# The Macroeconomic Effects of Large-Scale Asset Purchase Programs\*

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December 15, 2011

#### Abstract

The effects of asset purchase programs on macroeconomic variables are likely to be moderate. We reach this conclusion after simulating the impact of the US Federal Reserve second Large-Scale Asset Purchase program (LSAP II) in a DSGE model enriched with a preferred habitat framework and estimated on US data. Our simulations suggest that such a program increases GDP growth by less than half of a percentage point relative to the absence of intervention, but has a very persistent effect on the level of GDP. The marginal contribution of the asset purchase program to inflation is very small. One key reason behind our findings is that we estimate a small degree of financial market segmentation. If we enrich the set of observables with a measure of long-term debt, we find a substantially smaller semi-elasticity of the risk premium to the amount of debt in the hands of the private sector than the recent empirical literature. In this case, our baseline estimates of the effects of LSAP II on the macroeconomy decrease by at least a factor of two. Throughout the analysis, a commitment to an extended period at the zero lower bound for nominal interest rates increases the effects of asset purchase programs on GDP growth and inflation.

Keywords: quantitative easing, zero lower bound, unconventional monetary policy JEL codes: E43, E44, E52, E58

<sup>\*</sup>This paper was prepared for the conference on "Learning the Lessons from QE and Other Unconventional Monetary Policies," that took place at Bank of England on November 17-18, 2011. Special thanks to Richard Harrison for his discussion and to the conference participants for their comments. We have also benefited from comments by Hal Cole, Frank Diebold, Stefano Eusepi, John Fernald, Keith Kuester, James Nason, Ed Nelson, Ricardo Reis, Glenn Rudebusch, Frank Schorfheide, Eric Swanson, Michael Woodford, and participants in seminars at the University of Pennsylvania, Federal Reserve Banks of New York, Philadelphia and San Francisco, University of California Santa Cruz and at the European Economic Association 2011 Meetings in Oslo. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

# 1 Introduction

In response to the recent financial crisis, the major central banks around the world have engaged in multiple rounds of large scale asset purchase (LSAP) programs, typically financed with the creation of excess reserves. The Federal Reserve purchased a total of \$1.75 trillion in agency debt, mortgage-backed securities and Treasuries starting in early 2009, followed by a second Treasury-only program in the fall of 2010.¹ Also in early 2009, the Bank of England set up an asset purchase facility whose size is currently about £200 billion, approximately equivalent to \$315 billion. The European Central Bank initiated purchases of up to €60 billion (approximately \$43.5 billion) in Euro area covered bonds in mid-2009. In October of 2010, the Bank of Japan unveiled a ¥5 trillion (approximately \$61.5 billion) asset purchase program.

The objective of the various LSAP programs, often referred to as "Quantitative Easing" (or QE), is to support aggregate economic activity in periods when the traditional instrument of monetary policy (the short-term nominal interest rate) is not available due to the zero bound constraint. The general idea is that asset purchases operate directly on different segments of the yield curve, reducing rates at different maturities while the short-term rate is at zero.

Several papers find evidence that LSAP programs have indeed been effective in reducing long-term rates. For example, Gagnon et al. (2011) estimate that the first round of asset purchases by the Federal Reserve lowered the ten-year Treasury yield by 58 basis points (bp).<sup>2</sup>

Yet, agreement on the effectiveness of LSAP programs to boost the economy is far from

<sup>&</sup>lt;sup>1</sup>In September of 2011, the Fed announced an additional \$400 billion program, this time as a pure swap between short-term and long-term assets not involving the creation of additional reserves.

<sup>&</sup>lt;sup>2</sup>A selected sample of other estimates include 13 bp in Hamilton and Wu (2010), 39 bp in Doh (2010), 45 bp in D'Amico and King (2010), and 107 bp in Neely (2010). Krishnamurthy and Vissing-Jorgensen (2011) find that LSAP II reduced the ten-year yield by about 16 bp. See more details in Table 1.

universal. From a theoretical perspective, LSAP programs were criticized before their implementation, based on some version of the irrelevance result in Wallace (1983). Quantitative easing of this type is also irrelevant in the baseline New Keynesian model of Eggertsson and Woodford (2003). In this framework, injecting reserves in exchange for longer term securities is a neutral operation. To the extent that market participants take full advantage of arbitrage opportunities, LSAP programs should have no effect on real economic outcomes. Cúrdia and Woodford (2011) extend this result to a New Keynesian model with credit frictions. If households perceived the assets purchased (such as short-term government bonds) as equivalent to reserves, again LSAP programs have no effect on the macroeconomy.<sup>3</sup> Expost, the criticism has continued due to the difficulty of identifying empirically the effects of asset purchases from other macroeconomic forces (e.g. Cochrane, 2011).

In this paper, we estimate the effects of LSAP on macroeconomic variables in a dynamic stochastic general equilibrium (DSGE) model with segmented asset markets. General equilibrium effects are at the heart of Wallace's irrelevance theorem. By going beyond the effects of asset purchases on interest rates, we can evaluate the extent of the criticisms against this type of programs. At the same time, we want to give LSAP programs a chance. We introduce limits to arbitrage and market segmentation in a simple form that encompasses frictionless financial markets. Therefore, our strategy is to identify the degree of segmentation — and ultimately the effectiveness of asset purchases on macroeconomic activity — directly from the data, without assuming a priori that LSAP programs are bound to fail.

We augment a standard DSGE model with nominal and real rigidities, along the lines of Christiano, Eichembaum and Evans (2005) and Smets and Wouters (2007), with segmented

<sup>&</sup>lt;sup>3</sup>Asset purchase programs may be an effective tool to boost the economy if the government buys securities that are not equivalent to reserves, either because not all households can invest in those assets or because financial frictions impair the investment activity. Recent research along these lines, such as Cúrdia and Woodford (2011), Del Negro et al. (2011) and Gertler and Karadi (2011), has mostly focused on private credit markets. Here, instead, we study frictions that rationalize a role for government purchases of long-term bonds.

bond markets. In particular, we follow Andres, Lopez-Salido and Nelson (2004, henceforth ALSN) and assume that investors have heterogeneous preferences for assets of different maturities (a "preferred habitat" motive, similar to Vayanos and Vila, 2009). We do not model the details of why assets of different maturities are imperfect substitutes. Rather, we postulate that this type of market segmentation exists and estimate the importance of this friction for the transmission mechanism of monetary policy.

The form of asset market segmentation that we use in this paper implies that the long-term interest rate matters for aggregate demand distinctly from the expectation of short-term rates. In this world, even if the short-term rate is constrained by the zero lower bound (ZLB) for a long period of time, monetary policy can still be effective by directly influencing current long-term rates. In addition, we assume that the risk premium that arises in the model as a consequence of transaction costs is a positive function of the supply of long-term Treasury securities. This assumption captures, in reduced form, the notion that asset purchase programs are most effective in flattening the yield curve by reducing the risk premium (Gagnon et al., 2011).

We estimate the model with standard Bayesian methods for the post-war sample, including the recent years.<sup>4</sup> Our main experiment is a counterfactual evaluation of what would have happened to output, inflation and the other macroeconomic variables in the absence of LSAP programs.

First, we calibrate the asset purchase program in the model to match a reduction in the risk-premium by 30 annualized bp on impact. At the same time, we consider that the central bank annualized the commitment to hold the interest rate at the ZLB for four quarters.<sup>5</sup> The posterior median GDP growth increase is 0.4% (annualized), while the posterior median

<sup>&</sup>lt;sup>4</sup>Including the data since the Fall of 2008 presents us with the key challenge of incorporating the zero bound of nominal interest rates. We deal with this problem using the techniques developed in Cúrdia and Woodford (2011).

<sup>&</sup>lt;sup>5</sup>This assumption is consistent with the "extended period" language in the FOMC statements at the time of LSAP II and the market expectations as implied in surveys of private forecasters.

inflation increase is less than 5 bp (annualized). Importantly, the corresponding effect on the *level* of GDP is estimated to be very long lasting — six years after the start of the program the level of GDP is still 0.25% above the path that would have prevailed in the absence of the LSAP program.

Counterfactual simulations suggest that the commitment to hold the short-term nominal interest rate at the ZLB increases the response of real activity and inflation roughly by a factor of two and introduces upward skewness in the uncertainty surrounding the median estimates. Furthermore, the boost from the commitment to the ZLB is increasingly larger with the length of such a commitment. Overall, in our model, the effects of LSAP II are slightly smaller — and considerably more uncertain — than a 50 bp cut in the short-term rate.

These results take as given the effects on the risk premium from the empirical literature discussed above. When we estimate directly the elasticity of the risk premium to the supply of bonds using data on long-term US Treasuries, we find a much smaller number than the empirical literature suggests. Our estimates imply that an asset purchase program of \$600 billion leads to a posterior median reduction in the risk premium of 13 bp (compared with 30 bp in the baseline experiment). As a consequence, the effects on GDP and inflation are also smaller, by about a factor of two.

Our conclusion is that the effects of LSAP programs on macroeconomic variables, such as GDP and inflation, are likely to be moderate. We also consider several robustness exercises and find that the effects on GDP growth are unlikely to exceed half a percentage point. The inflationary consequences of asset purchase programs are consistently very small. As a comparison, using the FRB/US model, Chung et al. (2011) find that LSAP II induced a reduction in the risk premium of only 20 bp but increased the level of GDP by about 0.6% and the inflation rate by 0.1%. Baumeister and Benati (2011), using a VAR with time-varying coefficients, consider a change in the term premium of 60 bp and estimate a median

increase in GDP growth of 3% and on inflation of 1%. We thus conclude that our results are more moderate than in the existing literature, especially compared to the VAR methodology. Importantly, our results only touch upon the positive dimension of LSAP programs. Harrison (2010) evaluates the macroeconomic consequences of the optimal amount of asset purchases in a version of this model without capital. His findings are consistent with ours in the sense that asset purchases can improve aggregate welfare, but their quantitative relevance appears to be limited.

Our results do not depend on whether asset purchases are financed via reserves or sales of short-term debt, to the extent that these two assets are close to perfect substitutes. Therefore, according to our model, the effects of the Federal Reserve's last round of asset purchases (also known as "Operation Twist Again") should be in line with the estimates from LSAP II after controlling for the scale factor.

The rest of the paper proceeds as follows. The next section presents the model. Section 3 discusses the data and the priors for the estimation. The baseline results and some robustness exercises are in Section 4. Section 5 introduces the quantity of debt in the estimation. Section 6 concludes.

# 2 Model

Two types of households, unrestricted (denoted by u) and unrestricted (denoted by r), populate the economy and supply differentiated labor inputs. Competitive labor agencies combine these inputs into a homogeneous composite. Competitive capital producers transform the consumption good into capital. Monopolistic competitive firms hire the labor composite and rent capital to produce intermediate goods. Competitive final goods producing firms package intermediate goods into a homogeneous consumption good. Finally, the government sets monetary and fiscal policy.

### 2.1 Households

The key modification relative to a standard medium-scale DSGE model (Christiano, Eichembaum and Evans, 2005; Smets and Wouters, 2007) is the introduction of segmentation and transaction costs in bond markets, as in ALSN.

A continuum of measure one of households populate the economy. Household j = u, r enjoys consumption  $C_t^j$  (relative to productivity  $Z_t$ , as in An and Schorfheide, 2007) and dislikes hours worked  $L_t^{j,6}$  Households supply differentiated labor inputs indexed by i but perfectly share consumption risk within each group. The life-time utility function for a generic households j is

$$\mathbb{E}_{t} \sum_{s=0}^{\infty} \beta_{j}^{s} b_{t+s}^{j} \left[ \frac{1}{1 - \sigma_{j}} \left( \frac{C_{t+s}^{j}}{Z_{t+s}} - h \frac{C_{t+s-1}^{j}}{Z_{t+s-1}} \right)^{1 - \sigma_{j}} - \frac{\varphi_{t+s}^{j} (L_{t+s}^{j}(i))^{1+\nu}}{1 + \nu} \right], \tag{2.1}$$

where  $\beta_j \in (0,1)$  is the individual discount factor,  $b_t^j$  is a preference shock,  $\sigma_j > 0$  is the coefficient of relative risk aversion,  $h \in (0,1)$  is the habit parameter,  $\nu \geq 0$  is the inverse elasticity of labor supply and  $\varphi_t^j$  is a labor supply shock.<sup>7</sup> The preference and labor supply shocks both follow stationary AR(1) processes in logs.

Two types of bonds exist. Short-term bonds  $B_t$  are one-period securities purchased at time t that pay a nominal return  $R_t$  at time t + 1. Following Woodford (2001), long-term bonds are perpetuities that cost  $P_{L,t}$  at time t and pay an exponentially decaying coupon  $\kappa^s$  at time t + s + 1, for  $\kappa \in (0, 1]$ . We abstract from money and consider the limit of a cashless economy as in Woodford (1998).

<sup>&</sup>lt;sup>6</sup>We express utility as a function of detrended consumption to ensure the existence of a balanced growth path with constant relative risk aversion preferences. Imposing log-utility of consumption may be an excessively restrictive assumption in our model which is mainly concerned about asset pricing.

<sup>&</sup>lt;sup>7</sup>We allow for heterogeneity in preference shocks, discount factors and coefficient of relative risk aversions because these factors affect the household's consumption-saving decisions and financial market segmentation directly influences these optimality conditions. As such, this heterogeneity can potentially influence the simulation results in a substantial way.

<sup>&</sup>lt;sup>8</sup>If  $\kappa = 1$ , this security is a consol.

The fraction  $\omega_u$  of unrestricted households trade in both short-term and long-term government bonds. Unrestricted households, however, pay a transaction cost  $\zeta_t$  per-unit of long-term bond purchased. The remaining fraction of the population  $\omega_r = 1 - \omega_u$  consists of restricted households who only trade in long-term bonds but pay no transaction costs.

The flow budget constraint differs depending on whether the household is unrestricted or restricted. For an unrestricted household that can trade both short and long-term bonds, we have

$$P_{t}C_{t}^{u} + B_{t}^{u} + (1 + \zeta_{t})P_{L,t}B_{t}^{L,u} \leq R_{t-1}B_{t-1}^{u} + \sum_{s=1}^{\infty} \kappa^{s-1}B_{t-s}^{L,u} + W_{t}^{u}(i)L_{t}^{u}(i) + \mathcal{P}_{t}^{u} + \mathcal{P}_{t}^{cp,u} - T_{t}^{u}. \tag{2.2}$$

For a restricted household who can only trade in long-term securities but does not pay transaction costs, we have

$$P_{t}C_{t}^{r} + P_{L,t}B_{t}^{L,r} \leq \sum_{s=1}^{\infty} \kappa^{s-1}B_{t-s}^{L,r} + \mathcal{P}_{t}^{r} + \mathcal{P}_{t}^{cp,r} + W_{t}^{r}(i)L_{t}^{r}(i) - T_{t}^{r}.$$
(2.3)

In equations (2.2) and (2.3),  $P_t$  is the price of the final consumption good,  $W_t^j(i)$  is the wage set by a household of type  $j = \{u, r\}$  who supplies labor of type i,  $\mathcal{P}_t^j$  and  $\mathcal{P}_t^{cp,j}$  are profits from ownership of intermediate goods producers and capital producers respectively, and  $T_t^j$  are lump-sum taxes.

One advantage of assuming that the entire stock of long-term government bonds consists of perpetuities is that the price in period t of a bond issued s periods ago  $P_{L-s,t}$  is a function of the coupon and the current price

$$P_{L-s,t} = \kappa^s P_{L,t}.$$

 $<sup>^9{</sup>m We}$  discuss in more detail the implications of transaction costs and bond market segmentation in sections 2.6.1 and 2.6.2 below.

Using this relation, we can rewrite the household budget constraint in a more convenient recursive formulation.<sup>10</sup> In particular, the budget constraint of an unrestricted household becomes

$$P_{t}C_{t}^{u} + B_{t}^{u} + (1 + \zeta_{t})P_{L,t}B_{t}^{L,u} \leq R_{t-1}B_{t-1}^{u} + P_{L,t}R_{L,t}B_{t-1}^{L,u} + W_{t}^{u}(i)L_{t}^{u}(i) + \mathcal{P}_{t}^{u} + \mathcal{P}_{t}^{cp,u} - T_{t}^{u}, \quad (2.4)$$

while for a restricted household we have

$$P_{t}C_{t}^{r} + P_{L,t}B_{t}^{L,r} \leq P_{L,t}R_{L,t}B_{t-1}^{L,r} + \mathcal{P}_{t}^{r} + \mathcal{P}_{t}^{cp,r} + W_{t}^{r}(i)L_{t}^{r}(i) - T_{t}^{r}, \tag{2.5}$$

where  $R_{L,t}$  is the gross yield to maturity at time t on the long-term bond<sup>11</sup>

$$R_{L,t} = \frac{1}{P_{L,t}} + \kappa.$$

Household j consumption-saving decisions are the result of the maximization of (2.1) subject to (2.4) if j = u or (2.5) if j = r. See the technical appendix for details and section 2.6 for some discussion.

#### 2.1.1 Labor Agencies and Wage Setting Decision

Perfectly competitive labor agencies combine differentiated labor inputs into a homogenous labor composite  $L_t$  according to the technology

$$L_t = \left[ \int_0^1 L_t(i)^{\frac{1}{1+\lambda_w}} di \right]^{1+\lambda_w},$$

where  $\lambda_w \geq 0$  is the steady state wage markup.

<sup>&</sup>lt;sup>10</sup>See the technical appendix for the derivations.

<sup>&</sup>lt;sup>11</sup>We choose  $\kappa$  to match the duration of this bond, given by  $R_{L,t}/(R_{L,t}-\kappa)$ , to the average duration of ten-year US Treasury Bills.

Profit maximization gives the demand for the  $i^{th}$  labor input

$$L_t(i) = \left[\frac{W_t(i)}{W_t}\right]^{-\frac{1+\lambda_w}{\lambda_w}} L_t. \tag{2.6}$$

From the zero profit condition for labor agencies, we obtain an expression for the aggregate wage index  $W_t$  as a function of the wage set by the  $i^{th}$  household

$$W_t = \left[ \int_0^1 W_t(i)^{-\frac{1}{\lambda_w}} di \right]^{-\lambda_w}.$$

Households are monopolistic suppliers of differentiated labor inputs  $L_t(i)$  and set wages on a staggered basis (Calvo, 1983) taking the demand as given. In each period, the probability of resetting the wage is  $1 - \zeta_w$ , while with the complementary probability the wage is automatically increased by the steady state rates of inflation ( $\Pi$ ) and of productivity growth ( $e^{\gamma}$ ),

$$W_{t+s}^{j}(i) = (\Pi e^{\gamma})^{s} \tilde{W}_{t}^{j}(i), \tag{2.7}$$

for s>0, where  $\tilde{W}_t^j(i)$  is the wage chose at time t in the event of an adjustment. A household of type j who can reset the wage at time t chooses  $\tilde{W}_t^j(i)$  to maximize

$$\mathbb{E}_{t} \sum_{t=0}^{\infty} (\beta_{j} \zeta_{w})^{s} \left[ \Xi_{t+s}^{j,p} \tilde{W}_{t}^{j}(i) L_{t+s}^{j}(i) - \frac{\varphi_{t+s}^{j} (L_{t+s}^{j}(i))^{1+\nu}}{1+\nu} \right],$$

where  $\Xi_t^{j,p}$  is the marginal utility of consumption in nominal terms, subject to (2.6) and (2.7). The technical appendix presents the first order condition for this problem.

# 2.2 Capital producers

Competitive capital producers make investment decisions, choose the utilization rate and rent capital to intermediate good producing firms. By choosing the utilization rate  $u_t$ , capital

producers end up renting in each period t an amount of "effective" capital equal to

$$K_t = u_t \bar{K}_{t-1}$$

Capital producers accumulate capital according to

$$\bar{K}_t = (1 - \delta) \, \bar{K}_{t-1} + \mu_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t,$$
 (2.8)

where  $\delta \in (0,1)$  is the depreciation rate,  $\mu_t$  is an investment-specific technology shock that follows a stationary AR(1) process in logs and  $S(\cdot)$  is the cost of adjusting investment (with  $S'(\cdot) \geq 0$  and  $S''(\cdot) > 0$ ).<sup>12</sup>

Capital producers discount future profits at the marginal utility of the average shareholder

$$\Xi_{t+s}^p \equiv \omega_u \beta_u^s \Xi_{t+s}^{u,p} + \omega_r \beta_r^s \Xi_{t+s}^{r,p}.$$

This variable is the appropriate discount factor of future dividends because ownership of capital producing firms is equally distributed among all households.<sup>13</sup> Capital producers maximize the expected discounted stream of dividends to their shareholders

$$\mathbb{E}_{t} \sum_{s=0}^{\infty} \Xi_{t+s}^{p} \left[ R_{t+s}^{k} u_{t+s} \bar{K}_{t+s-1} - P_{t+s} a(u_{t+s}) \bar{K}_{t+s-1} - P_{t+s} I_{t+s} \right],$$

subject to the law of motion of capital (2.8), where  $R_t^k$  is the return per unit of effective capital. Note that we assume that utilization subtracts real resources measured in terms of the consumption good,  $a(u_t)\bar{K}_{t-1}$ .<sup>14</sup>

<sup>&</sup>lt;sup>12</sup>Furthermore, we assume that  $S(e^{\gamma}) = S'(e^{\gamma}) = 0$ .

 $<sup>^{13}</sup>$ The same consideration applies below to intermediate goods producers.

<sup>&</sup>lt;sup>14</sup>As in Christiano, Motto and Rostagno (2009), we choose an implicit functional form for  $a(u_t)$  such that u = 1 in steady state and a(1) = 0.

## 2.3 Final Goods Producers

Perfectly competitive final goods producers combine differentiated intermediate goods  $Y_t(f)$ , supplied by a continuum of firms f of measure 1, into a homogeneous good  $Y_t$  according to the technology

$$Y_t = \left[ \int_0^1 Y_t(f)^{\frac{1}{1+\lambda_f}} df \right]^{1+\lambda_f},$$

where  $\lambda_f \geq 0$  is the steady state price markup. The resulting demand for the  $f^{th}$  intermediate good is

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_t. \tag{2.9}$$

From the zero profit condition for intermediate goods producers, we obtain an expression for the aggregate price index  $P_t$  as a function of the price set by the  $f^{th}$  intermediate good producer

$$P_t = \left[ \int_0^1 P_t(f)^{-\frac{1}{\lambda_f}} df \right]^{-\lambda_f}.$$

# 2.4 Intermediate goods producers

A continuum of measure one of monopolistic competitive firms combine rented capital and hired labor to produce intermediate goods according to a standard Cobb-Douglas technology

$$Y_t(f) = K_t(f)^{\alpha} (Z_t L_t(f))^{1-\alpha}, \qquad (2.10)$$

where  $Z_t$  is a labor-augmenting technology process which evolves according to

$$\log\left(\frac{Z_t}{Z_{t-1}}\right) = (1 - \rho_z)\gamma + \rho_z \log\left(\frac{Z_{t-1}}{Z_{t-2}}\right) + \epsilon_{z,t}.$$

Cost minimization yields an expression for the marginal cost which only depends on aggregate variables

$$MC(f)_t = MC_t = \frac{(R_t^k)^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} Z_t^{1-\alpha}}.$$
 (2.11)

Intermediate goods producers set prices on a staggered basis (Calvo, 1983). In each period, a firm can readjust prices with probability  $1 - \zeta_p$  independently of previous adjustments. We depart from the basic formulation of staggered price setting in assuming that the firms that cannot adjust in the current period index their price to the steady state inflation rate  $\Pi$ . The problem for a firm that can adjust at time t is to choose the price  $\tilde{P}_t(f)$  that maximizes

$$\mathbb{E}_{t} \sum_{s=0}^{\infty} \zeta_{p}^{s} \Xi_{t+s}^{p} \left[ \tilde{P}_{t}(f) \Pi^{s} - \lambda_{f,t+s} M C_{t+s} \right] Y_{t+s}(f),$$

subject to (2.9) conditional on no further adjustments after t, where  $\lambda_{f,t}$  is a goods markup shock that follows a stationary AR(1) process in logs.

# 2.5 Government Policies

The central bank follows a conventional feedback interest rate rule similar to Taylor (1993), but with interest rate smoothing and using the growth rate in output, instead of the output gap,

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_m} \left[ \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t/Y_{t-4}}{e^{4\gamma}}\right)^{\phi_y} \right]^{1-\rho_m} e^{\epsilon_{m,t}},$$

where  $\Pi_t \equiv P_t/P_{t-1}$  is the inflation rate,  $\rho_m \in (0,1), \ \phi_{\pi} > 1, \ \phi_y \geq 0$  and  $\epsilon_{m,t}$  is an i.i.d. innovation.

The presence of long-term bonds modifies the standard government budget constraint

$$B_t + P_{L,t}B_t^L = R_{t-1,t}B_{t-1} + (1 + \kappa P_{L,t})B_{t-1}^L + P_tG_t - T_t.$$
(2.12)

The left-hand side of expression (2.12) is the market value, in nominal terms, of the total amount of bonds (short-term and long-term) issued by the government at time t. The right-hand side is the total deficit at time t, that is, the cost of servicing bonds maturing in that period plus spending  $G_t$  net of taxes.

We assume that the government controls the supply of long-term bonds following a simple autoregressive rule for their detrended market value in real terms

$$\frac{P_{L,t}B_t^L}{P_t Z_t} = \left(\frac{P_{L,t-1}B_{t-1}^L}{P_{t-1}Z_{t-1}}\right)^{\rho_B} e^{\epsilon_{B,t}},\tag{2.13}$$

where  $\rho_B \in (0,1)$  and  $\epsilon_{B,t}$  is an i.i.d. exogenous shock. We interpret LSAP programs as shocks to the composition of outstanding government liabilities compared to the historical behavior of these series.

Finally, the government adjusts the real primary fiscal surplus in response to the lagged real value of long-term debt, as in Davig and Leeper (2006) and Eusepi and Preston (2011),

$$\frac{T_t}{P_t Z_t} - \frac{G_t}{Z_t} = \Phi \left( \frac{P_{L,t-1} B_{t-1}^L}{P_{t-1} Z_{t-1}} \right)^{\phi_T} e^{\epsilon_{T,t}}, \tag{2.14}$$

where  $\phi_T > 0$  and  $\epsilon_{T,t}$  follows a stationary AR(1) process. All fiscal variables in rule (2.14) are cyclically adjusted (i.e. expressed relative to the level of productivity) and the constant  $\Phi$  is such that in steady state the fiscal rule is just an identity.

# 2.6 Equilibrium and Solution Strategy

In equilibrium, households and firms maximize their objectives subject to their constraints and all markets clear. In particular, the resource constraint is

$$Y_{t} = \omega_{u} C_{t}^{u} + \omega_{r} C_{t}^{r} + I_{t} + G_{t} + a(u_{t}) \bar{K}_{t-1}.$$

We solve the model by taking a first-order log-linear approximation about a steady state in which quantities are normalized by the level of productivity  $Z_t$  and relative prices are expressed as function of  $P_t$ . The technical appendix shows the full set of non-linear normalized equilibrium relations, characterizes the steady state solution, and presents the full set of log-linearized equations that constitute the basis for the estimation.

These conditions are standard in modern DSGE models (Christiano, Eichembaum and Evans, 2005; Smets and Wouters, 2007) with the exception of the households' consumption-saving decisions. Here, we focus on these Euler equations to sharpen the intuition about the effects of segmentation in the bond market. This discussion should also clarify the channels through which asset purchase programs can support macroeconomic outcomes.

Since only unrestricted households trade in short-term bonds, the pricing equation for these securities is

$$1 = \beta_u \mathbb{E}_t \left[ e^{-\gamma - z_{t+1}} \frac{\Xi_{t+1}^u}{\Xi_t^u} \frac{R_t}{\Pi_{t+1}} \right], \tag{2.15}$$

where  $\Xi_t^u$  is the marginal utility of detrended consumption in real terms for an unrestricted household and  $e^{-\gamma-z_{t+1}}$  is the correction factor due to productivity growth.

Both unrestricted and restricted households trade long-term bonds. For unrestricted households, the pricing equation of these securities is

$$(1+\zeta_t) = \beta_u \mathbb{E}_t \left[ e^{-\gamma - z_{t+1}} \frac{\Xi_{t+1}^u}{\Xi_t^u} \frac{P_{L,t+1}}{P_{L,t}} \frac{R_{L,t+1}}{\Pi_{t+1}} \right]. \tag{2.16}$$

For constrained households, the pricing condition is

$$1 = \beta_r \mathbb{E}_t \left[ e^{-\gamma - z_{t+1}} \frac{\Xi_{t+1}^r}{\Xi_t^r} \frac{P_{L,t+1}}{P_{L,t}} \frac{R_{L,t+1}}{\Pi_{t+1}} \right]. \tag{2.17}$$

Restricted households have a different marginal utility of consumption and do not pay the transaction cost.

#### 2.6.1 Transaction Costs and the Risk Premium

The presence of transaction costs for unrestricted households in the market for longterm bonds gives rise to a risk premium. Using equation (2.16), we define  $R_{L,t}^{EH}$  as the counterfactual yield to maturity on a long-term bond at time t in the absence of transaction costs, given the same path for the marginal utility of consumption of unrestricted households. No arbitrage implies that this fictitious bond should have the same risk-adjusted return as the long-term security actually traded. We measure the risk premium as the difference between these two yields to maturity, up to a first order approximation

$$\hat{R}_{L,t} - \hat{R}_{L,t}^{EH} = \frac{1}{D_L} \sum_{s=0}^{\infty} \left( \frac{D_L - 1}{D_L} \right)^s \mathbb{E}_t \zeta_{t+s}, \tag{2.18}$$

where  $D_L$  is the steady state duration of the two securities.<sup>15</sup> Expression (2.18) shows that the risk premium in this economy equals the present discounted value of current and expected future transaction costs.

In ALSN, the risk premium has two components, one endogenous and one exogenous. The endogenous component arises because households face a portfolio adjustment cost, function of the relative quantity of money relative to long-term assets. The idea is that long-term bonds entail a loss of liquidity that households hedge by increasing the amount of money in their portfolio. The transaction costs in the market for long-term bonds are treated as purely exogenous.

We retain the distinction between endogenous and exogenous component of the risk premium while abstracting from the portfolio adjustment cost component. Instead, we directly assume that transaction costs are function of the detrended real value of long-term

 $<sup>^{15}</sup>$  The details of the derivation are in the technical appendix. In defining the yield to maturity of a bond in the absence of transaction costs, we adjust the parameter  $\kappa$  to guarantee that the fictitious security has the same steady state duration as the actual long-term bond.

bonds plus an error

$$\zeta_t \equiv \zeta \left( P_{L,t} B_{z,t}^L, \epsilon_{\zeta,t} \right), \tag{2.19}$$

where  $B_{z,t}^L \equiv B_t^L/(P_t Z_t)$ . We do not take a stand on the explicit functional form of  $\zeta(.)$ . We only require the function and its first derivative to be positive when evaluated in steady state (i.e.  $\zeta(P_L B_z^L, 0) > 0$  and  $\zeta'(P_L B_z^L, 0) > 0$ ). The first assumption ensures the presence of a positive steady state risk premium, as in the data. The second assumption guarantees that the yield on long-term bonds drops following a reduction in their outstanding amount. This element gives LSAP programs a chance to work through the mechanism identified in the reduced form estimates (Gagnon et al., 2011).

Up to a log-linear approximation, our parsimonious formulation of transaction costs is observationally equivalent to the two frictions in ALSN. The idea that transaction costs depend directly on the aggregate stock of bonds captures the same intuition (i.e. a liquidity cost) of the adjustment cost function in the original formulation.

#### 2.6.2 Limits to Arbitrage

The assumption of market segmentation captures, in reduced form, the observation that in reality some fraction of the population mostly saves through pension funds and other types of long-term institutional investors. These financial intermediaries are specialists in certain segments of the market and their transaction costs are likely to be small. Conversely, households who invest in long-term bonds mostly for diversification motives may face higher transaction costs. The parameter  $\omega_u$  measures this segmentation and is one of the key objects of interest in our estimation results.

The key implication of bond market segmentation is that not all agents in the model can take full advantage of arbitrage opportunities. Unrestricted households can arbitrage away,

 $<sup>^{16}\</sup>mathrm{See}$  Andres et al. (2004) for a more detailed discussion of this interpretation.

up to some transaction cost, differences in risk-adjusted expected returns between short and long-term bonds (equations 2.15 and 2.16) but restricted households do not have this possibility. Equation (2.17) fully characterizes the savings behavior of restricted households.

This friction provides a rationale for asset purchase programs to influence macroeconomic outcomes, thus breaking the irrelevance result in Wallace (1983). In particular, in our model, a program targeted to purchases of long-term securities reduces the risk premium (equation 2.19). Absent segmentation, this program would affect the yield to maturity of the long-term bond (equation 2.16), but it would have no effects on the real allocation. The intuition for this result is that this program reduces the risk premium of the long bond, changing the expected return on the long bond. Because the unrestricted households can invest in both bonds, they will reallocate their portfolios until the two expected returns are equated again, implying a different yield to maturity on the long bond, so that in the end their expected return (accounting for transaction costs) is unchanged. Because the expected returns are unchanged, there is no need to change the stochastic discount factor, and thus nothing else in this economy changes.

Conversely, with segmented bond markets, LSAP programs do affect the real economy. The change in long-term yields induces a change in the expected return of the restricted households, which are not subject to the transaction costs. Because the expected return is different from the restricted households' perspective, then it requires an adjustment in their stochastic discount factor. This change in turn leads to changes in their intertemporal profile of consumption (equation 2.17) and indirectly influences both the pricing decisions of intermediate producing firms and the investment decisions of capital producers. Ultimately, general equilibrium forces imply that consumption for both types of agents, investment and production respond as well.<sup>17</sup> The simulations in section 4 illustrate the magnitude of the

<sup>&</sup>lt;sup>17</sup>In practice, another effect of asset purchase programs could be the incentive for households to shift their portfolios toward riskier assets, such as equity and corporate bonds. In the model, this mechanism is absent as the equity shares are non-tradable.

LSAP stimulus on aggregate demand and inflation.

# 3 Empirical Analysis

We estimate the model with Bayesian methods, as surveyed for example by An and Schorfheide (2007). Bayesian estimation combines prior information on the parameters with the likelihood function of the model to form the posterior distribution. We construct the likelihood using the Kalman filter based on the state space representation of the rational expectations solution of the model.<sup>18</sup> In the remainder of this section, we first describe the data used and then present parameter prior and posterior distributions.

### 3.1 Data

We use quarterly data for the United States from the third quarter of 1987 (1987q3) to the third quarter of 2009 (2009q3) for the following six series: real GDP per capita, hours worked, real wages, core personal consumption expenditures deflator, nominal effective Federal Funds rate, and the 10-year Treasury constant maturity yield. All data are extracted from the Federal Reserve Economic Data (FRED) maintained by the Federal Reserve Bank of St.

<sup>&</sup>lt;sup>18</sup>We impose a zero posterior density for parameter values that imply indeterminacy, which is equivalent to a truncation of the joint prior distribution.

<sup>&</sup>lt;sup>19</sup>We use an extended sample, starting in 1959q3, to initialize the Kalman filter, but the likelihood function itself is evaluated only for the period starting in 1987q3, conditional on the previous sample.

Louis. The mapping of these variables to the states is

$$\Delta Y_t^{obs} = 100(\gamma + \hat{Y}_{z,t} - \hat{Y}_{z,t-1} + \hat{z}_t),$$

$$L_t^{obs} = 100 \left( L + \hat{L}_t \right),$$

$$\Delta w_t^{obs} = 100(\gamma + \hat{w}_{z,t} - \hat{w}_{z,t-1} + \hat{z}_t),$$

$$\pi_t^{obs} = 100(\pi + \hat{\pi}_t),$$

$$r_t^{obs} = 100(r + \hat{r}_t),$$

$$r_{L,t}^{obs} = 100(r_L + \hat{r}_{L,t}),$$

where all state variables are in deviations from their steady state values,  $\pi \equiv \ln(\Pi)$ ,  $r \equiv \ln(R)$ , and  $r_L \equiv \ln(R_L)$ .

We construct real GDP by dividing the nominal GDP series by population and the GDP deflator. The observable  $\Delta Y_t^{obs}$  corresponds to the first difference in logs of this series, multiplied by 100. We measure the labor input by the log of hours of all persons in the nonfarm business sector divided by population. Real wages correspond to nominal compensation per hour in the non-farm business sector, divided by the GDP deflator. As for GDP,  $\Delta w_t^{obs}$  is the first difference in logs of this series, multiplied by 100. The quarterly log-difference in the personal consumption expenditures (PCE) core price index is our measure of inflation. We use the effective Federal Funds Rate as our measure of nominal short-term rate and the 10-year Treasury constant maturity rate as our measure of nominal long-term interest rate.

In the baseline estimation, we do not use data on the quantity of debt. The reason is that aggregate debt at quarterly frequency may not be the most appropriate data to capture the implications of asset purchases on financial variables. For example, Gagnon et al. (2011) consider high frequency time series and even studies' methods to infer these relations. Furthermore, D'Amico et al. (2011) argue that the use of aggregate data biases

the results, weakening the effects of asset purchases on yields. Therefore, instead of trying to estimate the elasticity for the transmission of asset purchases onto the risk premium, we assign a prior for this parameter consistent with the results from the vast literature on high frequency estimation (time series and event studies included) reported in Table 1.<sup>20</sup>

A comparison between the prior and the posterior distributions of the elasticity of the risk premium to the quantity of debt confirms our conjecture that the data contains little information value for this parameter. As a robustness check, section 5 presents our main experiment with parameter estimates obtained using the quantity of debt in the hands of the private sector as an observable variable.

#### 3.2 Prior Choice

Tables 2 and 3 (columns two to five) summarize the prior distributions of each parameter. We use a Gamma distribution for the parameters that economic theory suggests should be positive to constrain their support on the interval  $[0, \infty]$ . For those parameters that span only the unit interval, we use the Beta distribution. For the standard deviation of shock innovations, we use the Inverse-Gamma distribution.

The steady state value for inflation is centered at 2%, in line with the mandate-consistent level of inflation commonly assigned to the Federal Open Market Committee (FOMC). The steady state growth rate is centered at 2.5% (annualized). The discount factor has a prior that implies a real interest rate of about 2% (annualized). The steady state spread between the 10-year treasury yield and the federal funds rate has a prior centered at 0.75% (annualized), similar to the average in the data.

We follow Del Negro and Schorfheide (2008) for the priors of standard parameters. The

<sup>&</sup>lt;sup>20</sup>We could have alternatively calibrated this elasticity but the wide range of estimates in Table 1 suggests little consensus on a specific value. Since this parameter significantly impacts our simulations, we prefer to take a more agnostic approach and allow for some uncertainty that encompasses most of the estimates in the reduced form literature.

investment adjustment cost convexity parameter S'' has prior mean of 4 and standard deviation of 1. The utilization cost elasticity parameter a'' has prior mean 0.2 and standard deviation 0.1, implying that in response to a 1% increase in the return to capital, utilization rates rise by about 0.17%. We calibrate the share of capital in production  $\alpha$  to 0.33, and the capital depreciation rate  $\delta$  to 2.5% per quarter.

The habit formation coefficient for both types of agents has prior mean of 0.6 and standard deviation 0.1, also fairly common in the literature. The parameter controlling the labor supply elasticity  $\nu$  has a prior centered at 2. Similarly to Smets and Wouters (2007), we estimate the intertemporal elasticity of substitution of consumption for households, except that in our model we have two types of agents. The prior on  $\sigma_u$  and  $\sigma_r$  is relatively flat (centered at 2 with standard deviation of 1) and equal for both types, so that the data can be informative about their value.

The fraction of unrestricted agents  $\omega_u$  is the crucial parameter to identify the degree of bond market segmentation in the model. At the mean, our prior implies that 70% of the households are unrestricted. A standard deviation of 0.2, however, makes the distribution flat enough that the 90% prior interval is (0.32,0.96). The other key parameter is the elasticity of the risk premium to changes in the market value of long debt  $\zeta'$ . The prior for this parameter has a mean 1.5/100 and a standard deviation big enough to match the range of estimates shown in Table 1 (see discussion above). The cash-flow parameter that controls the duration of long-term bonds (given the yield to maturity) is calibrated to imply a duration of 30 quarters, similar to the average duration in the secondary market for 10-year US Treasury bills. We consider short-term debt to include both government bonds with less than one year to maturity as well as central bank liabilities in the form of reserves, vault cash and deposits. In the US, the average for this quantity since 1974 is about 16% of annual GDP. For long-term bonds, we consider all government bonds with maturity greater than one year, which in the US is also about 16% of annual GDP since 1974.

Table 2 contains three non-standard parameters ( $\Xi^u/\Xi^r$ ,  $C^u/C^r$ , and  $\chi_{wu}$ ) which refer to steady state ratios hard to pin down directly from the data. We decided not to calibrate these ratios to avoid biasing the estimation and the simulations in either direction. The posterior distribution for these three parameters turns out to deviate negligibly from our prior. Furthermore, the uncertainty in these ratios translates into uncertainty in the dynamics of the model and in the effects of asset purchases on macroeconomic variables.

The priors for the wage and price rigidity parameters  $\zeta_w$  and  $\zeta_p$  are centered at values that imply an expected frequency of price and wage changes of three quarters (Justiniano, Primiceri and Tambalotti, 2010). The fiscal rule parameter,  $\phi_T$  is centered at 1.5 and its posterior does not differ too much from the prior. For the monetary policy rule, we consider fairly standard parameter priors. The interest rate smoothing parameter  $\rho_r$  is centered at 0.7. The response to output growth  $\phi_y$  is centered at 0.4. The prior mean for the response to inflation  $\phi_{\pi}$ , centered at 1.75, is slightly higher than the usual value of 1.5 in Taylor (1993). The 90% prior interval, however, is completely above one, consistent with the Taylor principle.

The shocks follow AR(1) processes, with autocorrelation coefficient  $\rho_i$  centered at 0.75, except for the autocorrelations of productivity shocks (equal to 0.4 so that the growth rate shock is not too persistent) and of the risk premium and debt shocks (equal to 0.8). The prior mean of the innovations have standard deviations  $\sigma_i$  centered at 0.5, except for the innovation to the monetary policy shock and the risk premium shock whose standard deviation is smaller because these variables refer to quarterly changes in interest rates.

### 3.3 Parameter Posterior Distribution

In order to obtain the posterior distribution, we first get the posterior mode and then generate a sample of the posterior distribution using the Metropolis random walk Markov

Chain Monte Carlo (MCMC) simulation method.<sup>21</sup> The last five columns of tables 2 and 3 report the posterior distribution of each parameter.

The first result that emerges from these tables is that the measure of market segmentation is very small — the posterior 90% interval for  $\omega_u$  is (0.713,0.992) with a median of 0.934 and a mode of 0.987. Given our prior, the data pushes against a model with a significant degree of market segmentation. Ceteris paribus, we should expect small macroeconomic effects of asset purchases. In order to check the stability of the estimate of market segmentation (and, in general, of other parameters), we reestimated the model with alternative samples. While in our baseline estimation the sample ends in 2009q3 (just before the first US LSAP program), we considered three alternative endings: 2007q2 (before the recent financial turbulence), 2008q3 (before the federal funds rate reached the ZLB) and 2011q2 (the most recent available data). The parameter estimates always remain very comparable.<sup>22</sup>

The other key parameter is the elasticity of the risk premium to asset purchases  $\zeta'$ . If this elasticity were zero, asset purchases would affect neither the risk premium nor the real economy. While we estimate this parameter, the posterior and prior distributions are very similar, implying that the data are not very informative about this elasticity. This finding is not very surprising, given that in the baseline estimation we observe changes in the spread, but we do not use any observable for the quantity of long-term debt. In the baseline simulation, this elasticity is perhaps better interpreted as summarizing the empirical results from reduced-form estimates and event studies incorporated in the prior distribution. In section 5, we estimate this parameter augmenting our set of observables with data on the ratio between long-term debt and short-term debt. Conditional on these data and our

<sup>&</sup>lt;sup>21</sup>After getting four separate chains of 100,000 draws, we compute the covariance matrix (with a 25% burnin) and generate four new chains of 100,000 draws. We repeat this step two more times with 200,000 and 500,000 draws, respectively. At this stage, we use these last four chains to extract the parameter posterior distribution properties and to simulate the effects of asset purchases.

<sup>&</sup>lt;sup>22</sup>One caveat is that most of our sample corresponds to a period of relative macroeconomic and financial stability in the US. Because the recent crisis may have exacerbated financial frictions, we subject our main experiment to a robustness check where we allow for an (exogenous) increase in the degree of segmentation.

model, the posterior distribution is concentrated at low levels, with a median of 0.41/100, suggesting a fairly small impact of the quantity of debt on the risk premium and the 10-year yield.

The sensitivity of consumption to the interest rate is estimated to be 3.2 for the unrestricted type and 2.2 for the restricted type at the posterior median. These numbers suggest a specification of utility far enough from the usual log-utility assumption but also significant heterogeneity in the sensitivity to the interest rate for the two types. Finally, the posterior moments for the nominal rigidity parameters and policy rule coefficients are consistent with several contributions in the DSGE literature. Importantly, price rigidities are estimated to be quite high relative to the micro-evidence. These parameters may significantly influence the simulations. Therefore, in the robustness analysis, we repeat our baseline experiment with  $\zeta_p$  set at the prior mean.

# 4 Simulating LSAP II

Our baseline experiment corresponds to a simulation of the US LSAP II program. The central bank buys long-term bonds (in exchange for short-term bonds) over the course of four quarters, holds its balance sheet constant for the following two years and progressively shrinks its holdings of long-terms securities over the final two years of the simulation. We calibrate the asset purchase program in the model so that the median reduction of the risk premium is 30 bp (annualized), which corresponds to the mid-point of the range of estimates discussed in Section 1 and presented in Table 1.<sup>23</sup> We choose this calibration strategy for this part of the simulation because our focus is on the macroeconomic consequences of the asset purchase programs. Later, we reestimate the model using data also on the quantity of long-term Treasuries to shed some light on the elasticity of the risk premium to the quantity

 $<sup>^{23}</sup>$ Our model formalizes the finding of Gagnon et al. (2011) that LSAP programs reduce long-term rates via their effect on the risk premium.

of debt. Figure 1 illustrates the path of the market value of long-term bonds in the hands of the private sector following the government purchases. Based on survey evidence from Blue Chip, we impose that the FFR is at the ZLB for the first four quarters (the "extended period" language) after the beginning of the asset purchase program.<sup>24</sup>

We begin by showing our main simulation of LSAP II at the prior distribution and then repeat the same experiment at the posterior. The following two subsections discuss the role of the commitment to the zero lower bound and how LSAP compares to interest rate policy shocks. The last subsection offers some robustness results.

#### 4.1 Simulation at the Prior Distribution

This section illustrates how the choice of the priors constrains the macroeconomic effects of asset purchase programs via Monte Carlo simulations. Specifically, we get 1000 random draws for the parameter vector using the prior distribution. We then use these draws to solve the model and extract the path of the state variables in response to the LSAP experiment described above. Finally, we compute moments and percentiles of this sample of responses for the variables of interest.<sup>25</sup>

Figure 2 shows the response of output growth, output level, inflation, FFR, 10-year yield and risk premium to the simulated LSAP II experiment at the prior distribution, all in annualized percentage rates. The level of output corresponds to percentage deviations from trend, as opposed to a rate of change, and thus is not annualized. These plots represent the marginal contribution of LSAP II, i.e. the deviations of each variable relative to the path that would have prevailed absent the government intervention. The red continuous line is the prior median response while the grey shaded area corresponds to the  $50^{th}$ ,  $60^{th}$ ,  $70^{th}$ ,

<sup>&</sup>lt;sup>24</sup>Blue Chip has been asking the survey participants about the expected duration of the ZLB since the end of 2008. Until the recent change in the Federal Open Market Committee language that introduced a specific date for the expected liftoff, market participants had always maintained the expectation that the FFR would remain at the ZLB for the four/five quarters after the question was asked.

<sup>&</sup>lt;sup>25</sup>The results are not sensitive to increasing the number of draws.

80<sup>th</sup> and 90<sup>th</sup> prior probability intervals, from darker to lighter shading respectively.

The figure shows the width of the response of the risk premium to asset purchases (essentially mapping the reduced-form/high-frequency range of estimates into our model) with a median response at the peak of about 30 bp. As a consequence of the change in the risk premium, output growth, the output level and inflation are higher than in the absence of asset purchases. By construction, the asset purchase program achieves the desired effect in the model. The key question is how big these effects are. Our prior is fairly generous, encompassing very large effects, but also relative agnostic, as to extract as much information as possible from the data without imposing too many ex-ante restrictions. To be more precise, the median prior response of output growth is about 2% and the median response of inflation is 0.5%, roughly in line with the results in Baumeister and Benati (2011). Using a vector auto regression (VAR) model with time-varying coefficients, these authors find that a 60 bp reduction in long-term rates increases GDP growth by 3% and the inflation rate of the GDP deflator by 1% at the posterior median.

Our prior may be seen as too generous to the extent that we allow the effects of LSAP to be potentially quite extreme (for example the  $95^{th}$  percentile is above 12% for GDP growth and about 4% for inflation). The literature, however, does not rule out these extreme outcomes. For example, Kapetanios et al. (2011) present VAR evidence for the effects of similar policies on GDP growth in the United Kingdom that can be as high as 5% at mean, depending on the estimation method. Our choice of fairly uninformative priors gives the model a chance to generate such large effects.

In response to higher output and inflation, the central bank eventually increases the interest rate in accordance with the policy rule, but only after the end of the commitment to the ZLB. The evolution of the 10-year yield reflects the combined effect of the responses of the risk premium and the expected future short-term interest rate (expectations hypothesis). The former puts negative pressure on the long yield while the latter exerts the opposite

pressure. The outcome depends on how effective asset purchases ultimately are in boosting the economy. If LSAP programs have a significant effect on output and inflation, the policy rule dictates a strong response of the federal funds rate which can potentially dominate over the negative impact on the risk premium and lead to an equilibrium increase in the 10-year yield.

### 4.2 Simulation at the Posterior Distribution

In the previous subsection, we concluded that, according to this model and our choice of the priors for the parameters, LSAP programs can boost output and inflation while the effect on the 10-year yield is somewhat ambiguous, depending on the interplay between the risk premium and the expectation hypothesis. In this section, we combine the prior with the data for the past twenty years or so to form a posterior distribution of the parameters. We then use the posterior to revisit our simulations of the effects of LSAP II. Figure 3 shows the same variables and simulation as Figure 2, but now using parameter draws from the posterior distribution.

On impact, GDP growth increases by 0.4% at the posterior median. The uncertainty is skewed on the upside to almost 2%, partly due to the ZLB. After three quarters, the effect on output growth is less than a half of its peak (which occurs on impact) and completely vanishes after eight quarters. The effects on the level of output level are also modest. The peak in this case occurs after 8 quarters at about 0.3% (posterior median), but now the effects persist longer — after 24 quarters, the output level is still 0.25% higher than without asset purchases. This modest but persistent effect on GDP level is likely to be important from a welfare perspective — even more so if we consider that the 90<sup>th</sup> probability interval allows for an increase in the level of modest as high as 1.5%. The effect on inflation is very small, 3 bp (annualized) at the median. The effect on inflation is also skewed upward but

even the  $95^{th}$  percentile is only about 15 bp. Our results are at the lower end of the spectrum in the existing literature. Beside the estimates in Baumeister and Benati (2011) mentioned earlier, Chung et al. (2011), using the FRB/US model, assume that LSAP II induces a reduction in the risk premium of only 20 bp but increases the level of GDP by about 0.6% and inflation rate by 0.1%.

In spite of the small magnitudes, the positive boost of asset purchases to GDP growth and inflation puts upward pressure on the interest rate. After the four quarters at the ZLB which correspond to the commitment period, the FFR becomes positive but the median increase is only 8 bp. Because asset purchases introduce little stimulus, the central bank does not raise interest rates by much upon exiting the ZLB. The upward skewness in the FFR reflects the skewness in the effects on GDP growth and inflation. Later, we disentangle the effects of the non-linearity introduced by the ZLB from the pure estimation uncertainty.

The drop in the 10-year yield is a combination of the reduction in the risk premium, which is the assumption behind the experiment, and the expected increase in future short-term rates. The long-term rate decreases but less than the 30 bp drop in the risk premium, because the private sector correctly anticipates future short-term rates to increase. Notice that the posterior uncertainty on the term premium is skewed downward, which we will discuss further below.

To summarize, the effects of LSAP II on GDP and inflation are moderate, especially compared to the prior. Nevertheless, the effect on the output level is very persistent. The main reason for the modest effects is that the estimated degree of segmentation is small. The posterior median for  $\omega_u$  is 0.93, that is, less than 7% of the population is excluded from the short-term bond market. Given that in the model segmentation is crucial for asset purchases to have an effect, not surprisingly, the boost from government purchases of long-term bonds is small. The posterior distribution of the degree of segmentation, however, is skewed to the left, implying that we cannot completely discard the possibility of higher segmentation.

The posterior mean is below the median by two percentage points and the  $5^{th}$  percentile is 0.71 (Table 2). Together with the ZLB, this long left tail of the posterior estimates for  $\omega_u$  contributes to the upward skewness of the response of GDP growth and inflation in the baseline simulation.

The other parameter that potentially plays an important role in determining the effects of asset purchases on the macroeconomy is the elasticity  $\zeta'$  of the risk-premium to debt. Unfortunately, this parameter is not well identified in our baseline estimation. In section 5 we repeat our estimation augmenting the set of observables with the quantity of long-term debt to address this problem. For now, we note that our prior for the elasticity, which essentially coincides with the posterior, also contributes to the overall skewness of the response of macroeconomic variables documented above.

### 4.3 The Role of the ZLB

In this subsection, we show that the commitment of the central bank to keep the short-term nominal interest rate at the ZLB for an "extended period" amplifies the effects of LSAP II. According to our simulations, asset purchases boost GDP growth and increase inflation, thus leading the central bank to increase the FFR. This endogenous interest rate response mitigates the macroeconomic stimulus of asset purchase programs through the conventional monetary policy channel. A commitment to keep the short-term nominal interest rate at the ZLB for an "extended" period of time prevents the endogenous response of the monetary authority and magnifies the contribution of asset purchases on macroeconomic outcomes. Here, we quantify the magnitude of such a mitigation effect.

Figure 4 shows the responses in the case in which we do not impose the commitment to the zero lower bound. For reference, the dashed blue line corresponds to the baseline simulation with the commitment to the ZLB imposed. Quantitatively, the ZLB commitment roughly

doubles the effects of asset purchases. Absent this commitment, output growth increases by 0.2% and inflation by only 2 bp — which correspond to about half of the baseline effects.

Interestingly, while the profile for the FFR differs from the baseline, the 10-year yield is almost identical. The cumulative effect of the increase in short rates on long rates via the expectation-hypothesis component is the same. Without ZLB, the increase in nominal rates occurs earlier but is smoother.

Importantly, the responses to LSAP II remain skewed upward, regardless of whether the ZLB is imposed or not. This observation suggests that the role of the skewness in the posterior distribution of the degree of segmentation and of the semi-elasticity of the risk premium to the quantity of debt play a central role in explaining the upside uncertainty of the response of macroeconomic variables.

A ZLB commitment of four quarters essentially doubles the effects of LSAP on output and inflation. Recently, however, the Federal Reserve has extended its commitment to keep the nominal interest rate at zero for a longer period.<sup>26</sup> Figure 5 reproduces the main simulation considering a commitment to keep the interest rate at zero for six quarters. In this case, the effects on output and inflation are more than doubled, even though the commitment period only increased by two quarters. This result shows very clearly that in this model the ZLB commitment is very powerful in stimulating the economy, due to the strongly forward looking behavior of the agents in the economy, and its effects increase non-linearly with the number of quarters of the commitment.

The uncertainty bands also widen substantially. For example, the  $95^{th}$  percentile for output growth increased from just under 2% to just above 8%, and the same percentile at the peak effect on the output level increased from just above 1.5% to just under 7%. This more pronounced skewness further confirms the non-linear effects of the ZLB commitment in combination with the skewed distribution of the degree of segmentation and elasticity of

 $<sup>^{26}\</sup>mathrm{At}$  the time we are writing this paper, the commitment is for at least six quarters

risk premium to the asset purchases.

# 4.4 Comparison with a Standard Monetary Policy Shock

One of the motivations for central banks to engage in asset purchases is to support output and inflation at times in which the ZLB constrains conventional interest rate setting. To give a sense of the relative effectiveness of these two policies, this section compares the effects of asset purchase programs discussed so far with a standard monetary policy shock, that is, an unexpected reduction of the short-term nominal interest rate.

Figure 6 shows the response of the key macroeconomic variables to an unexpected reduction of 50 bp in the short-term interest rate. The median effects on GDP growth and inflation are somewhat stronger than those implied by the baseline simulation previously discussed. Furthermore, the effects of the interest rate shock on the output level are not only stronger but also more persistent than those of LSAP. GDP growth increases by 0.5% and inflation by 4 bp. The implied decrease in long-term rates, however, is much smaller, only 2 bp.<sup>27</sup> Moreover, the long-term rate quickly turns positive. This result is not surprising given that the risk premium does not change. Therefore, the expectation hypothesis component completely pins down the long-term interest rate in this case.

Another significant difference is the reduced uncertainty about the effects in the case of an interest rate shock. The absence of the ZLB constraint may in part explain why the posterior bands are more symmetric. Yet, as discussed in the previous subsection, even in the absence of a commitment to the ZLB, uncertainty remains skewed upward, mostly due to the skewness of the posterior estimates of the degree segmentation and of the semi-elasticity of the term premium to the quantity of debt. Because segmentation plays a smaller role in case of a shock to the short-term interest rate, the uncertainty in the response of GDP

<sup>&</sup>lt;sup>27</sup>Our estimates thus imply a much smaller sensitivity of long-term rates to shocks to the short-term rate. As a point of comparison, Gurkaynak, Sack and Swanson (2005) estimate that the typical response of long rates to a cut of 100 bp in the FFR is 15 bp.

growth and inflation becomes smaller and much more symmetric.

Overall, in this model, the effects of LSAP II on output and inflation are slightly smaller than those of a surprise reduction of the FFR by 50 bp, and much more uncertain. This conclusion stands in contrast with Furher and Moore (1995), who find that output is four times more sensitive to long-term than short-term rates. According to this metric, the 30 bp reduction in the risk premium should be equivalent to a reduction of the FFR of about 120 bp. Our results are thus much less generous to changes in the risk premium, confirming our previous finding that the model simulations yield weaker effects of LSAP II on output and inflation than what the VAR literature suggests.

#### 4.5 Robustness

This section considers three robustness exercises. First, we ask how sensitive the model is to the degree of market segmentation. Second, we study the role of nominal rigidities. Third, we consider the implications of extending the duration of the LSAP program.

#### 4.5.1 The Role of Market Segmentation

The baseline experiment suggests that the effects of LSAP II are fairly moderate on GDP and quite small on inflation. One reason why our results may underestimate the effects of asset purchase programs is that the degree of financial market segmentation may have recently increased due to the financial crisis.<sup>28</sup> As discussed earlier, our reduced-form friction for market segmentation aims at capturing a combination of preferences for certain asset classes and institutional restrictions on the type of investments certain financial intermediaries undertake. By shifting the true and perceived distribution of risk, the financial crisis

<sup>&</sup>lt;sup>28</sup>Baumeister and Benati (2010) estimate a VAR with time-varying coefficients and stochastic volatility to account for this type of effects, on top of other changes in the structural relations among macroeconomic variables potentially triggered by the financial crisis.

may have induced a fraction of investors previously active in multiple segments of financial markets to concentrate on one particular asset class.

For this purpose, we repeat the baseline experiment in the presence of a high degree of market segmentation. Figure 7 shows the results of the same simulated LSAP II experiment as in the baseline case. The difference is that, in this case, we only draw from the lower half of the posterior distribution of the parameter  $\omega_u$ . All other parameters are drawn from the same posterior distribution as before.

The median responses of GDP growth and inflation with the ZLB commitment are roughly twice as large as in the baseline case, at +0.7% and +0.06% respectively (compared to 0.4% and 0.03%). Upside posterior uncertainty is now more pronounced. The  $95^{th}$  percentile now nearly reaches 4% for GDP growth and 0.4% for inflation, compared to 2% and 0.15% before. The stronger response of macroeconomic variables requires the central bank to increase the short-term nominal interest rate by about 8 bp more. As a consequence, given that the drop in the risk premium is the same, long-term rates decrease slightly less (of the order of 4 bp).

The bottom line from this exercise is that allowing for a higher degree of segmentation does increase the response of GDP growth and inflation to the stimulus of asset purchase programs. However, unless segmentation becomes really extreme, the macroeconomic effects of LSAP remain relatively small, especially for inflation.

#### 4.5.2 The Role of Nominal Rigidities

One reason for the small response of inflation to LSAP II is that the estimated degree of nominal rigidities, especially for prices, is quite high. While our priors for the probability of holding prices and wages fixed in any given period ( $\zeta_p$  and  $\zeta_w$ ) are both centered at 0.75, the posterior medians for the two parameters are 0.94 and 0.88, respectively.

A high degree of stickiness in prices and wages is not an uncommon finding in the DSGE

literature, especially in the absence of real rigidities like in our case. Additionally, Hall (2011) has recently emphasized how prices have failed to fall substantially in the last recession. As in the case of segmentation, the financial crisis may have caused a structural change in the price setting process that the model interprets as an increase in price rigidities (the same consideration applies to wages).<sup>29</sup>.

Nevertheless, we want to quantify the sensitivity of our results to a lower degree of nominal rigidities, more in line with standard values from the empirical literature that uses micro data (e.g. Nakamura and Steinsson, 2008). Figure 8 shows the results of the baseline LSAP II experiment when we fix  $\zeta_p$  at its prior mean, equal to 0.75. All other parameters are drawn from the same posterior distribution as before.

The figure shows that nominal rigidities play an important quantitative role in the response of inflation to asset purchases. When prices are more flexible, the median response of inflation to LSAP II is three times larger on impact than when we use the estimated posterior distribution. The counterparts are a less persistent inflation process and a slightly smaller effect on GDP growth. In equilibrium (i.e., taking into account the endogenous response of monetary policy), the two effects roughly compensate each other. The increase in the short-term interest rates is almost the same in the two cases. Therefore, also the behavior of long-term rates is very similar. The increase in upside uncertainty for inflation is roughly proportional to the changes in the median. The  $95^{th}$  percentile of the response in inflation is 0.5%, compared to just above 0.15% in the baseline experiment.

In sum, higher price flexibility shifts the adjustment in response to asset purchase programs from GDP growth to inflation, by making its process more front-loaded.

<sup>&</sup>lt;sup>29</sup>Indeed, if we consider a sample that ends before the recent crisis (second quarter of 2007), the posterior median for  $\zeta_n$  is somewhat smaller

### 4.5.3 The Role of the Length of LSAP Programs

In our baseline simulation, the central bank accumulates assets over four quarters and holds the balance sheet constant for the next two years, before gradually winding down the program over two additional years. This assumption is fairly arbitrary. Depending on economic conditions, policy makers may change the length of the programs, as the recent US and UK experience suggests. Without undertaking an exhaustive analysis, this subsection considers one alternative path: the central bank still accumulates assets over the first year but then holds the balance-sheet constant for four years, instead of two, before gradually exiting. Figure 9 shows the corresponding responses, in the same format as the previous figures.

Not surprisingly, this change in the time profile of the asset holdings by the central bank induces a stronger response by the risk premium, with a median peak response of -43 bp (instead of -30 bp). As a result, output and inflation respond more strongly. However, while the inflation response is more than twice as big relative to the baseline scenario (median response at the peak of 0.07%, compared to 0.03%), the response of output is only modestly stronger (median response of 0.50%, instead of 0.37%, for GDP growth). However, the boost to the level of GDP becomes more persistent, peaking at 0.5% only after fifteen quarters (instead of eight quarters in the baseline). Not surprisingly, the uncertainty around the median is larger, with the 95<sup>th</sup> percentiles increasing proportionally.

In sum, if the central bank holds the purchased assets for longer, the stimulative impact on output and inflation increases and becomes more persistent. Moreover, the additional boost is stronger for inflation than for output. Nominal rigidities plays an important role in this respect. Because the shock lasts longer, more firms and workers are expected to change their prices and wages over time, which in turn leads the firms and workers who can change their prices and wages early to do so more aggressively.

# 5 Estimating the Model with Data on Debt Quantity

In the simulations presented so far, we have calibrated our LSAP II experiment so that the asset purchase program reduces the risk premium by 30 bp. This calibration is consistent with the effect of the quantity of debt on long-term rates estimated in the empirical literature (e.g. Gagnon et al., 2011). Because in the baseline estimation the semi-elasticity of rates to the quantity of debt is not well-identified, our results completely rely on the prior for this parameter, which is heavily informed by the reduced-form estimates available in the literature. By introducing the quantity of debt among the set of observables, we can hope to better identify this elasticity. In particular, the observed variable for debt in the model is the ratio between long-term and short-term US Treasury bonds, where the threshold maturity to distinguish between the two forms of liabilities is one year.<sup>30</sup>

For the new experiment, we maintain our baseline assumption about the evolution of government purchases of long-term bonds illustrated in Figure 1. The difference with the baseline simulation is that, in this case, we calibrate the size of the asset purchase program directly on the quantity, that is, we target a \$600 billion reduction of long-term debt in the hands of the private sector.

Figure 10 displays the results. The effects of LSAP II are smaller than in the baseline case by a factor of two for both GDP growth (+0.14%) and inflation (+0.015%). The results are entirely driven by a smaller semi-elasticity of the risk premium to the quantity of debt. At the posterior median, an asset purchase program of \$600 billion implies a reduction of about 13 bp in the risk premium, compared to the 30 bp of our previous experiment. Contrary to the baseline estimation, the semi-elasticity appears to be well-identified.

While uncertainty remains upward skewed, the posterior bands do not change exactly in proportion to the median. The  $95^{th}$  percentile for GDP growth is +0.6%, compared to about

<sup>&</sup>lt;sup>30</sup>This assumption is consistent with the announcement of LSAP II.

+2% in the baseline case. For inflation, the decline is from +0.15% to +0.06%. Because GDP growth and inflation move less, so does the FFR. A smaller increase of short-term rates, coupled with a smaller decrease in the risk premium, implies that the 10-year yield also drops less, from about 0.26% to about 0.12%.

Our results for the sensitivity of risk premia to the quantity of long-term debt in the hands of the private sector indicate that the estimates in the empirical literature may represent an upper bound. Of course, a structural change due to the disruption of financial markets associated with the recent crisis may explain why reduced-form estimates, especially if based on event studies, find higher values for this elasticity. Even allowing for this possibility, asset purchase programs of the size of LSAP II are probably unlikely to contribute for more than half a percentage point to GDP growth and five bp for inflation.

### 6 Conclusions

The effects of recent asset purchase programs on macroeconomic variables, such as GDP growth and inflation, are likely to be moderate but with a lasting impact on the level of GDP. We reach this conclusion after simulating LSAP II in an estimated medium-scale DSGE model, similar to Smets and Wouters (2007). Asset purchase programs are in principle effective at stimulating the economy because of limits to arbitrage and market segmentation between short-term and long-term government bonds. The data, however, provide little support for these frictions to be pervasive.

We consider several robustness exercises and find that the effects on GDP growth are not very likely to exceed half a percentage point. The inflationary consequences of asset purchase programs are even smaller. Combining LSAP programs with a commitment to keep interest rates low for some period of time allows these programs to be about twice as effective in boosting GDP growth and inflation.

Our results do not depend on whether asset purchases are financed via reserves or sales of short-term debt, to the extent that money and short-term bonds are close to perfect substitutes. Therefore, according to our model, the effects of the Federal Reserve's last round of asset purchases (also known as "Operation Twist Again") should be in line with the estimates from LSAP II after controlling for the scale factor.

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Table 1: Estimated Impact of LSAPs on the 10-Year Treasury Yield in the literature.

Papers	Total Impact	Impact per \$100 Bil
Hamilton and Wu (2010)	-13 bp	-3 bp
Doh (2010)	-39 bp	-4 bp
D'Amico and King (2010)	-45  bp	-15  bp
Bomfim and Meyer (2010)	-60 bp	-3 bp
Gagnon et al. (2011)	-58 bp to- $91$ bp	-3 bp to- $5$ bp
Neely (2011)	-107  bp	-6 bp
Krishnamurthy and Vissing-Jorgensen (2011)	-33 bp (LSAP2)	-5 bp
D'Amico et al. (2011)	-55 bp (LSAP2)	-9 bp
Swanson (2011)	-15 bp (Twist)	

Table 2: Parameter prior and posterior distribution: structural parameters.

	Prior				Posterior				
	Dist	5%	Median	95%	Mode	Mean	5%	Median	95%
$400\gamma$	G	1.7382	2.4667	3.3752	2.0317	2.0157	1.5722	2.0087	2.4814
$400\pi$	G	1.2545	1.9585	2.8871	2.5634	2.5838	1.9292	2.5807	3.2407
$400(\beta_u^{-1} - 1)$	G	0.6272	0.9792	1.4436	0.6994	0.7850	0.5190	0.7768	1.0745
$400\zeta$	G	0.3913	0.7224	1.2029	0.6261	0.7609	0.4040	0.7373	1.1995
S''	G	2.5090	3.9170	5.7743	4.3392	4.7148	3.2347	4.6481	6.4254
a''	G	0.0683	0.1836	0.3877	0.1725	0.2096	0.0850	0.1931	0.3892
h	В	0.4302	0.6029	0.7597	0.7388	0.7626	0.6216	0.7702	0.8781
$\sigma_u$	G	0.6832	1.8360	3.8768	3.2522	3.3099	1.8778	3.1781	5.2041
$\sigma_r$	G	0.6832	1.8360	3.8768	1.5767	2.3182	0.9159	2.1753	4.2176
$100\zeta'$	G	0.3067	1.2846	3.4294	0.8538	1.7312	0.3526	1.4929	3.9085
$\omega_u$	В	0.3214	0.7334	0.9646	0.9872	0.9051	0.7131	0.9336	0.9924
$\Xi^u/\Xi^r$	G	0.3416	0.9180	1.9384	0.7836	1.1429	0.4337	1.0726	2.0901
$C^u/C^r$	G	0.3416	0.9180	1.9384	0.7765	1.0620	0.3912	0.9920	1.9598
$\chi_{wu}$	В	0.2486	0.6143	0.9024	0.6914	0.5895	0.2494	0.6031	0.8877
u	G	1.2545	1.9585	2.8871	1.5728	1.8523	1.1603	1.8092	2.7035
$\zeta_w$	В	0.5701	0.7595	0.8971	0.8194	0.8751	0.7968	0.8780	0.9411
$\zeta_p$	В	0.5701	0.7595	0.8971	0.9402	0.9436	0.9294	0.9438	0.9570
$\dot{\phi_T}$	G	0.7825	1.4448	2.4058	1.2329	1.4772	0.7755	1.4209	2.3627
$ ho_r$	В	0.5242	0.7068	0.8525	0.8453	0.8557	0.8190	0.8563	0.8902
$\phi_\pi$	G	1.0164	1.7026	2.6453	1.7598	1.7790	1.4585	1.7711	2.1303
$\phi_y$	G	0.1366	0.3672	0.7754	0.2943	0.3318	0.2533	0.3270	0.4269

Table 3: Parameter prior and posterior distribution: shock process parameters.

		F	Prior		Posterior					
	Dist	5%	Median	95%	Mode	Mean	5%	Median	95%	
$\overline{ ho_z}$	В	0.0976	0.3857	0.7514	0.1441	0.1672	0.0506	0.1617	0.3039	
$ ho_{\mu}$	В	0.5701	0.7595	0.8971	0.8503	0.8676	0.8084	0.8691	0.9218	
$ ho_b$	В	0.5701	0.7595	0.8971	0.9697	0.9628	0.9366	0.9665	0.9794	
$ ho_\phi$	В	0.5701	0.7595	0.8971	0.5193	0.4707	0.3389	0.4714	0.5991	
$ ho_B$	В	0.6146	0.8135	0.9389	0.8465	0.8278	0.6298	0.8459	0.9592	
$ ho_{\zeta}$	В	0.6146	0.8135	0.9389	0.9454	0.9077	0.7724	0.9264	0.9656	
$ ho_g^{\cdot}$	В	0.5701	0.7595	0.8971	0.7889	0.7449	0.5690	0.7514	0.8977	
$\sigma_z$	IG1	0.1663	0.3433	1.2367	0.7451	0.7643	0.6737	0.7604	0.8671	
$\sigma_{\lambda_f}$	IG1	0.1663	0.3433	1.2367	2.5283	3.1068	1.7820	2.9146	5.0335	
$\sigma_{\mu}$	IG1	0.1663	0.3433	1.2367	2.6621	2.8190	2.0830	2.7858	3.6875	
$\sigma_b$	IG1	0.1663	0.3433	1.2367	4.7382	6.6897	3.7437	6.2078	11.2378	
$\sigma_{\phi}$	IG1	0.1663	0.3433	1.2367	1.4242	6.0173	1.1346	3.9569	18.2035	
$\sigma_B$	IG1	0.1663	0.3433	1.2367	0.2352	0.5506	0.1717	0.3883	1.5258	
$\sigma_T$	IG1	0.1663	0.3433	1.2367	0.2351	0.4774	0.1679	0.3452	1.3151	
$\sigma_m$	IG1	0.0819	0.1700	0.6217	0.1150	0.1164	0.1022	0.1159	0.1325	
$\sigma_{\zeta}$	IG1	0.0819	0.1700	0.6217	0.2385	0.2642	0.1321	0.2564	0.4178	
$\sigma_g$	IG1	0.1663	0.3433	1.2367	0.2446	0.4124	0.1723	0.3638	0.8140	

Figure 1: Simulated path of the market value of long-term debt, in percentage deviations from trend. (Calibrated to imply a median reduction of 30bp (annualized) in the risk premium.

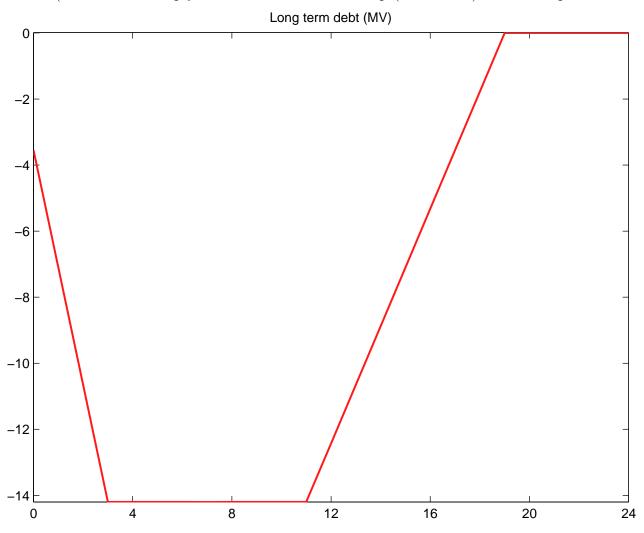


Figure 2: Responses to simulated shock to market value of long-term debt (shown in Figure 1), accounting for a commitment to the zero lower bound for four quarters, using the prior distribution. All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). Red line corresponds to the prior median response and the grey shades to different prior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading).

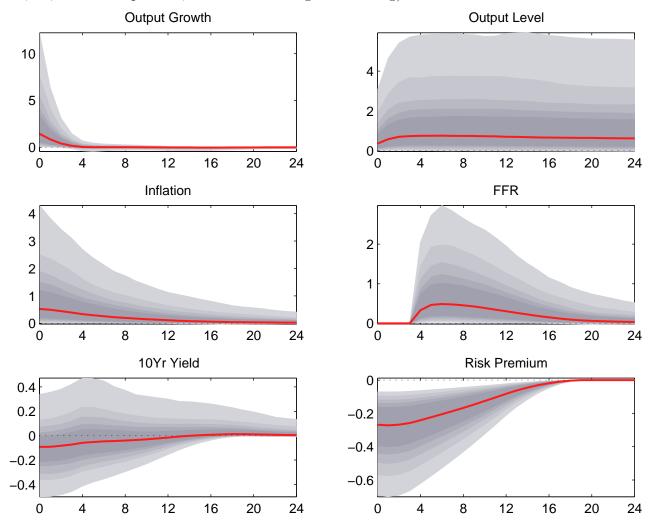


Figure 3: Responses to simulated shock to market value of long-term debt (shown in Figure 1), accounting for a commitment to the zero lower bound for four quarters, using the posterior distribution. All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). Red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading).

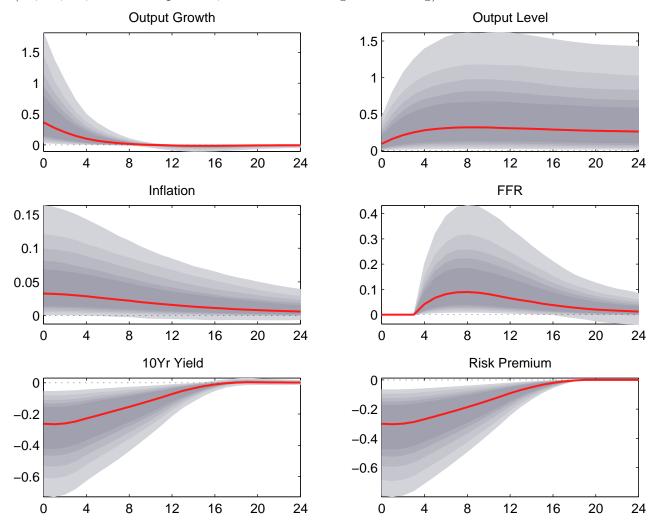


Figure 4: Responses to simulated shock to market value of long-term debt (shown in Figure 1), in the absence of a commitment to the zero lower bound. All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). Red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the median response of the variables in the baseline simulation, shown in 3.

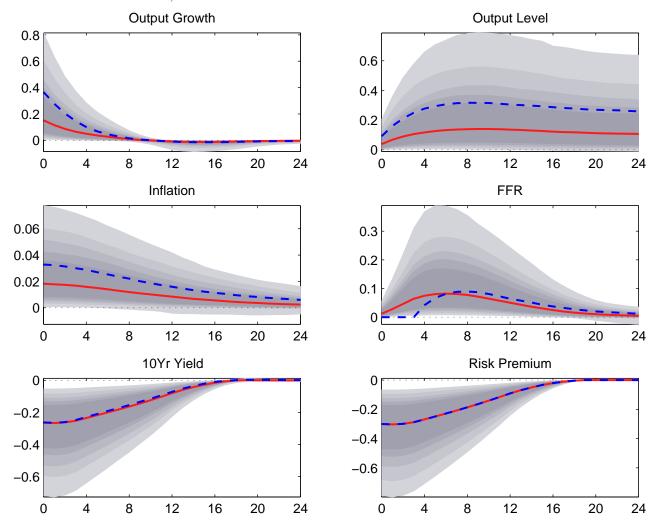


Figure 5: Responses to simulated shock to market value of long-term debt (shown in Figure 1), accounting for a commitment to the zero lower bound for six quarters. All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). Red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the median response of the variables in the baseline simulation, shown in 3.

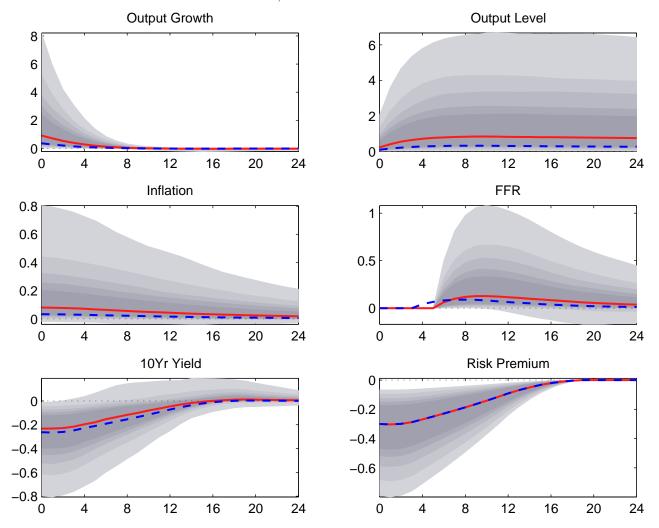


Figure 6: Responses to 50bp (annualized) temporary reduction in the FFR. All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). Red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the median response of the variables in the baseline simulation, shown in 3.

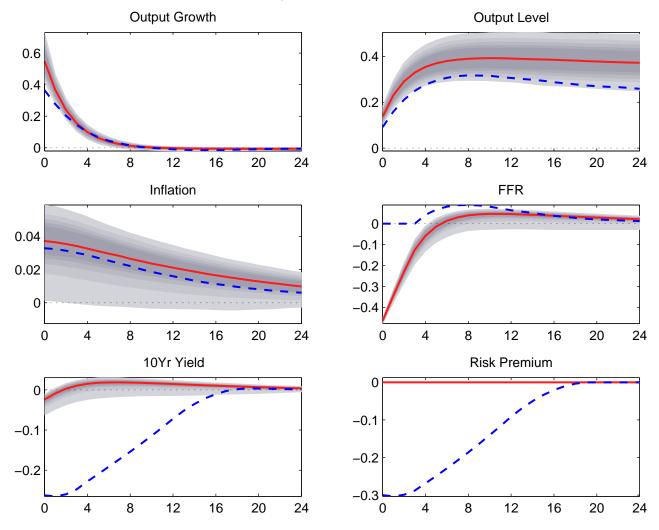


Figure 7: Responses to simulated shock to market value of long-term debt (shown in Figure 1) in the presence of a high degree of market segmentation (by considering the lower half of the distribution of  $\omega_u$ ). All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). Red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the median response of the variables in the baseline simulation, shown in 3.

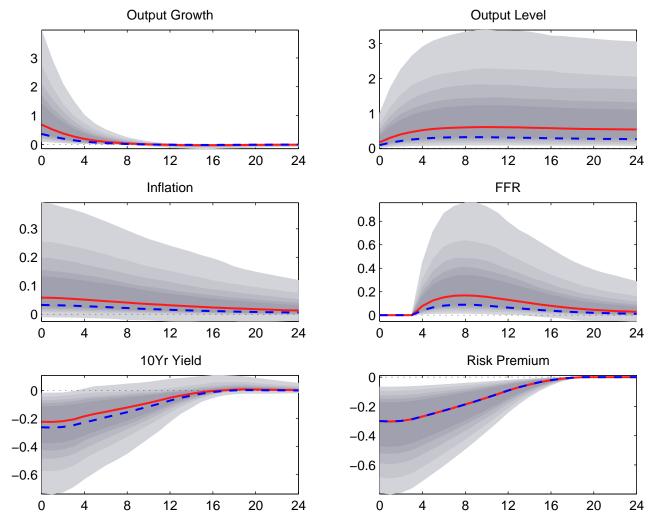


Figure 8: Responses to simulated shock to market value of long-term debt (shown in Figure 1) in the presence of lower price rigidities ( $\zeta_p = 0.75$ ). All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). Red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the median response of the variables in the baseline simulation, shown in 3.

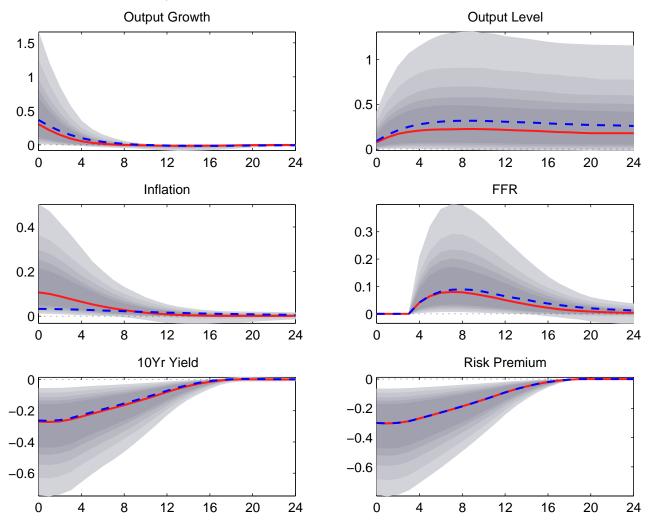


Figure 9: Responses to simulated shock to market value of long-term debt (shown in Figure 1) in the case in which the central bank keeps the purchased assets for four years (instead of two). All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). Red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the median response of the variables in the baseline simulation, shown in 3.

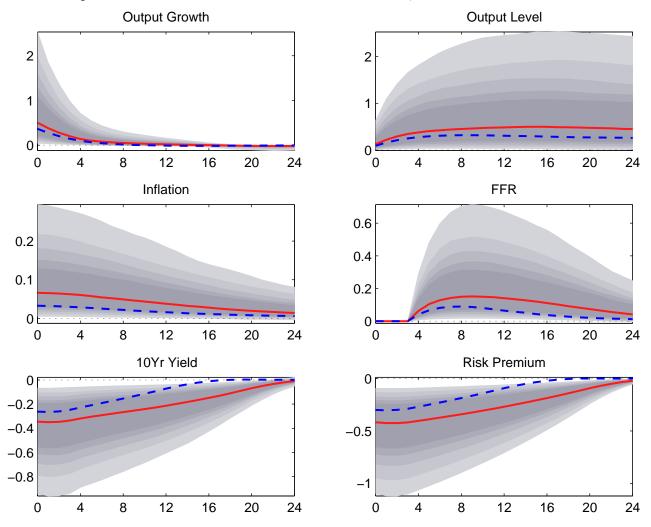


Figure 10: Responses to simulated shock to market value of long-term debt in the case of estimation with observed ratio between long and short debt. Path of shock is the same as in Figure 1) but the size of the shock is chosen to simulate a \$600bn reduction in long-term bonds available to the public. All responses are in annualized percentage rates (except the output level, shown in percentage deviations from the path in the absence of the shock). Red line corresponds to the posterior median response and the grey shades to different posterior probability intervals (50, 60, 70, 80 and 90 percent, from darker to lighter shading). The dashed blue line is the median response of the variables in the baseline simulation, shown in 3.

