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The Mathematics of Behavior

Mathematical thinking provides a clear, crisp way of defining problems. Our whole technology is based on it. What is less appreciated is that mathematical thinking can also be applied to problems in the social and behavioral sciences. This book illustrates how mathematics can be employed for understanding human and animal behavior, using examples in psychology, sociology, economics, ecology, and even marriage counseling.

Earl Hunt is Professor Emeritus of Psychology at the University of Washington in Seattle. He has written many articles and chapters in contributed volumes and is the past editor of *Cognitive Psychology* and *Journal of Experimental Psychology*. His books include *Concept Learning: An Information Processing Problem*, *Experiments in Induction*, *Artificial Intelligence*, and *Will We Be Smart Enough?*, which won the William James Book Award from the American Psychological Association in 1996. His most recent book is *Thoughts on Thought*.

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Preface

Many, many years ago, when I was a graduate student at Yale University, I attended Professor Robert Abelson's seminar on mathematical psychology. This was in the late 1950s, just as mathematical techniques were beginning to hit psychology. Subsequently I met Professor Jacob Marschak, an economist whose work on the economics of information was seminal in the field. After I received my doctorate in 1960 I had the great opportunity to work with Marschak's group at the University of California, Los Angeles. Marschak set a gold standard for the use of mathematics to support clear, precise thinking. It is now almost 50 years later, near the end of my own career, and I have yet to meet someone whose logic was so clear. I have had the opportunity to see some people come close to Marschak's standard, both in my own discipline of psychology and in other fields. This book is an attempt to let future students see how our understanding of behaviors, by both humans and non-humans, can be enhanced by mathematical analysis.

Is such a goal realistic today? Many people have deplored the alleged decline in mathematical training among today's college students. I do not think that that is fair. On an absolute level, students at the major universities arrive far better trained than they were 50 years ago. High school courses in the calculus are common today; they were rare even 25 years ago. It is true that on a comparative basis American students have slipped compared to their peers abroad, but on an absolute basis the better students in all countries are simply better prepared than they used to be. I have set my sights accordingly. This book should be easily accessible to anyone who has a basic understanding of the calculus, and most of the book will not even require that. It will require the ability (the willingness?) to follow a mathematical argument. I hope that the effort will be rewarded. Curious about the mathematics of love? Or how unprejudiced people can produce a segregated society? Read on!

And to those of you on college and university faculties, consider teaching a course that covers topics like this; mathematics used to analyze important issues in our day, or important issues in the history of science. Don't restrict it to your own discipline; think broadly. I hope you find this book helpful, but if you don't, get some readings and teach the course anyway. I have been fortunate to teach such a course in the University of Washington Honors Program for the past several years, and the discussions among students pursuing majors from philosophy to biology and engineering have been informative and enjoyable.

No one prepares a book without a great deal of support. I have had it. I thank the Honors Program and, most especially, the students in my classes, for letting me lead the course. I also thank the Psychology Department for letting me lead a predecessor of this course, focusing somewhat more on psychology. Cambridge University Press provided assistance in book preparation that was far superior to that of any other press with which I have ever worked. I thank Regina Paleski for production editing assistance, and I particularly thank Phyllis Berk for a superb job of copyediting a difficult manuscript. I also thank the editor, Philip Laughlin, for his assistance, and in particular for his obtaining very high-quality editorial reviews. Naturally, the people who wrote them are thanked, too! The final review, by Professor Jerome Busemeyer of the University of Indiana, was a model of constructive criticism.

Every author closes with thanks to family . . . or at least, he should. My wife, Mary Lou Hunt, has supported me in this and all my scholarly work. I could not accomplish any efforts without her loving aid and assistance.

Earl Hunt
Bellevue, Washington, and
Hood Canal, Washington
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