

THE LINE SPECTRA OF HELIUM IN GASEOUS NEBULAE

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SUMMARY

The level populations of neutral helium are calculated for conditions typical of gaseous nebulae. Full collisional redistribution of angular momentum and energy is included, and accurate helium atomic data are used throughout. Intensities of the most important singlet and triplet line series are presented for $n \leq 25$. It is suggested that the collisional excitation of 3^3D and 4^3D is overestimated in the calculations of Cox & Daltabuit.

I. INTRODUCTION

The recombination spectrum of helium is observed in many gaseous nebulae. By comparing the observed and theoretical line intensities, the physical characteristics of the emitting regions may be determined. Of particular importance is the determination of the helium abundance. At the present time there is some evidence of a small but important discrepancy between abundances derived by optical and by radio techniques (Churchwell & Mezger 1970; Seaton 1971). The slightly smaller values of $N(\text{He}^+)/N(\text{H}^+)$ obtained by radio observations may be due to the effects of masing (Seaton 1971). To verify this suggestion, accurate helium line intensities in the optical spectrum are required. Furthermore, recent work has established that many photographic line intensities may be in error (Miller 1971; Kaler, Czyzak & Aller 1971). New photoelectric techniques of much greater accuracy are being employed to measure the faint lines from states of high principal quantum number n .

The spectrum of helium differs from that of hydrogen in that transitions between the individual angular momentum states are observed as distinct lines. Calculations of the He I recombination spectrum have been made by Mathis (1957), Seaton (1960), Pottasch (1961), Pengelly (1963), Robbins (1968a, 1970) and Robbins & Robinson (1971).

Seaton (1960) considered only transitions of the type $2p\text{--}nd$. The upper state radial wave functions were assumed to be hydrogenic, and line intensities were calculated using hydrogen data. Mathis (1957) and Pottasch (1961) allowed only for a finite number of atomic levels. An infinite number of levels were included in the calculations of Pengelly (1963), but mainly hydrogen data was used. Robbins (1968a, 1970) and Robbins & Robinson (1971) extended the range of these calculations, but used many approximations to the accurate helium atomic data. No collisional redistribution of energy or angular momentum by electron impact is included (important for $n > 15$ at higher densities) and collisional redistribution of angular momentum by protons and He^+ ions is treated very approximately.

In addition, significantly more accurate radiative and recombination rate coefficients are now available.

The present calculations make extensive use of the level population calculations of hydrogen for highly excited states ($n > 40$) (Brocklehurst 1970, hereinafter referred to as Paper I). Solutions are obtained using methods which are essentially the same as those used for hydrogenic calculations (Brocklehurst 1971, hereinafter referred to as Paper II). Accurate atomic data for helium is used for all states with $l \leq 2$, and full allowance is made for the most important collisional transitions. Calculations of the most important singlet and triplet series line intensities are presented for a wide range of electron temperatures and electron densities. The tabulated values should be accurate to better than 5 per cent for the worst cases.

All triplet calculations are made assuming Case A (all radiation escapes from the nebulae). In the singlet spectrum, it is possible that some nebulae may be optically thick in the Lyman lines,* and we compute intensities for both Case A and Case B. No attempt is made to allow for the metastability of the levels 2^3S and 2^1S . Any collisional and self-absorption effects may be treated as separate problems using the present results. These effects are extensively discussed by Osterbrock (1964), Seaton (1968) and Robbins (1968b, 1970).

The paper concludes with a discussion of the collisional excitation of 3^3D and 4^3D ; the transitions from these levels are most frequently used to derive helium abundances. It is suggested that the collisional effects predicted by Cox & Daltabuit (1971) are considerably overestimated.

2. EQUILIBRIUM EQUATIONS

The equations of statistical equilibrium for a level (n, l) may be written as

$$N_e N_+ \alpha_{ne} + \sum_{n'=n+1}^{\infty} \sum_{l'=l\pm 1} A_{n'l',nl} N_{n'l'} + \sum_{l'=l\pm 1} N_c N_{nl'} C_{nl',nl} + \sum_{n'=n_0}^{\infty} \sum_{l'=l\pm 1} N_e N_{n'l'} C_{n'l',nl} = N_{nl} \left[A_{nl} + \sum_{l'=l\pm 1} N_c C_{nl,nl'} + \sum_{n'=n_0}^{\infty} \sum_{l'=l\pm 1} N_e C_{nl,n'l'} \right], \quad (2.1)$$

where N_e is the electron density, N_+ is the ion density and N_c is the density of electrons, ions or bare nuclei. The radiative recombination coefficient for nl is α_{nl} and the radiative transition probability for $n'l' \rightarrow nl$ is $A_{n'l',nl}$. A_{nl} is the total radiative transition probability for nl and is given by

$$A_{nl} = \sum_{n'=n_0}^{\infty} \sum_{l'=l\pm 1} A_{nl,n'l'}. \quad (2.2)$$

The collision rates $C_{nl',nl}$ for redistribution of angular momentum by electrons, ions or bare nuclei, and $C_{n'l',nl}$ for redistribution of energy by electrons are defined such that

$$C_{i,j} = \int Q_{i \rightarrow j} v f(v) dv \quad (2.3)$$

where $f(v)$ is a Maxwellian velocity distribution for an electron temperature T_e .

* 'Lyman lines' refers to transitions to 1^1S .

If we assume that the nebula is optically thin to radiation produced by transitions to the ground state (Case A), we have $n_0 = 1$ for singlets and $n_0 = 2$ for triplets. If the nebula is optically thick to Lyman line radiation (Case B), $n_0 = 2$ for the singlet series.

The level populations are conveniently described by the usual b factors. Thus, the Saha equation is written

$$N_{nl} = N_e N_+ \left(\frac{h^2}{2\pi m k T_e} \right)^{3/2} \frac{\omega_{nl}}{2\omega_+} \exp(x_{nl}) b_{nl} \quad (2.4)$$

where $x_{nl} = I_{nl}/kT_e$, I_{nl} is the ionization energy of level nl , ω_{nl} is the statistical weight of the state nl and ω_+ is the statistical weight of the recombining ion. Substituting (2.4) in (2.1) and using the detailed balancing relations, we obtain

$$\begin{aligned} & \sum_{n'=n+1}^{\infty} \sum_{l'=l\pm 1} \exp(x_{n'l'}) (2l'+1) A_{n'l',nl} b_{n'l'} - (2l+1) A_{nl} \exp(x_{nl}) b_{nl} \\ & + \exp(x_{nl}) (2l+1) \left(\sum_{n'=n_0}^{\infty} \sum_{l'=l\pm 1} N_e C_{nl,n'l'} b_{n'l'} \right. \\ & \quad \left. - b_{nl} \sum_{n'=n_0}^{\infty} \sum_{l'=l\pm 1} N_e C_{nl,n'l'} \right) \\ & + \exp(x_{nl}) (2l+1) \left(\sum_{l'=l\pm 1} N_e C_{nl,n'l'} b_{nl'} \right. \\ & \quad \left. - b_{nl} \sum_{l'=l\pm 1} N_e C_{nl,n'l'} \right) \\ & = -\alpha_{nl} \left(\frac{h^2}{2\pi m k T_e} \right)^{-3/2} S^{-1} \end{aligned} \quad (2.5)$$

where $S = \frac{3}{4}$ for the triplet series and $S = \frac{1}{4}$ for the singlet series.

3. ATOMIC DATA

(a) Term energies

For $l \leq 2$, the term values T listed by Martin (1960) are used. The levels $l > 2$ are reasonably hydrogenic. In these cases, term energies are evaluated using the polarization formula for hydrogen like terms (Edlén 1964). The correction to the hydrogen term value, T_H , is

$$\Delta T = T - T_H = \alpha_d Z^4 P(n, l) \quad (3.1)$$

where

$$P(n, l) = \frac{R \langle r^{-4} \rangle}{Z^4} = R \frac{3n^2 - l(l+1)}{2n^5 (l - \frac{1}{2}) l (l + \frac{1}{2}) (l+1) (l + \frac{3}{2})} \quad (3.2)$$

R is the Rydberg constant, Z is the net charge of the atomic core and α_d is the dipole polarizability of the atomic core. For hydrogenic ions we have $\alpha_d = 9/2 Z^4$ (Dalgarno 1962). The effective quantum number of the level nl is $\bar{\nu}_{nl} = n - \mu_{nl}$ where μ_{nl} , the quantum defect, is related to ΔT . These corrections are very small for l large. They are, however, of extreme importance in evaluating the cross-sections for redistribution of angular momentum among the states of large l (see Section (3(d))).

(b) *Radiative coefficients* $A_{n'l', nl}$

For transitions between states with $l \geq 3$ the term value corrections ΔT are very small and hydrogen rate coefficients are sufficiently accurate. Their calculation is described fully in Paper II. Many of the radiative coefficients for transitions between states $l \leq 2$ may be obtained from Wiese, Smith & Glennon (1966). When no value is quoted, the radiative coefficient is calculated by means of the Coulomb approximation, as formulated by Bates & Damgaard (1949).

Because of extreme numerical cancellation, the Coulomb approximation is unpracticable for $n' > 20$. For transitions from these levels, scaled hydrogen calculations are used, such that for $n \leq 19$,

$$A_{n'l', nl}(\text{He}) = A_{n'l', nl}(\text{H}) \times \frac{A_{20l', nl}(\text{He})}{A_{20l', nl}(\text{H})}. \quad (3.3)$$

For $n \geq 20$, we use the approximation,

$$A_{n'l', nl}(\text{He}) = A_{n'l', nl}(\text{H}) \times \frac{A_{20l', n-n'+20l}(\text{He})}{A_{20l', n-n'+20l}(\text{H})}. \quad (3.4)$$

These approximations use the fact that, for high series members, the ratios of the helium to hydrogen radiative coefficients become almost constant (but not unity).

(c) *Recombination coefficient* α_{nl}

For $l \geq 3$, the term values are effectively hydrogenic. Hydrogen recombination coefficients with the introduction of the appropriate spin statistical weights are used. Their efficient calculation is described in Paper II. We have

$$\begin{aligned} \alpha(\text{He I}, n^3L) &\approx \frac{3}{4}\alpha(\text{H I}, nl), \quad l \geq 3 \\ \alpha(\text{He I}, n^1L) &\approx \frac{1}{4}\alpha(\text{H I}, nl), \quad l \geq 3. \end{aligned} \quad (3.5)$$

The general formula for atomic photoionization cross-sections, as derived by Burgess & Seaton (1960a) is used for the calculation of all other recombination rates ($l \leq 2$). Peach (1965) has shown that some of the data given is in error, and that the results, as applied to helium (Burgess & Seaton 1960b) are in error for the higher quantum numbers ($n \geq 8$). The necessary data has been recalculated by Peach (1967), who also obtains the following revised general formula for the photoionization cross-section:

$$\begin{aligned} \alpha_{nl}(\epsilon', \bar{\nu}_{nl}) &= \frac{8\alpha a_0^2 \bar{\nu}_{nl}^3}{3Z^2} (1 + \epsilon' \bar{\nu}_{nl}^2)^{-3} \\ &\quad \sum_{l'=l \pm 1} C_{l' l} [G(\bar{\nu}_{nl} l; \epsilon' l') \cos \pi(\bar{\nu}_{nl} + \mu'(\epsilon') + \chi(\bar{\nu}_{nl} l; \epsilon' l'))]^2 \end{aligned} \quad (3.6)$$

where Z is the residual charge on the final ion, $\bar{\nu}_{nl}$ is the effective quantum number of the initial state, $k'^2 = Z^2 \epsilon'$ denotes the energy in Rydbergs of the ejected electron, $\mu'(\epsilon')$ is the extrapolated quantum defect of the final system, and G and χ are tabulated functions. For helium, the coupling coefficient $C_{l' l}$ is given by

$$C_{l' l} = \max(l, l') / (2l + 1). \quad (3.7)$$

The recombination coefficient is obtained on integrating the photoionization

cross-sections over a Maxwellian velocity distribution

$$\alpha_{nl} = \left(\frac{8\alpha^4 a_0^2 c}{3\pi^{1/2}} \right) y^{3/2} \frac{(2l+1)}{\bar{v}_{nl}} SZI(\bar{v}_{nl}, l, t) \quad (3.8)$$

where

$$I(\bar{v}_{nl}, l, t) = \int_0^\infty (1 + \bar{v}_{nl}^2 \epsilon')^{-1} \sum_{l'=\pm 1} C_{l,l'} [G \cos \pi (\bar{v}_{nl} + \mu' + \chi)]^2 \exp(-\epsilon' y) d\epsilon' \quad (3.9)$$

and

$$y = 15.778 \cdot 10^4 Z^2 / T_e. \quad (3.10)$$

The tables of G and χ extend to $n = 12$. Beyond this value we assume that the ratio of the helium to hydrogen recombination coefficients is a constant value. For $8 \leq n \leq 12$, the ratio decreases very slowly with increasing n . Hence we may put

$$\alpha_{nl}(\text{He}) = \alpha_{nl}(\text{H}) \times \frac{\alpha_{12,l}(\text{He})}{\alpha_{12,l}(\text{H})}, \quad l \leq 2. \quad (3.11)$$

(d) Cross-sections and collision rates

The most important cross-sections are of the type $nl \rightarrow nl \pm 1$ induced by electrons, ions and bare nuclei. In the regions emitting the H I spectrum, the electrons will come from the ionization of hydrogen atoms, and the single ionization of helium atoms: $N_e = N(\text{H}^+) + N(\text{He}^+)$. Churchwell & Mezger (1970) obtain helium to hydrogen abundance ratios of 0.1 for many nebulae. Using this value we have $N_c = N_e$ for electron impact, $N_c = 0.9 N_e$ for proton impact, and $N_c = 0.1 N_e$ for He^+ impact.

For all cross-sections we use the impact parameter formulation of Seaton (1962). Transitions involving appreciable transfers of energy ($l \leq 5$) are calculated using the full formula and collision rates are obtained from (2.3). The appropriate oscillator strengths are easily obtained from the radiative coefficients, the calculations of which are described in Section 3(b).

The relatively slow moving protons and He^+ ions are always much more efficient than electrons at redistributing angular momentum when the energy change is small (Paper II). For transitions between states of small l in helium, however appreciable energy transfer is also involved. Consequently it is found that electrons are of dominant importance, the collision rates for transitions $ns \rightarrow np$ being several orders of magnitude larger than the corresponding rates induced by protons. For $n = 20$, $l = 0$ and $N_e = 10^5$, the probability of collisional depopulation by electron impact is equal to the total radiative decay probability A_{nl} . Proton and He^+ impact dominates for $l \geq 3$. When l is not small, the formulation of Pengelly & Seaton (1964) is usually applicable. Collision rates may be evaluated directly provided that

$$\beta = \frac{L\Delta E}{\hbar^2 W} < 0.4 \quad (3.12)$$

where L is the mean angular momentum of the projectile, W is the mean energy of the projectile and ΔE is the transition energy. Condition (3.12) generally holds

for $l \geq 7$, when the transition energy is sufficiently small. Since ΔE is non-zero, we introduce a third upper bound on the impact parameter ρ_c . In the notation of Paper II.

$$2 \log_{10} \rho_c = \log_{10} \left(\frac{T_e m}{(\Delta E)^2 \mu} \right) - 11.22 \quad (3.13)$$

where μ is the reduced mass of the colliding system, m is the electron mass and ΔE is in units of cm^{-1} . In this approximate form, we use hydrogen wave functions throughout. Comparisons of calculations made using the approximate formulation and the full formulation indicate that condition (3.12) leads to maximum errors of 2 per cent in the collision rates.

Collisions of the type $nl \rightarrow n'l \pm 1$ ($n' \neq n$) are of less importance. As shown in Paper II, it is sufficient to include $n \rightarrow n \pm 1$ transitions for the higher levels ($n \geq 15$ for $N_e = 10^5 \text{ cm}^{-3}$). The impact parameter method is used (Seaton 1962).

4. RESULTS

The method of solution of the equilibrium equations (2.5) is similar to that described in Paper II. It is assumed, beyond a value of the principal quantum number n_c , that the level populations are such that

$$N_{nl} = N_n(2l+1)/n^2 \quad (4.1)$$

and we have $b_{nl} = b_n(n > n_c)$. The level populations are then described by the b_n factors for hydrogen, as tabulated in Paper I. For the triplet calculations, we assume that all radiation can escape from the nebula (Case A of Baker & Menzel 1938), and use the hydrogen Case B calculations of b_n ($n_0 = 2$ is the effective ground state). As pointed out by Robbins (1971), it is possible that some nebulae may be optically thick to helium Lyman line radiation. Calculations of the singlet spectrum are made for both Case A (using hydrogen Case A b_n values) and for Case B (using hydrogen Case B b_n values). Absorption of radiation and collisional excitations due to the metastability of the levels 2^3S and 2^1S may be treated as separate problems using the results presented below.

The calculations are most conveniently presented in terms of line intensities. We have the line emissivity $j_{nl, n'l'}$ such that

$$4\pi j_{nl, n'l'} = 0.902 \cdot 10^{-26} \frac{N_e N_+}{T_e^{3/2}} \left(\frac{1}{\bar{v}_{n'l'}^2} - \frac{1}{\bar{v}_{nl}^2} \right) \exp(x_{nl})(2l+1) A_{nl, n'l'} b_{nl} \text{ erg cm}^{-3} \text{ s}^{-1}. \quad (4.2)$$

The effective recombination coefficient $\alpha_{nl \rightarrow n'l'}$ for the line $nl \rightarrow n'l'$ is defined by

$$4\pi j_{nl, n'l'} = h\nu_{nl, n'l'} N_e N_+ \alpha_{nl \rightarrow n'l'} \quad (4.3)$$

where $\nu_{nl, n'l'}$ is the line frequency. It is usual to express the line intensity $I_{nl, n'l'}$ in terms of a reference intensity. We choose the reference lines 2^3P-4^3D ($\lambda 4471 \text{ \AA}$) and 2^1P-4^1D ($\lambda 4922 \text{ \AA}$) for the triplets and singlets respectively. These lines, whilst reasonably strong, are unlikely to be affected by collisional excitation and self absorption.

In Table I we present the relative intensities of the line series 2^3S-n^3P , 2^3P-n^3S and 2^3P-n^3D for $n \leq 25$ and for a wide range of electron temperatures and electron

TABLE I
Triplet line intensities (Case A) relative to $I(2^3P-4^3D)$
(a) 2^3P-n^3D

N_e (cm ⁻³)	$T_e = 2.0 \text{ } 10^4 \text{ } ^\circ\text{K}$				$T_e = 1.0 \text{ } 10^4 \text{ } ^\circ\text{K}$			
	10^7	10^6	10^4	10^2	10^7	10^6	10^4	10^2
3	2.540	2.559	2.578	2.579	2.691	2.726	2.764	2.765
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	0.489	0.488	0.487	0.487	0.478	0.476	0.474	0.474
6	0.276	0.275	0.274	0.274	0.267	0.265	0.264	0.264
7	0.171	0.170	0.170	0.170	0.165	0.163	0.163	0.163
8	0.114	0.112	0.113	0.113	0.110	0.108	0.108	0.107
9	0.0810	0.0787	0.0785	0.0785	0.0784	0.0755	0.0749	0.0749
10	0.0609	0.0573	0.0569	0.0569	0.0593	0.0550	0.0544	0.0543
11	0.0482	0.0432	0.0426	0.0426	0.0476	0.0416	0.0407	0.0407
12	0.0400	0.0336	0.0327	0.0327	0.0402	0.0324	0.0313	0.0313
13	0.0344	0.0269	0.0257	0.0257	0.0353	0.0261	0.0246	0.0246
14	0.0300	0.0222	0.0205	0.0205	0.0316	0.0218	0.0197	0.0197
15	0.0264	0.0189	0.0167	0.0167	0.0284	0.0187	0.0160	0.0160
16	0.0234	0.0165	0.0138	0.0137	0.0259	0.0166	0.0132	0.0132
17	0.0208	0.0146	0.0115	0.0114	0.0237	0.0150	0.0110	0.0110
18	0.0185	0.0132	0.0097	0.0096	0.0217	0.0138	0.0093	0.0093
19	0.0165	0.0120	0.0082	0.0082	0.0199	0.0128	0.0080	0.0079
20	0.0147	0.0110	0.0071	0.0070	0.0182	0.0119	0.0069	0.0068
21	0.0130	0.0100	0.0063	0.0060	0.0165	0.0112	0.0060	0.0058
22	0.0116	0.0092	0.0054	0.0053	0.0149	0.0105	0.0052	0.0051
23	0.0103	0.0085	0.0048	0.0046	0.0134	0.0098	0.0046	0.0045
24	0.0092	0.0078	0.0042	0.0040	0.0120	0.0093	0.0043	0.0039
25	0.0082	0.0071	0.0038	0.0036	0.0108	0.0087	0.0037	0.0035

N_e (cm ⁻³)	$T_e = 5.0 \text{ } 10^3 \text{ } ^\circ\text{K}$				$T_e = 2.5 \text{ } 10^3 \text{ } ^\circ\text{K}$			
	10^7	10^6	10^4	10^2	10^7	10^6	10^4	10^2
3	2.880	2.940	3.010	3.020	3.105	3.199	3.269	3.342
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	0.464	0.461	0.459	0.458	0.449	0.445	0.443	0.441
6	0.256	0.253	0.251	0.251	0.244	0.240	0.239	0.238
7	0.157	0.155	0.154	0.154	0.149	0.146	0.145	0.145
8	0.105	0.102	0.101	0.101	0.100	0.0963	0.0954	0.0951
9	0.0753	0.0716	0.0707	0.0707	0.0720	0.0673	0.0665	0.0662
10	0.0576	0.0523	0.0513	0.0513	0.0556	0.0493	0.0484	0.0481
11	0.0470	0.0396	0.0385	0.0385	0.0461	0.0376	0.0364	0.0362
12	0.0407	0.0311	0.0297	0.0296	0.0409	0.0297	0.0282	0.0279
13	0.0369	0.0253	0.0234	0.0233	0.0384	0.0243	0.0223	0.0220
14	0.0341	0.0213	0.0187	0.0187	0.0369	0.0207	0.0181	0.0177
15	0.0317	0.0186	0.0153	0.0152	0.0356	0.0185	0.0149	0.0144
16	0.0298	0.0169	0.0126	0.0126	0.0345	0.0172	0.0126	0.0120
17	0.0281	0.0157	0.0106	0.0105	0.0334	0.0165	0.0108	0.0100
18	0.0266	0.0149	0.0089	0.0089	0.0327	0.0162	0.0094	0.0085
19	0.0253	0.0142	0.0077	0.0074	0.0323	0.0160	0.0084	0.0072
20	0.0239	0.0136	0.0066	0.0065	0.0318	0.0158	0.0077	0.0062
21	0.0223	0.0131	0.0058	0.0056	0.0308	0.0155	0.0073	0.0054
22	0.0205	0.0125	0.0051	0.0049	0.0291	0.0153	0.0070	0.0047
23	0.0186	0.0121	0.0045	0.0043	0.0271	0.0152	0.0069	0.0041
24	0.0168	0.0117	0.0041	0.0038	0.0248	0.0151	0.0068	0.0037
25	0.0152	0.0113	0.0037	0.0034	0.0226	0.0150	0.0067	0.0032

TABLE I—continued

(b) 2^3S-n^3P

N_e (cm $^{-3}$)	$T_e = 2.0 \text{ } 10^4 \text{ } ^\circ\text{K}$				$T_e = 1.0 \text{ } 10^4 \text{ } ^\circ\text{K}$			
	10^7	10^6	10^4	10^2	10^7	10^6	10^4	10^2
n								
2	5.007	5.004	5.011	5.016	4.415	4.406	4.416	4.421
3	2.809	2.793	2.789	2.788	2.290	2.268	2.263	2.260
4	1.167	1.159	1.157	1.157	0.929	0.920	0.917	0.916
5	0.565	0.561	0.560	0.560	0.447	0.442	0.440	0.440
6	0.314	0.312	0.312	0.311	0.248	0.245	0.245	0.244
7	0.191	0.191	0.190	0.190	0.151	0.150	0.149	0.149
8	0.125	0.125	0.125	0.125	0.0989	0.0986	0.0983	0.0982
9	0.0850	0.0865	0.0864	0.0863	0.0676	0.0683	0.0682	0.0680
10	0.0594	0.0621	0.0622	0.0622	0.0474	0.0491	0.0492	0.0490
11	0.0420	0.0460	0.0463	0.0463	0.0337	0.0364	0.0366	0.0366
12	0.0297	0.0348	0.0354	0.0354	0.0240	0.0276	0.0280	0.0280
13	0.0211	0.0268	0.0277	0.0278	0.0174	0.0214	0.0220	0.0220
14	0.0152	0.0209	0.0221	0.0222	0.0128	0.0167	0.0176	0.0177
15	0.0112	0.0164	0.0179	0.0181	0.0098	0.0131	0.0143	0.0144
16	0.0085	0.0128	0.0147	0.0149	0.0079	0.0103	0.0117	0.0119
17	0.0067	0.0099	0.0121	0.0124	0.0066	0.0081	0.0097	0.0099
18	0.0055	0.0077	0.0100	0.0105	0.0058	0.0064	0.0080	0.0084
19	0.0046	0.0061	0.0083	0.0089	0.0051	0.0051	0.0067	0.0071
20	0.0040	0.0049	0.0070	0.0076	0.0046	0.0042	0.0056	0.0061

N_e (cm $^{-3}$)	$T_e = 5.0 \text{ } 10^3 \text{ } ^\circ\text{K}$				$T_e = 2.5 \text{ } 10^3 \text{ } ^\circ\text{K}$			
	10^7	10^6	10^4	10^2	10^7	10^6	10^4	10^2
2	3.954	3.943	3.961	3.975	3.703	3.692	3.725	3.749
3	1.930	1.902	1.895	1.894	1.684	1.649	1.639	1.645
4	0.763	0.750	0.747	0.748	0.647	0.631	0.627	0.630
5	0.363	0.357	0.355	0.356	0.306	0.298	0.296	0.297
6	0.202	0.198	0.197	0.197	0.169	0.165	0.164	0.164
7	0.123	0.121	0.120	0.121	0.104	0.101	0.100	0.101
8	0.0808	0.0798	0.0794	0.0795	0.0683	0.0667	0.0662	0.0664
9	0.0555	0.0554	0.0551	0.0552	0.0471	0.0464	0.0460	0.0462
10	0.0391	0.0399	0.0398	0.0398	0.0335	0.0335	0.0333	0.0334
11	0.0281	0.0297	0.0297	0.0297	0.0243	0.0250	0.0249	0.0250
12	0.0203	0.0226	0.0228	0.0228	0.0179	0.0191	0.0191	0.0191
13	0.0151	0.0176	0.0180	0.0180	0.0137	0.0150	0.0151	0.0152
14	0.0115	0.0138	0.0144	0.0145	0.0109	0.0118	0.0122	0.0122
15	0.0093	0.0110	0.0118	0.0118	0.0093	0.0096	0.0101	0.0101
16	0.0079	0.0087	0.0097	0.0098	0.0083	0.0077	0.0083	0.0084
17	0.0071	0.0070	0.0080	0.0082	0.0078	0.0063	0.0070	0.0070
18	0.0065	0.0056	0.0067	0.0069	0.0076	0.0052	0.0058	0.0060
19	0.0062	0.0047	0.0056	0.0059	0.0076	0.0045	0.0049	0.0051
20	0.0058	0.0040	0.0047	0.0051	0.0076	0.0040	0.0042	0.0044

densities. Note that collisional transitions produce negligible effects for $N_e < 10^4$ cm $^{-3}$ and $n < 20$. The intensities of series terminating on levels 3^3S and 3^3P are easily evaluated from these tables. Singlet line intensities for Cases A and B are presented in Tables II and III respectively. Since singlet line intensities are weaker than triplets by factors of at least three, the results are only presented for $n \leq 15$ and for $N_e \geq 10^4$ cm $^{-3}$.

The effective recombination coefficients of the reference lines are presented in Tables IV, V and VI for the same ranges of physical parameters. From these tables, and knowing the ion abundance N_+ , absolute line emissivities may be

TABLE II
Singlet line intensities (Case A) relative to $I(2^1P-4^1D)$

N_e (cm^{-3})	$T_e = 2.0 \times 10^4 \text{K}$			$T_e = 1.0 \times 10^4 \text{K}$			$T_e = 5.0 \times 10^3 \text{K}$		
	10^7	10^6	10^4	10^7	10^6	10^4	10^7	10^6	10^4
2	111.3	110.8	110.9	96.61	96.22	96.51	88.14	87.93	88.57
3	23.92	23.69	23.65	20.26	19.96	19.89	17.59	17.21	17.13
4	9.749	9.645	9.628	8.120	7.948	7.948	6.901	6.723	6.684
5	4.950	4.897	4.889	4.113	4.038	4.023	3.481	3.385	3.364
6	2.862	2.830	2.826	2.284	2.338	2.329	2.021	1.963	1.950
7	1.817	1.794	1.792	1.520	1.487	1.481	1.293	1.252	1.243
8	1.226	1.206	1.204	1.030	1.004	0.999	0.881	0.848	0.841
9	0.871	0.849	0.847	0.736	0.709	0.704	0.633	0.601	0.595
10	0.651	0.622	0.619	0.554	0.521	0.516	0.480	0.444	0.437
11	0.508	0.469	0.465	0.437	0.394	0.388	0.383	0.337	0.330
12	0.427	0.374	0.368	0.377	0.318	0.311	0.341	0.276	0.268
13	0.368	0.298	0.291	0.333	0.255	0.246	0.310	0.223	0.213
14	0.334	0.244	0.233	0.314	0.210	0.198	0.308	0.185	0.172
15	0.316	0.205	0.190	0.311	0.178	0.162	0.322	0.158	0.141
(a) 1^1S-n^3P									
(b) 2^1P-n^1S									
3	0.4341	0.4273	0.4261	0.2861	0.2797	0.2790	0.2039	0.1975	0.1973
4	0.1596	0.1571	0.1567	0.1054	0.1031	0.1028	0.0747	0.0724	0.0724
5	0.0749	0.0738	0.0736	0.0496	0.0485	0.0484	0.0352	0.0341	0.0341
6	0.0413	0.0407	0.0406	0.0275	0.0269	0.0268	0.0196	0.0190	0.0190
7	0.0252	0.0249	0.0248	0.0169	0.0165	0.0165	0.0121	0.0117	0.0117
8	0.0165	0.0163	0.0163	0.0111	0.0109	0.0109	0.0080	0.0078	0.0078
9	0.0113	0.0113	0.0113	0.0076	0.0076	0.0075	0.0055	0.0054	0.0054
10	0.0079	0.0080	0.0081	0.0054	0.0055	0.0055	0.0039	0.0039	0.0039
11	0.0057	0.0060	0.0061	0.0039	0.0041	0.0041	0.0028	0.0029	0.0029
12	0.0041	0.0046	0.0046	0.0028	0.0031	0.0031	0.0021	0.0022	0.0022
13	0.0029	0.0035	0.0036	0.0020	0.0024	0.0025	0.0015	0.0017	0.0018
14	0.0020	0.0028	0.0029	0.0014	0.0019	0.0020	0.0011	0.0014	0.0014
15	0.0014	0.0022	0.0024	0.0010	0.0015	0.0016	0.0008	0.0011	0.0012

TABLE II—continued

N_e (cm^{-3})	$T_e = 2.0 \times 10^4 \text{K}$		$T_e = 1.0 \times 10^4 \text{K}$		$T_e = 5.0 \times 10^3 \text{K}$			10^4
	10^7	10^6	10^7	10^6	10^7	10^6	10^4	
n				(c) 2^1S-n^1P				
2	0.0034	0.0034	0.0030	0.0030	0.0027	0.0027	0.0027	0.0027
3	0.0591	0.0584	0.0500	0.0493	0.0434	0.0425	0.0425	0.0423
4	0.0354	0.0350	0.0295	0.0290	0.0251	0.0244	0.0244	0.0243
5	0.0205	0.0202	0.0170	0.0167	0.0144	0.0140	0.0140	0.0139
6	0.0131	0.0129	0.0109	0.0107	0.0092	0.0090	0.0090	0.0089
7	0.0077	0.0076	0.0064	0.0063	0.0055	0.0053	0.0053	0.0053
8	0.0053	0.0052	0.0044	0.0043	0.0038	0.0037	0.0037	0.0036
9	0.0039	0.0038	0.0033	0.0032	0.0028	0.0027	0.0027	0.0027
10	0.0028	0.0027	0.0024	0.0022	0.0021	0.0019	0.0019	0.0019
11	0.0023	0.0021	0.0020	0.0018	0.0017	0.0015	0.0015	0.0015
12	0.0019	0.0016	0.0017	0.0014	0.0015	0.0012	0.0012	0.0012
13	0.0016	0.0013	0.0015	0.0011	0.0014	0.0010	0.0010	0.0009
14	0.0015	0.0011	0.0014	0.0009	0.0014	0.0008	0.0008	0.0008
15	0.0014	0.0009	0.0014	0.0008	0.0014	0.0007	0.0007	0.0006
3	2.649	2.688	2.800	2.861	2.992	3.084	3.160	3.160
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	0.4837	0.4813	0.4721	0.4686	0.4581	0.4536	0.4507	0.4507
6	0.2721	0.2701	0.2626	0.2597	0.2514	0.2478	0.2454	0.2454
7	0.1688	0.1674	0.1622	0.1600	0.1545	0.1514	0.1499	0.1499
8	0.1123	0.1112	0.1079	0.1061	0.1028	0.1001	0.0989	0.0989
9	0.0786	0.0778	0.0757	0.0741	0.0726	0.0699	0.0690	0.0690
10	0.0573	0.0566	0.0557	0.0540	0.0540	0.0510	0.0502	0.0502
11	0.0433	0.0425	0.0428	0.0406	0.0424	0.0385	0.0377	0.0377
12	0.0336	0.0328	0.0339	0.0314	0.0346	0.0300	0.0289	0.0289
13	0.0264	0.0258	0.0272	0.0248	0.0286	0.0238	0.0229	0.0229
14	0.0213	0.0207	0.0226	0.0202	0.0245	0.0196	0.0184	0.0184
15	0.0173	0.0170	0.0188	0.0168	0.0210	0.0167	0.0150	0.0150
				(d) 2^1P-n^1D				
				2.713	2.905	2.992	3.084	3.160
				1.000	1.000	1.000	1.000	1.000
				0.4798	0.4665	0.4581	0.4536	0.4507
				0.2689	0.2581	0.2514	0.2478	0.2454
				0.1665	0.1588	0.1545	0.1514	0.1499
				0.1106	0.1052	0.1028	0.1001	0.0989
				0.0773	0.0734	0.0726	0.0699	0.0690
				0.0562	0.0534	0.0540	0.0510	0.0502
				0.0422	0.0401	0.0424	0.0385	0.0377
				0.0325	0.0309	0.0346	0.0300	0.0289
				0.0255	0.0243	0.0286	0.0238	0.0229
				0.0205	0.0195	0.0245	0.0196	0.0184
				0.0167	0.0159	0.0210	0.0167	0.0150

TABLE III—continued

n	$T_e = 2.0 \times 10^4 \text{K}$		$T_e = 1.0 \times 10^4 \text{K}$		$T_e = 5.0 \times 10^3 \text{K}$	
	10^7	10^6	10^7	10^6	10^7	10^6
3	2.653	2.678	2.803	2.851	2.995	3.074
4	1.000	1.000	1.000	1.000	1.000	1.000
5	0.4833	0.4819	0.4719	0.4693	0.4583	0.4544
6	0.2716	0.2702	0.2623	0.2601	0.2515	0.2481
7	0.1687	0.1673	0.1623	0.1601	0.1548	0.1517
8	0.1129	0.1111	0.1086	0.1060	0.1037	0.1002
9	0.0802	0.0776	0.0774	0.0741	0.0743	0.0700
10	0.0602	0.0566	0.0586	0.0541	0.0568	0.0512
11	0.0475	0.0427	0.0468	0.0409	0.0463	0.0389
12	0.0391	0.0332	0.0392	0.0320	0.0397	0.0306
13	0.0326	0.0265	0.0333	0.0257	0.0345	0.0247
14	0.0279	0.0219	0.0290	0.0214	0.0311	0.0208
15	0.0239	0.0185	0.0255	0.0183	0.0281	0.0182

(c) 2^1P-n^1D

n	10^7	10^6	10^4
3	2.891	2.891	3.146
4	1.000	1.000	1.000
5	0.4677	0.4677	0.4519
6	0.2588	0.2588	0.2463
7	0.1592	0.1592	0.1504
8	0.1053	0.1053	0.0992
9	0.0734	0.0734	0.0691
10	0.0533	0.0533	0.0502
11	0.0400	0.0400	0.0377
12	0.0308	0.0308	0.0290
13	0.0241	0.0241	0.0228
14	0.0194	0.0194	0.0183
15	0.0158	0.0158	0.0149

TABLE IV

Effective recombination coefficients $\alpha_{4^3D \rightarrow 2^3P}$ (Case A)

(Units of $10^{-14} \text{ cm}^3 \text{ s}^{-1}$)				
$N_e \text{ (cm}^{-3}\text{)}$	$T_e \text{ (K)} = 2.0 \cdot 10^4$	$1.0 \cdot 10^4$	$5.0 \cdot 10^3$	$2.5 \cdot 10^3$
10^2	0.6617	1.363	2.607	4.693
10^4	0.6633	1.367	2.638	4.825
10^5	0.6647	1.374	2.667	4.927
10^6	0.6670	1.385	2.713	5.089
10^7	0.6689	1.395	2.759	5.277

TABLE V

Effective recombination coefficients $\alpha_{4^1D \rightarrow 2^1P}$ (Case A)

(Units of $10^{-14} \text{ cm}^3 \text{ s}^{-1}$)			
$N_e \text{ (cm}^{-3}\text{)}$	$T_e \text{ (K)} = 2.0 \cdot 10^4$	$1.0 \cdot 10^4$	$5.0 \cdot 10^3$
10^4	0.1919	0.4014	0.7819
10^5	0.1921	0.4028	0.7891
10^6	0.1923	0.4048	0.7997
10^7	0.1902	0.3993	0.7879

TABLE VI

Effective recombination coefficients $\alpha_{4^1D \rightarrow 2^1P}$ (Case B)

(Units of $10^{-14} \text{ cm}^3 \text{ s}^{-1}$)			
$N_e \text{ (cm}^{-3}\text{)}$	$T_e \text{ (K)} = 2.0 \cdot 10^4$	$1.0 \cdot 10^4$	$5.0 \cdot 10^3$
10^4	0.1976	0.4117	0.7998
10^5	0.1981	0.4137	0.8087
10^6	0.1989	0.4173	0.8233
10^7	0.1990	0.4173	0.8251

obtained using the tables of Burgess & Seaton (1960b). Robbins (1968a) assumed for several series ($2S-nP$, $3S-nP$), that the helium and hydrogen radiative coefficients, $A_{nl, n'l'}$, converged to the same values, and used hydrogen coefficients when no values were tabulated by Gabriel & Heddle (1960). The effects of these approximations are to underestimate the rates of radiative depopulation of S and D states, and to overestimate the rate of depopulation of P states.

5. COLLISIONAL EXCITATION OF 3^3D AND 4^3D

Transitions of the type 2^3P-n^3D have frequently been used to deduce helium abundances. The level populations of n^3D are reasonably approximated by hydrogen calculations with the introduction of appropriate statistical weight factors, and it is expected that the line intensities are not affected significantly by self-absorption and collisional excitation from the metastable 2^3S level. Comparing the present results with Seaton (1960) we see that the first assumption is reasonable. Pottasch (1961) demonstrates that self-absorption is small for $n = 3$ and $n = 4$.

Cox & Daltabuit (1971) have suggested that collisional excitation from 2^3S may increase the intensity of the lines 2^3P-3^3D and 2^3P-4^3D by significant amounts. This would lead to considerable overestimates of the helium abundances deduced

from observations of these lines. The suggestion has been questioned by Peimbert & Torres-Peimbert (1971b) on the basis of extensive observations of these lines in many planetary nebulae.

Cross-sections for $2^3S \rightarrow 3^3D$ and $2^3S \rightarrow 4^3D$ have been estimated by Ochkur & Bratsev (1966) using a modification of the Born–Oppenheimer approximation. The extrapolated fit to these calculations (Cox & Daltabuit 1971) shows, at impact energies near threshold, that the cross-sections for $2^3S \rightarrow 3^3D$ are several times larger than for $2^3S \rightarrow 3^3P$. As a result, the $2^3S \rightarrow 3^3D$ collision rate is not negligible.

The equivalent cross-sections in hydrogen have been evaluated by Burke, Ormonde & Whittaker (1967) using a 6-state close-coupling approximation. Although the two atoms are not identical, we compare the collision strengths, Ω , where

$$Q(i \rightarrow j) = \frac{\Omega(i, j)}{\omega_i k_i^2} \pi a_0^2 \quad (5.1)$$

and k_i^2 is numerically equal to the energy of the incident electron in Rydbergs. The statistical weight of the initial level is ω_i . At energies about 0.01 Ryd. above threshold, we have, for $2^3S \rightarrow 3^3D$ excitation of helium,

$$\Omega_{\text{He}}(k_i^2 = 0.25)/\omega_i = 0.88$$

and for $2s-3d$ excitation of hydrogen,

$$\Omega_{\text{H}}(k_i^2 = 0.15)/\omega_i = 0.67.$$

This is unexpected since the $2s-2p$ coupling in hydrogen is much greater than the 2^3S-2^3P coupling in helium where there is a finite energy difference.

There is strong evidence that $2s-2p$ coupling plays a dominant role in the transition $2s \rightarrow 3d$ in hydrogen. Mandelberg (1969), allowing only for dipole interactions, obtains values of $Q(2l \rightarrow 3l')/Q(2p-3d)$ which are almost identical to those obtained in the close coupling calculations of Burke *et al.* (1967). The calculations of Ochkur & Bratsev (1966) entirely neglect 2^3S-2^3P coupling. It is therefore to be expected for helium, that the probability of the excitation $2^3S \rightarrow 3^3D$ will be reduced compared to that of $2^3S \rightarrow 3^3P$. We define a quantity $R = \Omega(2S-3D)/\Omega(2S-3P)$. For helium the results of Ochkur & Bratsev give $R \sim 3.6$, at an electron impact energy of 4 eV. At the equivalent energy above threshold, we obtain $R \sim 1$ for hydrogen. While noting that 3^3P-3^3D coupling is also reduced in helium, it would appear that the Ochkur approximation overestimates the cross-sections for $2^3S \rightarrow 3^3D$ by at least a factor of three.

Comparison of the 2^3P-3^3D (λ 5876 Å) and 2^3P-4^3D (λ 4471 Å) calculated and observed line intensities also indicates that collisional excitation may be overestimated. We use the observations of Peimbert & Torres-Peimbert (1971a) and compare observed and calculated values of $I(5876)/I(4471)$. This procedure has the advantage of reducing observational errors to a minimum, and of not requiring an accurate knowledge of the level population of 2^3S . The observed ratios are corrected for reddening, and the calculated ratios are corrected for collisional excitation as described by Cox & Daltabuit (1971). The minimum deduced electron temperatures and densities (Peimbert 1971) are used to obtain the minimum correction for collisional excitation from 2^3S . It is found, for all nebulae listed, except IC 5217, that the calculated ratios with the minimum correction and neglecting

self-absorption, exceed the observed ratios. We conclude that the effects of collisional excitation are overestimated by Cox & Daltabuit (1971).

6. CONCLUSIONS

Helium recombination spectra have been calculated for a wide range of nebular electron densities and temperatures. Collisional redistributions of angular momentum and energy have been allowed for explicitly, and accurate helium radiative and recombination rates used throughout. The line intensities of the principal triplet and singlet series have been tabulated. A maximum error of 5 per cent in the intensities have a different density dependence from those calculated by Robbins (1968a, 1970) and Robbins & Robinson (1971) and differ considerably in magnitude for the higher lines of the various series. These variations are attributed to a more accurate method of solution for higher densities, and improved atomic data. Finally it is suggested that the collisional excitation of 3^3D and 4^3D is overestimated by Cox & Daltabuit (1971) in agreement with the conclusions of Peimbert & Torres-Peimbert (1971b).

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