (20) as a result of substitution $dw/d\tilde{v} = 0$, has the same asymptotic behaviour (28) for $\tilde{v} \to 0$ as the differential equation (20) itself. The physical meaning of condition $dw/d\tilde{v} = 0$ is as follows: *so long as $L_t > L^*$ one can neglect the radiative diffusion along the streamlines of the accretion flow.*

In the limiting case $\epsilon = 0$ equation (20) describes a well-known radiative shock which develops in the gas with a dominant pressure of radiation field (Zeldovich & Rayzer 1967), and that is why we call the zone $\tilde{v}_0 < \tilde{v} < 1$ a 'shock zone'. In this case (20) becomes integrable and the solution is

$$w_0(\tilde{v}) = -\tfrac{7}{4}\tilde{v}^2 + 2\tilde{v} - \tfrac{1}{4}. \tag{29}$$

From (29) one readily ascertains that the velocity of the gas passing through the shock drops to one-seventh of its initial value. The thickness of the shock front amounts to a few photon path lengths,

$$\Delta\tau = \int_{1/7+0.01}^{1-0.01} \frac{d\tilde{v}}{w_0(\tilde{v})} \simeq 6.$$

The radiation density behind the shock front is $u = -\frac{18}{7} s v_\infty$.

Thus, if the accretion rate is high and $\epsilon \ll 1$, then the structure of the shock zone is adequately represented by the solution (29) combined with (22)–(24). This means that all of the kinetic energy (more precisely $\frac{48}{49}$ of it) of the infalling material transforms within the shock into the radiation field and only a small fraction ($\sim \epsilon^2$) of it is radiated sideways. Beneath the shock, gas slowly ($\tilde{v} < \frac{1}{7}$) sinks downward, gradually liberating its gravitational energy and radiating it through the side walls of the accretion column.

4.4 *Sinking regime*

According to the last paragraph of the previous section we adopt the following simplifications under which we shall analyse the sinking zone.

(1) The radiative shock (which is no longer supposed to develop at the very surface of the neutron star) obeys the laws of mass, momentum and energy conservation and is described by solution (29).

(2) Just behind the shock one can neglect the following two terms in the total energy flux F_r: (a) the kinetic energy flux $sv^2/2$, and (b) the radiative energy flux $c/3\kappa\rho \, du/dr$.

Since the flow velocity drops in the shock at least by the factor of 7, the error introduced by assumption (2a) does not exceed 2 per cent. Assumptions (1) and (2b) introduce errors of the order of ϵ^2, i.e. they are accurate enough at high accretion rates $L_t \gg L^*$. In virtue of (2a) and (2b) a set of equations (12)–(13) takes the form

$$\begin{cases} v \dfrac{du}{d\xi} = \dfrac{3sGM}{R} \xi^{-n-2}, \\[2ex] \dfrac{1}{\xi^n} \dfrac{d}{d\xi} (\xi^n F_r) = \dfrac{4}{3} \gamma u v, \\[2ex] F_r = \dfrac{4}{3} uv + s \dfrac{GM}{R} \xi^{-n-1}. \end{cases} \tag{30}$$

Here we have introduced a new independent variable $\xi = r/R$ and a dimensionless constant

$$\gamma \equiv L^{**}/L_t = \epsilon R/d_0,$$

which serves as a measure of accretion rate. Expression (3) for the luminosity L^{**} is given in Section 3.2.

To single out the relevant solution of (30), one should assign two boundary conditions. One condition results from relations connecting the physical parameters on both sides of the shock front:

$$\begin{cases} v(\xi_s) = -\frac{1}{7}(2GM/R)^{1/2}\,\xi_s^{-1/2}, \\ u(\xi_s)\,v(\xi_s) = -\frac{3}{4}s(GM/R)\,\xi_s^{-n-1}. \end{cases} \tag{31}$$

Two equations (31) represent only one boundary condition because we do not know the radius $r_s = R\xi_s$ at which the shock stands out. In the second relationship from (31), that conveys the conservation of energy on the shock front, we have dropped the term corresponding to the kinetic energy flux behind the shock. For this reason $u(\xi_s)$ in (31) differs from that obtained from (29) by 2 per cent.

The second condition should be assigned at the lower boundary of the sinking gas column. However, it can no more be the condition $F_r = -\rho vGM/R$ which corresponds to all the energy released by accretion being emitted from the side walls of the accretion column. It is easy to show that in the sinking regime this condition can be satisfied if only $v = 0$ and $u = \infty$. In accordance with the argument of Section 3.3 we adopt a constant value of radiative energy density $u = u_0$ at the bottom of the accretion column. Since the height scale in the atmosphere of the neutron star is $\sim 10^2 \div 10^3$ cm, the pressure outside the accretion column reaches the value $p \sim u_0/3$ at a depth $\sim(10^{-3} \div 10^{-2})\,R$. On the other hand, the height of the sinking gas column can be of the same order of magnitude as R itself. Therefore, we can put $u(R) = u_0$.

The solution of system (30) is

$$\begin{cases} -u(\xi)\,v(\xi) = \dfrac{3}{4}\,s\,\dfrac{GM}{R}\,\dfrac{e^{\gamma\xi}}{\xi^n}\left[\dfrac{1}{\xi}E_2(\gamma\xi) + \beta\,e^{-\gamma} - E_2(\gamma)\right], \\ u(\xi) = u_0\left\{1 - \dfrac{e^{\gamma}}{\beta}\left[E_2(\gamma) - \dfrac{1}{\xi}E_2(\gamma\xi)\right]\right\}^4. \end{cases} \tag{32}$$

Here

$$E_k(x) = \int_1^\infty t^{-k}\exp(-tx)\,dt$$

is the integral exponential of order k. While solving (30), the following properties of $E_2(x)$ were used

$$dE_2(x)/dx = -E_1(x), \quad E_2(x) = e^{-x} - xE_1(x). \tag{33}$$

In (32) only one of the two boundary conditions, namely $u(1) = u_0$, is used and a new dimensionless constant

$$\beta = -\frac{4}{3}\,\frac{u(1)\,v(1)\,R}{sGM}$$

is introduced. To find β, one should make use of the second boundary condition (31). The constant β is that fraction of energy released by accretion which is carried in with the sinking gas under the surface of the neutron star. Having substituted (32) into (31), we arrive at the transcendental equation for ξ_s:

$$\eta\gamma^{1/4}\xi_s^{n/4+1/8} = 1 + \exp(\gamma\xi_s)[\xi_s E_2(\gamma) - E_2(\gamma\xi_s)] \tag{34}$$

and an expression for β:

$$\beta = 1 - \gamma e^{\gamma}[E_1(\gamma) - E_1(\gamma\xi_s)]. \tag{35}$$

The dimensionless constant η is given by

$$\eta = \left(\frac{8}{21}\frac{u_0 d_0^2 \kappa}{c\sqrt{2GMR}}\right)^{1/4}. \tag{36}$$

As one sees from (34), the height of the sinking gas column $r_s - R = R(\xi_s - 1)$ depends on two parameters γ and η. The quantity $\gamma^{-1} = L_t/L^{**}$ is a dimensionless accretion rate; the quantity η does not depend on the accretion rate and is determined by the parameters of the neutron star (its mass, radius, strength of magnetic field etc.), depending on them rather weakly. Equation (34) has a single solution $\xi_s > 1$ provided that $\eta\gamma^{1/4} > 1$. The latter condition means that the magnetic field must be strong enough to resist the ram pressure of the infalling material.

The calculated values of ξ_s are presented in Fig. 2 for a number of values of n and η. As fiducial cases we take the values $n = 3$ and $\eta = 18$ which correspond to a magnetic accretion channel with $u_0 = 3H^2/8\pi$, $H = 10^{13}$ gauss, $d_0 = 5 \times 10^3$ cm, $\kappa = 0{\cdot}4$ cm^2 g^{-1}, $M = M_\odot$, $R = 10^6$ cm.

The X-ray luminosity of two sinking gas columns is given by the expression

$$L_X \equiv L_t(1-\beta) = L^{**} e^{\gamma}[E_1(\gamma) - E_1(\gamma\xi_s)]. \tag{37}$$

Having analysed the behaviour of solutions of (34), one ascertains that in a wide range of parameters $E_1(\gamma\xi_s) \ll E_1(\gamma)$. Allowing for this fact, one readily arrives at expression (2) for the X-ray luminosity of the accreting neutron star. The plot of $L_X(\gamma)/L^{**}$ is given in Fig. 2.

4.5 *Variation of physical parameters in the sinking gas column*

In the framework of the model adopted it is impossible to calculate the cross-structure of the sinking gas column; however, one can easily establish the run of such quantities as flow velocity, gas density, temperature, etc., averaged over the width of the accretion column, with the radius. These quantities are plotted in Figs 4–6 for $L_t = L^{**}$ ($\gamma = 1$). In the illustrative case chosen, $\gamma = 1$, the shock stands out at the radius $r_S = 4{\cdot}6\,R$, the X-ray luminosity is

$$L_X = L_t(1-\beta) \simeq 0{\cdot}6\,L^{**} = 4{\cdot}8 \times 10^{38}\left(\frac{l_0/d_0}{40}\right)\left(\frac{\sigma_T}{\sigma_S}\right)\left(\frac{M}{M_\odot}\right) \text{erg s}^{-1}.$$

Fig. 4 presents the plot of $q(r)$—the fraction of the total energy released by accretion that is radiated by the section of a sinking gas column between r and r_S,

$$q(r) = \left[\left(\frac{r}{R}\right)^n F_r(r) - \left(\frac{r_S}{R}\right)^n F_r(r_S)\right]\left(\frac{sGM}{R}\right)^{-1}.$$

For comparison the quantity R/r—the fraction of gravitational energy released in the interval from r to ∞—is also plotted. Since these two curves are almost parallel, we can say that in the sinking regime some kind of a local energy balance establishes—the energy leaves accretion columns almost at the same rate as it is liberated there.

Once the radiative energy distribution (32) is found, one can evaluate the plasma temperature T_{int} inside the accretion column under an assumption of a full thermodynamic equilibrium; this quantity is plotted in Fig. 5. The density

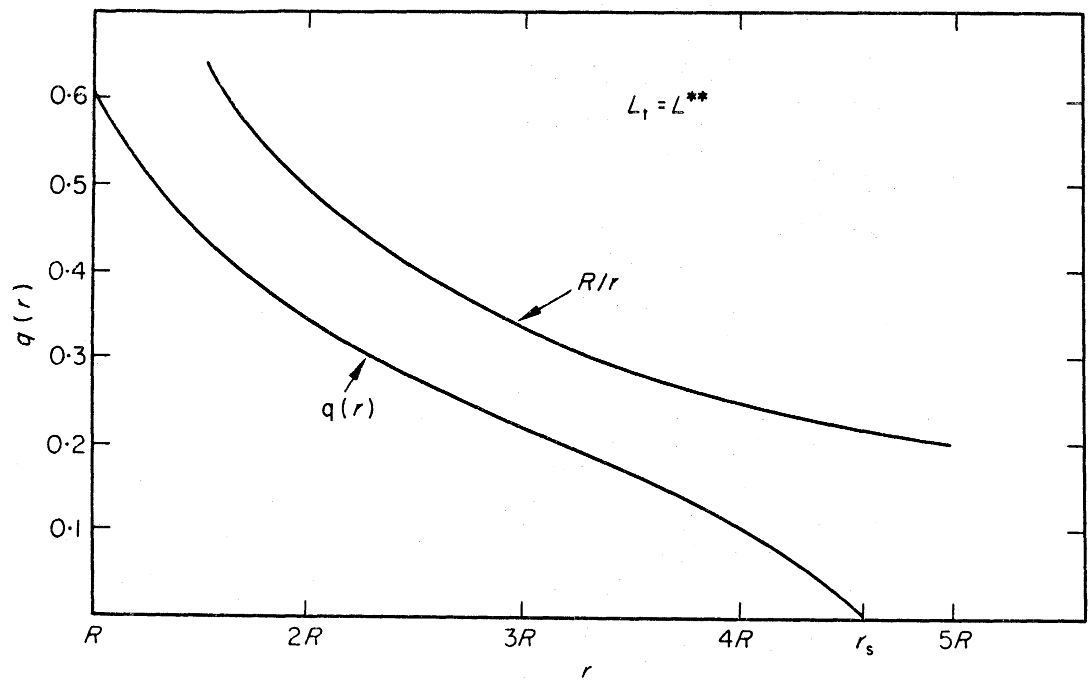

FIG. 4. *The plot of the quantity q(r) which is equal to the ratio of the energy emitted by the section of a sinking gas column lying above r to the total energy release by accretion L_t. For comparison the curve q = R/r is also drawn.*

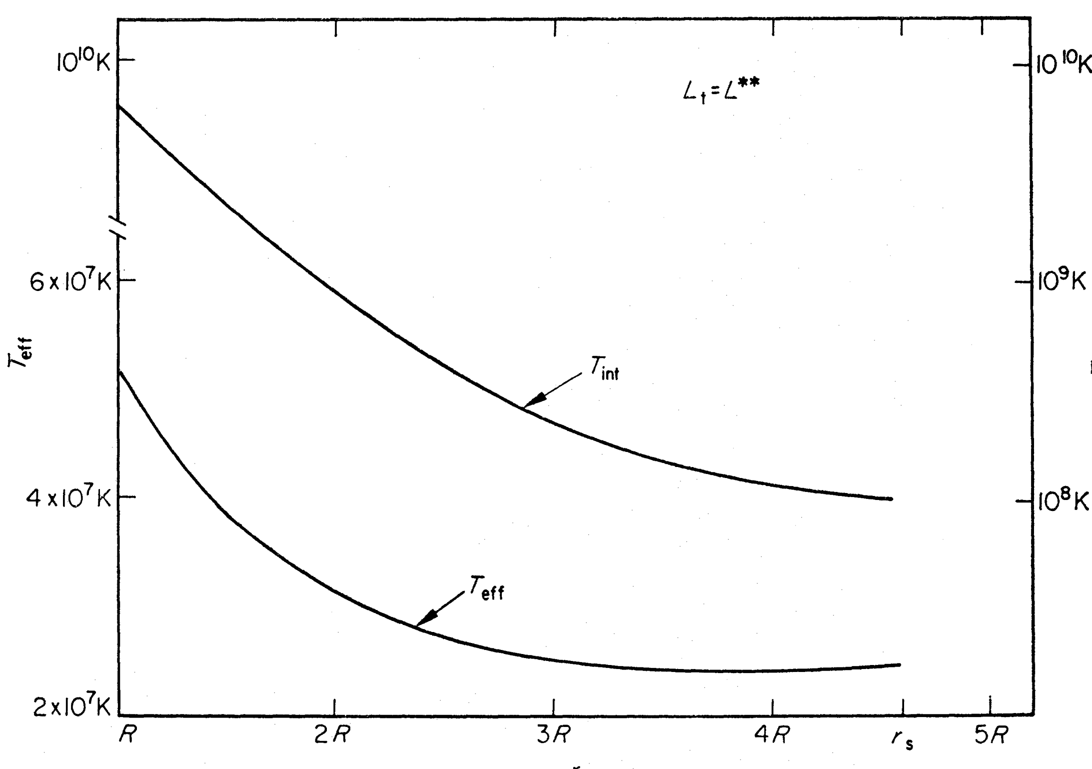

FIG. 5. *The run of the effective surface temperature T_{eff} (the left-ordinate scale) and the internal temperature T_{int} (the right-ordinate scale) in the sinking gas column. The abscissae are the distances from the centre of the neutron star of radius R.*

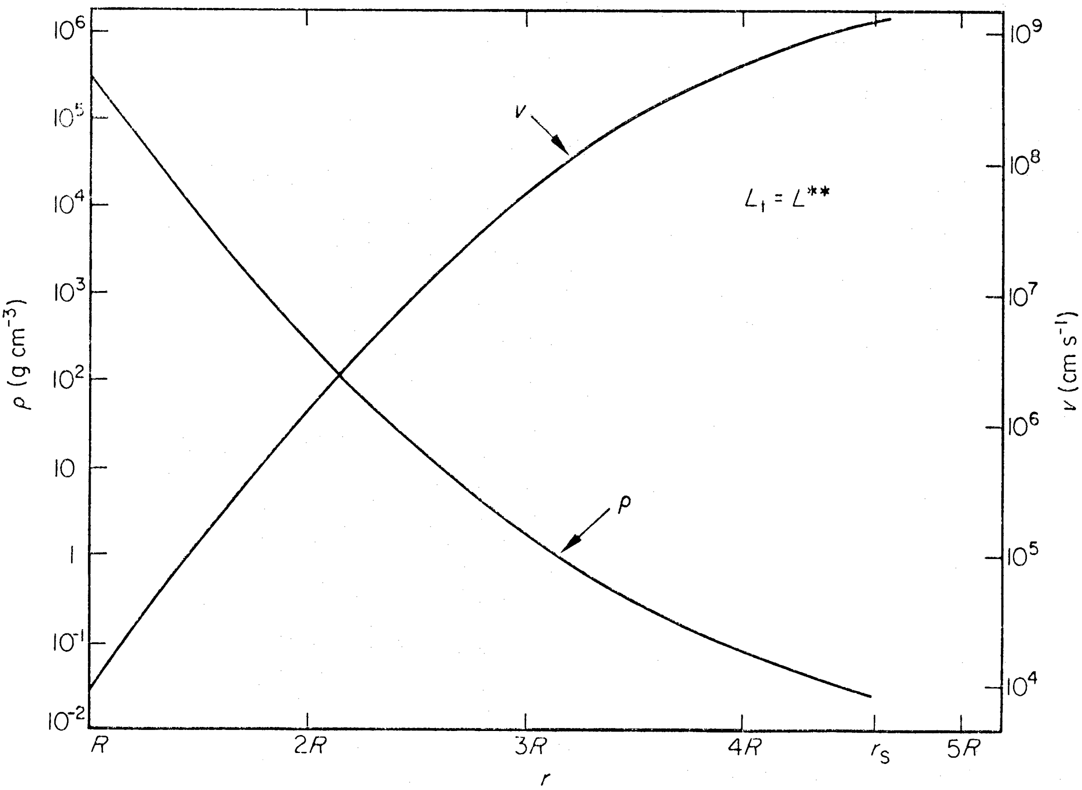

FIG. 6. *The run of the flow velocity v (the right-ordinate scale) and the gas density ρ (the left-ordinate scale) in a sinking gas column. The abscissae are the distances from the centre of the neutron star of radius R.*

and velocity of material in a sinking gas column calculated according to (32) are given in Fig. 6. Having calculated ρ and T_{int}, one readily verifies that

$$\sigma_{\mathrm{ff}}/\sigma_{\mathrm{T}}\big|_{h\nu=kT_{\mathrm{e}}} \simeq (0{\cdot}2 \div 5) \times 10^{-4} \ll 1,$$

thereby justifying the neglect of opacity mechanisms other than Compton scattering. On the other hand, since $(\sigma_{\mathrm{ff}}/\sigma_{\mathrm{T}})(\tau_{\mathrm{t}}/2)^2 > 1$ (τ_{t} is the optical depth across the accretion column), we can use the thermodynamic formulae when calculating T_{int} from $u(r)$. Once we know ρ and T_{int}, we can also evaluate the ratio of the gas pressure $p_{\mathrm{g}} = 2\rho kT_{\mathrm{int}}/m_{\mathrm{p}}$ to the radiative pressure $p_r = u/3$; it turns out to be $p_{\mathrm{g}}/p_r \simeq (0{\cdot}4 \div 8) \times 10^{-2}$, and we see that the neglect of gas pressure does not introduce a substantial error. The effective temperature T_{eff} of the side surface of sinking gas columns can be estimated from the Stefan–Boltzmann law, $T_{\mathrm{eff}} = (F_\theta/\mathscr{C})^{1/4}$, where $F_\theta(\xi) = -\frac{2}{3}\epsilon\xi^{n/2}u(\xi)\,v(\xi)$ is the energy flux radiated by the unit area of this surface. Fig. 5 shows that this temperature varies within the narrow interval $(2 \div 5) \times 10^7$ K.

Having estimated T_{eff}, one can say that the X-ray energy is mainly radiated at frequencies $h\nu \gtrsim 3kT_{\mathrm{eff}} \sim (6 \div 15)$ keV. However, a bit more detailed analysis immediately reveals that the spectrum of emergent X-rays should considerably differ from the Planckian one. As simple estimates show, an appreciable part of emergent X-rays is emitted from those regions of side surface area where $\sigma_{\mathrm{ff}}/\sigma_{\mathrm{T}} \ll 1$. According to Sunyaev & Shakura (1974) this effect causes the X-ray spectrum to flatten and the input of hard X-ray photons to increase. To carry out a detailed calculation of spectrum, one should find the density distribution across the accretion column, that is impossible in our formulation of the problem. We note also that besides the effect mentioned above one should take into account the dependence of the scattering cross-section on frequency (Canuto *et al.* 1971), since $kT_e \ll h\nu_H$ at the emitting surface. Both these effects will affect also the polarization of X-rays.

4.6 *Axisymmetric accretion channel*

In case of a hollow axisymmetric accretion channel (see Section 2) one should make the following alterations in the above discussion: (i) in the third equation from (12) the dissipative term takes the form F_θ/d, where $F_\theta = cu/3\kappa\rho d$; (ii) the total energy release by accretion (14) becomes $L_t = 4\pi a_0 d_0 s GM/R$. As a result of these alterations the expressions for L^* and L^{**} take the form

$$L^* = 2\pi \frac{a_0}{R}\frac{c}{\kappa} GM = 6{\cdot}2\times 10^{36}\left(\frac{\sigma_T}{\sigma_s}\right)\left(\frac{a_0}{10^5\ \mathrm{cm}}\right)\left(\frac{10^6\ \mathrm{cm}}{R}\right)\left(\frac{M}{M_\odot}\right)\ \mathrm{erg\ s^{-1}}, \tag{38}$$

$$L^{**} = \pi \frac{a_0}{d_0}\frac{c}{\kappa} GM = \frac{a_0}{4d_0} L_{\mathrm{Ed}} = 1{\cdot}3\times 10^{39}\left(\frac{a_0/d_0}{40}\right)\left(\frac{\sigma_T}{\sigma_s}\right)\left(\frac{M}{M_\odot}\right)\ \mathrm{erg\ s^{-1}}. \tag{39}$$

From these expressions one sees that all the above qualitative conclusions remain unaltered.

In the case of a solid axisymmetric accretion channel (Fig. 1(b)) with a radius $a(r) = a_0(r/R)^{n/2}$ the analogous alterations are as follows: (i) in the third equation from (12) the dissipative term takes the form $2F_\theta/a$, where $F_\theta = cu/3\kappa\rho a$; (ii) the total energy release by accretion is $L_t = 2\pi a_0{}^2 GM/R$. The expression for F_θ adopted here corresponds to the energy release localized along the axis of magnetic funnel.

The expressions for L^* and L^{**} take the form

$$L^* = \pi\sqrt{2}\frac{a_0}{R}\frac{c}{\kappa} GM = 4{\cdot}4\times 10^{36}\left(\frac{\sigma_T}{\sigma_s}\right)\left(\frac{a_0}{10^5\ \mathrm{cm}}\right)\left(\frac{10^6\ \mathrm{cm}}{R}\right)\left(\frac{M}{M_\odot}\right)\ \mathrm{erg\ s^{-1}}, \tag{40}$$

$$L^{**} = \pi\frac{c}{\kappa} GM = \frac{1}{4} L_{\mathrm{Ed}} = 3\times 10^{37}\left(\frac{\sigma_T}{\sigma_s}\right)\left(\frac{M}{M_\odot}\right)\ \mathrm{erg\ s^{-1}}. \tag{41}$$

From (41) and Fig. 2 one sees thet even when the shock rises up to the Alfvén surface, the X-ray luminosity of two sinking gas columns amounts to $(0{\cdot}5\div 0{\cdot}8)\,L_{\mathrm{Ed}}$. Note that if the transverse radiative diffusion were averaged according to the uniform distribution of energy sources, then the energy flux would be $F_\theta = 2cu/3\kappa\rho a$, the luminosity $L^{**} = \frac{1}{2}L_{\mathrm{Ed}}$ and the limiting X-ray luminosity would amount to $L_X \sim (1\div 1{\cdot}5)\,L_{\mathrm{Ed}}$. Thus, for the X-ray luminosity of a neutron star with this particular geometry of accretion to exceed the critical Eddington limit L_{Ed} corresponding to $\sigma_s = \sigma_T$, the effects reducing σ_s (see Section 3.4) should become of importance.

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NOTE ADDED IN PROOF

After this paper had been submitted to the journal, Lucke *et al.* (1975) discovered regular pulsations of X-rays from SMC X-1 with period $p = 0.716$ s and amplitude $A \sim 0.3$. This is new strong observational evidence that SMC X-1 is in fact an accreting neutron star with a luminosity $L_X > L_{Ed}$, and it makes the theoretical model developed above much more plausible and closer to reality than it seemed to be.