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The low-frequency structure of powerful radio sources and limits to departures from equipartition

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Summary. Recent interplanetary scintillation and spectral data have been combined to derive information about source structures at frequencies below 100 MHz. For an appreciable fraction of sources there is evidence of extended components which are prominent at low frequencies but which have not been detected in higher frequency interferometric observations. Sources having low-frequency spectral turnovers are studied in detail and in several cases it is shown that the total energy is within a factor 30 of the energy of equipartition between the particles and magnetic fields. In no case is there conclusive evidence of departure from equipartition.

1 Introduction

Studies of the relative magnetic and particle energies in radio galaxies and quasars are based on the assumption that the radiation originates from the synchrotron mechanism (*cf.* Ginzburg & Syrovatskii 1969). The conclusions of Williams (1963) and Bridle (1967) depended on the further assumptions that the structure of a source having a power-law spectrum with a constant spectral index is independent of frequency, and that the components responsible for most of the flux at frequencies <30 MHz are the same components whose sizes are measured at frequencies >150 MHz. This latter assumption was necessary because high angular resolution could only be obtained at high frequencies, and good limits to the magnetic field only from self-absorption at low frequencies, but it has been rightly criticized by Kellermann & Pauliny-Toth (1969) on account of the lack of substantiating evidence.

In this paper we examine the bearing on the equipartition question of the determinations of fine structure in radio sources (Readhead & Hewish 1974) by the method of interplanetary scintillation. Many of the more powerful radio sources with a steep power-law spectrum at high frequencies have a spectral cutoff near 81.5 MHz, so this frequency is a good one for such studies.

The physical arguments are discussed in Section 2. We then consider (in Section 3) a complete sample of radio sources, selected from the 3CR catalogue (Bennett 1962), to compare the spectral characteristics of those sources for which most of the radiation comes from 'hot spots', i.e. compact regions <1 arcsec in extent, with those of extended sources which do not contain prominent hot spots.

In Section 4 we discuss a subset of this complete sample containing only those sources for which there are reliable measurements of flux density at low frequencies and for which we can determine the angular sizes of hot spots to within ~0.1 arcsec. Most sources in the subset have also been observed at 151.5 MHz by the scintillation method, enabling us to allow for the effect of angular broadening due to interstellar scattering (Duffett-Smith & Readhead 1976). In Section 4.1 we consider sources for which synchrotron self-absorption occurs at, or very close to, the frequency at which the structure has been determined by observations of interplanetary scintillation. In such cases the observed angular sizes may properly be compared with the sizes expected if the sources are in equipartition (see Section 2). It is shown that all of these sources are consistent with the assumption of equipartition. In Section 4.2 we consider sources which have flat or convex spectra over a wide frequency range and find that the radiation at the lowest frequencies (< 30 MHz) must emanate from regions $\gtrsim 0.5$ arcsec in size. Such extended regions have not been detected by interferometric observations. Steep spectrum sources with little or no decrease in spectral index, α (defined in the sense $S \propto \nu^{-\alpha}$), near 81.5 MHz are discussed in Section 4.3. We find that in several cases the low-frequency emission must come from regions with extents greater than 1 arcsec, comparable with the overall sizes of the sources. It may be that these extended regions are related to the bridges of emission between the outer hot spots in double radio sources.

2 Physical arguments

Estimates of the magnetic field in a radio source may be obtained in two distinct ways. Firstly the assumption that there is equipartition of energy between the radiating particles and the magnetic field (e.g. Burbidge & Burbidge 1957) enables the field strength and total energy to be calculated from observations of the flux density and and angular size of the source if the distance to the source is known.

Secondly, even if there is not equipartition, the field strength may be estimated from the frequency, the flux density and the angular size at the turnover in the spectrum, assuming that this is due to synchrotron self-absorption (Scheuer & Williams 1968). The justification for this assumption is discussed in Section 3.

In principle, the two values for magnetic field may be compared (e.g. Bridle 1967) to see how far the source is from equipartition, but there are two objections to this procedure:

(1) the source structure may vary with frequency,
(2) the field in the self-absorption expression is very strongly dependent on the measured source parameters, and one is therefore comparing two quantities one of which has very large errors.

The approach we adopt here circumvents (1) and enables us to test for equipartition by comparing an observed quantity with a theoretical quantity in which the errors are small. In the discussion of whether the sources are near equipartition, the spectral data used are from a restricted range of frequencies near 81.5 MHz, at which measurements of interplanetary scintillation provide good angular resolution; frequency dependence of source structure is thus irrelevant. In addition, we make comparisons, not in terms of field strengths, but rather between the observed size, $\theta_{\rm scint}$, and the 'equipartition size', $\theta_{\rm eq}$. The latter is the size which a source with the given flux density and frequency at the synchrotron self-absorption turnover must have to be in equipartition, and it is obtained by equating the two values of field strength obtained by the arguments above and solving for the size. We thus have

$$\theta_{\rm eq} = F(\alpha)(1-(1+z)^{-1/2})^{-1/17}(1+z)^{(2\alpha+15)/34} S^{8/17} \nu^{(2\alpha-35)/34}, \qquad (1)$$

where z is the redshift, $\theta_{\rm eq}$ is the diameter in arcsec, S is in Jy and ν in MHz. This expression

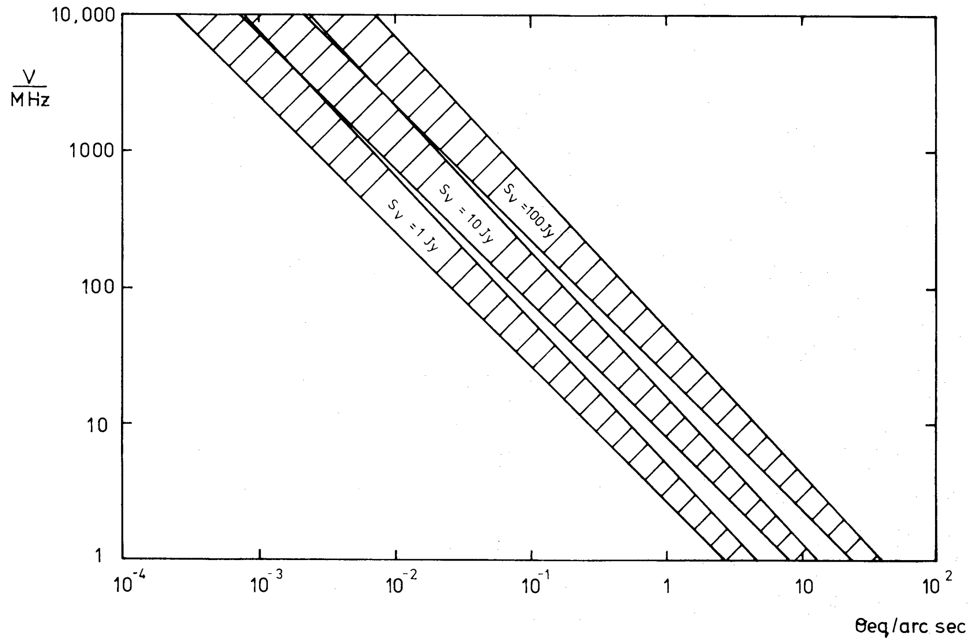

Figure 1. Angular size as a function of frequency for sources with given flux densities 1, 10 and 100 Jy at that frequency. The upper bound of the hatched region in each case is for a source with $\alpha = 1.5$ and $z = 0.7$ evaluated at the synchrotron self-absorption peak in the spectrum. The lower bound refers to a source with $\alpha = 0.3$ and $z = 0.7$ which is not self-absorbed. All sources in our sample lie between the lines.

for θ_{eq} is calculated assuming an Einstein–de Sitter cosmology with $H_0 = 50$ km/(s Mpc). $F(\alpha)$ has different values depending on whether S and ν refer to a point on the straight part of the spectrum, or to the peak in the spectrum of a source which is synchrotron self-absorbed (see Appendix). In the first case the value of θ_{eq} given by (1) is a lower limit. Fig. 1 shows plots of ν against θ_{eq} for various values of S and α. As is seen in (1) and Fig. 1, θ_{eq} depends only very weakly on z, S and α, and the dependence on $1/\nu$ is roughly linear. Thus the errors in these observables do not lead to large uncertainties in θ_{eq}.

The actual energy and magnetic field in the source may then be expressed simply as ratios with respect to the equipartition values, i.e. the values corresponding to $\theta_{scint} = \theta_{eq}$

$$U/U_{eq} = \tfrac{1}{2}\eta^{11}(1 + \eta^{-17}),$$

$$u/u_{eq} = \tfrac{1}{2}\eta^{8}(1 + \eta^{-17})$$

and

$$B/B_{eq} = \eta^{4},$$

where $\eta \equiv \theta_{scint}/\theta_{eq}$ and U, u, B are total energy, energy density and magnetic field respectively. We also have

$$u(\text{particle})/u(\text{field}) = \eta^{-17}.$$

Equation (1) refers strictly to a source with uniform surface brightness. However, numerical calculations show that θ_{eq} derived in this way for a source with any simple brightness distribution will not be more than ~ 20 per cent in error (Smith 1968). Sources with more complex geometries cannot be characterized by a single parameter, θ_{eq}, and the shape of the self-absorbed spectrum can vary substantially (de Bruyn 1976), nevertheless, the diameters which we *observe* are within a factor 2 of θ_{eq} calculated for the most simple case (see

Section 4). Throughout this paper we have used for $F(\alpha)$ in equation (1) the function appropriate for a tangled field. If the field is uniform, θ_{eq} is slightly larger, strengthening the arguments given below.

3 Special characteristics of compact and extended sources

The interplanetary scintillation survey by Readhead & Hewish (1974) at 81.5 MHz provides data on the fine structure of the complete sample of 199 extragalactic 3CR sources with $\delta > 10°$, $|b| > 10°$ and $S_{178} > 9$ Jy. We have used these data to divide the sources into two groups, namely those in which most of the radiation comes from hot spots of <1 arcsec in extent, and those in which the radiation is mainly from regions with low brightness temperatures. The former category contains sources for which the fraction, R, of the flux density in scintillating components is greater than or equal to 0.40, while the second group contains those for which $R \leqslant 0.25$. The numbers of sources in the two categories are 67 and 82 respectively (Readhead & Hewish 1976), the remainder of the sources being scintillators of intermediate strength.

Most of the sources in the complete sample have steep spectra ($\alpha > 0.6$) over the frequency range 10 MHz to 10 GHz. There are very few with flat spectra ($\alpha < 0.5$), or with

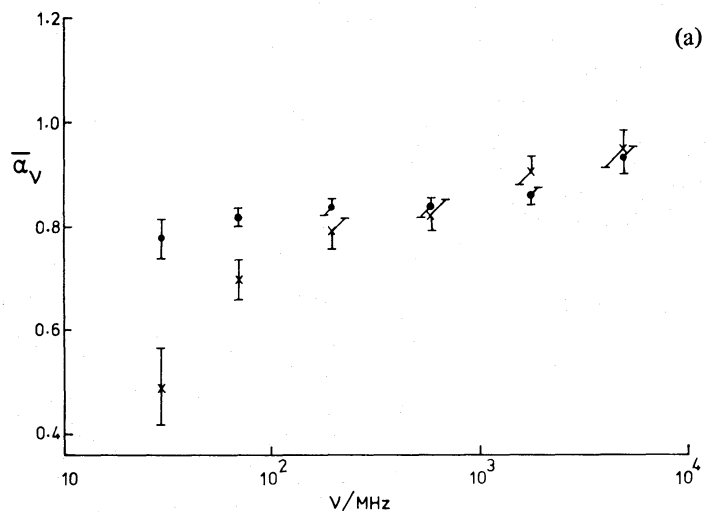

Figure 2(a). The variation of the mean spectral index with frequency. The crosses represent strong scintillations ($R \geqslant 0.40$), and the dots weak scintillations ($R \leqslant 0.25$).

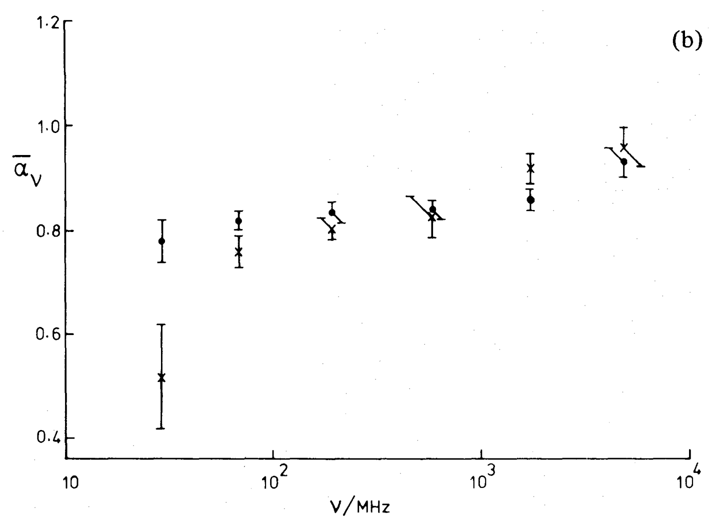

Figure 2(b). As Fig. 2(a) but excluding the most compact sources (i.e. those for which $R > 0.70$), so that the crosses represent sources with $0.40 \leqslant R \leqslant 0.70$.

concave spectra ($d\alpha/d\nu < 0$). Since the shapes of the spectra in the sample are so similar, it is reasonable to consider the mean forms of these spectra. To compare the spectral properties, we have selected sources for which flux density measurements have been made over the range 20 MHz to 10 GHz, 105 in all, of which 43 gave $R \geqslant 0.40$ and 62 have $R \leqslant 0.25$. This selection eliminates some of the weaker sources, but biasses the sample towards sources with constant α, and does not affect the conclusions which will be drawn. The tangential spectral indices at 30, 70, 200, 600, 1800 and 5000 MHz were taken from Roger, Bridle & Costain (1973) where possible, otherwise the spectra (corrected to the flux scales of Roger *et al.*) were plotted and the spectral indices derived from the plots.

The mean spectral indices, together with their standard errors, are shown in Fig. 2(a). We see that at high frequencies, above ~300 MHz, there are no significant differences between the compact and extended sources, but below about 200 MHz, the values for the two classes diverge, and the compact sources have significantly lower spectral indices below ~100 MHz. Similar results on a sample of sources selected at 408 MHz have been reported by Harris (1972). Synchrotron self-absorption is the obvious explanation of the low-frequency differences.

The similarity of the high-frequency spectral indices in these objects, which are morphologically very different (Readhead & Hewish 1976), suggests that synchrotron losses are either absent or very similar in both classes and that the particle acceleration processes are closely related. Although the mean spectra become steeper near 5000 MHz, this may not be due to synchrotron losses in the sources since the effect is the same for both strong and weak scintillators, whereas one might expect weak scintillators, which generally have larger overall sizes, to be older sources on average and thus to have steeper high-frequency spectra.

It is interesting to see if the effect of excluding the most compact sources from the sample of strongly scintillating sources is consistent with our interpretation of the reduction of the values of spectral index with frequency as due to synchrotron self-absorption. In Fig. 2(b) we show the mean spectral indices of the extended ($R \leqslant 0.25$) sources, and of those sources with $0.40 \leqslant R \leqslant 0.70$. There is no significant difference above 100 MHz for these two classes, but the point at 30 MHz is again significantly lower for the more compact sources. That the difference between the two classes is now apparent only at a lower frequency shows that sources of lower surface brightness have spectra which flatten at lower frequencies, strongly suggesting that synchrotron self-absorption is indeed the mechanism responsible for the spectral differences between the two classes.

The two classes of source in Fig. 2(b) have very different luminosities and sizes. The sources with $0.40 \leqslant R \leqslant 0.70$ are the most luminous radio sources ($P_{178} > 10^{27}$ W/(Hz sr)), and have overall physical sizes which seldom exceed 200 kpc. The $R \leqslant 0.25$ sources are much less luminous ($P_{178} < 10^{26}$ W/(Hz sr)) and are often greater than a few hundred kiloparsec in overall size (Readhead & Hewish 1976). Thus it appears that the injection spectra and synchrotron losses in these objects, apparently of different nature, are nevertheless very similar.

4 Detailed examination of sources with different types of spectra

In this section we consider those sources in our sample for which we have reliable flux density measurements at low frequencies from the work of Bridle & Purton (1968); Roger, Costain & Lacy (1969) and Viner & Erickson (1975), and for which the errors in the angular diameters at 81.5 MHz are ~0.1 arcsec.

We have divided the sources into three groups according to their spectral characteristics as follows:

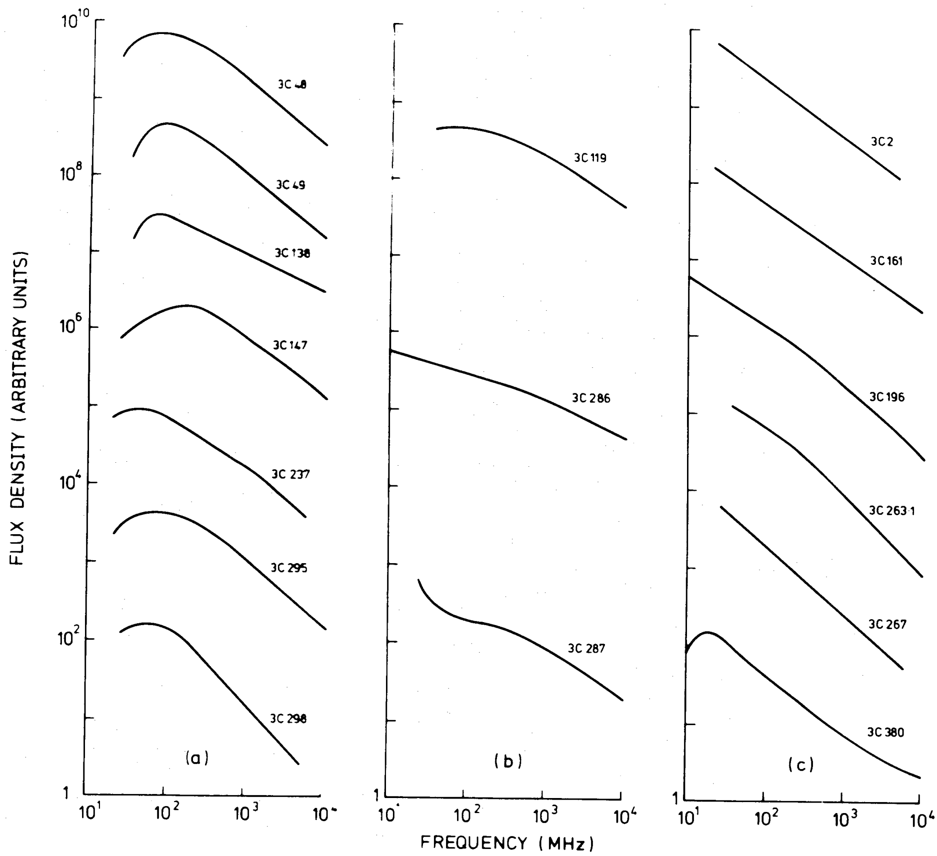

Figure 3. The spectra of the three groups of sources: (a) sources having a spectral turnover near 81.5 MHz, (b) sources with flat or convex spectra over a wide frequency range, (c) steep spectrum sources with $\alpha \sim$ constant around 81.5 MHz.

(1) sources with a well-defined turnover in the spectrum at or near 81.5 MHz;
(2) sources with spectra which flatten progressively as the frequency is decreased over a wide range (i.e. $d\alpha/d\nu > 0$ for a large frequency range);
(3) sources with steep spectra which are straight over a wide range of frequencies above and below 81.5 MHz.

The sources in these three groups are listed in Tables 1, 2 and 3 and the spectra are shown in Fig. 3(a), (b) and (c). Most of the sources in groups 1 and 2 are very compact ($R > 0.7$). The majority of sources in the complete sample discussed in Section 3 belong to group 3. We now consider the three groups in turn.

4.1 SOURCES FOR WHICH THERE IS STRONG SYNCHROTRON SELF-ABSORPTION NEAR 81.5 MHZ

The data for the seven sources in this group are summarized in Table 1. Those of 3C 295 come from observations made at the Mullard Radio Astronomy Observatory by Houminer (private communication), Readhead & Hewish (1974) and A. G. F. Brown (private communication). The flux densities quoted in Table 1 refer to the maxima in the spectra. In column 4 angular diameters θ_{scint}, corrected for interstellar scattering (Duffett-Smith & Readhead 1976), are given and in column 5 are shown the derived equipartition sizes, θ_{eq}. In these sources the turnover frequency is very close to the frequency of observation, and we are therefore justified in comparing the two sizes. Of the sources in this group, three have $\eta < 1$, but the errors in the data are still sufficient to permit $\eta = 1$, so that there is no definite

Table 1.

3C	ν_{peak} (MHz)	S_{peak} (Jy)	θ_{scint} (arcsec)	θ_{eq} (arcsec)	η	B_{eq} (10^{-5} G)	B_{est} (10^{-5} G)
48	70 ± 20	75 ± 10	0.40± 0.10	0.55 ± 0.10	0.7	50	15
49	100 ± 20	14 ± 2	0.55 ± 0.20	0.15 ± 0.05	3.5		
138	85 ± 20	33 ± 5	0.35 ± 0.15	0.35 ± 0.10	1.0		
147	140 ± 20	70 ± 5	0.45 ± 0.10	0.30 ± 0.05	1.5		
237	45 ± 10	40 ± 5	0.35 ± 0.10	0.65 ± 0.10	0.5	37	3
237★	45 ± 10	20 ± 5	0.35 ± 0.10	0.50 ± 0.10	0.7	40	11
295	70 ± 20	125 ± 10	1.00 ± 0.30	0.55 ± 0.15	1.8		
298	65 ± 15	90 ± 10	0.35 ± 0.10	0.85 ± 0.15	0.4	64	2
298★	65 ± 15	45 ± 10	0.35 ± 0.10	0.65 ± 0.15	0.6	70	8

★ Assuming an equal double at ν_{peak}.

evidence of departure from equipartition. Both 3C237 and 3C298 are doubles at high frequencies, and therefore the values of θ_{eq} have also been calculated assuming they are equal doubles at ν_{peak}. This interpretation is consistent with the scintillation data (Readhead, Kemp & Hewish 1977). Three of the remaining sources in the group have $\eta > 1$. In these cases we cannot conclude that the energy density of the magnetic field exceeds that of the particles, because it is possible that they contain features of higher surface brightness than are revealed by scintillation. For example, a line source of length L, with a very high brightness temperature, would scintillate much like a circularly symmetrical source of dimensions comparable to L.

For all except one of these sources the observed diameter differs from the equipartition diameter by a factor $\lesssim 2$. This provides useful limits on the total energies ($< 32\,U_{eq}$) and shows that the magnetic fields are within a factor 16 of the equipartition values.

In the cases where B_{est}, the magnetic field deduced from θ_{scint}, is less than B_{eq}, the field deduced for equipartition, the values are also given in Table 1. For these sources we have calculated the expected X-ray flux from inverse Compton scattering of the microwave background and synchrotron radiation. These X-ray fluxes are consistent with the upper limits of X-radiation from the fields (Giacconi *et al.* 1974).

4.2 SOURCES IN WHICH THERE IS EVIDENCE OF SYNCHROTRON SELF-ABSORPTION OVER A WIDE FREQUENCY RANGE

The sources in this group are listed in Table 2. If the flattening at low frequencies in their spectra is due to synchrotron self-absorption, their structures must be strongly dependent on frequency (Fig. 1). We have therefore calculated lower limits (Section 2) to the equi-

Table 2.

3C	ν (MHz)	S (Jy)	θ_{scint} (arcsec)	θ_{eq} (arcsec)	η
119	81.5	22 ± 3	<0.20	>0.18	<1.0
	40	22 ± 3		>0.35	<0.6
286	81.5	35 ± 5	0.40 ± 0.10	>0.20	<2.0
	10	85 ± 20		>2.6	<0.2
287	81.5	15 ± 3	0.30 ± 0.10	>0.15	<2.0
	26	~50 ± 20		>0.85	<0.4

partition angular sizes of the sources, using the flux densities at 81.5 MHz. We note that, as in the previous section, there are no large departures of θ_{scint} from θ_{eq}, so the same conclusions follow. In the case of 3C 287 we have assumed that the confusing source (4C 25.44) has $\alpha < 0.6$ down to 26 MHz (Smith 1968).

It is also of interest to consider the emission at very low frequencies from these sources. The actual sizes cannot be much smaller than the equipartition sizes because both energy and field depend strongly on η (Section 2). Knowledge of the flux densities of these sources at 20–30 MHz therefore enables us to derive lower limits to the sizes of the emitting regions at these frequencies (Table 2). In each case we find that the angular size at low frequencies is significantly greater than at 81.5 MHz and is much larger than any structure observed in these sources at higher frequencies. These more extended regions have therefore been missed by interferometry at decimetre wavelengths where they might contribute no more than ~10 per cent of the total flux density.

4.3 SOURCES WITH SPECTRA WHICH ARE OF A POWER-LAW FORM OVER A WIDE FREQUENCY RANGE

The majority of sources in the complete sample fall into this group. The data for those for which we also have accurate measurements of θ_{scint} and reliable flux density measurement at low frequencies are given in Table 3. For these sources we have calculated lower limits to the expected equipartition sizes both at 81.5 MHz and at the lowest frequency at which the flux density has been measured. In every case the measured angular size at 81.5 MHz is consistent with equipartition. However, we note that the angular size observed at 81.5 MHz is always significantly smaller than the lower limit on the size required for equipartition at the lowest frequency. If the source structure is actually the same at both frequencies, extremely large departures from equipartition are implied. For example, in the case of 3C196 the value of η would be $<¼$, implying $U/U_{eq} > 10^3$ and $u_p/u_B > 10^{10}$, which seems very unlikely. The X-ray flux expected from inverse Compton radiation in this case is only 0.2 *Uhuru* count/s, however, which is below current limits of detection (Giacconi *et al.* 1974), so we cannot definitely rule out this possibility.

Table 3.

3C	ν (MHz)	S (Jy)	θ_{scint} (arcsec)	θ_{eq} (arcsec)	Largest angular size (arcsec)	η
2	81.5	30 ± 3	0.70 ± 0.10	>0.20	5.7	<3.5
	22	70 ± 5		>1.25		<0.6
161	81.5	120 ± 5	0.60 ± 0.10	>0.35	4	<1.7
	22	270 ±30		>2.10		<0.3
196	81.5	130 ± 10	1.20 ± 0.10	>0.35	5	<3.4
	10	450 ± 50		>6.50		<0.2
263.1	81.5	33 ± 3	0.30 ± 0.10	>0.20	9	<1.5
	38	53 ± 10		>0.65		<0.5
267	81.5	27 ± 3	0.45 ± 0.20	>0.15	37	<3.0
	26	68 ± 5		>1.00		<0.5
380	81.5	120 ± 10	1.20 ± 0.30	>0.25	1.4	<4.8
	18	350 ± 20		>3.00		<0.4

A more likely explanation is that the low-frequency radiation comes from regions much larger than the hot spots scintillating at 81.5 MHz. The sources in this group are all known to be double or triple, and in some cases the size required for the low-frequency extended region is comparable to the overall extent of the source. The largest angular sizes, taken from Macdonald, Kenderdine & Neville (1968), Bash (1968), Pooley & Henbest (1974) and Scott (1976), are given in Table 3.

The extended regions which we propose as responsible for the low-frequency (< 30 MHz) radiation in sources such as 3C196 would need to have spectral indices roughly comparable with, or slightly greater than, the high-frequency spectral indices in these sources ($\alpha \simeq 1$, see Fig. 3). They would then contribute only 10–20 per cent of the flux density at high frequencies, and would therefore have low surface brightnesses easily missed in synthesis observations. Similarly, these extended regions would be difficult to detect in high-resolution interferometric observations at decimetric wavelengths.

5 Conclusions

The main points arising from this work are:

(1) The spectra of compact and extended sources are remarkably similar about a few hundred MHz. The differences observed at low frequencies are almost certainly attributable to flattening in the spectra of compact sources by synchrotron self-absorption. The mean electron-injection spectra of compact and extended sources are the same, and it seems unlikely that synchrotron losses are important in modifying the spectra of either group.

(2) In no case is there any significant evidence of departure from equipartition, and the magnetic fields are within a factor 16 of the equipartition values.

(3) For sources with straight spectra and prominent hot spots in outer components, which dominate at high frequencies, there is evidence that most of the radiation below ≈ 20 MHz comes from regions which are larger than the hot spots. These may be the bridges of emission which, at higher frequencies, are often observed to connect the outer two components with the central or optical component.

In order to check whether there are extended regions in sources like 3C196 or like 3C286, high-resolution observations at low frequencies are needed. A 1.6-hectare telescope recently constructed in Cambridge is now in use, making routine observations at 38 MHz of interplanetary scintillation in extragalactic sources. These data will provide a measure of the low-frequency emission from regions $\gtrsim 1.5$ arcsec.

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Appendix: the spectrum of a synchrotron self-absorbed source

In Section 4 we have used the value of S and ν on the peak of the spectrum to determine θ_{eq}. In so doing we have made allowance for the reduction in flux density due to the self-absorption of the synchrotron radiation (see Fig. 4). The following derivation of the reduction factor is based on an argument given to us by Dr P. A. G. Scheuer.

Consider synchrotron radiation from a uniform slab of plasma of thickness x_0, emissivity $J(\nu)$ and absorption coefficient $\kappa(\nu)$ at frequency ν. The flux density, S, is given by

$$S = A_1 \int_0^{x_0} J \exp(-\tau)\, dx$$

where τ $(=\kappa x)$ is the optical depth of the slab to a distance x along the line of sight through

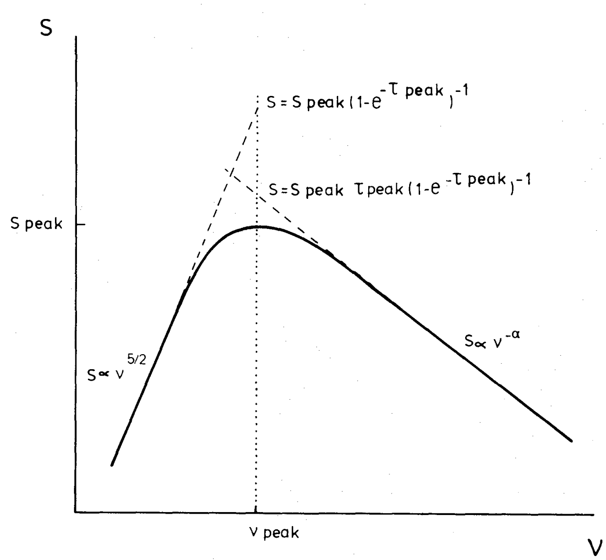

Figure 4. The spectrum of a uniform cube of plasma emitting synchrotron radiation which is self-absorbed at low frequencies.

the slab. The proportionality constant A_1 involves cosmological parameters, but we do not need these in explicit form since we are here concerned only with the shape of the spectrum.

Hence

$$S = A_1(J/\kappa)[1 - \exp(-\tau_0)], \quad \text{(A1)}$$

where τ_0 is the optical depth through the source. At high frequencies $\tau_0 \sim 0$ and $S = A_2\nu^{-\alpha}$, so that from (A1)

$$Jx_0 = (A_2\nu^{-\alpha})/A_1. \quad \text{(A2)}$$

At low frequencies $\tau_0 \to \infty$ and $S = A_3\nu^{5/2}$ which, together with (A1), gives

$$J/\kappa = (A_3\nu^{5/2})/A_1. \quad \text{(A3)}$$

Dividing (A2) by (A3) we have

$$\tau_0 = (A_2/A_3)\, \nu^{-(\alpha+5/2)} \quad \text{(A4)}$$

Thus (A1), (A3) and (A4) give

$$S = A_3\nu^{5/2}[\![1 - \exp[-(A_2/A_3)\, \nu^{-(\alpha+5/2)}]]\!],$$

which has a maximum where $\exp(\tau_{\text{peak}}) - 1 = [1 + (2/5)\,\alpha]\,\tau_{\text{peak}}$, where τ_{peak} is the optical depth at the peak of the spectrum. Solutions are tabulated below for typical values of α (see Table 4).

Table 4.

α	τ_{peak}	$1/(1 - \exp(-\tau_{\text{peak}}))$	$C(\alpha)$
0.0	0	∞	∞
0.1	0.078	13.33	2.91
0.2	0.152	7.09	2.25
0.4	0.290	3.97	1.78
0.6	0.416	2.94	1.58
0.8	0.53	2.43	1.46
1.0	0.64	2.11	1.38
1.2	0.74	1.91	1.34
1.4	0.83	1.77	1.30
1.6	0.92	1.66	1.26
1.8	1.00	1.58	1.24
2.0	1.08	1.51	1.22

Thus

$$S_{\text{peak}} = A_3\nu_{\text{peak}}^{5/2}\,[1 - \exp(-\tau_{\text{peak}})].$$

Similarly from (A1), (A2) and (A4) we have

$$S = A_2\nu^{-\alpha}\tau_0^{-1}\,[1 - \exp(-\tau_0)],$$

thus

$$S_{\text{peak}} = A_2\nu_{\text{peak}}^{-\alpha}\tau_{\text{peak}}^{-1}\,[1 - \exp(-\tau_{\text{peak}})].$$

In the derivation of θ_{eq} we equate the value of B given by the synchrotron self-absorption formula, with that of B_{eq} in the equipartition expression. These are of the form $B^2 \propto S^{-4}\theta^8$ and $B_{\text{eq}}^2 \propto S^{4/7}\theta^{-12/7}$ respectively. Thus in the case of a synchrotron self-absorbed source at the peak of the spectrum we have $B^2 \propto [\![S_{\text{peak}}/[1 - \exp(-\tau_{\text{peak}})]]\!]^{-4}\,\theta^8$ and

$$B_{\text{eq}}^2 \propto [\![S_{\text{peak}}/\tau_{\text{peak}}/[1 - \exp(-\tau_{\text{peak}})]]\!]^{4/7}\theta^{-12/7}.$$

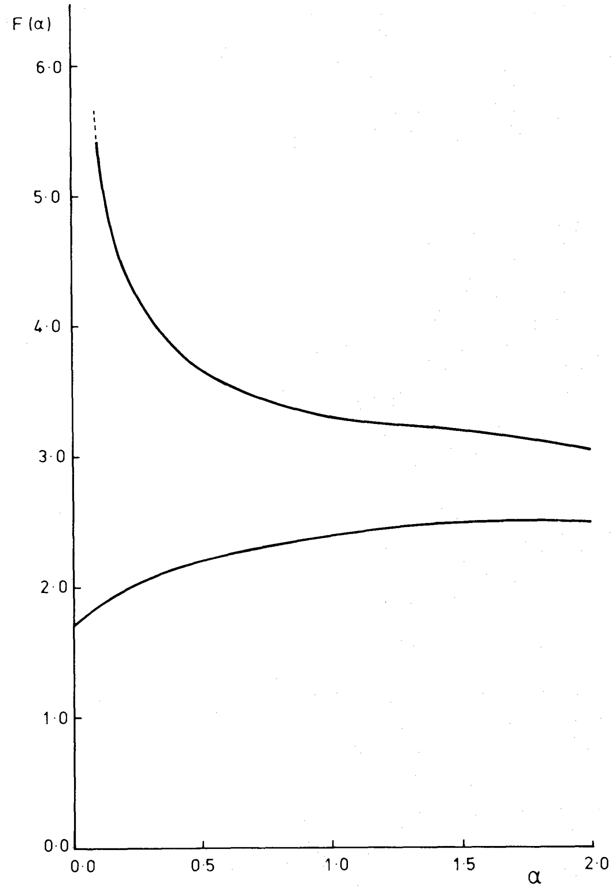

Figure 5. The value of $F(\alpha)$ for a point on the peak of the spectrum in a synchrotron self-absorbed source (upper curve), and for a point on the straight, unabsorbed, part of the spectrum (lower curve).

Equating, and solving for θ, we see that the required correction, C, to θ_{eq} is

$$C = \tau_{peak}^{1/17} [1 - \exp(-\tau_{peak})]^{-8/17}. \quad \text{(A5)}$$

We have assumed a uniform slab source in deriving this correction, but it is unlikely to be very much in error for simple source geometries, which have been shown to give spectra very similar in shape to the one derived here (Smith 1969).

Values of $1/[1 - \exp(-\tau_{peak})]$, and C for a range of α between 0 and 2 are given in Table 4.

Finally we turn to the factor $F(\alpha)$ in equation (1). This is defined by

$$F(\alpha) = 1.6\, C(\alpha) [\![[\nu_{low}^{1/2-\alpha} - \nu_{high}^{1/2-\alpha}] / [(2\alpha - 1)\, f_1(\alpha)]]\!]^{1/17} f_2(\alpha)$$

where ν_{low} and ν_{high} are in MHz, and are taken throughout this paper to be 10 and 10 000 MHz respectively. $f_1(\alpha)$ and $f_2(\alpha)$ are the factorial expressions defined in Scheuer & Williams (1968) by equations 5, 6 and 15(c).

In Fig. 5 we show two curves of $F(\alpha)$. The lower curve applies to a source which does not show synchrotron self-absorption, or to points on the spectrum of a self-absorbed source for which $\nu \gg \nu_{peak}$ (i.e. $C(\alpha) = 1$ in this case). The upper curve applies to the peak in the spectrum of a source exhibiting synchrotron self-absorption, i.e. $C(\alpha)$ given by (A5).