The Limits of Extended Kalman Filtering for Pulse Train Deinterleaving

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***Abstract*—Some signals, such as in radar systems, communication systems, and neural systems, are transmitted as periodic pulse trains. If more than one pulse train is transmitted over the same communication channel, a challenge is to separate them for source identification at the receiver. This is known as *pulse train deinterleaving* and is clearly a fundamental problem in the study of discrete-event systems. Frequently, the only relevant information at the receiver is the time of arrival (TOA) data, which is usually contaminated by jitter noise. Perhaps there are also missing or overlapping pulses.**

In this paper, we present an approach for deinterleaving pulse trains and estimating their periods using an extended Kalman filter (EKF). A naive application of EKF theory is not attractive because of discontinuities in the signal model. Here, a form of smoothing of the discontinuities is proposed so that the EKF approach becomes attractive. The advantage of this EKF approach is that it is less computationally expensive than most previously proposed methods, which are of order N^2, where N is the number of pulses being processed. The computation required here is of order N. The method proposed appears to give useful results for up to seven or so pulse trains, particularly when there is some *a priori* information on the pulse frequencies, which can be obtained using computations of order $N \log N$.

***Index Terms*—Deinterleaving, extended Kalman filtering.**

I. Introduction

SOME SIGNALS are transmitted as periodic pulse trains. Consider a situation where pulses from a number of different sources are being transmitted over a single communication channel. This leads to a series of pulse trains that are said to be *interleaved*. The process of *pulse train deinterleaving* is separating these pulses into the original trains. In order to do this, use is made of the fact that the different trains have different characteristics, such as period of pulse emission, phase, and pulse amplitude. Here, we restrict our attention to the case where only time of arrival (TOA) data for the received pulses is available or relevant. Deinterleaving is fundamentally a difficult problem, even in the ideal case with no jitter noise, missing or overlapping pulses, or period variations, but it is important that there be some robustness in the nonideal case. Pulse train deinterleaving is used in radar detection [1] and could potentially be studied in the areas of computer communications and neural systems.

Previously proposed techniques for pulse train deinterleaving include sequential search [2] and histogramming [2], [3]. These techniques work well when the interleaved signal is received in low noise. Another approach is to formulate the problem as a stochastic discrete-time dynamic linear model [4]. Here, the deinterleaving methods used are forward dynamic programming with fixed look-ahead and a probabilistic teacher.

A problem with all the above methods is that they are computationally expensive, typically of order N^2 or higher, where N is the number of pulses being processed. Optimal processing involves a full tree search, requiring computational effort of order M^N for M pulse trains. One method uses fast Fourier transform techniques to determine the number of pulse trains present in an interleaved signal and estimate their periods (their spectra) without actually deinterleaving them [5]. The computation required here is of order $N \log N$.

In this paper, the signal model from [4] is modified by a smoothing of its inherent discontinuities so that the deinterleaving task can be performed using the extended Kalman filter with computational effort of order N. It is assumed that the pulse trains are periodic and that the number of sources is finite and known. It is also desirable that the processing exploit *a priori* information as from spectra determined in [5].

This paper is structured as follows. In Section II, the problem is formulated in terms of a state space signal model, including a version with smooth nonlinearities. In Section III, the extended Kalman filter is presented. In Section IV, simulation examples are presented, and in Section V, some robustness issues are examined.

II. Signal Model—A μ Parameterization

Consider M periodic pulse train sources. Let $T^{(i)}$ and $t_0^{(i)}$ denote, respectively, the period and initial phase of the ith source. The received interleaved signal consists of the superposition of the M pulse trains produced by these sources. Let $t_1, t_2, \cdots, t_N$ denote the times of arrival of N consecutive pulses. The problem is as follows.

Problem: Given the pulses $t_1, \cdots, t_N$ and the number of sources present M, determine which source produced each pulse and estimate the periods $T^{(i)}$ and phases $t_0^{(i)}$ of each pulse train for $i = 1, 2, \cdots, M$.

This is a complex problem, as shown in Fig. 1. Here, pulses from interleaved pulse trains are shown with no information other than TOA data to identify their source. It is possible to deinterleave the two source train by eye, but this task quickly becomes impossible as more sources are added.

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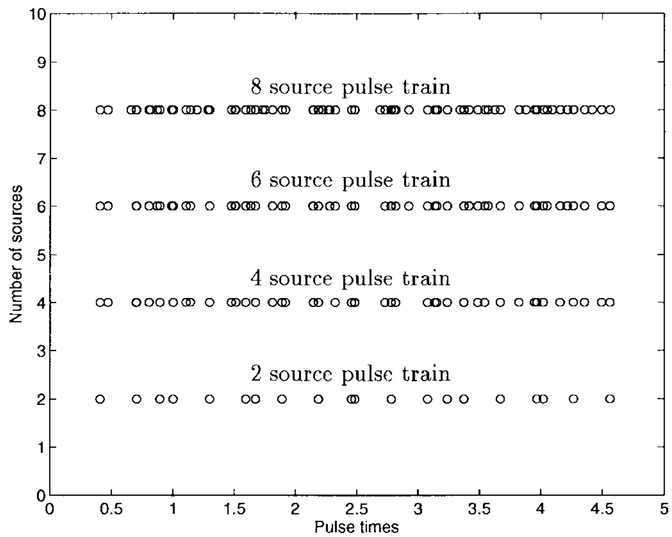


Fig. 1. Interleaved pulse trains.

The signal can be described by a discrete-state, discrete-time model where the index is pulse number and not time. That is, the model updates not at some discrete-time period but with each pulse received. We consider the model proposed in [4].

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} I & 0 \\ X_k^{\text{diag}} & I \end{bmatrix} x_k,\ x_0 \\ y_{k+1} &= [X_k' \quad X_k'] x_k + \omega_k \end{aligned} \tag{1}$$

where

- k pulse number;
- x_k state variable at k;
- X_k source indicator vector;
- y_k received signal at k.

Here, ω_k is zero-mean white Gaussian noise (WGN) on the received signal with known variance σ_ω^2, which is termed the *jitter noise*.

The state variable takes the form

$$x_k' = [T_k^{(1)}, \cdots, T_k^{(M)}, t_k^{(1)}, \cdots, t_k^{(M)}] \tag{2}$$

where there are M pulse train sources, $T_k^{(i)}$ is the period of train i, and $t_k^{(i)}$ is the time of arrival of the most recent pulse in train i at pulse number k. Clearly, $t_k^{(i)} \le y_k$.

The source indicator vector $X_k \in \{e_1, e_2, \cdots, e_M\}$, where e_i are unit column vectors in R^N with the 1 in the ith position, indicates the source-generated pulse k. If $X_k = e_i$, then the ith source is active at the kth pulse, and the value of $t_k^{(i)}$ for that train is increased by the period of the train, whereas all others remain constant. It is assumed in the first instance that only one source is active when a pulse is received. This eliminates the case when the sources are integer multiples, for example.

A key observation in this paper is that the source indicator vector X_k can be expressed in terms of the state as

$$X_k(x_k) = e_{i^*} \tag{3}$$

where i^* is the $\arg\min_i\{\cdots, (T_k^{(i)} + t_k^{(i)}), \cdots\}$. This indicator vector is discontinuous in x_k.

The signal model (1) can be fleshed out as a nonlinear state space model

$$\begin{aligned} x_{k+1} &= f_k(x_k),\ x_0 \\ y_{k+1} &= h_k(x_k) + \omega_k \end{aligned}$$

where

$$\begin{aligned} f_k(x_k) &= \begin{bmatrix} I & 0 \\ X_k^{\text{diag}}(x_k) & I \end{bmatrix} x_k \\ h_k(x_k) &= [X_k'(x_k) \quad X_k'(x_k)] x_k. \end{aligned} \tag{4}$$

Notice that this signal model has discontinuous nonlinearities $f_k(\cdot)$, $h_k(\cdot)$ and, thus, cannot be used without modification to derive an extended Kalman filter (EKF).

A. A Smooth Approximation of $X_k(x_k)$, Denoted $X_k^\mu(x_k)$

In order to use an extended Kalman filter for deinterleaving, we propose to approximate the source indicator vector function given in (3) by a smooth function as

$$\begin{aligned} X_k^\mu &= [(X_k^\mu)^1, \cdots, (X_k^\mu)^M]' \\ (X_k^\mu)^i &= \frac{(\tilde{t}_k^{(i)} + T_k^{(i)})^{-\mu}}{\sum_j (\tilde{t}_k^{(j)} + T_k^{(j)})^{-\mu}} \\ \tilde{t}_k^{(i)} &= t_k^{(i)} - \min_j (t_k^{(j)}). \end{aligned} \tag{5}$$

$\tilde{t}_k^{(i)}$ is simply a scaling of pulse time of arrival $t_k^{(i)}$ to the order of the pulse train period $T_k^{(i)}$ for all k. This is done to avoid ill conditioning of X_k^μ as k increases due to $t_k^{(i)}$ increasing with k, whereas $T_k^{(i)}$ remains constant. If this was not done, then for high k, X_k^μ would award the $(k+1)$th pulse to the train that was awarded the kth pulse, with no consideration given to the pulse train periods.

Notice that in (5)

$$\begin{aligned} &\lim_{\mu \to \infty} X_k^\mu = X_k \\ &\underline{1}' X_k^\mu = 1,\ (X_k^\mu)^i \ge 0 \quad \text{for} \quad i = 1, 2, \cdots, M \end{aligned}$$

where $\underline{1}' = [1, 1, \cdots, 1]$. That is, X_k^μ belongs to a simplex denoted Δ^{M-1} with vertices $e_1, e_2, \cdots, e_M$. We can think of X_k^μ as being a vector of probabilities, with $(X_k^\mu)^i$ as the probability of source i being active. This interpretation makes good sense when dealing with estimates of the states, which are denoted $\hat{x}_k$, leading to estimates of X_k or X_k^μ, which are denoted $\hat{X}_k$, $\hat{X}_k^\mu$, which also belong to Δ^{M-1}.

The source indicator vector from (5) for three sources is illustrated in Fig. 2, which depicts the simplex Δ^2 with vertices e_1, e_2, e_3. Note that the axes are labeled according to the notation used in (3). This illustrates the discontinuous nature of X_k. It can only take the three values that mark the vertices of the simplex, whereas X_k^μ can take any value on the simplex. Here, source 1 is active; therefore, X_k^μ lies closest to $X_k = e_1$.

The smoothed signal model is now

$$\begin{aligned} x_{k+1} &= f_k^\mu(x_k),\ x_0 \\ y_{k+1} &= h_k^\mu(x_k) + \omega_k \end{aligned} \tag{6}$$

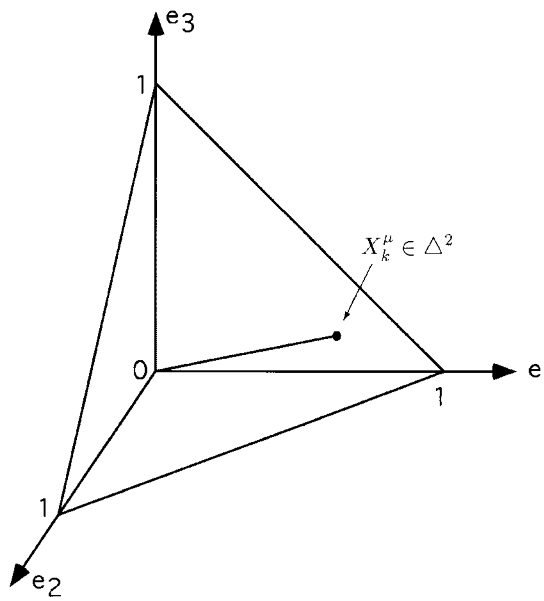


Fig. 2. Illustration of source indicator vector and simplex Δ^2.

where

$$\begin{aligned} f_k^\mu(x_k) &= \begin{bmatrix} I & 0 \\ X_k^{\mu\,\mathrm{diag}}(x_k) & I \end{bmatrix} x_k \\ h_k^\mu(x_k) &= [X_k^{\mu\prime}(x_k) \quad X_k^{\mu\prime}(x_k)]x_k. \end{aligned} \tag{7}$$

Calculation of the Smoothing Coefficient μ: It is desirable that the value of μ increase as the system states become better known. That is, when there is high certainty about the states, estimates $\hat{X}_k^\mu$ should be placed toward the simplex vertex indicated by X_k; therefore, μ should be large. This leads to an empirical calculation of μ using the change in the estimated periods over a set number of pulses.

To avoid ill conditioning, μ is constrained between fixed lower and upper values. An equation for μ is set up in such a way that when the estimated periods are fluctuating, μ is at its lower value, and once the estimated periods have reached their steady-state values, μ takes its upper value. This is done by observing the variation in the periods over, say, 50 pulses. If the maximum variation is of the order of five times the known noise variance, i.e., $5O(\sigma_\omega^2)$, then steady state has been reached. If the maximum variation is more than five times above this [$25O(\sigma_\omega^2)$], then the periods are still fluctuating. An intermediate value for μ is used when the maximum variation is between these values.

III. The Extended Kalman Filter

In order to construct an extended Kalman filter from the smoothed signal model given in (6) denoted EKF(μ), linearized versions of f_k^μ and h_k^μ (7) are needed. They are

$$\begin{aligned} F_k^\mu &= \begin{bmatrix} I & 0 \\ X_k^{\mu\,\mathrm{diag}} + S_{FT}^\mu & I + S_{Ft}^\mu \end{bmatrix} \\ H_k^\mu &= [X_k^{\mu\prime} + S_{HT}^\mu \quad X_k^{\mu\prime} + S_{Ht}^\mu]. \end{aligned} \tag{8}$$

The values of $S_{HT/t}^\mu$ in H_k^μ are called the sensitivity functions of H_k^μ. They are a measure of how much $[X_k^{\mu\prime}(x_k) \quad X_k^{\mu\prime}(x_k)]$ varies with small changes in T_k and t_k, respectively. This is similar for $S_{FT/t}^\mu$ in F_k^μ.

Let us now define two cases for the EKF(μ) deinterleaver. Case 1 [EKF(∞)] is where $\mu = \infty$; therefore, $X_k^\mu = X_k$ (3). Case 2 is where μ is allowed to take a range of values, and X_k^μ is defined in (5). Case 1 is not appropriate for use in an EKF deinterleaver as X_k is discontinuous and, therefore, cannot be linearized, except by ignoring aspects of the discontinuities, that is, by assuming that X_k does not vary with T_k or t_k or, equivalently, setting $S_{FT/t}^\mu$ and $S_{HT/t}^\mu$ to zero.

For case 2, the equations for the sensitivity functions of F_k^μ are

$$S_{FT}^\mu = \mathrm{diag}\left\{ \frac{\mu(A_k^{(i)})^{-(\mu+1)}\left[\sum_l (A_k^{(l)})^{-\mu} - (A_k^{(i)})^{-\mu}\right]}{\left[\sum_l (A_k^{(l)})^{-\mu}\right]^2} T_k^{(i)} \right\}$$

$$S_{Ft}^{\mu(i,i)} = S_{FT}^{\mu(i,i)} \tilde{t}_k^{(i)} \tag{9}$$

where $A_k^{(a)} = \tilde{t}_k^{(a)} + T_k^{(a)}$, and the equations for the sensitivity functions of H_k^μ are

$$\begin{aligned} S_{HT}^{\mu(i)} &= \sum_j \left[\frac{-\mu(A_k^{(i)})^{-(\mu+1)}(A_k^{(j)})^{-\mu}(t_k^{(j)} + T_k^{(j)})}{\left[\sum_l (A_k^{(l)})^{-\mu}\right]^2} \right] \\ &\quad + \frac{\mu(A_k^{(i)})^{-(\mu+1)}(t_k^{(i)} + T_k^{(i)})}{\sum_l (A_k^{(l)})^{-\mu}} \\ S_{Ht}^{\mu(i)} &= S_{HT}^{\mu(i)} \tilde{t}_k^{(i)} \end{aligned} \tag{10}$$

where $A_k^{(a)} = \tilde{t}_k^{(a)} + T_k^{(a)}$.

The introduction of $\tilde{t}_k^{(i)}$ in (9b) and (10b) is an approximation to remove a discontinuity introduced in the normalization procedure from (5). The exact equation for $S_{Ft}^{\mu(i,i)}$ is

$$S_{Ft}^{\mu(i,i)} = \begin{cases} S_{FT}^{\mu(i,i)}, & \text{if } t_k^i \neq \min_j(t_k^{(j)}) \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

Since $\tilde{t}_k^{(i)} = t_k^{(i)} - \min_j(t_k^{(j)})$ from (5), $\tilde{t}_k^{(i)}$ varies between zero and some number in the order of the train period. Therefore, $S_{Ft}^{\mu(i,i)}$ no longer takes exactly the values of zero or $S_{FT}^{\mu(i,i)}$ but varies so that the discontinuity is smoothed. This is similar for $S_{Ht}^{\mu(i)}$. Since the sensitivity terms are only used to give a feel for what is occurring (see below), this approximation is acceptable.

The sensitivity terms are approximately of the order of X_k^μ, except when a train is about to produce a pulse. Then, the sensitivity terms $S_{FT/t}^\mu$ become so large that they swamp the other terms in F_k^μ while $S_{HT/t}^\mu$ goes negative. It is important that this information be reflected in F_k^μ and H_k^μ but not dominate the other terms. For this reason, $S_{FT/t}^\mu$ and $S_{HT/t}^\mu$, although included in the calculations for F_k^μ and H_k^μ, are limited in magnitude.

A. The Riccati Equation

The Riccati equation for the extended Kalman filter is

$$\begin{aligned} K_k^\mu &= P_{k/k-1} H_k^\mu (H_k^{\mu\prime} P_{k/k-1} H_k^\mu + R_k)^{-1} \\ P_{k+1/k} &= F_k^\mu (P_{k/k-1} - K_k^\mu H_k^{\mu\prime} P_{k/k-1}) F_k^{\mu\prime} + Q_k \end{aligned} \tag{12}$$

where

- K_k^μ Kalman gain;
- $P_{k/k-1}$ error covariance at k, given measurements to $k-1$;
- R_k covariance of the noise on the measurement;
- Q_k covariance of the noise caused by smoothing.

The initialization here is provided by $P_{0/-1} = P_0$.

Now

$$\begin{aligned} R_k &= [T_k' \quad t_k'] \begin{bmatrix} \Sigma & \Sigma \\ \Sigma & \Sigma \end{bmatrix} \begin{bmatrix} T_k \\ t_k \end{bmatrix} + r_k^\mu \\ Q_k &= T_k^{\text{diag}} (\Sigma) T_k^{\text{diag}} + q_k^\mu I \end{aligned} \tag{13}$$

where $\Sigma = X_k^{\mu\,\text{diag}} - X_k^\mu X_k^{\mu\prime}$.

The constants r_k^μ and q_k^μ are added to the lower bound noise covariance equations to represent model errors and, therefore, to enhance robustness. For the case 2 EKF(μ) deinterleaver, they are tied to the value of μ and, hence, the certainty in the system. As μ increases, r_k^μ, q_k^μ decrease. See Section II-A1 for a description of the calculation of μ. For the case 1 EKF(∞) deinterleaver with $\mu = \infty$, r_k^μ, q_k^μ are constant, and $X_k^{\text{diag}} = X_k X_k'$; therefore, R_k and Q_k are constant.

B. The State Update Equations

The update equations for the extended Kalman filter are

$$\begin{aligned} \hat{x}_{k/k} &= \hat{x}_{k/k-1} + K_k^\mu [y_{k+1} - h_k(\hat{x}_{k/k-1})] \\ \hat{x}_{k+1/k} &= f_k(\hat{x}_{k/k}) \end{aligned} \tag{14}$$

where $\hat{x}_{k+1/k}$ is the filtered estimate of x_{k+1}, y_{k+1} is the input to the filter, f_k and h_k are defined in (4) and K_k^μ is defined in (12). The initialization here is provided by $\hat{x}_{0/-1} = \hat{x}_0$, where $\hat{x}_0 = [\hat{T}_0{}' \hat{t}_0{}']'$.

For both cases 1 and 2 deinterleavers, notice that the update equations work with f_k, h_k rather than the smoothed versions f_k^μ, h_k^μ. That is, there is no need to work with a model with smooth nonlinearities except for the calculation of K_k^μ.

IV. Some Examples

In this and the following section, results obtained using computer generated pulse trains are examined. The trains have randomly generated periods and phases, with the initial pulse from each train falling within one period of time zero. Here, it is assumed that there is no noise present on the EKF(μ) input signal. The effect of noise and other robustness issues are examined in Section V.

For the following discussion, the concept of the *ratio of periods* is needed. For any set of M pulse train sources that form the interleaved signal, the ratio of periods (RP) for that signal is defined as

$$RP = \frac{\max(T^{(1)}, \cdots, T^{(M)})}{\min(T^{(1)}, \cdots, T^{(M)})}$$

where RP must be greater than or equal to 1.

TABLE I
Comparison of the EKF(μ) Deinterleavers with $\hat{T}_0$ within 10% of T_0

RP	Number of sources deinterleaved	
	Case 1	Case 2
2	>7	>9
3	7	9
4	6	9
5	5	7
6	3	8
7	None	8
8	None	7
9	None	7
10	None	6
15	None	6
20	None	6

TABLE II
Comparison of the EKF(μ) Deinterleavers with $\hat{T}_0 = T_0$

RP	Number of sources deinterleaved	
	Case 1	Case 2
2	>9	>9
3-9	9	9
10	6	7
15	2	6
20	None	6

A. Comparison of Cases 1 and 2 EKF Deinterleavers

As stated in Section III, for the case 1 deinterleaver EKF(∞), it is assumed that F_k^μ and H_k^μ (8) have no sensitivity functions. This leads to simpler calculations. In this section, a comparison will be made between the extended Kalman filter deinterleaver using X_k (case 1) and X_k^μ (case 2).

Tables I and II show the results obtained with different initial conditions using the cases 1 and 2 EKF(μ) deinterleaver. Comparison of these results show that the case 2 EKF(μ) deinterleaver is the better of the two. The results are similar for low RP, but the effectiveness of the case 1 EKF(∞) deinterleaver falls off rapidly as RP increases beyond a threshold. For a 10% uncertainty in the periods, the case 1 EKF(∞) deinterleaver never works as well as the case 2 EKF(μ) deinterleaver. When the periods are known exactly, the threshold is an RP of 9. With any initial conditions for the train periods, there comes a point above the threshold where the case 1 EKF(∞) deinterleaver ceases to work, being unable to deinterleave even a two source pulse train.

The Effect of Initial Conditions: The results shown in Table I were generated with the same initial conditions for both EKF(μ) deinterleavers. It is assumed that the periods of the trains are known to within 10% (e.g., by order $N \log N$ spectral studies as in [5]) and that the first pulse in each train lies within one period of time zero. The initialization for the error covariance reflects this.

$$P_0 = \begin{bmatrix} (\hat{T}_0^{(i)})^2/100 & 0 \\ 0 & (\hat{T}_0^{(i)})^2/10 \end{bmatrix}.$$

Even with these constraints on the initial conditions for period and phase, it is found that both cases 1 and 2 deinterleavers are sensitive to the initial conditions chosen. For this reason, a bank of ten deinterleavers is considered with random phases, and the one leading to the least average prediction error squared is selected. For this deinterleaving method to retain its computational advantage over other methods of order N^2, the number of filters in the bank should be much less than N.

As can be seen from Tables I and II, it becomes increasingly difficult to deinterleave the signals as the number of sources or RP increase. This is because the set of possible initial phases increase rapidly with the number of sources M as well as with RP since phase has been defined as being linked to period. As this set increases, it contains more local minima that the EKF(μ) deinterleavers tend toward but does not lead to the correct deinterleaving of the pulse trains. Therefore, as the number of pulse trains increases, the choice of initial phase becomes more important. However, the EKF(μ) deinterleavers are not only convergent locally in the phase space; therefore, regardless of the number of sources present or the value of RP, the initial estimate for $\hat{t}_0$ does not necessarily need to be close to t_0. Therefore, the assumption that t_0 falls within one period of time zero that is made for choosing $\hat{t}_0$ does not need to be correct for the pulse trains to be deinterleaved.

When no *a priori* knowledge of the pulse train periods is assumed, the set of possible initial conditions that a filter bank must scan becomes much larger. A random generation of periods could be employed; however, the EKF(μ) deinterleavers are effective (convergent) only locally in the period space. Table I therefore shows the performance of both deinterleavers when there is no knowledge of the periods, assuming that there can be up to N randomized deinterleavers used.

B. The Case 2 EKF(μ) Deinterleaver

As the case 2 EKF(μ) deinterleaver is the best of the two presented, we will concentrate on it for the rest of the paper. Figs. 3 and 4 show the successful deinterleaving of pulse trains by this deinterleaver. Fig. 3 is an example of period estimation with a two-source input. It should be noted that the EKF(μ) takes longer to lock onto the pulse train with the highest period. This is due to less pulses from this train being present in the input signal.

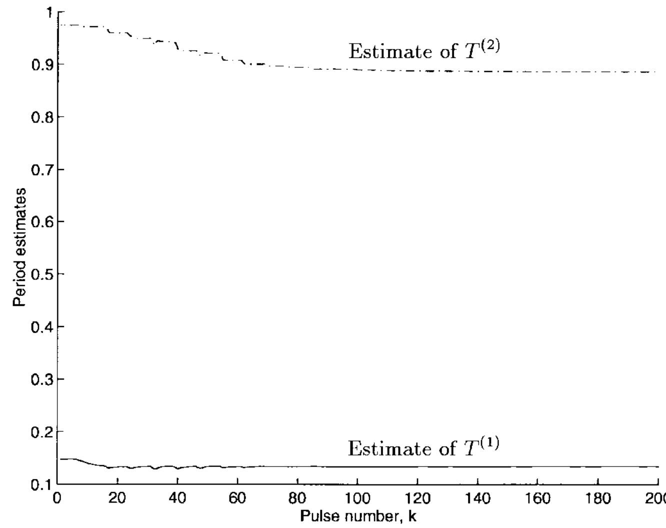
Fig. 4 is an example of period estimation with an eight-source input.

V. Robustness Issues

In this section, only the results from the case 2 EKF(μ) deinterleaver are examined.

A. Jitter Noise

Noise is present in all real-world situations, so for this method of pulse train deinterleaving to be useful, it must be robust to the effects of noise. Such noise is included in our



Parameters: $T_0' = [0.1340, 0.8858], t_0' = [0.0701, 0.1692]$
Initial estimates: $\hat{T}_0' = [0.1474, 0.9744], \hat{t}_0' = [0.0137, 0.1283]$

Fig. 3. Evolution of periods, case 2 EKF(μ) deinterleaver, two source input.

signal model. Table III shows how different levels of noise effect the case 2 deinterleaver with known pulse train periods. The error given is the percentage of pulses assigned to an incorrect train after a lock on the pulse train periods has been achieved. The maximum number of sources successfully deinterleaved is also given in each case.

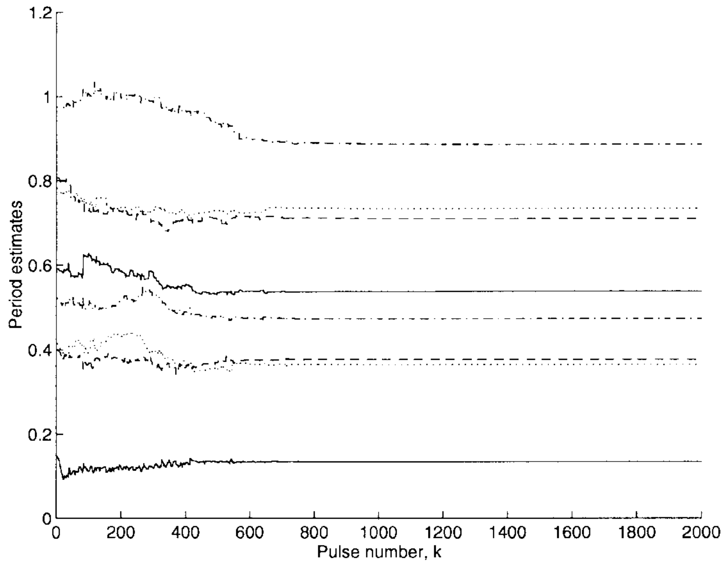
As the noise increases, the number of sources successfully deinterleaved decreases, and the number of errors increases. It should be noted that in a set of pulse trains, the EKF(μ) deinterleaver makes the most errors in the estimate of the train with the lowest period. This is because the pulses in this train come closest together and are therefore more easily disrupted by noise. In addition, when the periods of two trains are similar, noise can cause pulse train skipping; the pulse train estimates swap pulse train sources. This occurs when the periods differ by approximately the noise variance.

B. Missing Pulses

Table IV shows the effect of a percentage of pulses being removed from the input to the EKF(μ) deinterleaver. Of course, such missing pulses were not incorporated into the signal model so there is no *a priori* expectation of such by the EKF(μ). The same trains are used as in Section V-A. Here, there is no noise present on the EKF input, and each pulse in the input is given a 1, 5, or 10% probability of not being present.

The EKF(μ) deinterleaver is not robust to missing pulses. A dropoff in effectiveness is apparent with even 1% of pulses missing. It is also possible for the EKF(μ) deinterleaver to lose its lock on the pulse trains once it has been established. This occurs if there are a lot of pulses in the same region that are missing. With no missing pulses, this phenomenon is only observed under high noise conditions.

To cope with missing pulses, clearly, a modification of the present algorithm, such as in [4], is necessary. There, a comparison of the prediction errors is made, both assuming



$$\text{Parameters: } T_0' = [0.1340, 0.3644, 0.3769, 0.4736, 0.5377, 0.7099, 0.7332, 0.8858],$$
$$t_0' = [0.0706, 0.0159, 0.3198, 0.4201, 0.0677, 0.1255, 0.2343, 0.0773]$$
$$\text{Initial estimates:}$$
$$\hat{T}_0' = [0.1474, 0.4009, 0.4146, 0.5210, 0.5915, 0.7809, 0.8065, 0.9744],$$
$$\hat{t}_0' = [0.1356, 0.3467, 0.1769, 0.1337, 0.1930, 0.3089, 0.1706, 0.7894]$$

Fig. 4. Evolution of periods, case 2 EKF(μ) deinterleaver, eight source input.

TABLE III
EFFECT OF NOISE ON THE CASE 2 EKF(μ) DEINTERLEAVER

Noise σ_ω^2	Number of sources (% error) RP3	RP7
0.001	8 (1.5%)	7 (2.1%)
0.01	8 (9.6%)	7 (12.5%)
0.02	7 (14.6%)	6 (18.2%)

TABLE IV
EFFECT OF MISSING PULSES ON THE CASE 2 EKF(μ) DEINTERLEAVER

Missing pulses	Number of sources RP3	RP7
1%	6	6
5%	2	3
10%	2	2

that there is no missing pulse and assuming there is. Such an approach adapted to the EKF(μ) setting is beyond the scope of this paper.

VI. CONCLUSION

The most important aspect of the method for pulse train deinterleaving presented here is its computational efficiency. The use of an extended Kalman filter allows computations of order N, rather than N^2, which is typical for other deinterleaving methods: N is the number of pulses to be processed. It is advantageous to use information about the pulse train periods that can be obtained using computations of order $N \log N$ [5]. The EKF(μ) deinterleaving method can therefore use a bank of much less than N deinterleavers with different initial conditions and still be more efficient than other methods. This decreases the sensitivity to initial conditions observed in a single EKF(μ) deinterleaver.

The EKF(μ) deinterleaver is robust to noisy pulse time of arrival data but not to missing pulses. It is possible that a modification to this method needing more computational effort could improve its response when missing pulses are present.

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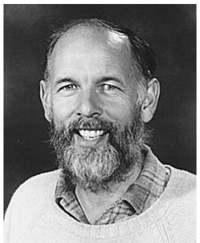

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