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**The Magnitude and Cyclical Behavior of Financial  
Market Frictions**

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# The Magnitude and Cyclical Behavior of Financial Market Frictions\*

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## Abstract

We quantify the cross-sectional and time-series behavior of the wedge between the cost of external and internal finance by estimating the structural parameters of a canonical debt-contracting model with informational frictions. For this purpose, we construct a new dataset that includes balance sheet information, measures of expected default risk, and credit spreads on publicly-traded debt for about 900 U.S. firms over the period 1997Q1 to 2003Q3. Using nonlinear least squares, we obtain precise time-specific estimates of the bankruptcy cost parameter and consistently reject the null hypothesis of frictionless financial markets. For most of the firms in our sample, the estimated premium on external finance was very low during the expansionary period 1997–99, but rose sharply in 2000—especially for firms with higher ratios of debt to equity—and remained elevated until early 2003.

JEL CLASSIFICATION: D82, E22, G32

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# 1 Introduction

In light of the Modigliani–Miller (1958) theorem, macroeconomic modeling has largely abstracted from the influence of firms’ financing decisions on the evolution of the real economy. As demonstrated by Bernanke and Gertler (1989), however, the magnitude and persistence of business cycle fluctuations can be amplified by informational asymmetries in credit markets that induce a wedge between the cost of external and internal funds—the external finance premium. Empirical research on the financial accelerator mechanism has analyzed the influence of cash flow and net worth on firm-level investment spending but has provided no direct evidence on the magnitude or cyclical properties of financial market frictions.<sup>1</sup>

In this paper, we quantify the cross-sectional and time-series behavior of the external finance premium by estimating the structural parameters of a canonical debt-contracting model with asymmetric information. For this purpose, we construct a dataset that includes balance sheet variables, measures of expected default risk, and credit spreads on publicly-traded debt for about 900 U.S. nonfinancial firms over the period 1997Q1 to 2003Q3. Our sample is representative of the broader economy: When we aggregate the key variables in our dataset—including sales growth and the debt-equity ratio—the time-series pattern of each sample aggregate tracks closely its corresponding series for the U.S. nonfinancial business sector as a whole.

This new dataset enables us to estimate the microeconomic debt-contracting framework underlying the financial accelerator model of Bernanke, Gertler, and Gilchrist (1999).<sup>2</sup> In this framework, the size of the external finance premium depends on the bankruptcy cost parameter—which quantifies the fraction of the firm’s value that is lost in the event of default—as well as on the firm’s debt-equity ratio and expected default probability. An appealing feature of the BGG formulation is that frictionless financial markets correspond to the special case of zero bankruptcy costs.

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<sup>1</sup>See Hubbard (1998) for extensive discussion of research regarding the influence of financial frictions on capital spending, and Kashyap, Lamont, and Stein, (1994) for a related analysis of inventory investment.

<sup>2</sup>BGG embed the costly state verification debt-contracting problem of Townsend (1979, 1988) and Gale and Hellwig (1985) into a dynamic stochastic general equilibrium model with nominal inertia. Other recent formulations of financial market frictions in general equilibrium models include, for example, Fuerst (1995), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Kvarn (2002), and Cooley, Marimon, and Quadrini (2004). Extensions to open-economy settings include Krugman (1999), Aghion, Bacchetta, and Banerjee (2000), Cespedes, Chang, and Velasco (2000), and Gertler, Gilchrist, and Natalucci (2003).

Using nonlinear least squares, we obtain precise time-specific estimates of the bankruptcy cost parameter and consistently reject the null hypothesis of frictionless financial markets. For the expansionary period through late 1999, these estimates imply expected bankruptcy costs between 10 to 20 percent, roughly in line with the calibrated parameter values used by BGG and others. Our estimates also fall within the relatively wide range of liquidation costs (ranging from 5 to 70 percent) associated with actual bankruptcy proceedings (cf. Altman (1984) and Alderson and Betker (1995)). Although the estimated magnitude of bankruptcy costs is statistically different from zero, we find that the 1997–99 period is associated with a very low model-implied external finance premium for nearly all firms in our sample. This outcome mainly reflects the relatively low levels of leverage and expected default probabilities observed over this period.<sup>3</sup>

In contrast to the results for the 1997–99 period, we find that the magnitude of financial market frictions rose dramatically during 2000—prior to the onset of the recession—and remained at elevated levels until early 2003. Our estimates of the bankruptcy cost parameter for this period lie in the range of 30 to 50 percent, more than twice as high as those for the preceding expansionary period. According to our analytical framework, the pronounced run-up in expected bankruptcy costs is necessary to explain the marked widening of corporate credit spreads observed at a time when market-based measures of expected default probability rose only moderately.

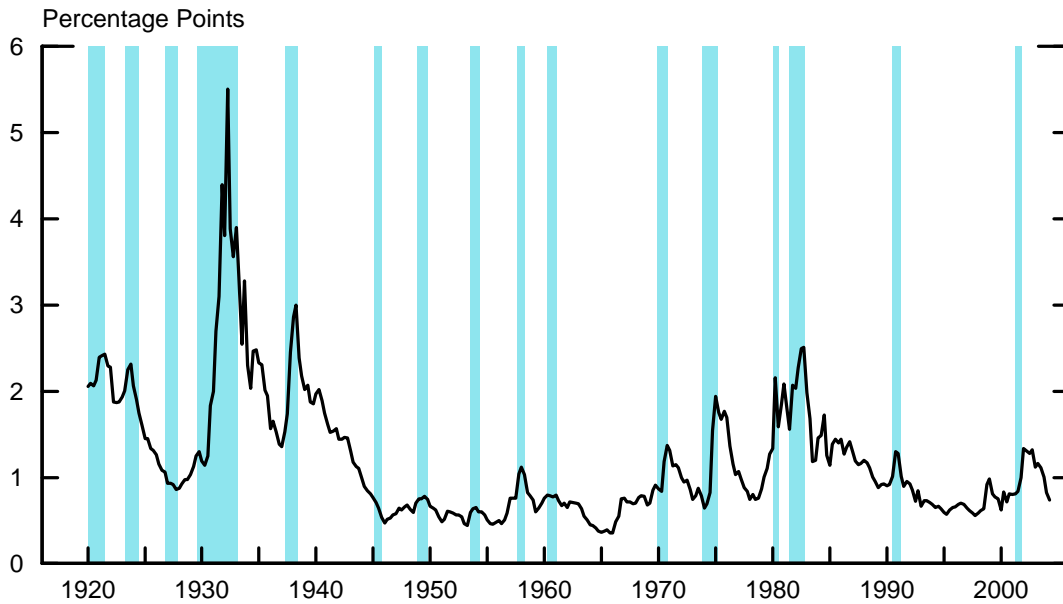
Our analysis indicates that the rise in expected bankruptcy costs was the key factor behind the sharp increase in the model-implied external finance premium in 2000. Over the course of the year, the premium increased more than 100 basis points for the sales-weighted median firm, and it jumped at least 300 basis points for one-fourth of the firms in our sample. The subsequently elevated level of the external finance premium may have contributed to the sharp contraction in capital expenditures during the 2001–02 period, which occurred despite the substantial decline in long-term real interest rates associated with the easing of monetary policy.

Given that our sample consists of relatively large firms with publicly-traded equity

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<sup>3</sup>The model-implied external finance premium for the 1997–99 period is noticeably smaller than the steady-state value obtained by BGG. This difference arises because the sales-weighted median firm in our sample had a debt-equity ratio of about 20 percent and an expected year-ahead default probability of about 0.5 percent during this period, whereas the representative firm in BGG was assumed to have a debt-equity ratio of unity and an annualized default probability of 3 percent.

Figure 1: Historical Evolution of U.S. Corporate Credit Spreads



NOTES: The solid line depicts the difference in yields between the lowest-rated (Baa) and highest-rated (Aaa) investment-grade corporate bonds. The shaded vertical bars denote NBER-dated recessions.

and debt, these results provide strong support for the macroeconomic significance of financial market frictions. Previous empirical research on this mechanism has typically proceeded under the assumption that such firms have relatively unimpeded access to external financing, especially compared with smaller firms that rely on bank-intermediated credit and that may sometimes be shut out of credit markets altogether.<sup>4</sup> Thus, to the extent that firms in our sample experienced substantial movements in the external finance premium in the period surrounding the most recent cyclical downturn, it is likely that financial market frictions had pervasive effects across the entire business sector. Indeed, one may presume that small and medium-sized firms faced even larger swings in the external finance premium or in the extreme case, a loss of access to credit.

Finally, while our sample only includes a single economic downturn, the available evidence suggests that the recent behavior of credit spreads was not unusual

<sup>4</sup>See Gertler and Gilchrist (1994) for evidence of how the composition of external financing varies with firm size.

by historical standards. For example, Figure 1 depicts the evolution of the spread between the yields of the highest-rated (Aaa) and lowest-rated (Baa) categories of investment-grade securities.<sup>5</sup> During the 2000–01 period, this yield spread rose about 100 basis points, an increase comparable to the rise in the model-implied external finance premium for the sales-weighted median firm in our sample. Furthermore, the magnitude of this swing is not particularly large compared with other post-war business cycles and looks quite mild relative to the widening of the spread during the Great Depression.<sup>6</sup>

The remainder of the paper is organized as follows. Section 2 specifies the microeconomic debt-contracting framework and examines the implications of changes in bankruptcy costs. Section 3 provides an overview of the data, and Section 4 outlines the estimation methodology. Section 5 presents our estimates of the key structural parameters, while Section 6 analyzes the behavior of the model-implied external finance premium. Section 7 discusses several outstanding issues revealed by our analysis. Section 8 concludes.

## 2 The Theoretical Framework

In this section, we present the theoretical framework used to assess the empirical magnitude and cyclical behavior of financial market frictions. While closely following the microeconomic debt-contracting model of BGG, our notation allows for a fairly general degree of heterogeneity across borrowers, which we incorporate into our empirical methodology. As in BGG, we abstract from considerations related to the issuance of equity or multi-period debt.<sup>7</sup>

### 2.1 The Debt-Contracting Problem

*The Entrepreneur’s Expected Return.* At the end of period  $t$ , the entrepreneur who manages firm  $i$  purchases physical capital  $K_{it}$  at price  $Q_t$  for use in production in the following period. Realized revenues in period  $t + 1$  are given by  $\omega_{i,t+1} R_{it}^k Q_t K_{it}$ , where

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<sup>5</sup>Further information on the cyclical properties of various yield spreads may be found in Stock and Watson (1989), Friedman and Kuttner (1992, 1998), and Gertler and Lown (1999).

<sup>6</sup>The role of financial factors in the Great Depression was originally emphasized by Fisher (1933) and has recently been considered by Bernanke (2000) and Christiano, Motto, and Rostagno (2004).

<sup>7</sup>See Gertler (1992) and von Thadden (1995) for theoretical analysis of optimal debt contracts in multi-period settings.

$R_{it}^k$  is the entrepreneur-specific expected gross rate of return on capital investment,  $Q_t$  is the price of capital (identical for all entrepreneurs), and  $\omega_{i,t+1}$  is the idiosyncratic productivity shock. The productivity disturbance is distributed according to the probability density function  $f(\omega | \theta_{it})$  parameterized by the vector  $\theta_{it}$ , and it has an expected value of unity; that is,  $\int_0^\infty \omega f(\omega | \theta_{it}) d\omega = 1$ , for all  $i$  and  $t$ . Note that we allow the parameter vector  $\theta_{it}$  to vary across entrepreneurs and time.

The expected return  $R_{it}^k$  is taken as given by the individual entrepreneur but may differ across projects due to cross-sectional variation in expected total factor productivity. Because each entrepreneur can alternatively deposit her net worth with a financial intermediary, the active investment projects must have expected return  $R_{it}^k$  that exceed the gross risk-free real interest rate  $R_t$ .

In addition to investing her own net worth, the entrepreneur can resort to external financing to leverage the project:

$$Q_t K_{it} = N_{it} + B_{it}, \quad (1)$$

where  $N_{it}$  denotes the entrepreneur's net worth and  $B_{it}$  denotes the amount borrowed from a risk-neutral financial intermediary; the resulting leverage ratio is then given by  $B_{it}/N_{it}$ . The financial intermediary observes the entrepreneur's expected return  $R_{it}^k$  and the parameter vector  $\theta_{it}$  but cannot directly observe the idiosyncratic disturbance  $\omega_{i,t+1}$ . Under these informational assumptions, the optimal financing arrangement specifies the loan amount  $B_{it}$  along with the gross contractual interest rate  $R_{it}^b$ ; importantly, the terms of this debt contract do not involve the realization of the idiosyncratic productivity shock  $\omega_{i,t+1}$ .

Given the terms of the debt contract, the entrepreneur's realized profit is given by  $\omega_{i,t+1} R_{it}^k Q_t K_{it} - R_{it}^b B_{it}$ . Whenever the revenue from the project is insufficient to cover the debt obligation, the entrepreneur defaults on the loan and walks away with zero profit; that is, default occurs when the idiosyncratic productivity shock falls below the threshold  $\omega_{it}^*$  satisfying the following condition:

$$\omega_{it}^* R_{it}^k Q_t K_{it} = R_{it}^b B_{it}. \quad (2)$$

Evidently, the debt contract is incentive-compatible: When the idiosyncratic shock exceeds the threshold  $\omega_{it}^*$ , the entrepreneur earns positive profit by repaying the loan and keeping the remaining revenue from the project.

Using condition (2), the entrepreneur's realized profit is  $(\omega_{i,t+1} - \omega_{it}^*)R_{it}^k Q_t K_{it}$  in the absence of default, and zero otherwise. Thus, *ex ante* expected profit can be expressed as  $\psi_{it} R_{it}^k Q_t K_{it}$ , where  $\psi_{it}$  denotes the entrepreneur's expected return as a fraction of total proceeds from the project:

$$\psi_{it} \equiv \psi(\omega_{it}^* | \theta_{it}) = \int_{\omega_{it}^*}^{\infty} (\omega - \omega_{it}^*) f(\omega | \theta_{it}) d\omega. \quad (3)$$

Note that  $\psi_{it}$  depends on the default threshold  $\omega_{it}^*$  and the parameter vector  $\theta_{it}$ .

Finally, it is useful to consider the extent to which external finance raises the entrepreneur's expected return on her net worth. In particular, her expected profit is given by  $\psi_{it} R_{it}^k (1 + B_{it}/N_{it}) N_{it}$  when she leverages her investment, while in the absence of borrowing, the expected profit from the project is simply  $R_{it}^k N_{it}$ . Thus, the entrepreneur chooses to borrow as long as  $\psi_{it}(1 + B_{it}/N_{it})$  exceeds unity.

*The Lender's Expected Return.* When the realization of the idiosyncratic productivity shock  $\omega_{i,t+1}$  exceeds the threshold  $\omega_{it}^*$ , the entrepreneur satisfies the terms of the debt contract by paying  $R_{it}^b B_{it}$  to the lender; note that this outcome occurs with probability  $\int_{\omega_{it}^*}^{\infty} f(\omega | \theta_{it}) d\omega$ . Using equation (2), the loan payment can also be expressed in relation to the total proceeds from the project, namely,  $\omega_{it}^* R_{it}^k Q_t K_{it}$ .

If the entrepreneur defaults on the debt contract, then the lender takes over the project and incurs bankruptcy costs associated with accounting and legal fees, asset liquidation, and interruption of business. These bankruptcy costs are assumed to be proportional to the realized return on the project; in particular, the lender receives residual revenue  $(1 - \mu_t)\omega_{i,t+1} R_{it}^k Q_t K_{it}$ , where the bankruptcy cost parameter  $\mu_t$  satisfies  $0 \leq \mu_t < 1$ , for all  $t$ . With this specification,  $\mu_t = 0$  represents the special case of frictionless financial markets.

Thus, the *ex ante* return to the lender can be expressed as  $\xi_{it} R_{it}^k Q_t K_{it}$ , where  $\xi_{it}$  denotes the lender's expected return (net of bankruptcy costs) as a fraction of total proceeds from the project:

$$\xi_{it} \equiv \xi(\omega_{it}^* | \mu_t, \theta_{it}) = (1 - \mu_t) \int_0^{\omega_{it}^*} \omega f(\omega | \theta_{it}) d\omega + \omega_{it}^* \int_{\omega_{it}^*}^{\infty} f(\omega | \theta_{it}) d\omega. \quad (4)$$

Note that  $\xi_{it}$  involves the bankruptcy cost parameter  $\mu_t$  as well as the parameter vector  $\theta_{it}$  and the default threshold  $\omega_{it}^*$ .



In equilibrium, perfect competition among risk-neutral financial intermediaries ensures that the expected return on each debt contract is equated to the opportunity cost of funds:

$$\xi_{it} R_{it}^k Q_t K_{it} = R_t B_{it}. \quad (5)$$

To ensure that the lender cannot obtain unbounded profits by entering in a debt contract with probability of default equal to one, the project's expected return must also satisfy the condition  $(1 - \mu_t) R_{it}^k \leq R_t$ .

*The Optimal Contract.* Each entrepreneur chooses the loan amount that maximizes her expected profit subject to the constraint that the lender's expected return equals the risk-free rate. In particular, the entrepreneur recognizes that the contractual loan rate  $R_{it}^b$  will depend on the loan amount  $B_{it}$  and on the various factors that influence the likelihood of default and the expected recovery rate, namely, her own net worth  $N_{it}$ , the bankruptcy cost parameter  $\mu_t$ , and the parameter vector  $\theta_{it}$  characterizing the probability density function  $f(\omega | \theta_{it})$  of the idiosyncratic productivity shock.

In light of equation (5), it is also apparent that the optimal debt contract depends crucially on the external finance premium  $\rho_{it}$ , that is, the deviation of the project's expected return from the risk-free rate:

$$\rho_{it} = \frac{R_{it}^k}{R_t} - 1. \quad (6)$$

It is important to distinguish the external finance premium  $\rho_{it}$  from the contractual credit spread,  $(R_{it}^b/R_t) - 1$ . For example, in the frictionless case with no bankruptcy costs ( $\mu_t = 0$ ), the external finance premium equals zero, whereas the credit spread is positive to compensate the lender for the incidence of default associated with low realizations of the idiosyncratic productivity shock.

Because the debt contract is incentive compatible, it is convenient to analyze the entrepreneur's optimization problem in terms of the default threshold  $\omega_{it}^*$ . By substituting the lender's zero-profit condition into the entrepreneur's expected profit, the optimization problem can be expressed as

$$\max_{\omega_{it}^*} \left[ \frac{(1 + \rho_{it}) \psi_{it}}{1 - (1 + \rho_{it}) \xi_{it}} \right] R_t N_{it}. \quad (7)$$

The optimal default threshold  $\omega_{it}^*$  satisfies the following first-order condition:

$$\frac{\psi'_{it}(1 - \xi_{it}) + \psi_{it}\xi'_{it}}{\psi'_{it}\xi_{it} - \psi_{it}\xi'_{it}} = \rho_{it}, \quad (8)$$

where  $\psi'_{it}$  denotes the derivative of  $\psi_{it}$  with respect to  $\omega_{it}^*$ , and  $\xi'_{it}$  denotes the derivative of  $\xi_{it}$  with respect to  $\omega_{it}^*$ ; note that these derivatives satisfy  $\psi'_{it} < 0$  and  $\xi'_{it} > 0$ .<sup>8</sup>

The optimal debt contract reflects the extent to which the entrepreneur faces a tradeoff between the degree of leverage (which determines the overall scale of the project) and the contractual interest rate (which influences the entrepreneur's share of the proceeds). This tradeoff becomes evident by recalling that the entrepreneur's expected return can be expressed as  $\psi_{it}(1 + B_{it}/N_{it})R_{it}^k N_{it}$ , and noting that the first two terms depend on the default threshold while  $R_{it}^k$  and  $N_{it}$  are taken as given by the entrepreneur. Consequently, since the optimization problem is invariant to monotonic transformations, the optimal default threshold can be obtained by maximizing the objective function  $\log(\psi_{it}) + \log(1 + B_{it}/N_{it})$  subject to the lender's zero-profit constraint, namely, that  $1 + B_{it}/N_{it} = [1 - (1 + \rho_{it})\xi_{it}]^{-1}$ .

Thus, the entrepreneur's first-order condition can be equivalently expressed as

$$\frac{\partial \log(\psi_{it})}{\partial \omega_{it}^*} + \frac{\partial \log(1 + B_{it}/N_{it})}{\partial \omega_{it}^*} = 0. \quad (9)$$

Note that the first term corresponds to the elasticity of the entrepreneur's expected return with respect to the default threshold, while the second term corresponds to the elasticity of the entrepreneur's gross leverage with respect to this threshold. Evidently, the optimal default threshold equates the marginal benefit of raising the scale of the project to the marginal cost of reducing the entrepreneur's share of the total proceeds.

*The Log-Normal Distribution.* To obtain analytical solutions for the optimal debt contract, we follow BGG and assume that the idiosyncratic productivity disturbance  $\omega_{it}$  has a log-normal distribution:

$$\log \omega_{it} \sim N(-0.5\sigma_{it}^2, \sigma_{it}^2); \quad (10)$$

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<sup>8</sup>To ensure that the equilibrium debt contract results in finite leverage, it must be the case that the external finance premium  $\rho_{it} < [1 - \xi(\bar{\omega}_{it})]/\xi(\bar{\omega}_{it})$ , where  $\bar{\omega}_{it}$  satisfies  $\xi'(\bar{\omega}_{it}) = 0$ . In terms of the notation used in the appendix of BGG,  $\psi = (1 - \Gamma)$ ,  $\xi = (\Gamma - \mu G)$ , and the Lagrange multiplier on the lender's expected return  $\lambda = -\psi'/\xi'$ .

that is, the parameter vector  $\theta_{it}$  is simply the scalar  $\sigma_{it}$ . Given this distributional assumption, it is convenient to express the default threshold in the standardized form:

$$z_{it}^* = \frac{\log \omega_{it}^* + 0.5\sigma_{it}^2}{\sigma_{it}}. \quad (11)$$

Letting  $\phi(\cdot)$  and  $\Phi(\cdot)$  denote the standard normal density function and cumulative distribution function, respectively, we can express  $\psi_{it}$ ,  $\xi_{it}$ , and their derivatives as follows:

$$\psi_{it} = 1 - \Phi(z_{it}^* - \sigma_{it}) - \omega_{it}^*[1 - \Phi(z_{it}^*)]; \quad (12)$$

$$\xi_{it} = 1 - \psi_{it} - \mu_t \Phi(z_{it}^* - \sigma_{it}); \quad (13)$$

$$\psi'_{it} = \frac{\phi(z_{it}^*)}{\sigma_{it}} + \Phi(z_{it}^*) - 1 - \frac{\phi(z_{it}^* - \sigma_{it})}{\sigma_{it} \omega_{it}^*}; \quad (14)$$

$$\xi'_{it} = -\psi'_{it} - \mu_t \frac{\phi(z_{it}^* - \sigma_{it})}{\sigma_{it} \omega_{it}^*}, \quad (15)$$

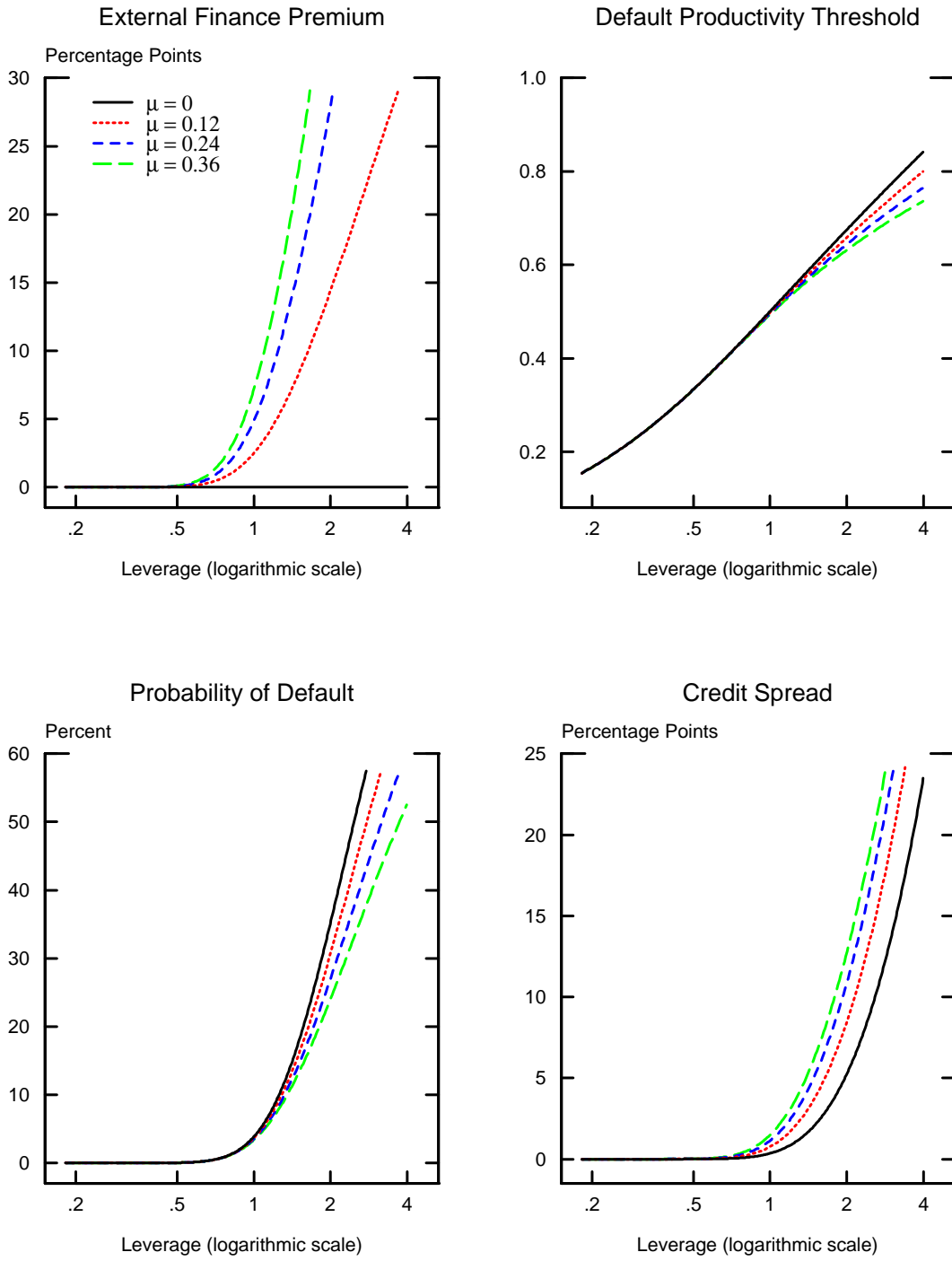
where  $\Phi(z_{it}^*)$  quantifies the probability of default, and the expected realization of the productivity disturbance in the event of default is given by  $\Phi(z_{it}^* - \sigma_{it})$ .

Using these analytical expressions, we can obtain the terms of the optimal debt contract for specified values of the external finance premium  $\rho_{it}$ , idiosyncratic shock variance  $\sigma_{it}^2$ , and bankruptcy parameter  $\mu_t$ . In particular, to solve for the optimal default threshold  $\omega_{it}^*$ , we substitute equations (11) through (15) into equation (8). The resulting solution, along with equations (1) and (5), can then be used to obtain the equilibrium leverage ratio  $B_{it}/N_{it}$ , which in turn implies the loan amount for a given level of net worth. Finally, equation (2) yields the value of the credit spread  $(R_{it}^b/R_t) - 1$ , which determines the contractual loan interest rate for a given risk-free rate.

## 2.2 The Bankruptcy Cost Parameter

We now proceed to examine the influence of bankruptcy costs on the terms of the optimal debt contract. For this purpose, we use the BGG calibration ( $\mu = 0.12$  and  $\sigma = 0.28$ ) as a benchmark and then consider higher bankruptcy costs ( $\mu = 0.24$  and  $0.36$ ). For given  $\mu$  and  $\sigma$ , and for values of the external finance premium ranging from 0 to 30 percentage points, we solve the model using the analytic expressions given

Figure 2: Varying the Bankruptcy Cost Parameter



above. We also obtain results for the special case of no financial market frictions ( $\mu = 0$ ), in which the external finance premium equals zero and the loan amount—and hence the leverage ratio—is indeterminate; in this case, we simply solve the model for selected values of the leverage ratio ranging from 0 to 4.

For each value of  $\mu$ , Figure 2 depicts the relationship between the leverage ratio, external finance premium, default threshold, probability of default, and contractual credit spread. For example, consider an entrepreneur with an external finance premium of 15 percentage points. For the benchmark case with  $\mu = 0.12$ , this entrepreneur chooses a debt contract with a leverage ratio of 2. The entrepreneur defaults on the loan whenever the realized idiosyncratic productivity is 35 percent below the mean, or equivalently, falls short of the default threshold of about 0.65. Given the assumed distribution of the idiosyncratic shock, this threshold is associated with a default probability of 30 percent. Finally, the terms of the debt contract imply a credit spread of about 8 percentage points, thereby compensating the lender for expected bankruptcy costs as well as for the relatively low value of the project in instances of default.

Tripling the bankruptcy cost parameter ( $\mu = 0.36$ ) creates a strong incentive for the same entrepreneur to select contractual terms that alleviate the deadweight loss associated with bankruptcy costs. Thus, the entrepreneur chooses a debt contract with a leverage ratio of about 1.3 and a credit spread of about 3 percentage points. Under these terms, the default probability is only one-fourth that of the benchmark case, thereby offsetting the effect of higher bankruptcy costs in the event of default.

In the frictionless case with no bankruptcy costs ( $\mu = 0$ ), the entrepreneur earns the risk-free rate regardless of the leverage ratio. Nevertheless, as noted above, the terms of the debt contract may involve a positive credit spread to compensate the lender for the incidence of default associated with low realizations of the idiosyncratic productivity shock. For example, if this entrepreneur chooses to borrow twice her net worth, then the credit spread is close to 5 percentage points, reflecting the probability of default of about 35 percent.

### 3 Data Description

Our dataset is an unbalanced quarterly panel for 918 publicly-traded firms in the U.S. nonfarm nonfinancial corporate sector over the period 1997Q1 to 2003Q3. The

distinguishing feature of the firms in our sample is that a significant part of their long-term debt is in form of bonds that are actively traded in the secondary market. For these firms, we have linked market prices of their outstanding securities and market-based measures of default risks to quarterly balance sheet statements.<sup>9</sup> We now turn to the construction of our key variables: credit spreads, leverage, and expected probabilities of default.

### 3.1 Sources and Methods

*Credit Spreads.* Daily market prices of corporate bonds were obtained from the Merrill Lynch database, which includes prices of dollar-denominated corporate bonds publicly issued in the U.S. market. Qualifying securities must have a remaining term-to-maturity of at least one year, a fixed coupon schedule, and a minimum amount outstanding of \$100 million for below investment-grade and \$150 million for investment-grade issuers.

To calculate an overall credit spread on the firm’s outstanding bonds, we matched the daily effective yield on *each* individual security issued by the firm to the estimated yield on the Treasury coupon security of the same maturity.<sup>10</sup> Treasury yields were taken from a smoothed yield curve estimated on a large sample of off-the-run Treasury coupon securities using the technique proposed by Svensson (1994).<sup>11</sup> The resulting spread between corporate and Treasury securities, however, is distorted by the differential tax treatment of corporate and government debt—coupons on corporate bonds are subject to taxes at the state level whereas coupons on Treasury securities are not. Because investors compare returns across instruments on an after-tax basis, yields on corporate bonds will be systematically higher than yields on government securities to compensate for the payment of state taxes. Indeed, Elton et al. (2001) estimate that, on average, these tax factors can account for as much as 20 percent of corporate credit spreads.

We used the method proposed by Cooper and Davydenko (2002) to estimate the

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<sup>9</sup>The membership in our panel is limited to firms that reported at least 4 consecutive quarters of income and balance sheet data. The availability of price data on individual corporate securities (January 2, 1997) determined the starting date of our sample.

<sup>10</sup>To avoid extrapolating the Treasury yield curve, we dropped from our sample a small number of corporate issues with maturities greater than 30 years.

<sup>11</sup>On-the-run Treasuries were excluded from the sample because yields on those securities are strongly influenced by liquidity premiums, which can affect the shape of the estimated yield curve and, moreover, can shift the curve around the auction cycle.

distortionary effect of the state-level taxation on corporate bond spreads. According to Elton et al. (2001), the relevant tax rate for the tax-adjusted spread between corporate and government securities is given by  $\tau = t_s(1 - t_g)$ , where  $t_s$  and  $t_g$  are the state and the federal tax rates, respectively. As suggested, we set  $\tau$  equal to 4.875% and compute for each corporate security the portion of the spread due to taxes according to

$$\Delta y^\tau = \frac{1}{t_M} \log \left[ \frac{1 - \tau}{1 - \tau \exp(-r_{t_M} t_M)} \right],$$

where  $t_M$  is the corporate security’s maturity and  $r_{t_M}$  is the corresponding Treasury coupon yield (see Cooper and Davydenko (2002) for further details). To calculate an overall firm-specific credit spread, we averaged the tax-adjusted spreads on the firm’s outstanding bonds, using the product of market values of bonds and their effective durations as weights.<sup>12</sup> We matched the firm-specific daily spreads to quarterly balance sheet information by averaging the daily spreads over the first month of the quarter.<sup>13</sup>

*Leverage.* Our measure of the firm’s leverage is constructed using Compustat balance sheet information. **Leverage is defined as the ratio of the book value of long-term debt to the market-value of common equity.** Long-term debt includes all debt obligations due in more than one year from the firm’s balance sheet at the last day of the quarter.<sup>14</sup> We use the book value of debt, as opposed to the market value, because the book value is the amount that the firm must repay to avoid default. Market capitalization is computed by multiplying the number of common shares outstanding by the closing stock price, both measured at the last day of the quarter.

*Default Probabilities.* To measure a firm’s probability of default at each point in time, we employ a monthly indicator that is widely used by financial market participants. In particular, the “Expected Default Frequency” (EDF)—constructed and marketed

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<sup>12</sup>The use of the dollar duration of bonds as a weight in computing the yield on a portfolio of bonds represents a first-order Taylor series approximation to the portfolio yield; see Choi and Park (2002) for details. Our results were virtually identical when portfolio spreads were averaged using market values of bonds as weights only.

<sup>13</sup>That is, credit spreads matched to any first quarter of balance sheet data are averages of the daily spreads in January, spreads during the second quarter are averages of the daily spreads during April, and so on. We also converted daily spreads to a quarterly frequency by averaging over the entire quarter. All of the results reported in this paper were robust to this alternative timing assumption.

<sup>14</sup>We restrict the numerator of the leverage ratio to long-term debt because our secondary-market prices pertain to long-term corporate securities. In addition, firms often maintain a stock of liquid assets to cover their short-term liabilities.

by the Moody's/KMV Corporation (MKMV)—gauges the probability of default over the subsequent 12-month period. In contrast to traditional measures of default risk based on credit rating transitions, the EDF moves primarily in response to changes in equity values and hence reacts rapidly to deterioration in equity investors' assessment of a firm's credit quality.

The MKMV methodology builds on the seminal work of Merton (1973, 1974). In particular, this approach assumes that the firm defaults—and its equity becomes worthless—if the market value of its assets falls below a specific “default point.” Thus, given the current level and recent volatility of the firm's stock price, option theory can be used to derive the (unobserved) level and volatility of the market value of assets, which in turn determines the likelihood of default.

In constructing firm-specific EDFs, the MKMV procedure utilizes several refinements intended to capture the complexity of financial markets and the firm's choice of capital structure (cf. Crosbie and Bohn (2003)). For example, rather than simply setting the default point equal to the book value of total liabilities, MKMV calibrates the default point to reflect the finding that most defaults occur when the market value of the firm's assets drops below the sum of its current liabilities and one-half of its long-term liabilities. Furthermore, after deriving the default probability implied by the option-pricing framework, MKMV makes adjustments based on an extensive proprietary database of historical defaults and bankruptcies.<sup>15</sup>

In constructing the dataset used in our empirical analysis, we converted the EDF to a quarterly basis to reflect the one-period nature of the debt-contracting framework.<sup>16</sup> Finally, the EDF at the end of the previous quarter serves as the indicator of the expected probability of default during the current quarter.

## 3.2 Descriptive Statistics

Table 1 contains several summary statistics for our panel. Despite our focus on firms that have both equity and a portion of their debt traded in open markets, firm size—measured by sales or market capitalization—varies widely in our sample. Not surprisingly, though, most of the firms in our dataset are quite large. The median firm

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<sup>15</sup>It should also be noted that MKMV imposes a lower bound of 0.02 percent and an upper bound of 20 percent in constructing each EDF.

<sup>16</sup>This conversion employs the simplifying assumption of a constant hazard rate over each four-quarter horizon.



Table 1: Summary Statistics

Variable	Minimum	Median	Maximum
Sales (\$ millions)	1.6	674	56,701
Mkt. Capitalization (\$ millions)	6.1	2,173	308,998
Leverage Ratio	0.02	0.50	15.9
Credit Spread <sup>a</sup> (p.p.)	0.27	2.27	29.14
No. of Issues Traded	1	2	59
Avg. Portfolio Maturity (years)	1	8	30
Share of Traded Debt <sup>b</sup> (%)	2	51	100
S&P Credit Rating	C2	BBB3	AAA
Year-Ahead EDF (%)	0.02	0.55	19.9

*Panel Dimensions*

Observations = 14,124 Firms = 918

Min. Tenure = 4 Median Tenure = 14 Max. Tenure = 27

NOTES: Sample period: 1997Q1–2003Q3. In every period, the sample excludes firms with leverage ratios below the 2.5<sup>th</sup> percentile and above the 97.5<sup>th</sup> percentile, firms with credit spreads above the 97.5<sup>th</sup> percentile, and firms with EDFs at exactly 20%. Sales and market capitalization are in real (2000) chain-weighted dollars.

<sup>a</sup>Adjusted for the differential tax treatment of corporate and Treasury securities.

<sup>b</sup>The book value of traded bonds relative to the book value of total long-term debt.

has sales of \$674 million and a market capitalization of almost \$2.2 billion. About one-half of observations are associated with leverage ratios greater than 50 percent. The relatively high leverage in our sample is due in part to the steep fall in equity prices that started in the spring of 2000, which significantly reduced the market capitalization of firms, thereby driving up their leverage ratios.

Despite the fact that firms in our sample generally have only a few bond issues trading at any given point in time, this publicly-traded debt represents a significant portion of their long-term debt. The median ratio of the book value of traded bonds outstanding to the book value of total long-term debt on firms' balance sheet is about one-half, suggesting that market prices on outstanding securities likely provide an accurate gauge of the marginal cost of external finance for most of the firms. Our sample also spans nearly the entire corporate credit quality spectrum—from C2, just about the “junkiest junk,” to AAA, the highest grade. In terms of credit quality, the median observation is at the bottom rung of the investment-grade ladder, and it is associated with a tax-adjusted spread of 227 basis points over the risk-free rate and

an expected year-ahead default frequency of 55 basis points.

Our sample consists of 918 nonfinancial corporations, but various indicators confirm that it is reasonably representative of the broader economy. The upper panel of Figure 3 compares the aggregate growth rate of real sales for the firms in our dataset (here denoted as the “LNZ dataset”) with the corresponding series for all nonfinancial firms in Compustat and for the entire nonfarm nonfinancial sector.<sup>17</sup> The three series are highly correlated and exhibit very similar business cycle dynamics.

The lower panel compares the sales-weighted median leverage of firms in our sample with the corresponding statistic for all nonfinancial firms in Compustat as well as with a measure of long-term leverage in the nonfinancial business sector obtained from the Flow of Funds accounts.<sup>18</sup> The three measures paint a very similar picture of the state of corporate balance sheets over time. Clearly evident is the sharp run-up in corporate leverage during the late 1980s, followed by a steady decline over most of the past decade. Leverage in the nonfinancial business sector bottomed out at a very low level in the late 1990s and then rose noticeably after the bursting of the stock market bubble in the spring of 2000.

Credit spreads and expected default probabilities for firms in our dataset also indicate that our sample is reasonably representative of the broader economy. For example, as shown in the upper panel of Figure 4, the weighted median credit spread for BBB-rated firms in our sample provides a very close match to the corresponding statistic for all BBB-rated nonfinancial issuers in the Merrill Lynch database. As shown in the lower panel, the evolution of the weighted median year-ahead EDF for the firms in our sample also tracks the corresponding statistic for all nonfinancial firms in the MKMV database.<sup>19</sup>

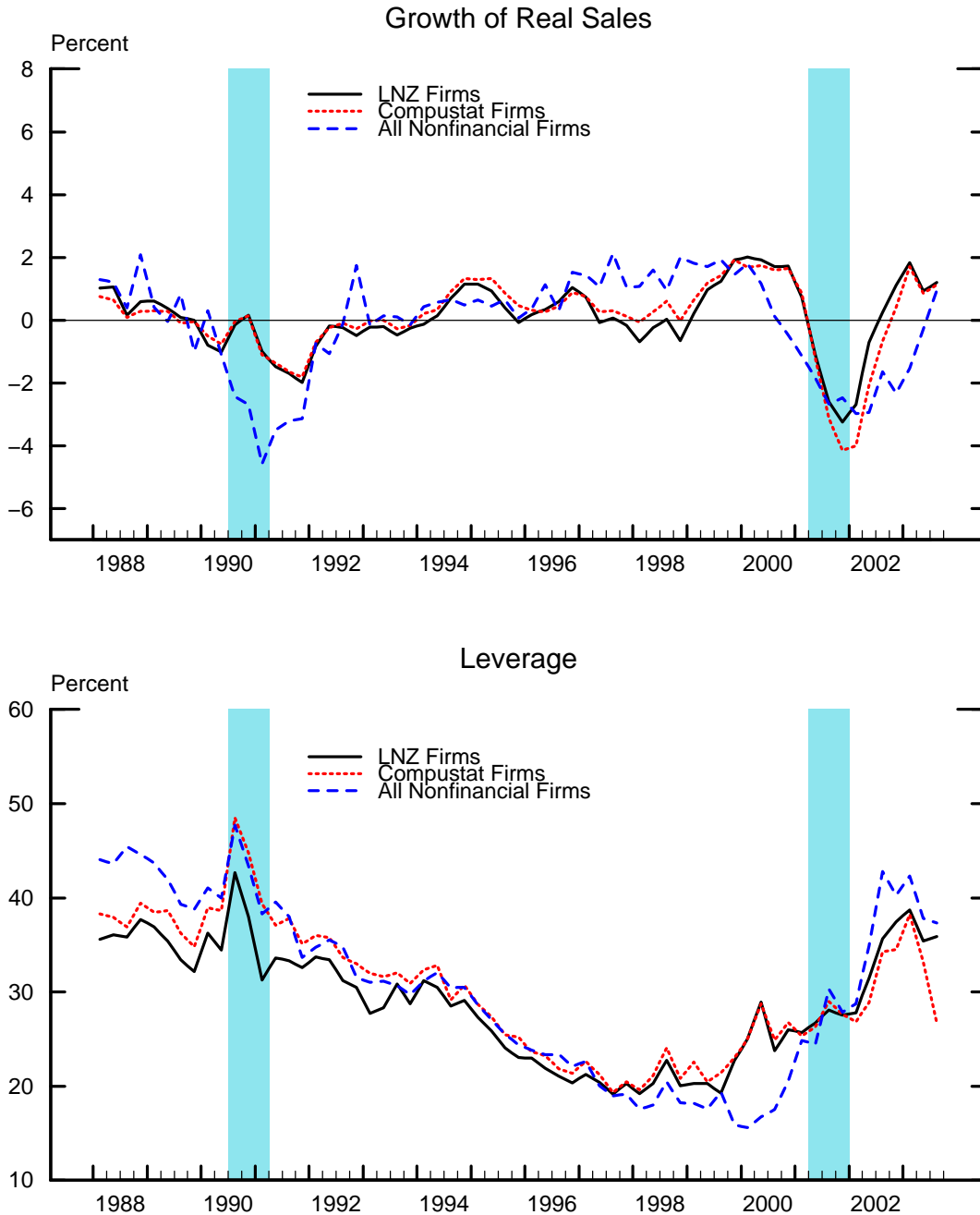
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<sup>17</sup>The nominal series from our dataset and from Compustat have been deflated using the chain-weighted GDP price index. All three series have been seasonally adjusted and demeaned.

<sup>18</sup>As discussed above, we define leverage as the ratio of the book value of long-term debt to the market value of equity. Because the Flow of Funds accounts do not contain a measure of long-term debt for the entire nonfinancial business sector, we utilize the total book value of bonds and mortgages as a proxy.

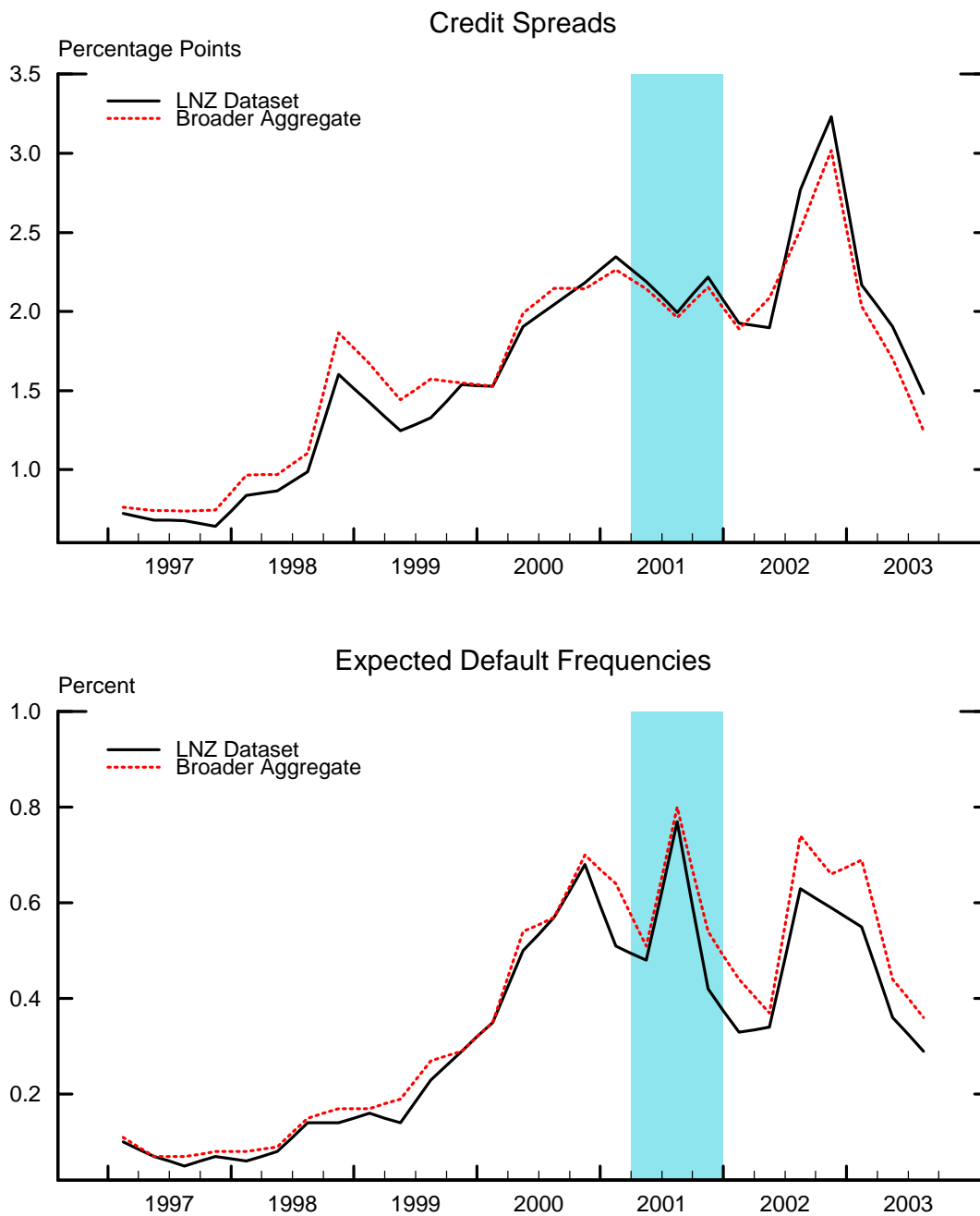
<sup>19</sup>The median credit spread is constructed using the the market value of bonds outstanding as weights, while the median EDF is constructed using the book value of total liabilities as weights.

Figure 3: Comparing the Data with Broader Aggregates



NOTES: This figure compares the four-quarter average growth rate of aggregate real sales (upper panel) and the ratio of equity to long-term debt (lower panel) for three samples: the 918 firms in our dataset (solid line), all nonfinancial firms in Compustat (dotted line), and the entire nonfinancial business sector (dashed line).

Figure 4: Comparing the Data with Broader Aggregates (contd.)



NOTES: The upper panel compares the weighted median credit spread for BBB-rated firms in our sample with the corresponding statistic for all BBB-rated nonfinancial firms in the Merrill Lynch dataset. The lower panel compares the weighted median year-ahead EDF for all firms in our sample with the corresponding statistic for all nonfinancial firms in the MKMV dataset.

## 4 Estimation Methodology

As discussed in Section 2, the magnitude of financial market frictions in the BGG framework is determined by the bankruptcy cost parameter  $\mu$ . Recognizing that this parameter may exhibit substantial temporal variation, we utilize the cross-section of firm-level observations in each period  $t$  to obtain a time-varying estimate of the bankruptcy cost parameter  $\mu_t$ . Although we impose the same bankruptcy cost parameter on all firms in a given period, we allow the remaining structural parameters of the model—the default threshold  $\omega_{it}^*$  and the volatility of idiosyncratic risk  $\sigma_{it}$ —to vary across firms as well as time.

Our nonlinear least-squares (NLLS) procedure consists of two steps. First, for a given value of the bankruptcy cost parameter  $\mu_t$ , we use the conditions characterizing the optimal debt contract, along with the observed leverage and the expected probability of default, to solve for the firm-specific default threshold  $\omega_{it}^*$  and standard deviation  $\sigma_{it}$  of the idiosyncratic shock. In the second step, we use these two solutions to derive the model-implied contractual credit spread. Our NLLS estimate of the bankruptcy cost parameter in period  $t$  is the value of  $\mu_t$  that minimizes the sum of squared deviations between observed credit spreads and those predicted by the model.

Specifically, for a given value of the bankruptcy cost parameter  $\mu_t$ , we solve the following two equations for  $\omega_{it}^*$  and  $\sigma_{it}$ :

$$\left[\frac{B}{N}\right]_{it} = -\frac{\psi'(\omega_{it}^* | \sigma_{it}) \xi(\omega_{it}^* | \mu_t, \sigma_{it})}{\psi(\omega_{it}^* | \sigma_{it}) \xi'(\omega_{it}^* | \mu_t, \sigma_{it})}, \quad (16)$$

$$\text{EDF}_{it} = \Phi\left(\frac{\log \omega_{it}^* + 0.5\sigma_{it}^2}{\sigma_{it}}\right), \quad (17)$$

where  $[B/N]_{it}$  is firm  $i$ 's leverage at the beginning of quarter  $t$ , and  $\text{EDF}_{it}$  is the probability that firm  $i$  will default during quarter  $t$ . Equation (16) is obtained by substituting the first-order condition of the debt-contracting problem, equation (8), into the lender's zero profit condition, whereas equation (17) is the definition of the expected default probability (see Section 2.1). Under the assumption of log-normality, the functions  $\psi$  and  $\xi$  are given by equations (12) and (13), respectively.

For each firm/quarter observation, the solutions to equations (16) and (17), denoted by  $\widehat{\omega}_{it}^*$  and  $\widehat{\sigma}_{it}$ , can be substituted into equation (8) to obtain the model-implied

(gross) external finance premium:

$$1 + \widehat{\rho}_{it} = \frac{\psi'(\widehat{\omega}_{it}^* | \widehat{\sigma}_{it})}{\psi'(\widehat{\omega}_{it}^* | \widehat{\sigma}_{it}) \xi(\widehat{\omega}_{it}^* | \mu_t, \widehat{\sigma}_{it}) - \psi(\widehat{\omega}_{it}^* | \widehat{\sigma}_{it}) \xi'(\widehat{\omega}_{it}^* | \mu_t, \widehat{\sigma}_{it})}. \quad (18)$$

We then use equation (2) to derive the model-implied contractual (gross) credit spread for firm  $i$  in period  $t$ :

$$\left[ \frac{R^b}{R} \right]_{it} = \widehat{\omega}_{it}^* \left( 1 + \left[ \frac{B}{N} \right]_{it}^{-1} \right) (1 + \widehat{\rho}_{it}). \quad (19)$$

In our empirical implementation, we assume that the difference between the actual and model-implied credit spreads can be decomposed as

$$\left[ \frac{R^b}{R} \right]_{it} - \left[ \frac{R^b}{R} \right]_{it}^* = \mathbf{x}_{it}^\top \beta_t + \epsilon_{it}, \quad (20)$$

where  $\mathbf{x}_{it}$  is a vector of firm characteristics that includes firm  $i$ 's industry classification and the average bond portfolio credit rating at the beginning of quarter  $t$ .<sup>20</sup> The stochastic disturbance  $\epsilon_{it}$  is assumed to have zero mean and to be independent across firms, though it may exhibit time-varying heteroscedasticity:  $E[\epsilon_{it}] = 0$ ,  $E[\epsilon_{it}^2] = \nu_{it}^2$ , and  $E[\epsilon_{it}\epsilon_{jt}] = 0$ , for all  $i \neq j$ .

We include the vector  $\mathbf{x}_{it}$  of credit rating and industry fixed effects in our benchmark empirical specification, because the stylized nature of the BGG debt-contracting problem abstracts from various other frictions such as risk, liquidity, and term premiums. In particular, Jones, Mason, and Rosenfeld (1984), Elton et al. (2001), Delianedis and Geske (2001), and Huang and Huang (2003) report that default risks accounts for a relatively small portion of observed credit spreads on corporate bonds, and that these spreads include an important risk premium in addition to compensation for the expected default loss. We thus include credit rating fixed effects in an attempt to control for the influence of possibly time-varying risk premiums on the size of credit spreads.

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<sup>20</sup>Industry fixed effects are based on the 3-digit North American Industry Classification System (NAICS). Credit rating fixed effects are based on the average of the S&P ratings of the firm's outstanding bond issues at the beginning of quarter  $t$ , weighted by the market value of bonds. The resulting portfolio credit ratings were condensed into nine categories: AAA, AA, A, BBB, BB, B, CCC, CC, and C.

Credit rating and industry effects may also pick up the distortionary effect of liquidity factors that arise from the fact that certain corporate bonds trade rather infrequently, implying a relatively thin secondary markets for some securities.<sup>21</sup> In such a case, a credit spread will include a premium to compensate investors for the risk of having to sell or hedge a position in an illiquid market. Indeed, using information on actively traded credit-default swaps, Longstaff, Mithal, and Neis (2004) find that the nondefault component of corporate credit spreads is strongly related to individual bond and market-wide measures of liquidity. In addition, controlling for industry differences is potentially important because our dataset, though rich in the cross-sectional dimension, spans a single business cycle dominated by the bursting of the high-tech bubble.

Given our assumptions, we can compute the residual vector  $(\epsilon_{1,t}, \dots, \epsilon_{n_t,t})^\top$  for any value of the bankruptcy cost parameter  $\mu_t$ , given a sample of  $n_t$  observations on leverage, credit spreads, and EDFs in period  $t$ . To obtain an estimate of the bankruptcy cost in period  $t$ , we start with an initial guess for  $\mu_t$  and then utilize a standard optimization algorithm to minimize the sum of the squared residuals.<sup>22</sup> Statistical inference of the resulting NLLS estimator  $\hat{\mu}_t$  is based on standard errors computed using an asymptotic heteroscedasticity-consistent covariance matrix (see White (1980)).

## 5 Bankruptcy Costs

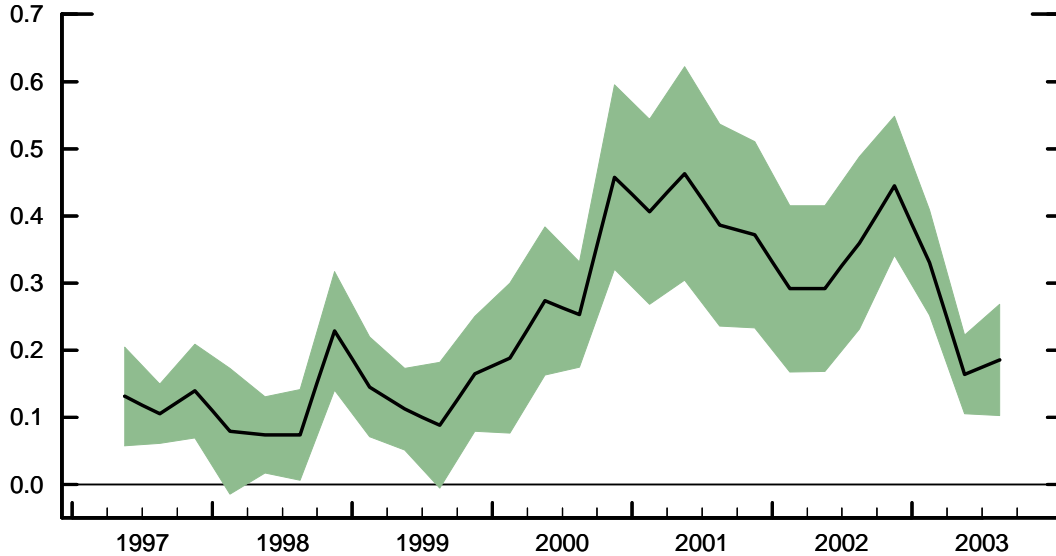
Figure 5 shows the evolution of the estimated bankruptcy cost parameter  $\mu_t$  over our sample period. As indicated by the shaded region, we clearly reject the null hypothesis of no financial market frictions in all but two periods (1998Q1 and 1999Q3). Importantly, the estimates of  $\mu_t$  appear to vary systematically over the course of the business cycle, suggesting an important temporal dimension to the magnitude of

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<sup>21</sup>See Warga (1991) for a discussion of problems associated with high-frequency corporate bond prices and the use of “grid-based” pricing. Relatedly, Collin-Dufresne, Goldstein, and Martin (2001) find that a significant portion of monthly *changes* in credit spread on straight industrial bonds can be attributed to local supply/demand shocks that are unrelated to the fundamentals.

<sup>22</sup>To ensure that our estimates are not driven by a small number of extreme observations, we exclude from the estimation firms with leverage ratios below the 2.5<sup>th</sup> percentile and above the 97.5<sup>th</sup> percentile, firms with credit spreads above the 97.5<sup>th</sup> percentile, and firms with EDFs at exactly 20 percent. To guarantee that the final estimate of  $\mu_t$  corresponds to the global minimum of the objective function, we chose the initial guess by employing an extensive grid search over the relevant parameter space.

Figure 5: Benchmark Results for the Bankruptcy Cost Parameter



NOTES: The solid line denotes the time-specific estimate of the bankruptcy cost parameter  $\mu_t$ . The shaded region represents the 95 percent confidence interval, computed using White's (1980) heteroscedasticity-consistent asymptotic covariance matrix.

financial market frictions.

From early 1997 to the end of 1999, the point estimates for  $\mu_t$  appear to be quite stable, moving in a narrow range between 0.08 and 0.16. One clear exception during this period is the substantially larger estimate for 1998Q4. This jump in estimated bankruptcy costs likely reflected the turbulence in financial markets following the Russian default and the collapse of the Long-Term Capital Management (LTCM) hedge fund. Smoothing through the 1998Q4 spike, the average estimate of  $\mu_t$  during this period is remarkably close to 0.12, the value chosen by BGG in the steady-state calibration of their model. Our estimates are also within the range of bankruptcy costs estimated by Altman (1984) for a sample of industrial firms that declared bankruptcy in the mid-1970s.<sup>23</sup>

The bursting of the stock market bubble in the spring of 2000 significantly de-

<sup>23</sup>Altman's (1984) estimates of bankruptcy costs include both the direct and indirect costs and average between 11 percent and 17 percent of the value of the firm. Direct costs—explicit administrative costs paid by the debtor during the reorganization/liquidation process—were taken from the bankruptcy records of individual firms. Measures of indirect costs, namely lost profits, were estimated.



pressed equity valuations, causing an increase in corporate leverage. By the onset of the last NBER-dated recession in March 2001, credit spreads also had widened significantly. In the context of the BGG model, however, the rise in leverage apparently was insufficient to account fully for the run-up in credit spreads. A part of the increase in credit spreads during this period reflected a jump in the external finance premium, as our estimates of the default threshold  $\omega_{it}^*$  only rose slightly (see equation (2)).<sup>24</sup> The sharp increase in the external finance premium also did not come about from an increase in the volatility of idiosyncratic risk ( $\sigma_{it}$ ), a point to which we return later. Instead, it stemmed from an increase in the expected bankruptcy costs, as evidenced by our estimates of  $\mu_t$ , which more than doubled over this period.

After declining moderately over the course of the 2001 downturn and into early 2002, the estimates of  $\mu_t$  rose sharply in the latter half of that year. This increase likely reflected the post-Enron wave of corporate governance scandals that rattled investors' confidence and may have led to perceptions of greater losses in the event of bankruptcy. For this period as a whole, our estimates of bankruptcy costs are much closer to the average liquidation costs calculated by Alderson and Betker (1995) for a sample of firms that completed Chapter 11 bankruptcy proceedings between 1982 and 1993. In 2003, as the economy recovered, stock prices started to rise, and credit spreads narrowed, the point estimate of  $\mu_t$  declined back to the range that prevailed at the beginning of our sample period.

## 6 The External Finance Premium

Using the parameter estimates obtained above, we now proceed to characterize the cross-sectional and time-series behavior of the external finance premium implied by the optimal debt-contracting framework. We present benchmark estimates and analyze the sources of time variation, and then consider the macroeconomic implications of our findings.

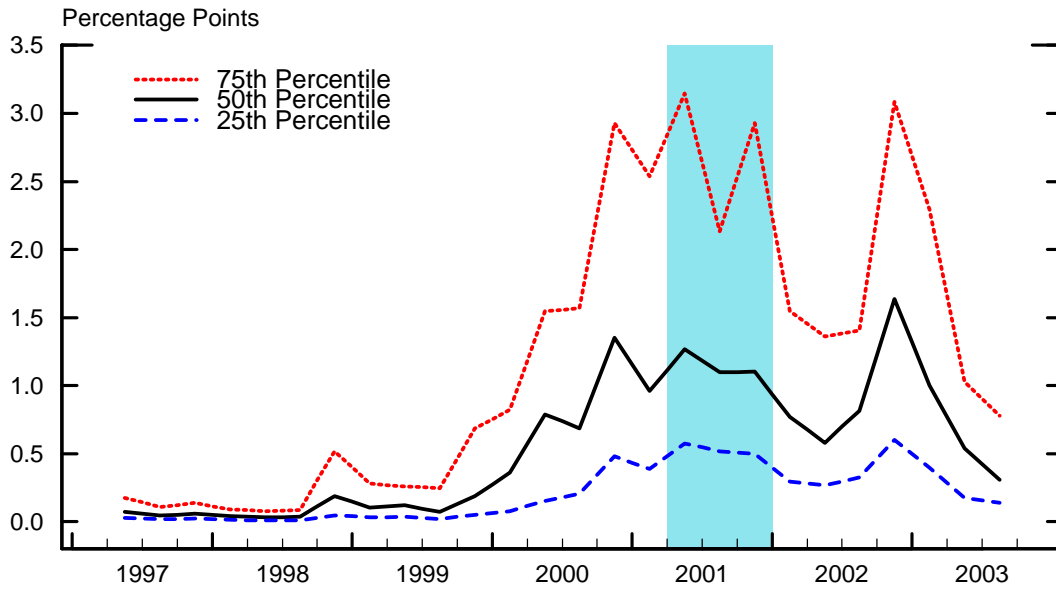
### 6.1 Benchmark Estimates

To compute the model-implied external finance premium  $\hat{\rho}_{it}$ , we use equation (18), together with the estimated bankruptcy cost parameter  $\mu_t$  and the corresponding so-

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<sup>24</sup>For example, the sales-weighted median  $\hat{\omega}_{it}^*$  rose from 0.218 to 0.236 in 2000.

Figure 6: Cross-Sectional Distribution of the External Finance Premium



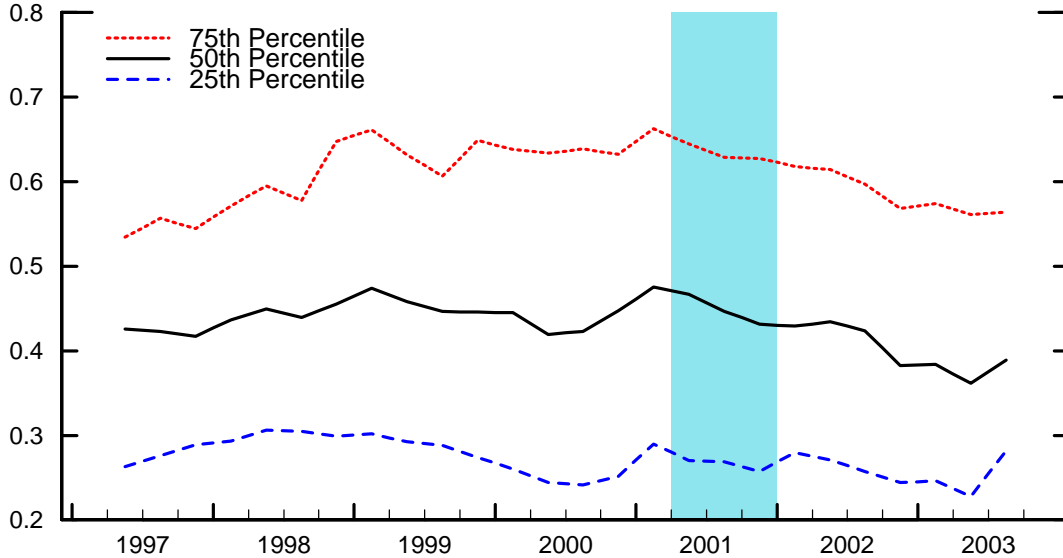
NOTES: Each line denotes the specified sales-weighted percentile for the model-implied external finance premium constructed using our benchmark estimates of the bankruptcy cost parameter  $\mu_t$ .

lutions for the idiosyncratic risk parameter  $\hat{\sigma}_{it}$  and the default threshold  $\hat{\omega}_{it}^*$ . Figure 6 depicts the time-series behavior of the external finance premium at the sales-weighted 25<sup>th</sup>, 50<sup>th</sup>, and 75<sup>th</sup> percentiles of the cross-sectional distribution of firms.

During the expansionary period from 1997 to 1999, the model-implied external finance premium was close to zero for most of the firms in our sample, apart from a transitory rise in the fall of 1998 following the Russian default and the collapse of LTCM. The marked absence of an economically significant premium on external financing during this period reflects relatively small estimates of expected bankruptcy costs and is consistent with the rapid pace of capital spending during the late 1990s.

As the estimated bankruptcy costs started to trend higher in early 2000, the external finance premium rose sharply. The increase in the external finance premium during the 2000–01 period was also economically significant. In particular, the results for the sales-weighted median indicate that firms that account for one-half of aggregate sample sales experienced an increase in the external finance premium of more than 100 basis points. Furthermore, as shown by the results for the sales-weighted 75<sup>th</sup> percentile, firms that account for one-quarter of aggregate sample sales faced an

Figure 7: Time Variation in Idiosyncratic Shock Volatility



NOTES: Each line denotes the specified sales-weighted percentile for the idiosyncratic risk parameter  $\sigma_{it}$ .

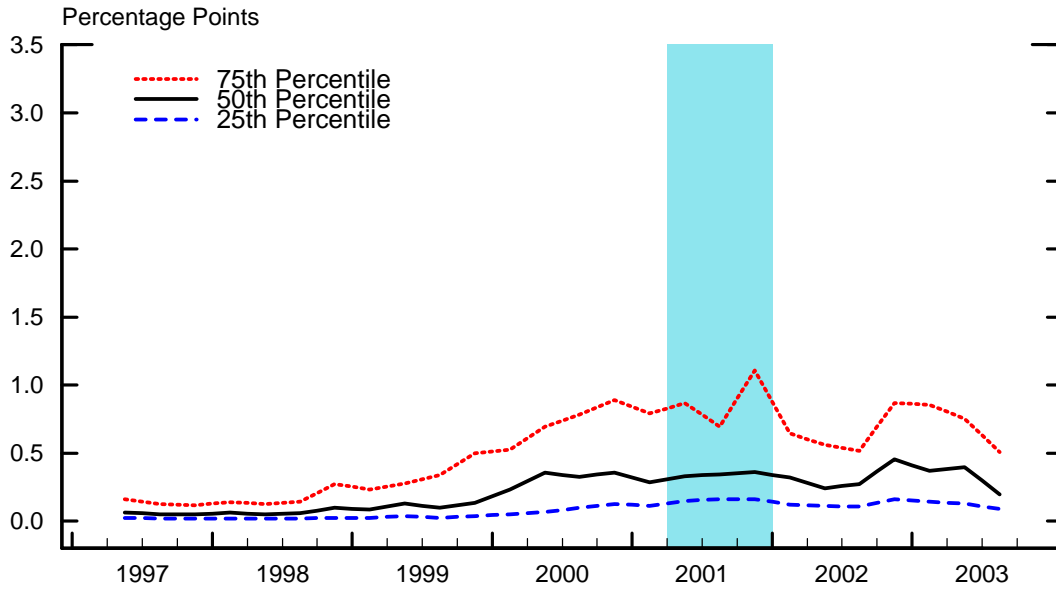
increase in the external finance premium of about 300 basis points. As the recession ended, the external finance premium started to move lower but then jumped up again at the end of 2002, in response to a concomitant increase in estimated bankruptcy costs, which—as argued above—likely reflected investors’ ongoing concerns about the veracity of corporate balance sheets.

## 6.2 Sources of Time Variation

In principle, these relatively large swings in the external finance premium could be due to shifts in the volatility of idiosyncratic risk, but the evidence suggests otherwise. In particular, as shown in Figure 7, our benchmark results for the idiosyncratic volatility parameter  $\sigma_{it}$  indicate relatively little time variation across the entire distribution of firms. For the sales-weighted median firm, for example,  $\hat{\sigma}_{it}$  remained within a fairly narrow range around an average value of about 0.4 over the entire sample period.

To gauge the influence of time variation in the bankruptcy cost parameter, we now consider a counterfactual scenario in which we set  $\mu_t$  at a constant value of 0.12, roughly its average over the 1997–99 period. In implementing this scenario,

Figure 8: The External Finance Premium under the Counterfactual Scenario

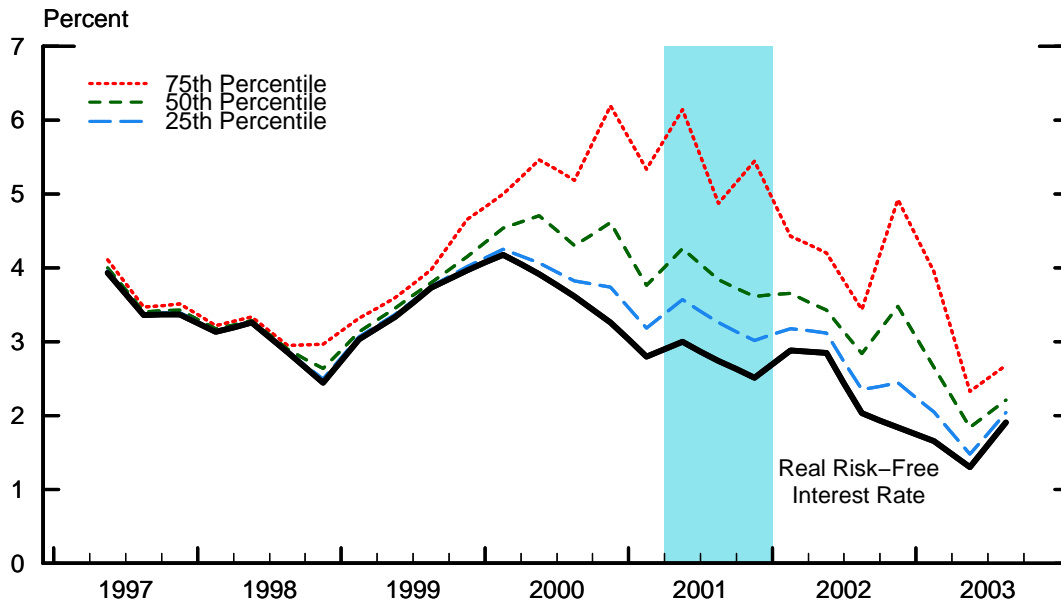


NOTES: Each line denotes the specified sales-weighted percentile for the model-implied external finance premium obtained for the counterfactual exercise with a constant bankruptcy cost parameter  $\mu = 0.12$ .

we assume that each firm’s idiosyncratic volatility parameter follows the same time path  $\hat{\sigma}_{it}$  as in the benchmark case. We further assume that each firm’s leverage ratio follows the observed path  $[B/N]_{it}$ , thereby abstracting from any endogenous response of equity prices or the book value of long-term debt. We then solve equation (16) for the value of  $\omega_{it}^*$  and then use equation (18) to obtain the model-implied external finance premium.

As shown in Figure 8, the counterfactual scenario with a constant bankruptcy cost parameter implies relatively small movements in the external finance premium, especially in comparison with the benchmark results depicted in Figure 6. Evidently, while equity prices fell sharply during the 2000–01 period, the resulting run-up in corporate leverage would not have generated very marked changes in the external finance premium unless accompanied by a substantial increase in expected bankruptcy costs.

Figure 9: Benchmark Results for the Cost of External Finance



NOTES: The solid line denotes the risk-free real interest rate, that is, the 10-year nominal Treasury yield less expected inflation as measured by the Philadelphia Fed’s Survey of Professional Forecasters. The other three lines denote the specified sales-weighted percentiles for the cost of external finance, that is, the risk-free rate plus the model-implied external finance premium.

### 6.3 Macroeconomic Consequences

Finally, in considering the potential macroeconomic consequences of financial market frictions, it is helpful to evaluate the expected cost of external finance for the cross-section of firms in our sample, because this cost plays a fundamental role in determining the level of capital investment in models with imperfect capital markets. For this purpose, we compute the risk-free real interest rate as the 10-year nominal Treasury yield less the median long-term inflation expectations taken from the Philadelphia Fed’s Survey of Professional Forecasters. We then construct the expected cost of external finance for each firm by adding its external finance premium to the risk-free rate.

As shown in Figure 9, the expected cost of funds for most firms in our sample remained close to the long-term risk-free rate from 1997 through 1999, a period associated with very low levels of the model-implied external finance premium. During

the year 2000, long-term Treasury yields declined by about 150 basis points, presumably reflecting market expectations of an imminent easing in short-term rates due to slowing macroeconomic activity. In contrast, the cost of external finance only declined slightly during 2000 for the sales-weighted median firm in our sample, indicating that the expected stimulus from monetary policy was largely offset by a rise in the external finance premium. Furthermore, the cost of external finance *rose* more than 100 basis points for the upper quartile of the cross-sectional distribution, that is, for firms representing 25 percent of total sales in our sample. Together with other factors (such as perceptions of a capital overhang), these results may help explain why investment spending remained relatively weak despite the aggressive easing of monetary policy during the 2001–02 period.

## 7 Outstanding Issues

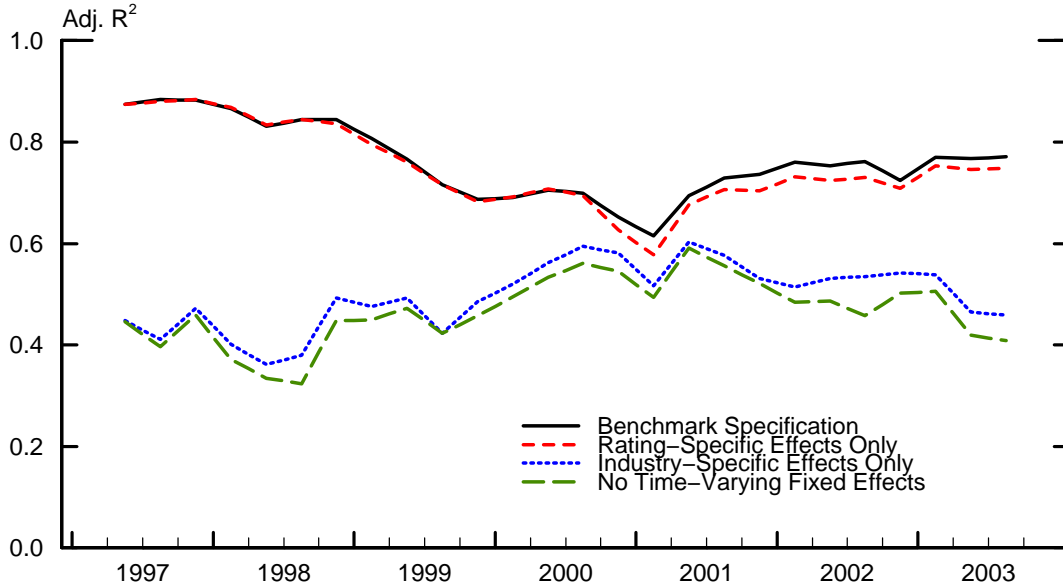
In this section, we discuss several issues raised by our analysis. First, we explore the influence of credit rating and industry fixed effects on our benchmark parameter estimates. Second, we compare recovery rates implied by the BGG model with the actual recovery rates on defaulted corporate bonds. Finally, we examine the cross-sectional relationship between leverage and the volatility of idiosyncratic risk in the context of the equilibrium debt contract.

### 7.1 The Role of Credit Rating and Industry Effects

Our benchmark empirical specification (20) included time-varying rating-specific and industry-specific fixed effects to control for the influence of risk, liquidity and other premiums on the size of observed credit spreads. According to likelihood ratio tests, we overwhelmingly reject the exclusion of credit rating effects in every period ( $p$ -values less than 0.001), while industry effects are statistically significant only in the aftermath of the Russian default in late 1998 and from the end of 2000 forth.

To determine whether these proxies for risk, liquidity, term, and other premiums explain a significant component of credit spreads and to shed some light on their interaction with our measure of financial market frictions, we now consider estimates of the time-varying bankruptcy parameter  $\mu_t$  under three alternative specifications for the time-varying fixed effects: (*i*) only rating-specific fixed effects; (*ii*) only industry-

Figure 10: Goodness-of-Fit Comparison of Alternative Specifications



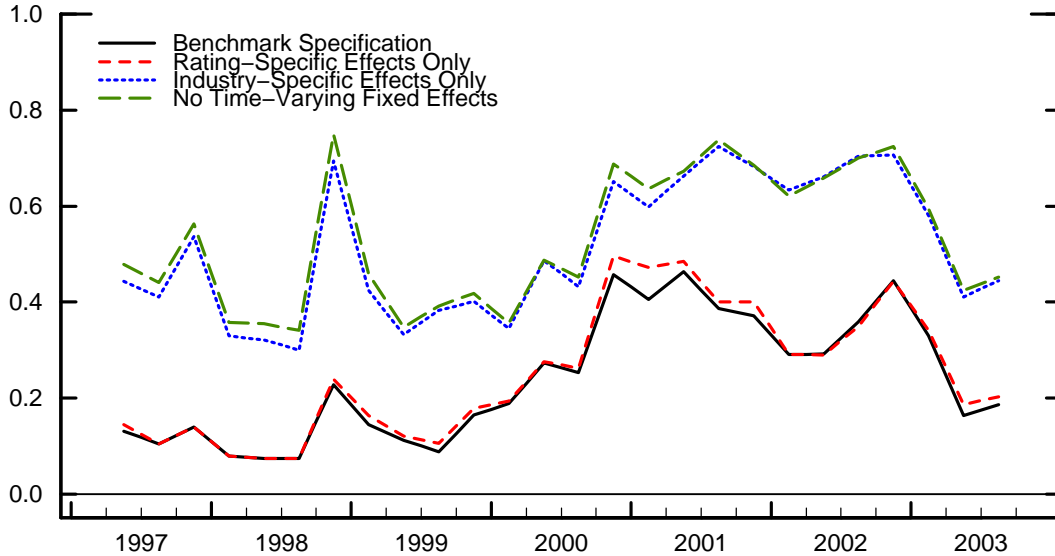
NOTES: For each specification of the time-varying fixed effects, this figure depicts the adjusted  $R^2$  obtained for each time period.

specific fixed effects; and *(iii)* a time-varying intercept, but no rating-specific or industry-specific fixed effects.

According to Figure 10, our benchmark specification—which includes time-varying fixed effects to control for both credit rating and industry differences—consistently explains the highest proportion of the cross-sectional variation in credit spreads. Although the fit of the model declines somewhat in 1999 and 2000, this specification explains, on average, about 80 percent of the variance in credit spreads during our sample period. Furthermore, while the industry-specific fixed effects are statistically significant during the latter half of our sample, these effects only generate a marginal improvement in the fit of the model.

The exclusion of credit rating fixed effects, in contrast, results in a significant deterioration in the fit of the model. In this case, the adjusted  $R^2$ —with or without controls for industry characteristics—is, on average, only about half as high as in our benchmark specification. These results indicate that credit rating effects are explaining a substantial fraction of the residual spread in equation (20), that is, the component that cannot be explained solely by the expected default probabilities

Figure 11: A Comparison of Estimates of the Bankruptcy Cost Parameter



NOTES: For each specification of the time-varying fixed effects, this figure depicts the time-specific estimates of the bankruptcy cost parameter  $\mu_t$ .

derived from the option-theoretic framework underlying the MKMV approach.

As shown in Figure 11, the exclusion of credit rating effects leads to significantly larger estimates of the bankruptcy cost parameter  $\mu_t$ , although the cyclical pattern of the estimates is essentially the same as in our benchmark specification. Thus, when we exclude portfolio credit rating fixed effects from the vector  $\mathbf{x}_{it}$ , the model-implied credit spread presumably incorporates a combination of bankruptcy costs as well as risk, liquidity, and other premiums, yielding, in turn, a larger estimate of  $\mu_t$ , for all  $t$ . The systematically higher estimates of the bankruptcy cost parameter  $\mu_t$  do not affect the time-series dynamics of the model-implied external finance premium, though they do lead to a substantial increase in the level of the premium across the entire cross-section of firms.

## 7.2 Recovery Rates

Our empirical methodology allows us to derive firm-specific recovery rates implied by the estimated structural parameters  $\mu_t$ ,  $\omega_{it}^*$ , and  $\sigma_{it}$ . Comparing the model-implied recovery rates with the actual recovery rates on corporate debt provides a useful



metric by which to evaluate the quantitative significance of bankruptcy costs during the latest economic downturn.

Conditional on default, the recovery rate is the ratio of the value of the firm (net of bankruptcy costs) to the face value of debt. As discussed in Section 2.1, the realized value of the firm is  $\omega_{i,t+1}R_{it}^k(B_{it} + N_{it})$ . Given the definition of the external finance premium (6) and the first-order condition (8), the expected return to capital  $R_{it}^k$  can be expressed in terms of explicit functions of the default threshold  $\omega_{it}^*$ . Furthermore, under the log-normality assumption, the expected value of the idiosyncratic disturbance  $\omega_{i,t+1}$  conditional on default is given by  $E[\omega_{i,t+1} | \omega_{i,t+1} < \omega_{it}^*] = \Phi(z_{it}^* - \sigma_{it})/\Phi(z_{it}^*)$ .

Thus, the expected recovery rate for firm  $i$  in period  $t$  is given by

$$\text{Recovery Rate} = (1 - \hat{\mu}_t) \left[ \frac{\Phi(\hat{z}_{it}^* - \hat{\sigma}_{it})}{\Phi(\hat{z}_{it}^*)} \right] [(1 + \hat{\rho}_{it})R_t] \left( 1 + \left[ \frac{B}{N} \right]_{it}^{-1} \right), \quad (21)$$

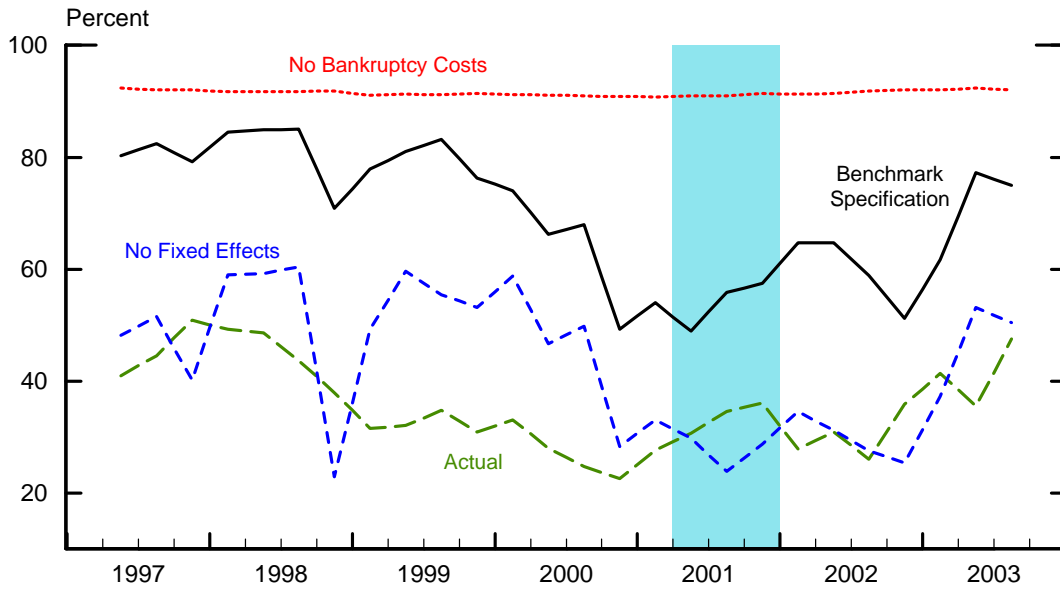
where the first term nets out the estimated bankruptcy costs, the second term corresponds to the mean of the idiosyncratic productivity disturbance in the case of default, the third term is the estimated rate of return on capital, and the final term is the ratio  $(B + N)/B$ .<sup>25</sup> In each period, we average the firm-specific model-implied recovery rates using the book value of firms' bonds outstanding as weights. This resulting series is then compared with the actual recovery rate on nonfinancial corporate issues, computed as an average recovery rate at default, weighted by the book value of defaulted bond issues.

As shown in Figure 12, the average recovery rate implied by our structural estimates consistently exceeded the actual recovery rate on corporate debt during our sample period. Despite this gap, the cyclical pattern of the two series is quite similar, and the divergence narrowed noticeably between 2000 and 2001, a period associated with a considerable increase in the estimated bankruptcy cost parameter  $\mu_t$ . According to this metric, the fit of the model is greatly improved if we drop dummy variables associated with credit ratings (and industries) from estimation, as the average model-implied recovery rate is much closer to the actual recovery rate on defaulted bonds, particularly since 2001. Importantly, the average model-implied recovery rate in the

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<sup>25</sup>In computing the expected return to capital  $R_{it}^k$ , we set the risk-free real interest rate equal to 3 percent; because  $R_t$  is a gross interest rate, the model-implied recovery rates are not sensitive to the choice of a proxy for the real risk-free rate.

Figure 12: Model-Implied vs. Observed Recovery Rates



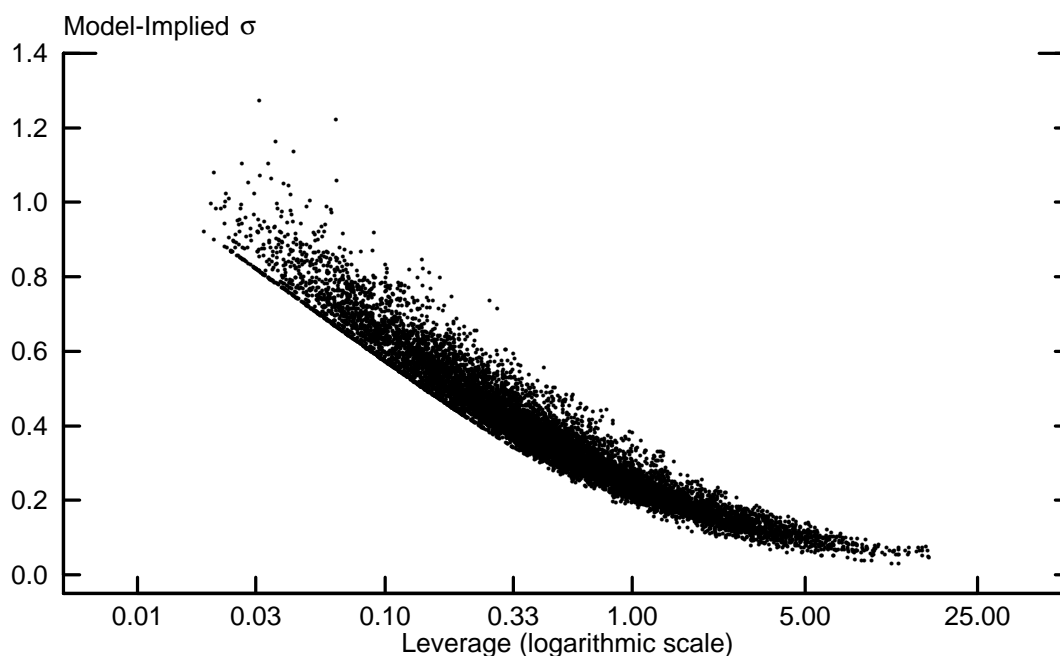
NOTES: This figure depicts the model-implied recovery rate obtained using our benchmark specification (solid line), compared with the actual recovery rate on defaulted corporate debt (long-dashed line). The figure also shows the model-implied recovery rate obtained when the bankruptcy cost parameter is estimated without including any time-varying fixed effects (short-dashed line), and the rate obtained when the bankruptcy cost parameter  $\mu$  is set equal to zero (dotted line).

case of no bankruptcy costs ( $\mu = 0$ ) is unrealistically high and displays no cyclical pattern. Taken together, these results suggests that time-varying bankruptcy costs of substantial magnitude may be needed to match the business cycle dynamics of actual recovery rates on corporate bonds.

### 7.3 Cross-Sectional Implications

Our results indicate a significant variation in the magnitude of financial market frictions over the course of a business cycle. This cyclical variation in expected bankruptcy costs could be related directly to changes in economic and financial conditions, but it could also reflect the somewhat unrealistic specification of the stochastic environment. Although our current framework allows for considerable heterogeneity—in both the cross-sectional and time-series dimension—in the second moment of the distribution of productivity shocks, it could be also important to allow

Figure 13: The Implied Relation between Leverage and Idiosyncratic Volatility



NOTES: This figure plots the model-implied value of the idiosyncratic shock volatility parameter  $\sigma_{it}$  against the leverage ratio  $[B/N]_{it}$  for all observations in our sample.

other moments of idiosyncratic risk to vary over time. For example, while log-normal during periods of normal economic activity, the probability distribution of idiosyncratic shocks could become heavy tailed during economic downturns or periods of financial turmoil. In this case, the rise in credit spreads—and the decline in observed recovery rates—could be due to an increased likelihood of a very bad outcome, as opposed to an increase in the magnitude of financial market frictions.

Indeed, this distributional assumption likely explains a somewhat counterintuitive cross-sectional relationship between leverage and the model-implied value of the idiosyncratic shock volatility parameter  $\sigma_{it}$ . As shown in Figure 13, idiosyncratic volatility exhibits strong negative correlation with the leverage ratio, a result contrary to the usual view that highly leveraged firms are associated with relatively greater risk. Although useful to obtain closed-form solutions for the optimal debt contract, the log-normality assumption implies a very steep leverage-spread schedule (as depicted in the lower-right panel of Figure 2). Because changes in leverage are associated with moderate changes in credit spreads, the theoretical framework can fit

the data only by assigning less variable returns to a firm that experienced an increase in leverage.

## 8 Conclusion

In this paper, we estimated the structural parameters of a canonical debt-contracting model with asymmetric information, employing balance sheet information, market-based measures of expected default risk, and credit spreads on publicly-traded debt for about 900 U.S. nonfinancial firms over the period 1997Q1 to 2003Q3. The parameter estimates characterizing the optimal debt contract, in turn, allowed us to quantify the cross-sectional and time-series behavior of the external finance premium, thus providing direct evidence on the magnitude or cyclical properties of financial market frictions.

We find that the cyclical variation in corporate leverage and the probability of default is insufficient to explain the pronounced widening of credit spreads that preceded the most recent economic downturn. To explain the rise in credit spreads in 2000, our results imply a significant increase in the expected bankruptcy costs during this period. Moreover, the increase in the magnitude of financial market frictions is the key factor behind the concurrent run-up in the external finance premium for a significant portion of firms in our sample. Given that our sample consists of relatively large firms with publicly-traded equity and debt and that the recent behavior of corporate credit spreads was not unusual by historical standards, these results provide strong support for the macroeconomic significance of financial market frictions.

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