The Lorentz Transformation at the Maximum Velocity for a Mass

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Abstract

Haug [1, 2] has recently shown there is a speed limit for fundamental particles just below the speed of light given by $v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}$. This speed limit means that the mass of a fundamental particle not will go towards infinity as v approaches c in the Einstein relativistic mass equation. The relativistic mass limit for a fundamental particle is the Planck mass. In this paper we use the same velocity limit in the Lorentz transformation. This leads to what we think could be significant results with some interesting interpretations. In addition we look at rapidity as well as relativity of simultaneity for subatomic particles at this maximum velocity for masses.

Key words: Lorentz transformation, maximum velocity, fundamental particles, Planck length, Planck mass, rapidity, relativity of simultaneity, reduced Compton wavelength.

1 The Lorentz Transformation at the Maximum Velocity for Fundamental Particles

The Lorentz [3, 4] transformation is given by the following equation; see also [5], [6] and [7].

$$\hat{x} = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}},}, \qquad \hat{y} = y \qquad \hat{z} = z, \qquad \hat{t} = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Here we will look at the limit of the Lorentz transformation for fundamental particles based on the maximum velocity recently given by Haug. The length transformation for the reduced Compton wavelength must be

$$\hat{x} = \frac{\bar{\lambda} - tv}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

In the case where a fundamental particle moves at the maximum velocity v_{max} at which it can travel before it bursts into energy we get:

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$$
\begin{aligned}
\hat{x} &= \frac{\bar{\lambda} - tv_{max}}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} \\
\hat{x} &= \frac{\bar{\lambda} - \frac{\bar{\lambda}}{c} c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{\sqrt{1 - \frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} \\
\hat{x} &= \frac{\bar{\lambda} - \frac{\bar{\lambda}}{c} c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{\frac{l_p}{\bar{\lambda}_e}} \\
\hat{x} &= \frac{\bar{\lambda} - \bar{\lambda}\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{\frac{l_p}{\bar{\lambda}}} \\
\hat{x} &= \frac{\bar{\lambda}^2 - \bar{\lambda}^2\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{l_p} \qquad (2)
\end{aligned}
$$

We can estimate $\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}$ by using a series expansion, here given by the first three terms:

$$
\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \approx 1 - \frac{l_p^2}{\bar{\lambda}^2}\frac{1}{2} - \frac{l_p^4}{\bar{\lambda}^4}\frac{1}{8} - \frac{l_p^6}{\bar{\lambda}^6}\frac{1}{16}... \qquad (3)
$$

It can be shown that the first term of the series expansion gives a very accurate result and that further terms are negligible as long as $\frac{l_p^2}{\bar{\lambda}^2} << 1$. This is the case for all observed subatomic particles, including even the Higgs mass, but not for the Planck mass, since the reduced Compton wavelength of the Planck mass is l_p. Replacing the series expansion into equation 2 we get

$$
\begin{aligned}
\hat{x} &\approx \frac{\bar{\lambda}^2 - \bar{\lambda}^2\left(1 - \frac{l_p^2}{\bar{\lambda}^2}\frac{1}{2}\right)}{l_p} \\
\hat{x} &\approx \frac{\bar{\lambda}^2 - \bar{\lambda}^2 + l_p^2\frac{1}{2}}{l_p} \\
\hat{x} &\approx \frac{1}{2}l_p \qquad (4)
\end{aligned}
$$

And the inverse Lorentz length transformation is given by

$$
\begin{aligned}
\hat{x} &= \frac{\bar{\lambda} + tv_{max}}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} \\
\hat{x} &= \frac{\bar{\lambda} + \frac{\bar{\lambda}}{c} c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{\sqrt{1 - \frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} \\
\hat{x} &= \frac{\bar{\lambda} + \bar{\lambda}\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{\frac{l_p}{\bar{\lambda}_e}} \\
\hat{x} &= \frac{\bar{\lambda}^2 + \bar{\lambda}^2\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{l_p} \qquad (5)
\end{aligned}
$$

again using a series expansion we get

$$\begin{aligned}
\hat{x} &\approx \frac{\bar{\lambda}^2 + \bar{\lambda}^2\left(1 - \frac{l_p^2}{\bar{\lambda}^2}\frac{1}{2}\right)}{l_p} \\
\hat{x} &\approx \frac{\bar{\lambda}^2 + \bar{\lambda}^2 - l_p^2\frac{1}{2}}{l_p} \\
\hat{x} &\approx 2\frac{\bar{\lambda}^2}{l_p} - \frac{1}{2}l_p \qquad (6)
\end{aligned}$$

Very interesting is the special case of $\bar{\lambda} = l_p$; in this case both equation 2 and 5 will return l_p. That is a Planck length object has the same length no matter what frame it is observed from. This makes the Planck length very unique. Based on Haug's atomism the Planck length is likely the diameter of the indivisible particle. We can only observe an indivisible particle at the moment of counter-strike with another indivisible particle. This again is the Planck mass that is the same as observed from any reference frame; see [1].

The round trip distance we get from the Lorentz transformation is

$$\frac{\bar{\lambda} - tv_{max}}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} + \frac{\bar{\lambda} + tv_{max}}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} = \frac{1}{2}l_p + 2\frac{\bar{\lambda}^2}{l_p} - \frac{1}{2}l_p = 2\frac{\bar{\lambda}^2}{l_p} \qquad (7)$$

In the special case of $\bar{\lambda} = l_p$, we get $2l_p$. Bear in mind that under atomism we assert that twice the Planck length is the length of a Planck mass, which consists of two indivisible particles with diameter l_p laying next to each other in the instant of counter-strike.

Lorentz time transformation

Similarly, for the Lorentz time transformation we get:

$$\begin{aligned}
\hat{t} &= \frac{t - \frac{\bar{\lambda}v_{max}}{c^2}}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} \\
\hat{t} &= \frac{\frac{\bar{\lambda}}{c} - \frac{\bar{\lambda}c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{c^2}}{\sqrt{1 - \frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} \\
\hat{t} &= \frac{\frac{\bar{\lambda}}{c} - \frac{\bar{\lambda}\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{c}}{\frac{l_p}{\bar{\lambda}}} \qquad (8)
\end{aligned}$$

Again, we use the series expansion for $\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \approx 1 - \frac{l_p^2}{\bar{\lambda}^2}\frac{1}{2}$ and get

$$\begin{aligned}
\hat{t} &\approx \frac{\frac{\bar{\lambda}^2}{c} - \frac{\bar{\lambda}^2\left(1-\frac{l_p^2}{\bar{\lambda}^2}\frac{1}{2}\right)}{c}}{l_p} \\
\hat{t} &\approx \frac{\frac{\bar{\lambda}^2}{c} - \frac{\bar{\lambda}^2}{c} + \frac{l_p^2}{c}\frac{1}{2}}{l_p} \\
\hat{t} &\approx \frac{1}{2}\frac{l_p}{c} \qquad (9)
\end{aligned}$$

For a signal going in the opposite direction we get

$$
\begin{aligned}
\hat{t} &= \frac{t + \frac{\bar{\lambda} v_{max}}{c^2}}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} \\
\hat{t} &= \frac{\frac{\bar{\lambda}}{c} + \frac{\bar{\lambda} c \sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{c^2}}{\sqrt{1 - \frac{\left(c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} \\
\hat{t} &= \frac{\frac{\bar{\lambda}}{c} + \frac{\bar{\lambda}\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{c}}{\frac{l_p}{\bar{\lambda}}} \qquad (10)
\end{aligned}
$$

and again we use a series expansion approximation:

$$
\begin{aligned}
\hat{t} &\approx \frac{\frac{\bar{\lambda}^2}{c} + \frac{\bar{\lambda}^2\left(1 - \frac{l_p^2}{\bar{\lambda}^2}\frac{1}{2}\right)}{c}}{l_p} \\
\hat{t} &\approx \frac{\frac{\bar{\lambda}^2}{c} + \frac{\bar{\lambda}^2}{c} - \frac{l_p^2}{c}\frac{1}{2}}{l_p} \\
\hat{t} &\approx 2\frac{\bar{\lambda}^2}{l_p c} - \frac{1}{2}\frac{l_p}{c} \qquad (11)
\end{aligned}
$$

In the special case of $\bar{\lambda} = l_p$ we have that both equation 8 and 11 simply return a Planck second: that is $\frac{l_p}{c}$. It is interesting to note that the transformation in this special case does not change the value from its input. The Planck second seems to be invariant.

Further, the round trip time is given by

$$
\frac{t - \frac{\bar{\lambda} v_{max}}{c^2}}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} + \frac{t + \frac{\bar{\lambda} v_{max}}{c^2}}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} = \frac{1}{2}\frac{l_p}{c} + 2\frac{\bar{\lambda}^2}{l_p c} - \frac{1}{2}\frac{l_p}{c} = 2\frac{\bar{\lambda}^2}{l_p c} \qquad (12)
$$

In the Appendix we also show how this is linked to the invariant Minkowski space-time interval.

2 Rapidity

In 1908, Minkowski [8] showed how the Lorentz transformation could be seen as a hyperbolic rotation of the space-time coordinates. This leads to the development of rapidity, also known as the hyperbolic parameter; see [9, 10, 11].

Rapidity is given by

$$
w = \operatorname{artanh}\left(\frac{v}{c}\right) \qquad (13)
$$

where artanh is the inverse hyperbolic tangent function. Further, we have

$$
\cosh(w) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (14)
$$

and

$$
\sinh(w) = \frac{\frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (15)
$$

Based on this, the Lorentz transformation can be written as

$$
\hat{x} = \cosh(w)x - \sinh(w)ct = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (16)
$$

and

$$\hat{t} = \cosh(w)t - \sinh(w)\frac{x}{c} = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{17}$$

Further, we must have

$$\frac{v}{c} = \tanh(w) \tag{18}$$

For small values of v we have, $w \approx \frac{v}{c}$, but for large values of v this is no longer true. In addition, we have the following link between rapidity and Einstein's relativistic Doppler shift

$$e^w = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} \tag{19}$$

and

$$e^{-w} = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \tag{20}$$

All this is well known, but here we will look at equations linked to rapidity when using the maximum velocity for a mass (fundamental particles) v_{max}. This leads to

$$w = \text{artanh}\left(\frac{v_{max}}{c}\right) = \text{artanh}\left(\frac{c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{c}\right) = \text{artanh}\left(\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}\right) \tag{21}$$

Again we use a series expansion approximation, $\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \approx 1 - \frac{l_p^2}{\bar{\lambda}^2}\frac{1}{2}$, and this gives

$$w \approx \text{artanh}\left(1 - \frac{l_p^2}{\bar{\lambda}^2}\frac{1}{2}\right) \tag{22}$$

Further, we have the following relationships

$$e^w = \sqrt{\frac{1 + \frac{v_{max}}{c}}{1 - \frac{v_{max}}{c}}} \approx \sqrt{\frac{2 - \frac{lp^2}{\bar{\lambda}^2}\frac{1}{2}}{\frac{lp^2}{\bar{\lambda}^2}\frac{1}{2}}} = \sqrt{\frac{4 - \frac{lp^2}{\bar{\lambda}^2}}{\frac{lp^2}{\bar{\lambda}^2}}} \approx 2\frac{\bar{\lambda}}{l_p} \tag{23}$$

and

$$e^{-w} = \sqrt{\frac{1 - \frac{v_{max}}{c}}{1 + \frac{v_{max}}{c}}} \approx \sqrt{\frac{\frac{lp^2}{\bar{\lambda}^2}\frac{1}{2}}{2 - \frac{lp^2}{\bar{\lambda}^2}\frac{1}{2}}} = \sqrt{\frac{\frac{lp^2}{\bar{\lambda}^2}}{4 - \frac{lp^2}{\bar{\lambda}^2}}} \approx \frac{l_p}{\bar{\lambda}}\frac{1}{2} \tag{24}$$

This also means, see [1], that we must have

$$e^w \approx 2\frac{\bar{\lambda}}{l_p} = \frac{2}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} \tag{25}$$

and

$$e^{-w} \approx \frac{l_p}{\bar{\lambda}}\frac{1}{2} = \frac{1}{2}\sqrt{1 - \frac{v_{max}^2}{c^2}} \tag{26}$$

We can, for example, calculate the maximum rapidity for an electron. We are using $\bar{\lambda} = \bar{\lambda}_e \approx 3.861593 \times 10^{-13}$ and Planck length of $l_p \approx 1.61619910^{-35}$, this gives

$$w_{max} = \text{artanh}\left(\frac{v_{max}}{c}\right) = \ln\left(\sqrt{\frac{1 + \frac{v_{max}}{c}}{1 - \frac{v_{max}}{c}}}\right) \approx \ln\left(2\frac{\bar{\lambda}_e}{l_p}\right) \approx 52.22102184 \tag{27}$$

This means we get a rapidity limit for an electron equal to $-52.22102184 > v < 52.22102184$. We can also use this to find the maximum velocity of an electron:

$$\frac{v_{max}}{c} = \tanh(w_{max}) \approx \tanh(52.22102184) \approx 0.999999999999999999999999999999999999999999999999991241561704 \tag{28}$$

which, as expected, is the same maximum velocity derived for a electron as given by [1, 2]. The rapidity of a Planck mass, $\bar{\lambda} = l_p$, must be

$$w = \text{artanh}\left(\sqrt{1-\frac{l_p^2}{l_p^2}}\right) = 0 \tag{29}$$

This again is consistent with the concept that a Planck mass is at rest as observed from any reference frame. Further, we have

$$E = m_e c^2 \cosh(w_{max}) = m_p c^2 \tag{30}$$

and

$$E = m_p c^2 \cosh(w_{max}) = m_p c^2 \tag{31}$$

Again we see that the Planck mass must be the same as observed from any reference frame. This basically just shows that our theory of the maximum velocity leads to a maximum mass of fundamental particles equal to the Planck mass. When the fundamental particles travel at the maximum speed v_{max} they will turn into a Planck mass. This Planck mass is, based on the atomist view, highly unstable and will turn into pure energy instantaneously. In our theory there is a maximum velocity for each so-called "elementary" particle, a maximum Doppler frequency, and a maximum mass for fundamental particles that is equal to the Planck mass, and a minimum reduced Compton wavelength of l_p. What is remarkable is that the Planck mass and the Planck length are the same as observed from any reference fame.

3 Relativity of Simultaneity at the Maximum Velocity for Masses

The time difference between two distant events in Einstein's relativity of simultaneity is given by

$$\Delta t = \frac{Lv}{c^2\sqrt{1-\frac{v^2}{c^2}}} \tag{32}$$

Equation 32 can be derived directly from the Lorentz transformation and is well known from a series of sources in the special relativity theory literature; see, for example, [12, 13, 14, 15, 16]. When we replace the length L with the reduced Compton wavelength of a particle and v with the maximum velocity for a mass v_{max} we get

$$\begin{aligned}
\Delta t &= \frac{\bar{\lambda} v_{max}}{c^2\sqrt{1-\frac{v_{max}^2}{c^2}}} \\
\Delta t &= \frac{\bar{\lambda} c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{c^2\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} \\
\Delta t &= \frac{\bar{\lambda} c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{c^2\frac{l_p}{\bar{\lambda}}} \\
\Delta t &= \frac{\bar{\lambda}^2\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{c l_p}
\end{aligned} \tag{33}$$

Again using a series expansion for $\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \approx 1 - \frac{l_p^2}{\bar{\lambda}^2}\frac{1}{2}$ we get

$$\begin{aligned}
\Delta t &= \frac{\bar{\lambda}^2\left(1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}\right)}{c l_p} \\
\Delta t &= \frac{\bar{\lambda}^2 - \frac{1}{2}l_p^2}{c l_p} \\
\Delta t &= \frac{\bar{\lambda}^2}{c l_p} - \frac{1}{2}\frac{l_p}{c}
\end{aligned} \tag{34}$$

In the special case where $\bar{\lambda} = l_p$ we cannot use the series expansion approximation result 34, because it only holds for $\frac{l_p^2}{\bar{\lambda}^2} << 1$. Going back to equation 33 and inputting $\bar{\lambda} = l_p$ we get

$$\Delta t = \frac{l_p^2\sqrt{1 - \frac{l_p^2}{l_p^2}}}{c l_p} = 0 \tag{35}$$

That is to say, for a Planck mass (the collision of two indivisible particles) there is no time difference between the two events; in reality they are one event. This is not a surprise in the view of atomism, because the Planck mass event is the collision of two indivisible particles: the collision happens in a single point and is therefore simultaneous. Still, the Planck mass itself has a length of $2l_p$. That is the length of two indivisible particles with diameter l_p laid out next to each other. Further, a Planck mass has a reduced Compton wavelength of $\bar{\lambda} = l_p$, which under atomism is the distance center to center between two indivisible particles. For all non-Planck mass particles there is a time difference related to the reduced Compton wavelength of the particle in question.

4 Summary

We have presented the Lorentz transformation as well as rapidity and the relativity of simultaneity in the limit for a maximum speed of particles with mass. This points towards the idea that the Planck length is the same from any reference frame and the Planck mass is as well. This makes the Planck mass very unique. The Planck mass is actually at rest in every reference frame. The Planck mass can be seen as the turning point of "light". Doesnt light stand still as observed from any reference frame just in the instant it turns around (is reflected) in a reflection point? This is best understood under atomism, where the Planck mass is simply two indivisible particles at the moment of counter-strike. The two indivisible particles always move at the speed of light and each one has a potential mass of half the Planck mass when it is not counter-striking. At the instant of counter-strike they each have a rest mass equal to half the Planck mass, and together they form the Planck mass.

Appendix: Minkowski Space-Time Interval

Minkowski [8] showed that the space-time interval was invariant:

$$ds^2 = c^2t^2 - dx^2 - dy^2 - dz^2 \tag{36}$$

Replacing t and x with the Lorentz transformation at the maximum velocity for mass we get

$$\begin{aligned} ds^2 &= c^2\left(\frac{1}{2}\frac{l_p}{c}\right)^2 - \left(\frac{1}{2}l_p\right)^2 \\ ds^2 &= c^2\frac{1}{4}\frac{l_p^2}{c^2} - \frac{1}{4}l_p^2 \\ ds^2 &= 0 \end{aligned} \tag{37}$$

This is as expected. And in relation to the inverse Lorentz transformation we have

$$\begin{aligned} \hat{x}^2 &= \left(2\frac{\bar{\lambda}^2}{l_p} - \frac{1}{2}l_p\right)^2 \\ \hat{x}^2 &= \left(2\frac{\bar{\lambda}^2}{l_p} - \frac{1}{2}l_p\right)\left(2\frac{\bar{\lambda}^2}{l_p} - \frac{1}{2}l_p\right) \\ \hat{x}^2 &= 4\frac{\bar{\lambda}^4}{l_p^2} - \bar{\lambda}^2 - \bar{\lambda}^2 + \frac{1}{4}l_p^2 \\ \hat{x}^2 &= 4\frac{\bar{\lambda}^4}{l_p^2} - 2\bar{\lambda}^2 + \frac{1}{4}l_p^2 \end{aligned} \tag{38}$$

and

$$
\begin{aligned}
c^2\hat{t}^2 &= c^2\left(2\frac{\bar{\lambda}^2}{l_p c} - \frac{1}{2}\frac{l_p}{c}\right)^2 \\
c^2\hat{t}^2 &= c^2\left(4\frac{\bar{\lambda}^4}{l_p^2 c^2} - 2\frac{\bar{\lambda}^2}{c^2} + \frac{1}{4}\frac{l_p^2}{c^2}\right) \\
c^2\hat{t}^2 &= 4\frac{\bar{\lambda}^4}{l_p^2} - 2\bar{\lambda}^2 + \frac{1}{4}l_p^2
\end{aligned}
\tag{39}
$$

which means that the Minkowski space-time interval for a fundamental particle traveling at its maximum speed is

$$
\begin{aligned}
ds^2 &= c^2\hat{t}^2 - \hat{x}^2 \\
ds^2 &= 4\frac{\bar{\lambda}^4}{l_p^2} - 2\bar{\lambda}^2 + \frac{1}{4}l_p^2 - 4\frac{\bar{\lambda}^4}{l_p^2} + 2\bar{\lambda}^2 - \frac{1}{4}l_p^2 \\
ds^2 &= 0
\end{aligned}
\tag{40}
$$

This is as expected.

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