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Economics Working Paper 74

**The Maximum Capture Problem with  
Uncertainty\***

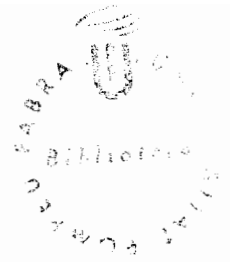
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March 28, 1994



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*Keywords:* Competitive Location, Discrete Network Location, Scenarios.

*Journal of Economic Literature classification:* C61, R30, L11.

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\* This project was partially funded by DGICYT grants PB92-1036 and PB92-1037, Ministry of Education, Spain.

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## **Abstract**

The strategic location of servers by a firm in a competitive environment is basic in the definition of market shares. Suppose that a firm wants to locate  $p$  servers so as to maximize market capture in a region where competitors are already located but where there is uncertainty –there are several possible future scenarios with respect to demand and/or the location of competitors. The firm will want a strategy of positioning that will do as “well as possible” over the future scenarios. This paper presents a discrete location model formulation to address this Maximum Capture Problem under uncertainty.

# 1 Introduction

The problem of market capture has become a classic since the visualization by Hotelling, of ice cream vendors on a beach competing for market share. That simple paradigm with its remarkable result of co-location has underlain and informed investigations since.

In the modern era, at least two lines of work can be discerned. On the one hand, a number of models have focused on the profit objective, using production and pricing decisions of the competing firms (see, for example, Lederer 1986, Tobin and Friesz 1986 , Friesz, Miller and Tobin 1987 , and Dobson and Karmakar 1987, Lederer and Thisse 1990 , Labbé and Hakimi 1991 , Miller, Tobin and Friesz 1992 ). On the other hand, a line of research has focused solely on location and distance; assuming the products are the same and sold at the same price everywhere in the network (see, for example, Hakimi 1983 , ReVelle 1986 , Eiselt and Laporte 1989 , Karkazis 1989 , Serra, Marianov and ReVelle 1992 , and Serra and ReVelle 1994).

This paper follows the direction set out by Hakimi (1983) and ReVelle (1986) and continued in Serra and ReVelle ((forthcoming)). Serra and ReVelle sought the positions of  $p$  servers of firm  $A$  at nodes of a network in such a way that the  $q$  servers of firm  $B$  who respond to the  $p$  locations would be limited to the least possible capture from firm  $A$ . That is, firm  $A$ 's goal is to site its servers to minimize the maximum capture that firm  $B$  can achieve by its siting. Serra and ReVelle utilize a heuristic procedure to produce good solutions to this problem.

The present paper recognizes uncertainty in the demand or population at the nodes of the network. That is, the demand or population that firm  $A$  is setting out to capture is not a known quantity but can assume different values depending on community growth or the community's economic vitality, among other factors. Uncertainty is treated by the classic scenario approach in which different patterns of demand are realized in different scenarios. The approach is two-pronged. First, over a range of possible demand scenarios, facilities are deployed to sites in such a way as to maximize the minimum demand captured. Second, over that same range of scenarios, facilities are positioned in such a way to minimize the maximum regret. Regret is the difference between the demand that might have been captured had the decision maker planned its sites for the scenario that actually occurred and the value of demand that was

actually captured.

## 2 Problem Formulation

Consider a spatial market that is represented by discrete points in a connected network. Each node can represent a local market and has a parameter that can represent population or local demand for the product offered in that market. On the other hand, some -but not all- of the nodes in the network have servers that offer the product. These servers belong to different firms that compete for consumers. Each one of these firms can have more than one server located. The product that is sold is homogeneous and its price is the same across the market. It is assumed that all firms bear the same unit costs. Consumers will travel to their closest facility to obtain the desired product. A consumer node is captured by a firm if it has a server closer to it than any of its competitor's servers. If some population has two or more servers at the same distance, they will divide in equal share the captured population.

Suppose that a Firm (from now on Firm A) wants to enter in this spatial market by locating  $p$  servers in order to capture the maximum market share possible. It will be assumed without loss of generality that only one firm (Firm B) has already  $q$  servers operating in the market. The problem is that Firm A faces some possible scenarios, where populations (or local demands) are possible. On the other hand, Firm A knows with certainty where its competitors are located. Given the assumptions on the market, when there is only one scenario the objective to optimize is clear cut: maximize the market share by locating a given number of servers, as in the Maximum Capture Problem (MAXCAP, ReVelle 1986). The problem now is that it depends on the location of Firm A's servers that some scenarios will be better captured than others.

Two different objectives can be used to obtain the final locations. The first one consists on the maximization of minimum the capture that can be achieved accross scenarios (from now on Max Objective) The second one uses the regret approach, that is, minimize the maximum regret accross scenarios (from now on, Regret Objective).

The mathematical formulation of the model using the Maximin Objective, based on the Maximum Capture Problem (1986), is as follows:

$$\max Z = m$$

subject to:

$$\sum_{i \in I} a_{ik} y_i + \sum_{i \in I} (a_{ik}/2) z_i \geq m \quad \forall k = 1, \dots, s \quad (1)$$

$$y_i \leq \sum_{j \in N_i} x_j \quad \forall i \in I \quad (2)$$

$$z_i \leq \sum_{j \in O_i} x_j \quad \forall i \in I \quad (3)$$

$$y_i + z_i \leq 1 \quad \forall i \in I \quad (4)$$

$$\sum_{j \in J} x_j = p \quad (5)$$

$$y_i, z_i, x_j = (0, 1) \quad \forall i \in I, \forall j \in J$$

where:

- $i, I$  = index and set of demand nodes
- $j, J$  = index and set of potential facility sites
- $k, s$  = Index and total number of of scenarios
- $a_{ik}$  = Population at node  $i$  in scenario  $k$
- $d_{ij}$  = distance between node  $i$  and node  $j$
- $b_i$  = Closest Firm B server to node  $i$
- $d_{ib_i}$  = Distance from node  $i$  to its closest Firm B's server
- $N_i$  =  $\{\forall j \in J, d_{ij} < d_{ib_i}\}$
- $O_i$  =  $\{\forall j \in J, d_{ij} = d_{ib_i}\}$
- $p$  = number of servers to locate by Firm A
- $y_i$  =  $\begin{cases} 1, & \text{if node } i \text{ is captured by Firm A} \\ 0, & \text{otherwise} \end{cases}$
- $z_i$  =  $\begin{cases} 1, & \text{if node } i \text{ is divided between A and B} \\ 0, & \text{otherwise} \end{cases}$
- $x_j$  =  $\begin{cases} 1, & \text{if Firm A locates a server at node } j \\ 0, & \text{otherwise} \end{cases}$

The first set of constraints is directly related to the objective. Since we want to maximize the minimum capture across scenarios, we want to find a set of locations that will give the largest minimum capture possible in a given scenario. The left side of each constraint (one for each scenario) represents the capture that will be achieved in the corresponding scenario. The right hand side,  $m$ , is the same in each constraint. The objective of the model is to maximize  $m$ , that is, the model will try to find a set of locations that maximizes the lowest capture achieved in a given scenario.

The rest of the constraints are the same as the constraint set of the MAX-CAP problem. The second set of constraints allows the capture of node  $i$  by Firm A if and only if Firm A has a server located closer to  $i$  than the closest Firm B server to node  $i$ . The third set of constraints examines the situation where there is a tie in the capture of a node. The variable  $z_i$  will be allowed to be 1 if and only if the distance from  $i$  to the closest Firm A server and to the closest Firm B server is equal. Therefore, the capture of node  $i$  will be divided between both firms, as stated in the objective function. Observe that for any node  $i \in I$  can be captured, or half captured, or lost to the competitor. Constraints in group (4) will enforce one of these three states. Finally, the number of servers to be located by Firm A is determined by constraint (4).

If the Regret Objective is used, constraint set (1) is replaced by the following set:

$$Z_k - \sum_{i \in I} a_{ik} y_i + \sum_{i \in I} (a_{ik}/2) z_i \leq U \quad \forall k = 1, \dots, s \quad (1')$$

Where  $Z_k$  is the optimal objective found when Firm A locates  $p$  facilities in each scenario independently using the MAXCAP Problem. The new objective is:

$$\min Z = U$$

This model will try to find a set of locations for Firm A that will minimize the maximum regret that can be achieved across scenarios. For a given set of locations, the left side of each one of the constraints (there is again one constraint for each scenario) computes the difference between the optimal capture that can be achieved in the scenario and the capture given the current set of locations, that is, the regret. The objective improves as the largest

regret found in a given scenario is reduced. The final solution will be found when no smaller regret is obtained in a given scenario for a given set of locations. The rest of the constraints remain equal.

The basic model (Max Objective) can be adapted to consider scenarios where not only demands differ, but also the location of the competitors. In this case, the new formulation is as follows:

$$\max Z = m$$

subject to:

$$\sum_{i \in I} a_{ik} y_{ik} + \sum_{i \in I} (a_{ik}/2) z_{ik} \geq m \quad \forall k = 1, \dots, s \quad (10)$$

$$y_{ik} \leq \sum_{j \in N_{ik}} x_j \quad \forall i \in I, \forall k = 1, \dots, s \quad (7)$$

$$z_{ik} \leq \sum_{j \in O_{ik}} x_j \quad \forall i \in I, \forall k = 1, \dots, s \quad (8)$$

$$y_{ik} + z_{ik} \leq 1 \quad \forall i \in I, \forall k = 1, \dots, s \quad (9)$$

$$\sum_{j \in J} x_j = p \quad (5)$$

$$y_{ik}, z_{ik}, x_j = (0, 1) \quad \forall i \in I, \forall j \in J, \forall k = 1, \dots, s$$

where additional notation is:

$$\begin{aligned} b_{ik} &= \text{Closest Firm B server to node } i \text{ in scenario } k \\ d_{ib_{ik}} &= \text{Distance from node } i \text{ to its closest Firm B's server in scenario } k \\ N_{ik} &= \{\forall j \in J, d_{ij} < d_{ib_{ik}}\} \\ O_{ik} &= \{\forall j \in J, d_{ij} = d_{ib_{ik}}\} \\ y_{ik} &= \begin{cases} 1, & \text{if node } i \text{ is captured by Firm A in scenario } k \\ 0, & \text{otherwise} \end{cases} \\ z_{ik} &= \begin{cases} 1, & \text{if node } i \text{ is divided between A and B in scenario } k \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

Since competitors are located differently in each scenario, there is a different set  $N_i$  and  $O_i$  for each scenario. Therefore, the capture of a node has



to be redefined since it depends on the location of its closest competitor. To take into account this new situation, capture variables  $y_i$  and  $z_i$  are replaced by  $y_{ik}$  and  $z_{ik}$ . Observe that now the number of constraints in sets (2) and (3) has increased. There is one constraint for each node and each scenario.

Again, if the regret approach is used the objective and constraint set (1) are replaced by:

$$\min Z = U$$

$$Z_k - \sum_{i \in I} a_{ik} y_{ik} + \sum_{i \in I} (a_{ik}/2) z_{ik} \leq U \quad \forall k = 1, \dots, s \quad (1')$$

respectively. Observe that if only the location of the competitors differ across scenarios, and the local demands remain the same, the demand parameter  $a_{ik}$  can be replaced by  $a_i$  in the formulation

### 3 Computational Experience

The 55-node Swain network was used with five different population scenarios (see Appendix). MINOS, a linear, integer and nonlinear programming software was used on a HP Apollo 710 workstation with a Risc Processor. The problem was solved on this network using standard Linear Programming with Branch and Bound when necessary (LP+BB). Five servers of Firm B were already located and Firm A wants to enter this market by also locating 5 servers. Table 1 presents the initial locations of Firm B's servers in each scenario.

The value of the objective  $m$  found for the Max model is equal to 1989, and the final locations for Firm A are 5,8,16,29,41. In the Regret model is used, the final objective  $U$  is equal to 217.5 and final locations are 5,8,16,31 and 41. Table 2 presents the captures that could be achieved independently in each scenario. Other results are shown in the following table. The second column shows the total population in the region of interest for each scenario. The location of Firm A's servers and the optimal capture that the firm could achieve if each scenario was considered independently when locating 5 servers, that is, if a MAXCAP problem was used in each scenario, is presented in the third column. Therefore, for each scenario a MAXCAP is used. The fourth

Table 1: Firm B Locations in each Scenario

| loc-<br>ations  | Scenarios |    |    |    |    |
|-----------------|-----------|----|----|----|----|
|                 | 1         | 2  | 3  | 4  | 5  |
| 1 <sup>st</sup> | 4         | 3  | 1  | 3  | 4  |
| 2 <sup>nd</sup> | 21        | 5  | 10 | 10 | 21 |
| 3 <sup>rd</sup> | 22        | 17 | 16 | 17 | 22 |
| 4 <sup>th</sup> | 36        | 26 | 29 | 25 | 36 |
| 5 <sup>th</sup> | 38        | 38 | 36 | 38 | 38 |

and fifth columns show the total capture obtained in each scenario with the locations found by the Max Model and the difference from the optimal capture if each scenario is considered independently. The same applies for the Regret model in columns six and seven. Observe that except for the fourth scenario, Firm A obtains better results if the Regret model is used.

Table 2: Final solutions: Max and Regret Objectives

| Scen-<br>ario | Total<br>Pop. | Optimal Solution<br>in each scenario |        | Max. Capture<br>across scenarios |       | Min. Regret<br>across scenarios |       |
|---------------|---------------|--------------------------------------|--------|----------------------------------|-------|---------------------------------|-------|
|               |               | Locations                            | Obj.   | Obj.                             | Diff. | Obj.                            | Diff. |
| 1             | 3575          | 5,22,26,38,42                        | 2243.5 | 1989.0                           | 254.5 | 2026.0                          | 217.5 |
| 2             | 3526          | 11,29,33,42,45                       | 2240.5 | 2009.0                           | 231.5 | 2024.0                          | 216.5 |
| 3             | 3579          | 2,13,21,33,41                        | 2135.0 | 2021.5                           | 113.5 | 1949.5                          | 185.5 |
| 4             | 3575          | 8,9,21,31,33                         | 2341.0 | 2080.0                           | 261.0 | 2128.0                          | 213.0 |
| 5             | 3614          | 5,9,22,26,42                         | 2532.5 | 2284.0                           | 248.5 | 2327.0                          | 205.5 |

The formulation presented here can involve many variables and constraints as the number of nodes and scenarios increase. In the example presented in this section, 5 scenarios and 55 candidate nodes, there were 1155 integer variables and 1661 constraints. For both objectives, it was needed more than 2:00 of real computer time. The model can become intractable from an optimization viewpoint if the number of nodes and scenarios increase. In this case, it is necessary to find an alternative method to find solutions, that if

not optimal, are near-optimal. Following a heuristic procedure is proposed to tackle the problem.

The heuristic algorithm proposed is based on the well known one-opt Teitz and Bart procedure and involves two phases. In the first one, an initial solution is obtained using the MAXCAP problem. In the second phase a one-opt trade is used to try to improve the initial objective.

A good starting solution (phase 1) can be obtained as follows: for each scenario a MAXCAP problem is used to find the optimal location of Firm A's servers. Once the the optimal locations are obtained for each individual scenario, they can be used to compute the capture that could be achieved in the other scenarios. This can be represented in a matrix form, where each row represents a given scenario, and each column represents the capture that is achieved given the optimal locations given in each scenario (row). For example, the value of the matrix element  $c_{ij}$  (that is row  $i$ , column  $j$ ) corresponds to the capture that would be achieved in scenario  $j$  if the optimal locations for Firm A in scenario  $i$  were true. Therefore, the coefficients in the diagonal represent the maximum capture that can be achieved in each scenario. Once the table is obtained the regret from optimal capture can be computed for each scenario. The initial location for the heuristic can be therefore chosen depending on the objective used. If the Max objective is used, the initial locations for Firm A's servers will correspond to scenario (row)  $i$  where the smallest  $c_{ij}$  is maximum. If the Regret objective is used, the initial solution will correspond to the locations in scenario  $i$  where the largest regret is minimum. Therefore, for each scenario  $j$  the regret from optimal capture is computed, and we choose the scenario  $i$  that gives the largest regret. The locations corresponding to the scenario  $i$  where the largest regret is minimum will define the initial solution.

Once the initial locations are obtained, the second phase of the heuristic algorithm is used to improve the objective. At each iteration one server's location is traded. The new objective is computed and it is stored as the best solution so far if there is an improvement. Otherwise, the relocation will be ignored and the previous solution is restored. The one-opt trade will be done for all nodes and Firm A servers. A step-by-step description of the algorithm when the max objective is used follows:

1. For each scenario, find the locations of Firm A's servers where the maximum capture is achieved (a MAXCAP problem for each scenario).

2. Compute, again for each scenario, the capture that is achieved if the locations found in step 1 are true.
3. Choose the locations that give the smallest capture on the other scenarios is largest. These locations will be the starting solution.
4. Trade the location of one of the  $p$  servers of Firm A.
5. Compute the new captures that are achieved in each scenario. When the Max objective is used, if the smallest scenario's capture is larger than before the trade, keep the solution. If not, restore the old solution. Repeat steps 4-5 until all of Firm A's facilities and nodes have been traded.
6. If the objective after steps 4-5 has improved, go to step 4 and restart the procedure. When no improvement is achieved on a complete set of one-at-a-time trades, stop.

If the Regret approach is used, the heuristic is modified as follows:

1. For each scenario, find the locations of Firm A's servers where the maximum capture is achieved (a MAXCAP problem for each scenario).
2. Compute, again for each scenario, the capture that is achieved if the locations found in step 1 are true.
3. Find, for each scenario  $j$ , the regret from optimally locating Firm A's servers in all scenarios. For each scenario  $i$  choose the scenario  $j$  that would give the maximum regret. The initial solution -the initial locations and regret- will be the scenario where the largest regret is minimum.
4. Trade the location of one of the  $p$  servers of Firm A.
5. Compute the new captures and the regrets that are achieved in each scenario. Choose the locations corresponding to the scenario that gives the smallest maximum regret on the other scenarios. If this regret is smaller than before the one-opt trade, store the solution. If not, restore the old locations and objective. Repeat steps 4-5 until all of Firm A's facilities and nodes have been traded.

6. If the objective after steps 4-5 has improved, go to step 4 and restart the procedure. When no improvement is achieved on a complete set of one-at-a-time trades, stop.

Both heuristics described above have been applied in the example presented. Table 3 shows the results obtained in phase 1. Row  $i$  in the table corresponds to the capture obtained when the servers are located optimally with the MAXCAP problem in scenario  $i$ . In parenthesis the regret from the optimal capture is computed. For example, the capture that would be achieved in scenario 4 if Firm A locates optimally five servers in scenario 3 is equal to 1836.0. On the other hand, the regret will be equal to 505.0, since the maximum capture that can be obtained in scenario 4 is 2341.0 (row 4 , column 4).

Table 3: Firm A's capture and regret in each scenario

|   | scenarios <sup>a</sup> |                 |                   |               |                 |
|---|------------------------|-----------------|-------------------|---------------|-----------------|
|   | 1                      | 2               | 3                 | 4             | 5               |
| 1 | 2243,5<br>(0)          | 1954,5<br>(286) | 1782<br>(353)     | 1853<br>(488) | 2403,5<br>(129) |
| 2 | 2052<br>(191,5)        | 2240,5<br>(0)   | 1899<br>(236)     | 2053<br>(288) | 2240,5<br>(292) |
| 3 | 1650,5<br>(593)        | 1596,5<br>(644) | 2135<br>(0)       | 1836<br>(505) | 2023<br>(509.5) |
| 4 | 1681<br>(562,5)        | 1895<br>(345,5) | 1698,5<br>(436,5) | 2341<br>(0)   | 2047,5<br>(485) |
| 5 | 2231<br>(12,5)         | 1788<br>(452,5) | 1562<br>(573)     | 1779<br>(562) | 2532,5<br>(0)   |

<sup>a</sup>Regret from optimal capture in parenthesis

From this table we can choose now our starting solution for both objectives. If the Max objective is used, the initial solution will correspond to the optimal locations in scenario 2 (see Table 2) and the value of the objective  $m$  will be equal to 1899, since it is the largest minimum capture accross scenarios. If the regret scenario is used, the initial objective  $U$  equal to 292.0, that is, the smallest maximum regret that can be achieved accross scenarios, given the optimal locations in scenarios 1 to 5. Therefore the initial locations correspond to the ones found when locating optimally the second scenario.

Once the initial conditions are established, the second phase is used to improve the objective. Standard FORTRAN 77 was used to implement the heuristic. Results are presented in Table 4 and Table 5.

Table 4: Final results, Heuristic and Optimal methods

| Objective | Initial solution |      | Heuristic method |      | Optimal method |       |
|-----------|------------------|------|------------------|------|----------------|-------|
|           | Locations        | Obj. | Locations        | Obj. | Locations      | Obj.  |
| Max.      | 11,29,33,42,45   | 1899 | 11,21,29,24,45   | 1915 | 5,8,16,29,41   | 1989  |
| Regret    | 11,29,33,42,45   | 292  | 13,21,29,33,42   | 262  | 5,8,16,31,41   | 217.5 |

Table 5: Comparison between heuristic and optimal solutions

| Scenario | Optimal Capture | Max. Obj. |           | Regret Obj. |           |
|----------|-----------------|-----------|-----------|-------------|-----------|
|          |                 | LP + BB   | Heuristic | LP + BB     | Heuristic |
| 1        | 2243.5          | 1989.0    | 2048.5    | 2026.0      | 2088.5    |
| 2        | 2240.5          | 2009.0    | 2032.5    | 2024.0      | 1954.0    |
| 3        | 2135.0          | 2021.5    | 1967.5    | 1949.5      | 1915.0    |
| 4        | 2341.0          | 2080.0    | 2079.0    | 2128.0      | 1977.0    |
| 5        | 2532.5          | 2284.0    | 2357.0    | 2327.0      | 2217.5    |

Second phase of the heuristic achieved to find a better set of locations for both objectives. It did not obtain the optimal solutions found using linear programming plus branch and bound.

An interesting result when the max. objective is used can be observed in Table 5. Even if the heuristic did not find an optimal solution, the final locations obtained with this method outperform the capture in three out of five scenarios (1,2 and 5) and are very close to the ones obtained when using LP+BB in scenarios 3 and 4. This is not observed when the regret objective is considered. Except for scenario 1, the capture achieved by the final solutions using LP+BB outperform significantly the ones obtained with the heuristic.

## 4 Conclusions

A formulation has been presented to tackle the issue of uncertainty in a competitive environment, and where it is possible to define different scenarios that account for differences in the location of competitors and/or the demand in the region of interest. Two different objectives were used: maximize the minimum capture across scenarios (max. objective) and minimize the maximum regret across scenarios (regret objective). Both an optimal solution method (LP+BB) and a heuristic algorithm were described and used in an example. From this example it has been observed that the regret objective outperformed significantly the max objective

## Appendix: Population Scenarios

| Node<br>number | Coordinates |    | Scenarios |      |      |      |      |
|----------------|-------------|----|-----------|------|------|------|------|
|                | X           | Y  | 1         | 2    | 3    | 4    | 5    |
| 1              | 32          | 31 | 120       | 110  | 129  | 136  | 123  |
| 2              | 29          | 32 | 114       | 106  | 109  | 125  | 130  |
| 3              | 27          | 36 | 110       | 120  | 115  | 112  | 117  |
| 4              | 29          | 29 | 108       | 123  | 98   | 108  | 119  |
| 5              | 32          | 29 | 105       | 107  | 94   | 119  | 98   |
| 6              | 26          | 25 | 103       | 100  | 115  | 118  | 90   |
| 7              | 24          | 33 | 100       | 91   | 89   | 114  | 106  |
| 8              | 30          | 35 | 94        | 90   | 94   | 100  | 81   |
| 9              | 29          | 27 | 91        | 78   | 80   | 78   | 103  |
| 10             | 29          | 21 | 90        | 88   | 90   | 81   | 99   |
| 11             | 33          | 28 | 88        | 92   | 87   | 77   | 95   |
| 12             | 17          | 53 | 87        | 86   | 75   | 84   | 79   |
| 13             | 34          | 30 | 87        | 94   | 96   | 87   | 85   |
| 14             | 25          | 60 | 85        | 74   | 93   | 83   | 77   |
| 15             | 21          | 28 | 83        | 88   | 82   | 75   | 83   |
| 16             | 30          | 51 | 82        | 76   | 93   | 89   | 90   |
| 17             | 19          | 47 | 80        | 70   | 90   | 75   | 85   |
| 18             | 17          | 33 | 79        | 77   | 79   | 82   | 84   |
| 19             | 22          | 40 | 79        | 84   | 87   | 82   | 73   |
| 20             | 25          | 14 | 77        | 72   | 80   | 74   | 75   |
| 21             | 29          | 12 | 76        | 72   | 73   | 70   | 78   |
| 22             | 24          | 48 | 74        | 78   | 64   | 65   | 79   |
| 23             | 17          | 42 | 72        | 76   | 77   | 66   | 66   |
| 24             | 6           | 26 | 70        | 72   | 61   | 78   | 68   |
| 25             | 19          | 21 | 69        | 76   | 69   | 73   | 75   |
| 26             | 10          | 32 | 69        | 70   | 64   | 61   | 61   |
| 27             | 34          | 56 | 64        | 67   | 67   | 56   | 71   |
| 28             | 12          | 47 | 63        | 69   | 55   | 57   | 63   |
| 29             | 19          | 38 | 62        | 64   | 63   | 55   | 65   |
| 30             | 27          | 41 | 61        | 66   | 64   | 70   | 62   |
| 31             | 21          | 35 | 60        | 63   | 60   | 55   | 58   |
| 32             | 32          | 45 | 58        | 50   | 53   | 58   | 52   |
| 33             | 27          | 45 | 57        | 51   | 62   | 52   | 59   |
| 34             | 32          | 38 | 55        | 53   | 54   | 52   | 61   |
| 35             | 8           | 22 | 54        | 49   | 47   | 54   | 48   |
| 36             | 15          | 25 | 53        | 49   | 54   | 49   | 51   |
| 37             | 35          | 16 | 51        | 49   | 48   | 46   | 46   |
| 38             | 36          | 47 | 49        | 45   | 55   | 51   | 47   |
| 39             | 46          | 51 | 47        | 52   | 47   | 46   | 41   |
| 40             | 50          | 40 | 44        | 43   | 45   | 40   | 47   |
| 41             | 23          | 22 | 43        | 41   | 46   | 47   | 46   |
| 42             | 27          | 30 | 42        | 38   | 47   | 42   | 39   |
| 43             | 38          | 39 | 41        | 45   | 43   | 44   | 39   |
| 44             | 36          | 32 | 40        | 38   | 36   | 36   | 43   |
| 45             | 32          | 41 | 39        | 34   | 44   | 42   | 42   |
| 46             | 42          | 36 | 37        | 37   | 41   | 41   | 37   |
| 47             | 36          | 26 | 35        | 33   | 33   | 31   | 39   |
| 48             | 15          | 19 | 34        | 29   | 38   | 38   | 37   |
| 49             | 19          | 14 | 33        | 30   | 29   | 37   | 33   |
| 50             | 45          | 19 | 33        | 34   | 33   | 37   | 33   |
| 51             | 27          | 5  | 32        | 33   | 32   | 32   | 34   |
| 52             | 52          | 24 | 26        | 24   | 27   | 23   | 25   |
| 53             | 40          | 22 | 25        | 24   | 25   | 25   | 27   |
| 54             | 40          | 52 | 24        | 24   | 25   | 24   | 27   |
| 55             | 42          | 42 | 21        | 22   | 23   | 23   | 23   |
| <b>Total</b>   |             |    | 3575      | 3526 | 3579 | 3575 | 3614 |



## References

- DOBSON, G. AND V. KARMAKAR (1987), "Competitive location on a network," *Operations Research*, 4(35):565-574.
- EISELT, H. AND G. LAPORTE (1989), "The maximum capture problem in a weighted network," *Journal of Regional Science*, 3(29):433-439.
- FRIESZ, T., T. MILLER, AND R. TOBIN (1987), "Algorithms for spatially competitive network facility location," *Environment and Planning B*, 15:191-203.
- HAKIMI, S. (1983), "On locating new facilities in a competitive environment," *European Journal of Operational Research*, 12:29-35.
- KARKAZIS, J. (1989), "Facilities location in a competitive environment: A promethee-based multiple criteria analysis," *European Journal of Operational Research*, 42:294-304.
- LABB, M. AND L. HAKIMI (1991), "Market and locational equilibrium for two competitors," *Operations Research*, 5(39):749-756.
- LEDERER, P. (1986), "Duopoly competition in networks," *Annals of Operations Research*, 6:99-109.
- LEDERER, P. AND J.F. THISSE (1990), "Competitive location on networks under delivered pricing," *Operations Research Letters*, 9:147-153.
- MILLER, T., R. TOBIN, AND T. FRIESZ (1992), "Network facility location models in stackelberg-nash-cournot spatial competition," *Papers in Regional Science: The Journal of the RSAI*, 3(71):277-291.
- REVELLE, C. (1986), "The maximum capture or sphere of influence location problem: Hotelling revisited on a network," *Journal of Regional Science*, 26:343-358.
- SERRA, D., V. MARIANOV, AND C. REVELLE (1992), "The maximum capture hierarchical location problem," *European Journal of Operational Research*, 62:363-371.

SERRA, D. AND C. REVELLE ((forthcoming)), "Market capture by two competitors: The pre-emptive location problem," *Journal of Regional Science*.

TOBIN, R. AND T. FRIESZ (1986), "Spatial competition facility location models: Definition, formulation and solution approach," *Annals of Operations Research*, 6:49-74.

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