

## BOOK REVIEWS

semiempirical procedure for estimating an allowable fatigue strength of rigid plastics and plastic composites.

In summary, this proceeding contains many experimental results of practical interests to the practicing as well as research oriented engineers. Like many similar publications, a lack of forceful editorship is noted by numerous awkward usages of English.

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**MHD Flows and Turbulence.** Edited by H. Branover. John Wiley & Sons, Inc., New York, 1976. Pages 161. Price \$25.

REVIEWED BY S. LEIBOVICH<sup>9</sup>

This book contains accounts of most of the lectures presented at the Bat-Sheva Seminar on MHD-Flows and Turbulence held at the Ben-Gurion University during March 17–20, 1975. The title suggests a focus on turbulence in MHD flows, but few of the papers integrate the two topics. In fact, the book contains an odd miscellany of subject matter.

The topics given greatest attention are theoretical papers (5) on laminar MHD duct flows, and descriptions of experimental MHD facilities or programs (5 papers). In addition, there is one paper each on homogeneous MHD turbulence theory, on turbulence in ordinary fluid dynamics, and on the Toms effect, and abstracts of two other papers.

Most of the papers are very short, highly condensed reviews of recent progress made by the contributors. The book is therefore not one that individuals are likely to buy for their personal bookshelves.

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**Turbulence in Liquids.** Proceedings of the Fourth Biennial Symposium on Turbulence in Liquids, September 1975. Edited by J. L. Zakin and G. K. Patterson. Science Press, Princeton, N. J. 1977. Pages 366. Price \$22.

REVIEWED BY P. A. LIBBY<sup>10</sup>

The field of turbulence contains many facets and is rich in problems of an applied and fundamental nature. Consequently there are each year a variety of meetings, symposia, workshops, etc., on various aspects of turbulence. This volume presents the contributions to the Fourth Symposium on Turbulence in Liquids held at the University of Missouri-Rolla in 1975. Generally the papers are concerned with measurements employing various techniques, hot film, and laser anemometry and electrochemical techniques to provide data on turbulent phenomena in water but well known in gases as well. In fact some papers are devoted to measurements in air, clearly not consistent with the title of the symposium, in air-water mixtures and in blood. Five papers involve pressure measurements at a wall. Many papers are followed by written discussions which aid considerably in placing the proceeding contribution in some perspective, relative to the extensive relevant literature.

Workers in the field of experimental turbulence will want at the least peruse this volume in order to keep track of developments in their field.

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**The Mathematics of Finite Elements and Applications II.** Edited by J. R. Whiteman, Academic Press, London, England. 1977. Pages 573. Price \$41.

REVIEWED BY J. TINSLEY ODEN<sup>11</sup>

Over the past five years, there have been a number of national and

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international meetings designed to bring engineers and mathematicians together who have a common interest in finite-element methods. By enlarge, these meetings have been failures. There has either been too much emphasis on mathematics to interest those engineers who attended or the flavor has been so applied as to repulse the mathematicians who have attended. Also these meetings are frequently burdened by poor communication between one camp and another.

There have been, however, one or two of these meetings that were of notable successes, and the MAFFELAP Meetings are among them. This is particularly true of the Second MAFFELAP Meeting which, in the reviewer's opinion, surpassed the first in not only the quality of papers presented, but also the breadth of the topics covered and the importance of many of the new results. The present volume contains the proceedings of this conference which was held in April of 1975 at Brunel University in Uxbridge, England. John Whiteman, Director of the Institute for Computational Mathematics at Brunel, organized the meeting which highlighted invited lecturers and a number of contributed papers. Having attended the meeting myself, I believe that there was an unusually good flow of information between the theorists and those with more applied orientation. Perhaps it was the choice of the speakers or perhaps the hospitality provided at the meeting were conducive to this, but there was more. For it seems that many people saved some of their better results to report at this meeting.

The present volume contains forty-three (43) papers presented at the meeting, some of them representing very important developments in the theory and application of finite-element methods. Let me mention a few: Two independent papers, one by A. R. Mitchell and another by R. McLeod, discussed various technical problems encountered in using curved finite elements. Mitchell's paper, in particular, showed how essential boundary conditions can be matched exactly on curved linear triangular elements. In McLeod's paper, he represented an enlargement and extension of the main ideas of Mitchell's paper. This paper, together with an interesting paper by R. E. Barnhill, "Blending Function Finite Elements for Curved Boundaries," represent what is commonly referred to as the interpolation theories surrounding finite-element methods: i.e., the technique of using finite elements for curve fitting.

There were some other important developments: D. S. Watkins reported on a new Hermite cubic blending finite element which involves  $C^1$  interpolation functions. This element should prove to be quite competitive for plate bending problems.

Among the most exciting papers in the volume were two on a self-adaptive approach for finite-element methods, one by Ivo Babuska and another by G. Sewell. Here the idea is to specify *a priori* a degree of accuracy that the user desires and to allow the computer to refine the mesh accordingly. Babuska's paper represents a significant step in producing adaptive finite-element mesh generators. A similar paper was presented by G. Sewell in which a quite different method was proposed for adaptive mesh refinement. Sewell's method employed triangular elements and was confined to Poisson-type problems.

Frequently in the mathematical theory of finite elements, the question arises as to whether or not the constants appearing in the error bounds can be evaluated. An important step toward answering this question was proposed in the paper by J. A. Gregory which estimates on the constants for linear interpolation on triangles was provided. There were several papers on parabolic and hyperbolic problems—one of some interest by M. Zlamal on heat conduction problems and another by A. Cella on accuracy and stability of finite-element methods for parabolic and hyperbolic equations.

An important paper describing nonconforming elements for solving transport equations was presented by P. Lesaint and another intriguing paper on degenerate interface problems was contributed by G. H. Meyer and G. Sewell.

Variational methods for free-boundary problems were studied by M. J. O'Carroll and H. T. Harrison and on discretization in time for parabolic equations by N. R. Nassif, finite-element solutions in space and time for heat transfer problems by M. M. Cecchi, viscoelastic problems by J. Brilla, and semidiscrete Galerkin techniques with time

interpolation for plasma simulations by R. England, J. P. Hennart, and J. G. Martin.

For many years, an open question in the theory of finite elements has been that surrounding the manner in which a noneuclidean surface can be approximated which is the domain of a solution of a partial differential equation: in other words, the approximation of the equations of thin elastic shells. In a very important paper, P. G. Ciarlet obtained *a priori* estimates for finite-element approximations in which the surface and the solution of the shell equations are approximated simultaneously. Ciarlet's analysis employs the Koiter theory of shells.

There were several papers on applications of finite elements to practical problems. Among these we mention a survey article by R. W. Clough and H. Petersson; a paper on "Finite-Element Solution of Two-Dimensional Unsteady and Unsaturated Flow in Porous Media," by G. Zvoloski and J. C. Bruch, Jr.; and work on "Coupled Convective/Conductive Heat Transfer Problems," by C. Taylor and A. Ijam. Applications in reactor physics were discussed by F. A. R. Schmidt, H. P. Franke, and E. Sapper; convection/diffusion problems by R. Piva and A. DiCarlo; curved beams by A. B. Sibir; and the nonlinear behavior of thick and thin plates by E. Hinton, D. R. J. Owen, and D. Shantaram.

There are a number of good papers on fluid dynamics applications and theory. Among these were a paper by O. C. Zienkiewicz, R. H. Gallagher, and P. Hood on "Newtonian and Non-Newtonian Viscous Incompressible Flow and Temperature Induced Flows." I also mention a paper on water pressure and soil-water content distributions in drained land by A. B. Gureghain and E. C. Youngs; a paper on least-squares methods for compressible flow by W. S. Blackburn; through-flow calculations by Ch. Hirsch and G. Warzee; and "Prediction of Fully Developed Turbulent Flow," by H. Barrow, R. P. Hornby, and J. Mistry.

A paper on "cruciform elements" for the analysis of fabric structures was presented by I. Torbe and "Thermal and Stress Analysis of Prismatic Nuclear Fuel Elements," by H. Cords and W. Diemont.

There were a number of papers on the solution of equations generated by finite-element methods. Among these we mention a paper on the "Natural Factor Approach," by J. R. Argyris and O. Brønlund; a paper on "Iterative Solution of Linear Systems," by D. M. Young; a paper "On the Solution of Some Nonlinear Equations Arising in Finite-Element Methods," by W. C. Rheinboldt; "Approach for Deriving Better Conditions Stiffness Matrices," by P. G. Bergan and L. Hanssen; a paper on "General Solution Routines for Symmetric Equations," by N.-E. Wiberg and H. Tagnfors; "Numerical Integration Considerations for Two and Three Dimensional Isoparametric Elements," by T. K. Hellen; "Removal of Truncation Error," by C. W. Martin and A. J. Harrold; "Element Resequencing for Frontal Solutions," by J. E. Akin and R. M. Pardue.

Also in the proceedings was an interesting survey paper by John Whiteman on some aspects of the mathematics of finite elements, which contained some new results on formulation of nonlinear boundary-value problems. Another paper on nonlinear boundary-value problems was contributed by W. Froidevaux.

A paper by J. T. Oden and L. C. Wellford, Jr., discussed the use of discontinuous finite elements for the analysis of acceleration emission waves and shock waves. By discontinuous elements, we do not mean the usual nonconforming elements; rather, the elements described here have a built-in discontinuities to depict shocks and second-order discontinuities in finite-element methods used in wave problems.

One point of much concern during the meeting itself was the use of finite-element methods on convective diffusion problems. Various finite-element techniques for handling these problems were proposed in the paper by Zienkiewicz, Gallagher, and Hood and in the paper by Piva and DiCarlo. In the former paper, an up-wind and differencing element is used, while in the latter, a "mesa-type" element is proposed. Encouraging results have been obtained in both papers.

This is a handsome book; the paper is of high quality and it has a sturdy hard-cloth cover. The format is a good one and Whiteman has

done a very good job of editing the work. A subject index is also provided.

There are a number of important papers in the book that should be of interest to not only those working in the theory of finite-element methods, but also to those interested in applications of the method to important problems in contemporary mechanics and physics.

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**Construction of Integration Formulas for Initial Value Problems.** By P. J. Van der Houwen. North-Holland Publishing Company, Amsterdam, The Netherlands. 1977. Pages xii-276. Price \$33.95.

REVIEWED BY K. C. PARK<sup>12</sup>

This monograph is intended for those who are seeking a balanced treatment of both theory and practice on numerical initial value problems. The author has done much of the original work that is presented in the text, especially on generalized Runge-Kutta methods. One unique feature of the book is the classification of different integration techniques within the context of generalized Runge-Kutta formulas. For example, iterating the trapezoidal rule by Richardson's method or by Newton-Raphson's method leads to formulas which are equivalent to generalized Runge-Kutta methods. The other important contribution of the book is a unified construction technique of different integration formulas from their associated stability polynomials.

Chapter 1 covers some preliminary material: notation, brief mathematical considerations, an introduction to stiff differential equations, and some well known iterative techniques for nonlinear problems. In Chapter 2, one-step methods are treated as pertaining to consistency and convergence conditions. Then, the concept of propagation of numerical errors is utilized to derive the stability functions of various one-step methods, i.e., Taylor, Padé, and generalized Runge-Kutta methods. As the numerical stability requirement is often dictated by the problem characteristics, the task of constructing methods which possess the same stability characteristics is presented. This is one of the practical aspects of the stability functions, which the author illustrates remarkably well.

The construction of multistep methods is briefly treated at the beginning of Chapter 3. The rest of this chapter is devoted to several high-order integration algorithms which possess either extended stability ranges or minimized extraneous roots. Finally, Chapter 4 deals with four classes of stability functions:

- 1 Polynomial Padé approximations which lead to either implicit or explicit Taylor methods.
- 2 Polynomials with maximal negative stability ranges for parabolic problems.
- 3 Polynomials with maximal imaginary stability ranges which are crucial for the effective solution of hyperbolic equations.
- 4 Polynomials with widely separated subregions of stability which can appeal to problems with widely separated time constants.

The overall format of the book is concise, and where appropriate, tables and figures are interspersed for augmenting the theory and practical aspects presented in the text. The extensive numerical experiments in Chapter 2 should prove useful regarding comparison criteria for different methods and stepsize control strategies for nonstiff problems.

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**Analytical Dynamics of Discrete Systems.** By Reinhardt M. Rosenberg. Plenum Press, New York. 1977. Pages 424. Price \$25.

REVIEWED BY W. E. SCHMITENDORF<sup>13</sup>

This book is written as a textbook in dynamics for seniors and beginning graduate students.

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