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THE LINEAR QUADRATIC OPTIMIZATION PROBLEM FOR A CLASS OF SINGULARLY PERTURBED STOCHASTIC SYSTEMS

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Abstract. *In this note the asymptotic structure of the stabilizing solution of Riccati equation and the asymptotic structure of the optimal feedback gain for a linear quadratic optimization problem associated to a singularly perturbed stochastic linear system were investigated. To obtain the dominant part of the stabilizing solution of algebraic Riccati equation and the dominant part of the optimal feedback gain, the solutions of two linear quadratic optimization problems of lower dimension and independent of the small parameter ε are involved.*

Keywords: Linear quadratic optimization problem, Singular perturbations, Linear stochastic systems, Algebraic Riccati equations, Asymptotic structure

1. **Introduction.** The linear quadratic optimization problem for linear stochastic systems both in finite time horizon and infinite time horizon was investigated starting with Wonham's work [21]. Among the main contribution in this area we mention [2, 6, 13, 14, 19, 20] and the references therein.

In [1, 17, 18] the linear quadratic optimization problem for linear stochastic systems and quadratic cost functional without definite sign was considered.

It is known that in many cases the mathematical model of some physical phenomena is described by deterministic or stochastic systems of differential equations containing one or more small time constants. Such systems of differential equations are known as singularly perturbed systems of differential equations.

Rich lists of works dealing with applications of singular perturbations techniques in various control problems may be found in [8, 15, 16].

It is known that the presence in the mathematical model of the small time constants may produce ill conditioning of numerical computations. In the case of linear quadratic optimization problem associated to a singularly perturbed linear system such ill conditioning occurs in the numerical computation of the stabilizing solution of the corresponding Riccati equation.

Based on some techniques of singular perturbation theory, the difficulties due to the presence of the small parameter, may be avoided replacing the original problem by two reduced problems which are independent of the small parameter ε. In the case of linear quadratic optimization problem for deterministic singularly perturbed linear systems there are a lot of works where the asymptotic behavior of stabilizing solution of corresponding