The Magnification for Optical Parametric Image Conversion in Thin Nonlinear Materials*

Ichiro SAKURABA**
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Contents

Abstract

The relation between the longitudinal, transverse and angular magnifications in optical parametric image conversion was presented. These obey relations quite similar to the Maxwell elongation and Smith-Helmholtz formulas in geometrical optics. The fundamental equation for analyses of spherical aberrations was also given.

1. Introduction

The up-conversion of infrared light to the visible by means of an optical parametric process in a nonlinear crystal has been studied in detail by several investigators.

In the range of infrared wavelengths that can be upconverted for a given pump wavelength, Warner[1] discussed the directional characteristics and the required geometry for phase-matched up-conversion in terms of the orientations of the various wave vectors with respect to the crystalline axis. Midwinter[2] proposed a means of broad-band, high resolution and high efficiency image conversion and pointed out that an image up-converter competitive with existing scanning photo-detectors (such as InSb detectors) in the 1 to 10 μm band can be built. Firester[3] demonstrated that the effect of the pump beam divergence did not degrade the resolution but changed the transverse and longitudinal magnification of the image by a paraxial ray-tracing analysis. Also, he discussed the dependence on the system parameters of the converted image resolution, location and chromatic aberration in the case where the pump is at infinity by the Fourier transform theory[4]. The factors that limit the resolution of the upconverted image, the efficiency of up-conversion and several characteristics of six nonlinear crystals were analyzed by Andrews[5].

However, the general questions of aberrations have not been considered[6]. The purpose of this paper is to show a fundamental equation and the relation between the longitudinal, transverse and angular magnifications for optical parametric interaction in thin nonlinear materials.

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** Department of Electronic Engineering, Hokkaido University, Sapporo, Japan.

2. Optical Parametric Image Conversion in Infinitesimally Thick Nonlinear Materials

Image up-conversion is possible in materials where the dielectric susceptibility coefficients which are frequency-dependent tensor elements, are sensitive to directions of the various optical field and propagation vectors with respect to crystal axes. The first order nonlinearity $\chi^{(2)}$ couples signal and pump waves and generates a nonlinear polarization P_i at the idler frequency f_i

$$P_i = \chi^{(2)} E_p E_s, \quad (1)$$

in the case of perfectly phase-matched up-conversion

$$f_i = f_p + f_s, \quad (2)$$

$$\boldsymbol{k_i} = \boldsymbol{k_p} + \boldsymbol{k_s}, \quad (3)$$

and E_p and E_s are the electric fields at pump frequency f_p and signal frequency f_s, respectively, and $\boldsymbol{k}$ is the corresponding wave vector. For a crystal of a width w the phase-matched up-conversion takes place as long as[7]

$$|\Delta \boldsymbol{k}| = |\boldsymbol{k_p} + \boldsymbol{k_s} - \boldsymbol{k_i}| \leqq 2\pi/w \quad (4)$$

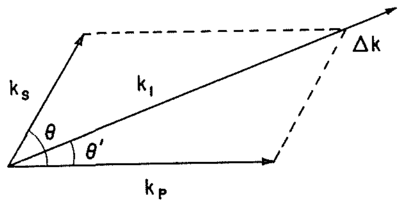

Fig. 1 Schematic diagram of up-conversion geometry.

where for convenience it is assumed that $\Delta \boldsymbol{k}$ is along the direction of $\boldsymbol{k_i}$ (see Fig. 1). This polarization radiates a wave at a frequency f_i which produces the upconverted image.

For paraxial signal waves, the angle θ, which the signal wave makes with the optic axis (the pump beam direction), and the angle θ' which the idler wave makes with the optic axis are related by[3]

$$\theta' = \frac{\lambda_i}{\lambda_s}\theta \quad (5)$$

where λ_i and λ_s are the optical wave lengthes at frequencies f_i and f_s, respectively.

Consider an infinitesimal thickness of isotropic, perfectly phase-matched, nonlinear material that is perpendicular to a line connecting a point object and point-source pump. This thin plate is a distance x_p from the pump of frequency f_p and a distance x_s from the object of frequency f_s as shown in Figs. 2 and 3. The distance from O to the point of interaction Q is y. It is assumed that the spherical wavefronts originating at points P and S intersect at point Q and they

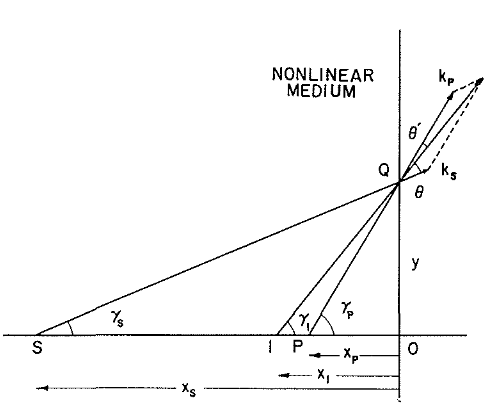

Fig. 2 Imaging of an axial object point.

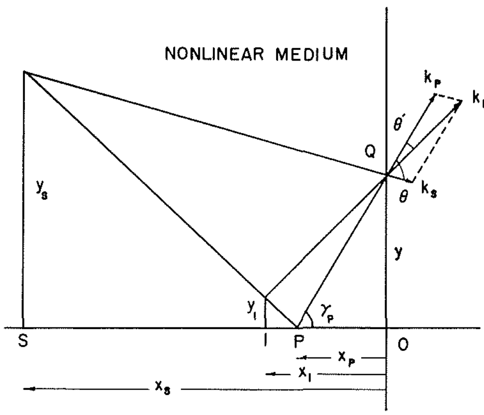

Fig. 3 Imaging of an off axis object point.

can be represented by wave vectors $\boldsymbol{k_p}$ and $\boldsymbol{k_s}$, respectively. These wave vectors are normal to the respective wavefronts at point Q. The quantities $\boldsymbol{k_p}$ and $\boldsymbol{k_s}$ make angles of γ_p and γ_s, respectively, with the line $\overline{SPO}$. The parametric interaction produces a third wavefront with wave vector $\boldsymbol{k_i}$ as given in Eq. (3). The parametric interaction is assumed to be phase-matched for all angles and the nonlinear material is assumed to have an infinitesimal thickness negligible as compared to the other dimensions inverted. Using the geometry shown in Figs. 1 and 2 this gives

$$x_i = \frac{\lambda_p x_s \sqrt{x_p^2+y^2} + \lambda_s x_p \sqrt{x_s^2+y^2}}{\lambda_p \sqrt{x_p^2+y^2} + \lambda_s \sqrt{x_s^2+y^2}}. \tag{6}$$

This result shows that the idler wave is not a spherical wave. Assuming all angles are small, it can be seen from the geometry of Figs. 2 and 3 that the following relations hold

$$\gamma_p - \theta' = \frac{y-y_i}{x_i}, \qquad \gamma_p - \theta = \frac{y-y_s}{x_s}, \tag{7}$$

$$\theta = \gamma_p - \frac{y}{x_s}, \qquad \gamma_p = \frac{y}{x_p}, \qquad \frac{y}{x_i} = \frac{y}{x_s} + \theta - \theta', \tag{8}$$

Combining these equations it follows that

$$\frac{y_i}{y_s} = \frac{\lambda_i x_i}{\lambda_s x_s} = \frac{x_p \lambda_i}{x_p \lambda_i + x_s(\lambda_s - \lambda_i)}, \tag{9}$$

which shows that the position x_i is independent of y and hence the image point and the parametric interaction produces an idler spherical wave front. Therefore, it may be seen that within the Gaussian geometric optics, axial and non-axial object points are indeed reimaged by a nonlinear upconverter employing a divergent point pump source and that by ray-tracing techniques the longitudinal, transverse and angular magnification of an upconverter can be defined. From Eq. (9) it follows that, for fixed objects and image planes[8],

$$\left(\frac{dy_i}{dy_s}\right)_{x=\mathrm{const}} = \frac{x_p \lambda_i}{x_p \lambda_i + x_s(\lambda_s - \lambda_i)}. \tag{10}$$

This quantity is the transverse magnification M_t. Further, the longitudinal magnification M_l becomes

$$M_l = \frac{dx_i}{dx_s} = \frac{x_p^2 \lambda_i \lambda_s}{[x_p \lambda_i + (\lambda_s - \lambda_i) x_s]^2}. \tag{11}$$

Since the transverse magnification depends on x but not on y it follows that an object which is situated in a plane perpendicular to the axis will be transformed into one which is geometrically similar to it. The angular magnification M_a in the Gaussian approximation can be defined as

$$M_a = \frac{\tan \gamma_i}{\tan \gamma_s} = \frac{\gamma_i}{\gamma_s} = \frac{x_s}{x_i} = \frac{x_p \lambda_i + x_s(\lambda_s - \lambda_i)}{\lambda_s x_p}, \tag{12}$$

where γ_s is the angle which the signal ray makes with the axis and γ_i is the angle which the idler ray makes with the axis.

3. The Relation of Magnifications

From Eqs. (10) and (11), it may be seen that the two magnifications are related by the formula

$$M_l = M_t^2 \frac{\lambda_s}{\lambda_i}. \tag{13}$$

It implies that the longitudinal magnification is equal to the square of the transverse magnification multiplied by the ratio of the signal wavelength to the idler wavelength, λ_s/λ_i. This is quite similar to Maxwell's elongation formula[8] in the geometrical theory of optics when an equivalent refractive index is defined as $n_{si} = \lambda_s/\lambda_i$. From Eqs. (10) and (12) it also follows that

$$M_t M_a = \frac{\lambda_i}{\lambda_s} = \frac{1}{n_{si}}. \tag{14}$$

The product of the transverse and the angular magnification is equal to the inverse of the equivalent refractive index. It should be noted that this relation is quite similar to the result from the Smith-Helmholtz formula in the geometrical optics[8]. From Eqs. (13) and (14) we obtain

$$M_l M_a = M_t. \tag{15}$$

The similarity between the above results in parametric interactions and equations from the Newton equation in the geometrical optics is apparent.

4. Resolution

When the optical systems and the structure of pump and signal wavefronts are given it is possible to analyze the effects that tend to limit the resolution of an image up-converter[2),5)]. In the case where the pump can be thought as a simple point source, there is a signal pump wavefront. At each point in the nonlinear crystal the pump wave vector has a unique direction. Hence at each point in the crystal the pump divergence is equal to zero and the angular resolution approaches infinity. The resolution due to the thickness of the nonlinear crystal also equals infinity in the infinitesimal thickness of nonlinear materials.

The resolution limits are due to higher order effects[5]. In Eqs. (6) and (9) it was shown that in general two spherical wavefronts interacting in a plane produce an idler spherical wavefront only in the paraxial ray-tracing analysis. Hence there will be spherical aberrations due to nonplane wave interactions. Since for a finite aperture all waves have a divergence due to the diffraction limit, spherical wavefronts are always present and there will always be a finite value of the linear resolution for some finite thickness of nonlinear crystals. The dispersion in the nonlinear material will cause a distortion similar to that produced by the crystal thickness. Each of these effects will produce higher order limits on resolution.

5. Conclusions

The relation between M_l, M_t and M_a in optical parametric image conversion was shown as $M_l = M_t^2 n_{si}$, $M_t M_a = 1/n_{si}$ and $M_l M_a = M_t$, where n_{si} is the equivalent refractive index defined as the ratio of λ_s to λ_i. These are quite similar to the Maxwell elongation and Smith-Helmholtz formulas. The fundamental equation for analyses of spherical aberrations was also presented.

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