The Matter Bounce Scenario in Loop Quantum Cosmology

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Motivation

It is generally expected that quantum gravity effects will only become important when

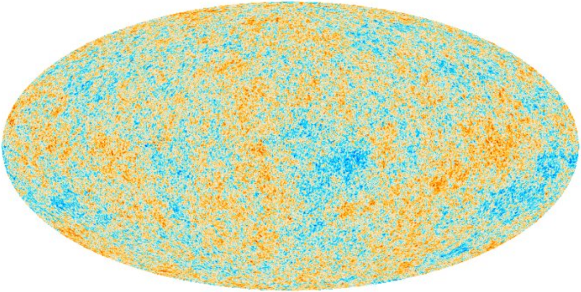
- the space-time curvature becomes very large,
- or at very small scales / very high energies.

Since we cannot probe sufficiently small distances with accelerators, or even with cosmic rays, the best chance of testing any theory of quantum gravity is to observe regions with high space-time curvature.

The two obvious candidates are black holes and the early universe. However, since the strong gravitational field near the center of astrophysical black holes is hidden by a horizon, it seems that observations of the early universe are the best remaining option.

Observational Data

The Planck collaboration has released a wealth of data on the temperature anisotropies in the cosmic microwave background (CMB), and more recently the BICEP2 collaboration has claimed detection of primordial gravitational waves.



These precision observations probe the high space-time curvature regime of the early universe.

Loop Quantum Cosmology

Following these arguments, it seems that the best way to test any theory of quantum gravity is to study its cosmological sector and determine what imprints it would leave on the CMB, and on primordial gravitational waves. This is what we shall do now for loop quantum cosmology (LQC).

In LQC, the same variables and quantization procedures are used as in loop quantum gravity in order to study cosmological space-times, giving a well-defined quantum theory. [Bojowald, Ashtekar, Lewandowski, Pawłowski, Singh, . . .]

One main result of LQC is that quantum gravity effects become important in the very early universe and resolve the big-bang singularity, replacing it by a "big bounce".

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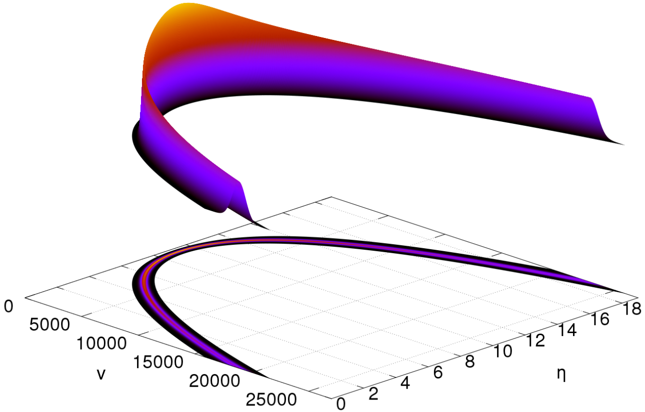
Does the pre-bounce era have any impact on the CMB or primordial gravitational waves?

Outline

1. Brief Overview of Loop Quantum Cosmology
2. Cosmological Perturbation Theory in Loop Quantum Cosmology
3. The Matter Bounce Scenario

The Bounce

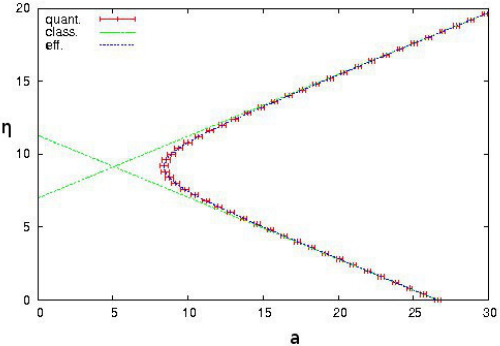
Using a matter field as a relational clock, we can plot the "wave function of the universe" $\Psi(v)$ as a function of the matter field acting as time. Here v is related to the scale factor by $v = a^3$.



[Pawłowski, Pierini, WE]

The Effective Theory

The effective equations of loop quantum cosmology (LQC) provide quantum-gravity corrections to the classical solutions. [Taveras, Willis]



[Pawłowski, Pierini, WE]

The dynamics of a sharply-peaked state are very well approximated by the effective Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right),$$

with $\rho_c \sim \rho_{\rm Pl}$.

Furthermore, the wave function remains very sharply peaked, even at the bounce point.

Perturbations in Loop Quantum Cosmology

Cosmological perturbations have been studied in LQC for some time now, and following several different approaches:

- Inverse triad corrections using effective equations, [Bojowald, Hossain, Kagan, Shankaranarayanan]
- Holonomy corrections using effective equations, [WE; Cailleteau, Mielczarek, Barrau, Grain, Vidotto]
- Lattice loop quantum cosmology, [WE]
- Hybrid quantization. [Fernández-Méndez, Mena Marugán, Olmedo; Agulló, Ashtekar, Nelson]

Here we will follow the lattice LQC approach.

Lattice Loop Quantum Cosmology

Lattice LQC comes from taking a Friedmann universe with linear perturbations and discretizing it on a lattice. In this approximation, all cells in the lattice are homogeneous. Then an LQC quantization is possible in each cell. [WE; cf. Salopek, Bond; Wands, Malik, Lyth, Liddle]

a(1) φ(1)	a(2) φ(2)	a(3) φ(3)
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From the resulting theory, it is possible to derive effective equations for the perturbations.

These effective equations are expected hold for perturbation modes that remain larger than the Planck length. [Rovelli, WE]

The effective LQC-corrected Mukhanov-Sasaki equation is [WE; Cailleteau, Mielczarek, Barrau, Grain]

$$v'' - \left(1 - \frac{2\rho}{\rho_c}\right) c_s^2 \nabla^2 v - \frac{z''}{z} v = 0, \qquad z = \frac{a\sqrt{\rho + P}}{c_s H}.$$

Scale-invariant Perturbations

The spectrum of scalar perturbations in the CMB, determined from the temperature anisotropies, has been observed to be almost scale-invariant. LQC by itself is not enough to generate scale-invariant perturbations: it is also necessary to choose an appropriate matter field.

A common choice is an inflaton field that gives $\sim 60 - 70$ e-foldings of inflation, and this generates almost scale-invariant scalar perturbations, as observed in the CMB. [Ashtekar, Sloan, Agulló, Nelson, Linsefors, Cailleteau, Barrau, Grain, . . .]

However, there are some alternatives to inflation, and in particular some where the scale-invariant perturbations are generated in the contracting branch of the universe and thus rely on the existence of a bouncing universe. These alternatives seem particularly interesting from the perspective of LQC.

Review of the Matter Bounce Scenario

The matter bounce scenario is one such alternative to inflation.

In a contracting, matter-dominated universe ($P = 0$), if perturbations are initially in the quantum vacuum state, as they exit the Hubble radius the perturbations become scale-invariant. [Wands]

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Note however that there is a priori no guarantee that the scale-invariance of these perturbations will survive the bounce. There exist some heuristic matching conditions that argue that scale-invariance will be preserved across the bounce [Finelli, Brandenberger], but this must be checked in realizations of the matter bounce scenario.

Questions in the Matter Bounce

There are two main questions that all realizations of the matter bounce scenario must address.

1. What causes the bounce?

   It is possible to obtain a bounce either by violating energy conditions in general relativity [Brandenberger, Cai, Easson, Qiu, Zhang, . . .] or modified gravity, as arises naturally in LQC.

   Interestingly, many of the qualitative predictions are similar in both types of realizations. [Cai, WE]

2. Is scale-invariance preserved across the bounce?

   Due to the presence of a non-singular bounce, it will be possible to calculate how the perturbations evolve across the bounce and determine explicitly the resulting spectrum.

The Matter Bounce in Loop Quantum Cosmology

Solving the effective Friedmann equations for a matter-dominated cosmology, we find

$$a(t) = a_o \left(6\pi G\, t^2 + \frac{1}{\rho_c} \right)^{1/3},$$

and from this it is possible to solve the LQC-corrected Mukhanov-Sasaki equation and thus determine the evolution of the perturbations through the non-singular bounce.

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In the classical regime, where corrections from LQC are negligible,

$$\mathcal{R}_k \sim \frac{1}{(-\eta)^{3/2}} \cdot H_{\frac{3}{2}}(-k\eta).$$

Via the asymptotics of the Hankel function, we can verify that the perturbations are initially in the quantum vacuum state, and become scale-invariant for $|k\eta| \ll 1$.

Evolution Through the Bounce

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During the bounce, the evolution of the long wavelength modes goes as

$$\mathcal{R}_k \sim k^{-3/2} \cdot \sqrt{\frac{\rho_c}{\rho_{\rm Pl}}} \cdot \left(\arctan \sqrt{6\pi G \rho_c} t + \frac{1}{6\pi G \rho_c t^2 + 1} \right),$$

where only the dominant term is given here.

Clearly, $\mathcal{R}_k$ remains finite at all times, and the dominant term is scale-invariant and approaches a constant amplitude for $t \to \infty$.

Radiation-Dominated Bounce

On the previous slide, we did the calculations for a "pure" matter bounce, which is not very realistic.

Instead, let's consider a universe composed of cold dark matter with a sound speed $c_s \ll 1$, and radiation. At early times, the cold dark matter dominates the dynamics, and long wavelength modes are scale-invariant.

Then, after the "equality time" t_e when $\rho_{\rm CDM} = \rho_{\rm rad}$, the radiation field dominates the dynamics. t_e is reached well before the bounce.

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For the modes that reach the long-wavelength limit during the CDM-dominated era, the power spectrum is

$$\Delta^2_{\mathcal{R}}(k) \sim \frac{\sqrt{G\hbar}|H_e|}{c_s^3} \cdot \sqrt{\frac{\rho_c}{\rho_{\rm Pl}}},$$

where $H_e = H(t_e)$.

An Asymmetric Bounce

For all of the observed modes to be scale-invariant, they must have all exited the Hubble radius before radiation-domination. However, in the current expanding branch of our universe, many of the observed modes re-entered the Hubble radius during the radiation-dominated era.

Therefore the bounce must be asymmetric: matter-domination must "last longer" in the contracting branch. More precisely, $|H_e|$ before the bounce must be significantly larger than H_e after the bounce.

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Interestingly, it has been suggested that particle production during the LQC bounce may be important, and this could cause an asymmetry of precisely the type required here. [Mithani, Vilenkin]

Primordial Gravitational Waves

The matter bounce scenario presented here, in the context of loop quantum cosmology and with matter fields consisting of cold dark matter and radiation, predicts a very small tensor-to-scalar r,

$$r = 24 c_s^3 < 0.01,$$

assuming a conservative upper bound on the sound speed of cold dark matter of $c_s < 0.1$. Such an r is not expected to be detected in the near future.

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However, there do exist other realizations of the matter bounce where r can be larger. In these other models, the bounce is caused by a matter field which violates energy conditions, and for some choices of parameters in the matter field Lagrangian, r can be of the order [Cai, Quentin, Saridakis, WE]

$$r \sim O(0.1).$$

Conclusions

- There exist several complementary approaches to cosmological perturbation theory in loop quantum cosmology, including lattice LQC.
- The matter bounce is an alternative to inflation that generates scale-invariant perturbations.
- Assuming a matter content of cold dark matter and radiation, the matter bounce scenario is naturally realized in loop quantum cosmology.
- In this particular realization of the matter bounce, the predicted tensor-to-scalar ratio is very small.

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Thank you for your attention!