BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 3, May 1975

RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of outstanding new results, with some indication of proof. Research announcements are limited to 100 typed lines of 65 spaces each. A limited number of research announcements may be communicated by each member of the Council who is also a member of a Society editorial committee. Manuscripts for research announcements should be sent directly to those members of the Publications Committees listed on the inside back cover.

THE MAXIMUM SIZE OF AN INDEPENDENT SET IN A NONPLANAR GRAPH

BY MICHAEL O. ALBERTSON[1] AND JOAN P. HUTCHINSON

Communicated by W. Wistar Comfort, February 4, 1975

Suppose G is a simple graph with V vertices that embeds on S_n, a surface of genus n. A set of vertices H is independent in G if no pair of vertices in H is adjacent in G. Let $\alpha(G)$ be the maximum number of vertices in any independent set and $\mu(G)$ be $\alpha(G)/V$, the *independence ratio.*

Set

$$U(n) = \{\mu(G): G \text{ embeds on } S_n\}$$

and

$$L(n) = \{\text{limit points of } U(n)\}.$$

REMARK. It is known that $U(0) \subset (2/9, 1]$ [1], [2] and $U(n) \subset [1/\chi, 1]$ where χ is the Heawood number of S_n $(n \geq 1)$ [4]. Erdős [3] has asked if $U(0) \subset [1/4, 1]$, a result implied by the four color conjecture.

CONJECTURE. $L(n) = L(0)\ \forall n.$

THEOREM 1. $L(n) \subset [1/5, 1]\ \forall n.$

In the proof of Theorem 1 the small elements of $U(n)$ are examined and shown to be isolated points with relatively few graphs corresponding to each point. Thus we attempt to characterize those graphs with small independence ratios. In each of the following we assume that G embeds on S_n, $n \geq 1$, and

AMS (MOS) subject classifications (1970). Primary 05C10, 55A15; Secondary 05C15.

[1] Research supported in part by the Research Corporation through a Cottrell College Science Research Grant.

Copyright © 1975, American Mathematical Society

that χ is the chromatic number of S_n.

THEOREM 2. *If $G \neq K_\chi$ then $\mu(G) \geqslant 1/(\chi - 1)$.*

THEOREM 3. *If $G \neq K_\chi, K_{\chi-1}$ and $n \geqslant 4$ then $\mu(G) > 1/(\chi - 1)$.*

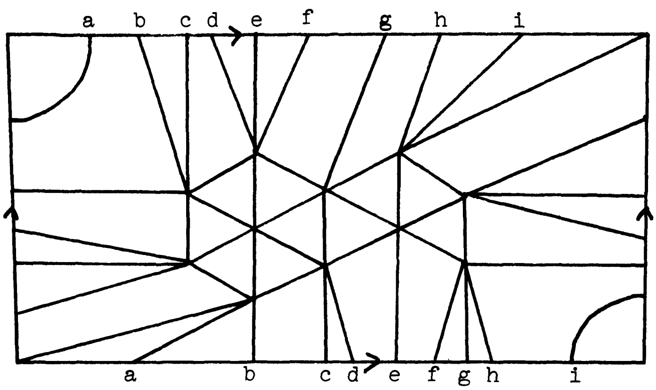
REMARK. Two vertex disjoint copies of K_7 embed on S_2.

THEOREM 4. *Given any positive integer l there exists $N(l)$ such that if $n > N(l)$ and $\mu(G) < 1/(\chi - l)$ then G contains $K_{\chi-l+1}$.*

The results on the torus are more specific and stronger.

THEOREM 5. *If G embeds on the torus, then $\mu(G) \geqslant 1/5$ unless $G = K_7$, K_6, the graph J pictured below, or a vertex disjoint union of K_7 and K_4.*

REMARK. The graph J has $\mu = 2/11$ and does not contain K_5.



THEOREM 6. $L(1) \subset [2/9, 1]$.

Proofs will appear elsewhere.

REFERENCES

1. M. O. Albertson, *Finding an independent set in a planar graph*, Graphs and Combinatorics (R. Bari and F. Harary, editors), Springer-Verlag, New York, 1974.

2. ———, *A lower bound for the independence number of a planar graph*, J. Combinatorial Theory Ser. B (to appear).

3. C. Berge, *Graphs and hypergraphs*, Dunod, Paris, 1970.

4. G. Ringel and J. W. T. Youngs, *Solution of the Heawood map-coloring problem*, Proc. Nat. Acad. Sci. U. S. A. 60 (1968), 438–445. MR 37 #3959.

DEPARTMENT OF MATHEMATICS, SMITH COLLEGE, NORTHAMPTON, MASSACHUSETTS 01060

DEPARTMENT OF MATHEMATICS, DARTMOUTH COLLEGE, HANOVER, NEW HAMPSHIRE 03755