The Mean Age of Mantle and Crustal Reservoirs

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Mantle and crust evolution is discussed in terms of two simple transport models. In model I, continents ($j = 3$) are derived by melt extraction over the history of the earth from undepleted mantle ($j = 1$), and the residue forms a depleted mantle ($j = 2$), which today is the source of mid-ocean ridge basalts. In model II, new additions to continents are derived from a mantle reservoir 2, which becomes more depleted through time by repeated extraction of melts. Transport equations were solved for stable s, radioactive r, and daughter d isotopes for arbitrary mass growth curves $M_j(\tau)$. For both models the isotopic composition and concentrations of trace elements are shown to reduce to simple mathematical expressions which readily permit calculations of basic evolutionary parameters from the data. For long-lived isotopes ($\lambda^{-1} \gg 4.5$ aeons) for model I the deviations in parts in 10^4 of the ratio of a daughter isotope to a stable reference isotope of the same element in reservoirs $j = 2, 3$ from that of 1 is given by $\epsilon_{d_j}{}^* = Q_d{}^* \bar{t}_{M_j} f_j^{r/s}$. Here $\bar{t}_{M_j}$ is the mean age of the mass of j, $f_j^{r/s}$ is the enrichment factor of the ratio of a radioactive isotope to a stable isotope relative to that in 1, and $Q_d{}^* \simeq$ const. Thus for long-lived isotopes such as ^{147}Sm and ^{87}Rb the only time information that can be obtained from model I from measurement of the relative chemical enrichment factors and isotopic ratios at a single time is the mean age of the mass of the continental crust and the complementary depleted mantle reservoir. This mean age is independent of the long-lived parent-daughter system. An analogous result is obtained for model II, where $\epsilon_{d,2}{}^* = Q_d{}^* \langle f_2^{r/s} \rangle \tau$. Here $\langle f_2^{r/s} \rangle$ is the weighted time average of the enrichment factor, and τ is the time measured from the origin of the earth. The mean age of the mass of the crust ($\bar{t}_{M,3}$) and the time parameter $\bar{t}_{r/s} = \tau \langle f_2^{r/s} \rangle / f_2^{r/s}$ for the crust in model II will for long-lived parent-daughter systems be different depending on the element fractionation during partial melting. Decay systems with small parent-daughter fractionations during partial melting may, however, be used to estimate the mean age of the continental crust. Sm-Nd and Rb-Sr isotopic data for continental crust, depleted and undepleted mantle, have been used to evaluate both models and yield young mean ages for the mass of the continental crust of 1.8 and 1.5 aeons for model I and model II, respectively. Both models also suggest that the rate of growth of the continents for the last 0.5 aeon is much less than the average growth rate. The young mean age of the continents implies either rapid refluxing of crustal materials to the mantle in the period from 4.5 to 3.6 aeons or that very little early crust ever formed. Mass balance calculations for both models show that the continents were only formed from ~30% of the total mantle, leaving 70% of the mantle as undepleted. The major difference in the two models lies in the difference in the compositions of newly derived crust. For model I the trace element concentrations in new additions to the crust is constant, and the isotopic values are those of the undepleted mantle reservoir, in agreement with recent Nd isotopic studies. The correlation line between ϵ_{Nd} and ϵ_{Sr} for young basalts can be explained with model I by mixing depleted and undepleted mantle, but it cannot in any simple way be explained by model II. Model II implies that new additions to the continents have the isotopic characteristics of depleted mantle and that the concentration of Rb, U, Ba, and other highly incompatible trace elements in newly added material have changed by a factor of ~10 through time, for which there is no evidence. For both models the simple analytical expressions derived herein permit calculations of earth models with great facility without requiring a computer calculation.

INTRODUCTION

Over the past few years a substantial amount of new data has become available on the Sm-Nd and Rb-Sr isotope characteristics of mantle and crustal reservoirs. These data have been used to determine the times of additions of new continental crustal materials [*McCulloch and Wasserburg,* 1978] and in discussions of the evolution of the mantle sources of modern continental and oceanic basalts. These studies have demonstrated that the mid-ocean ridge basalts (MORB) have been derived from a depleted mantle source [*DePaolo and Wasserburg,* 1976a, b; *Richard et al.,* 1976] and that young continental flood basalts and new additions to the continents throughout most of the earth's history appear mainly to have been derived from undepleted mantle reservoirs [*DePaolo and Wasserburg,* 1976b, 1979b]. Recently, data on the initial isotopic composition of 0.5-aeon-old (1 aeon = 1 AE = 10^9 years) oceanic crust have become available which permit an estimate of the composition of the oceanic mantle in the early paleozoic [*Jacobsen and Wasserburg,* 1979]. The simplest model for explaining the observed isotopic heterogeneities in the mantle

would be to assume that they formed in a single event [*Brooks et al.,* 1976a, b; *Brooks and Hart,* 1978]. However, as was discussed by *O'Nions et al.* [1978], *Tatsumoto* [1978], and *Jacobsen and Wasserburg* [1979], available Pb, Nd, and Sr isotopic data on young basalts do not appear to fit such a single-stage model, and it is not compatible with our knowledge that the continental crust has evolved in many stages over geologic time [see, for example, *Moorbath,* 1978; *O'Nions and Pankhurst,* 1978; *McCulloch and Wasserburg,* 1978]. Because of these considerations it is necessary to analyze the data in terms of continuous or multiepisodic mantle-crust transport models.

In this study we present an analysis of two simple transport models for trace elements and relate them to the isotopic abundance patterns that can be observed. The general transport equations for transfer of stable, radioactive, and daughter isotopes between n reservoirs were given by *Wasserburg* [1964]. The models presented here assume that the transport is solely from mantle to continental crust and do not treat the flux back to the mantle of continental crustal material for simplicity, but the treatment may be simply extended to include this. In the first model (model I) the mechanism for crustal

growth is by deriving melts over the age of the earth by equilibrium partial melting from undepleted mantle. In the second model (model II), melts that form new continental crust are being derived from a mantle reservoir that is continuously depleted through time. We will show that although the models involve rather heavy algebra, they lead to exceedingly simple expressions for the isotopic ratios and trace element abundances in terms of the mass of continental crust and depleted mantle which are evolved. The models apply to arbitrary crustal growth rates in contrast to previous transport models [*Patterson*, 1964; *Patterson and Tatsumoto*, 1964; *Wasserburg*, 1966; *Armstrong*, 1968; *Hart and Brooks*, 1970; *Russell*, 1972; *Russell and Birnie*, 1974; *DePaolo*, 1978; *O'Nions et al.*, 1979]. This approach is a generalization of the treatment of nucleosynthetic time scales given by *Schramm and Wasserburg* [1970]. The results may be used directly to discuss earth models using available isotopic data and concentration data for mantle and crustal rocks. This treatment appears to give a deep and direct insight into the problems of mantle evolution.

MANTLE-CRUST TRANSPORT MODELS

Model I

We first consider a model of crustal growth from an undifferentiated mantle reservoir (reservoir 1) with a given segment of the mantle being differentiated to form two reservoirs: a depleted mantle (reservoir 2) and a continental crust (reservoir 3). The model is shown schematically in Figure 1. We define M_j, $N_{i,j}$, and $C_{i,j}$ to be the total mass of reservoir j, number of atoms of species i in reservoir j, and the concentration of species i in reservoir j, respectively. Differential masses added to or subtracted from the total reservoir j are denoted by δM_j. The number of species i and the concentration of i in the differential mass δM_j are denoted by $\delta N_{i,j}$ and $c_{i,j}$, respectively. Let us assume that a mass δM_1 is removed from reservoir 1 and a partially melted fraction F is extracted from this differential mass to form new crust with added mass δM_3 and a new residual mass δM_2 is added to the depleted mantle. This gives $\delta M_3/\delta M_1 = F$ and $\delta M_2/\delta M_1 = 1 - F$. We imagine that the crust is thus increased by the amount δM_3 and the undifferentiated mantle reservoir is diminished in amount by mass δM_1. The mass δM_2 is added to the depleted mantle, which we assume to be otherwise isolated from the remaining undifferentiated reservoir 1.

The number of atoms of species i in each differential mass j in this unit process is $\delta N_{i,j}$, which gives

$$\delta N_{i,1} = \delta N_{i,2} + \delta N_{i,3} \tag{1}$$

The number of species i in the differential mass δM_j may be written $\delta N_{i,j} = c_{i,j}\delta M_j$. Conservation of species and mass gives

$$c_{i,1} = c_{i,2}(1 - F) + c_{i,3}F \tag{2}$$

Trace elements are fractionated between melts and residue with an effective partition coefficient. The effective partition coefficient D_i for element i between the residue (δM_2) and the partial melt (δM_3) is defined by $D_i \equiv c_{i,2}/c_{i,3}$. The concentration in δM_3 is

$$c_{i,3} = \frac{c_{i,1}}{F + D_i(1 - F)} = \frac{c_{i,2}}{D_i} \tag{3}$$

We may thus express the number of species i in the differential masses j in terms of their masses δM_j and the concentration in reservoir 1, since $c_{i,1} = C_{i,1}$:

$$\delta N_{i,j} = \frac{D_{i,j}C_{i,1}}{F + D_i(1 - F)}\,\delta M_j \tag{4}$$

$$D_{i,j} = D_i \quad j = 2$$

$$D_{i,j} = 1 \quad j = 3$$

We consider the following species: (1) a stable index isotope s of an element which is not affected by radioactive decay, (2) a radioactive isotope r with decay constant λ, and (3) a daughter isotope d which is the decay product of r and of the same chemical element as s.

The transport equations for the total reservoir j containing the number $N_{i,j}$ of each isotope $i = s, r, d$ are

$$\frac{dN_{i,j}(\tau)}{d\tau} = \frac{\delta N_{i,j}}{\delta \tau} - \lambda_i N_{r,j}(\tau) \tag{5}$$

$$\lambda_i = 0 \quad i = s$$

$$\lambda_i = \lambda \quad i = r$$

$$\lambda_i = -\lambda \quad i = d$$

where $dN_{i,j}/d\tau$ is the time derivative and $\delta N_{i,j}/\delta\tau$ is the flux of species i into reservoir j. In this treatment, $\delta\tau$ is the time over which mass δM_1 is differentiated at time τ. For simplicity we have chosen to ignore transport between 2 and 3. Using the expressions for $\delta N_{i,j}$ in terms of $C_{i,1}$ and δM_j and calling $\delta M_j/\delta\tau \equiv M_j'(\tau)$, which is the rate of mass growth of reservoir j, we obtain the basic transport equation for model I,

$$\frac{dN_{i,j}}{d\tau} = \frac{D_{i,j}C_{i,1}(\tau)}{F + D_i(1 - F)}M_j'(\tau) - \lambda_i N_{r,j}(\tau) \tag{6}$$

where $D_{i,j}$ and λ_i are defined as in (4) and (5). The first term on the right-hand side of (6) is the transport term, and the second term is due to either radioactive decay or production.

The general solution of (6) for a stable index isotope is

$$N_{s,j}(\tau) = \frac{D_{s,j}}{F + D_s(1 - F)} \int_0^\tau C_{s,1}(\xi)M_j'(\xi)\,d\xi \tag{7}$$

The equation was integrated with the assumption that the time $\tau = 0$ at the time of initial chemical differentiation of the earth and $N_{s,j}(0) = 0$. The earth is taken to be initially homogeneous.

The undifferentiated reservoir is characterized by a closed system behavior for intensive quantities, so that the concentration for species s, r, and d are

$$C_{s,1}(\tau) = C_{s,1}(0) \tag{8}$$

$$C_{r,1}(\tau) = C_{r,1}(0)e^{-\lambda\tau} \tag{9}$$

$$C_{d,1}(\tau) = C_{d,1}(0) + C_{r,1}(0)[1 - e^{-\lambda\tau}] \tag{10}$$

The relationship between the masses of reservoirs 2 and 3 is given by $M_3(\tau) = [F/(1 - F)]M_2(\tau)$. Substituting (8) into (7) gives the following general solutions for stable isotopes in reservoirs 2 and 3:

$$N_{s,2}(\tau) = \frac{D_s C_{s,1}(0)}{F + D_s(1 - F)}M_2(\tau) \tag{11}$$

$$N_{s,3}(\tau) = \frac{C_{s,1}(0)}{F + D_s(1 - F)}M_3(\tau) \tag{12}$$

The above equation for a stable isotope in the crust may also be written as $C_{s,3}(\tau) = C_{s,1}(0)/[F + D_s(1 - F)]$, which is equivalent to the equation for equilibrium partial melting derived by *Consolmagno and Drake* [1976] which showed that previous equations given by *Shaw* [1970], *Schilling* [1971], and *O'Nions and Clarke* [1972] all reduce to this general form.
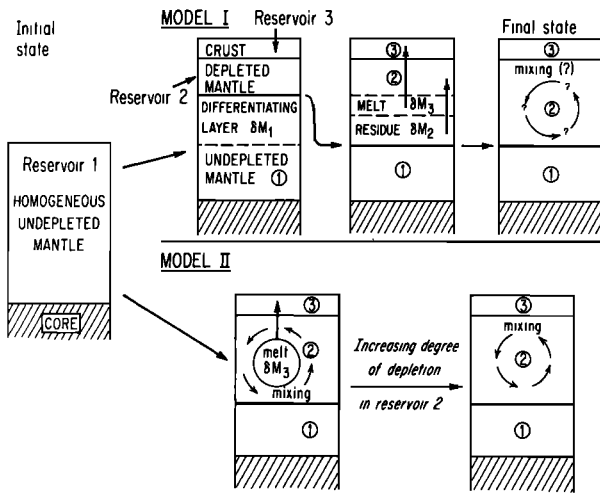
Fig. 1. Cartoon showing the processes in model I and model II. Model I is a three-layer model consisting of undepleted mantle (1), depleted mantle (2), and continental crust (1), as indicated under the final state. In model I a given segment (the differentiating layer δM_1) of undepleted mantle (1) is being differentiated by equilibrium partial melting and added to two reservoirs: a depleted mantle (2) and a continental crust (3). We assume that a differential mass δM_1 is removed from reservoir 1 and a partially melted fraction F is extracted from this differential mass to form new crust with added mass δM_3 and that a new residual mass δM_2 is added to the depleted mantle. In this differentiation process the masses of reservoirs 1, 2, and 3 are conserved. The masses of the depleted mantle and the crust increase with time, and the undepleted mantle decreases with time. The upper boundary of the undepleted mantle is thus lowered, and the volumes of both depleted mantle and crust increased. These stages are shown in the second and third columns. In model II the crust grows from an initially undifferentiated mantle reservoir (2), which is depleted at all times subsequent to the initial chemical differentiation as a result of continuing crustal growth. This reservoir may make up all or part of the mantle. Layers 2 and 3 are thus the only active regions in this model. The remaining undifferentiated mantle is passive and is still indicated as layer 1. In this model we assume that a differential mass δM^* of reservoir 2 is being differentiated into a melt δM_3, which is added to the continental crust and a residue δM_R^*. The degree of melting F is taken as finite. The residue after removal of a melt fraction from reservoir 2 is assumed to be homogenized instantaneously with the remaining part of the depleted mantle. The total mass of reservoirs 2 and 3 is conserved; the continental crust increases in mass with time, and the depleted mantle decreases in mass with time. In both models the depleted mantle is taken to include the basaltic part of the oceanic crust, since it is derived from the depleted mantle and is subducted back to the depleted mantle at a short time scale (<0.2 aeon).

Solutions of (6) analogous to (7) may also be obtained for radioactive isotopes. Substituting (9) into these gives the following result for reservoirs 2 and 3:

$$N_{r,2}(\tau) = \frac{D_r e^{-\lambda\tau} C_{r,1}(0)}{F + D_r(1 - F)} M_2(\tau) \qquad (13)$$

$$N_{r,3}(\tau) = \frac{e^{-\lambda\tau} C_{r,1}(0)}{F + D_r(1 - F)} M_3(\tau) \qquad (14)$$

Similarly, by substituting (10), (13), and (14) into (6) we obtain the following equations for daughter isotopes in reservoirs 2 and 3:

$$N_{d,2}(\tau) = \left\{ D_d \left[C_{d,1}(\tau)M_2(\tau) + \lambda C_{r,1}(0) \right. \right.$$

$$\left[\frac{F + D_d(1 - F)}{F + D_r(1 - F)} \left(\frac{D_r}{D_d} \right) - 1 \right]$$

$$\left. \left. \int_0^\tau M_2(\xi) \exp[-\lambda\xi] \, d\xi \right] \right\} [F + D_d(1 - F)]^{-1} \qquad (15)$$

$$N_{d,3}(\tau) = \left\{ \left[C_{d,1}(\tau)M_3(\tau) + \lambda C_{r,1}(0) \left[\frac{F + D_d(1 - F)}{F + D_r(1 - F)} - 1 \right] \right. \right.$$

$$\left. \left. \int_0^\tau M_3(\xi) \exp[-\lambda\xi] \, d\xi \right\} [F + D_d(1 - F)]^{-1} \qquad (16)$$

where $D_s = D_d$, since d and s are isotopes of the same element, and $C_{d,1}(\tau)$ is given by (10).

The enrichment factor of the ratio of a radioactive isotope to a stable isotope in a reservoir j relative to that in reservoir 1 is called $f_j^{r/s}$. This enrichment factor is the same both for new additions to j and for the total reservoir j, and it is defined by

$$f_j^{r/s} \equiv \frac{N_{r,j}(\tau)/N_{s,j}(\tau)}{N_{r,1}(\tau)/N_{s,1}(\tau)} - 1 \qquad (17)$$

For this model the enrichment factor is independent of time and is given by

$$f_2^{r/s} + 1 = \left(\frac{D_r}{D_s} \right) \frac{F + D_s(1 - F)}{F + D_r(1 - F)} = \left(\frac{D_r}{D_s} \right)(f_3^{r/s} + 1) \qquad (18)$$

As $D_s = D_d$, the ratio of a daughter isotope to a stable reference isotope of the same element (with no parent) in each reservoir is given by

$$\frac{N_{d,j}(\tau)}{N_{s,j}(\tau)} = \frac{N_{d,1}(\tau)}{N_{s,1}(\tau)} + \frac{N_{r,1}(\tau)}{N_{s,1}(\tau)} \frac{f_j^{r/s}\lambda}{M_j(\tau)}$$

$$\cdot \int_0^\tau M_j(\xi) \exp[\lambda(\tau - \xi)] \, d\xi \qquad (19)$$

In terms of deviations $\epsilon_{d,j}^*(\tau)$ in parts in 10^4 from the ratios in the undifferentiated reference reservoir we obtain the final equations to be used in calculations for this model for arbitrary rates of growth of the crust and mantle:

$$\epsilon_{d,j}^*(\tau) = \frac{Q_d^*(\tau) f_j^{r/s}}{M_j(\tau)} \int_0^\tau M_j(\xi) \exp[\lambda(\tau - \xi)] \, d\xi \qquad (20a)$$

and

$$\frac{\epsilon_{d,3}^*(\tau)}{\epsilon_{d,2}^*(\tau)} = \frac{f_3^{r/s}}{f_2^{r/s}} \qquad (20b)$$

where

$$Q_d^*(\tau) \equiv \left[10^4\lambda \left(\frac{N_{r,1}(\tau)}{N_{s,1}(\tau)} \right) \right] \Big/ \left(\frac{N_{d,1}(\tau)}{N_{s,1}(\tau)} \right) \qquad (20c)$$

The time parameter τ is the time running forward from the initial state (cf. equation (7)), and the time measured backward from the present will as usual be called T (see Figure 2). Here $\epsilon_{d,j}^*(\tau) = \epsilon_{d,j}^*(T_0 - T) \equiv \epsilon_{d,j}(T)$, where $T = T_0 - \tau$, T_0 is the age of the earth today, and $\epsilon_{d,j}(T)$ corresponds to ϵ_{Nd} and ϵ_{Sr} as defined by DePaolo and Wasserburg [1976a, 1977]. The asterisks denote the functional form of the deviations 'ϵ' relative to a time variable with an origin at the time of formation of the earth. Similarly, we define $Q_d^*(\tau) = Q_d^*(T_0 - T) \equiv Q_d(T)$ and note that $Q_d(0)$ is equal to the constants Q_{Nd} and Q_{Sr} used by DePaolo and Wasserburg [1976a, 1979a] for the Sm/Nd and Rb/Sr systems, respectively. If $\lambda\tau \ll 1$, then we have a good approximation that $Q_d^*(\tau) = Q_d(0)$. We note that the
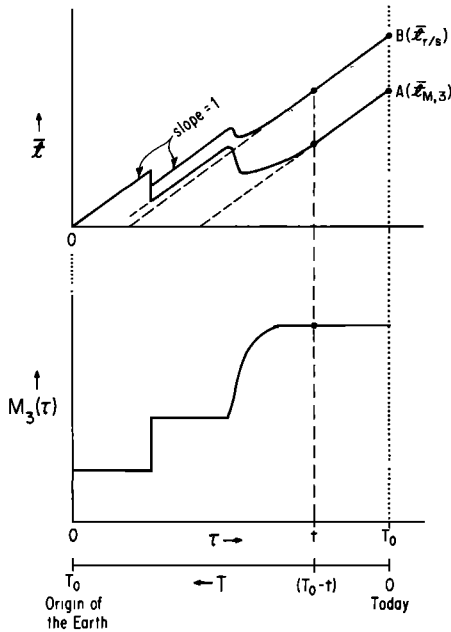
Fig. 2. Cartoon showing the relationship between a mean age $\bar{t}$ and the corresponding mass growth curve $M_3(\tau)$ for the crust. Two time variables are shown, where τ is the time measured from the origin of the earth and T is the time measured backward from today. The vertical dashed line is shown for a time $\tau = t$ years after the origin of the earth, which corresponds to a time $T_0 - t$ years ago. Curve A shows the mean age of the mass of the crust $\bar{t}_{M,3}$ versus time τ. The breaks in curve A correspond to the breaks in $M_3(\tau)$. Rapid addition of new material implies a rapid decrease in $\bar{t}_{M,3}$ of the continental crust. For periods where no significant new mass is added, $(d\bar{t}/d\tau) = 1$. This is valid for any model. For models such as model I, where the concentration of stable isotopes does not vary with time, $\bar{t}_{M,3}$ is also the mean age of any stable isotope $\bar{t}_{s,3}$ in the crust. For parent-daughter systems with $\lambda \tau \ll 1$, $\bar{t}_{M,3}$ is also the mean age of the parent and daughter isotopes in the crust (i.e., $\bar{t}_{M,3} = \bar{t}_{d,3} = \bar{t}_{r,3}$). Curve B shows the case where a stable isotope is strongly enriched in the crust earlier in time and the concentration in the bulk crust decreases monotonically with time. This weights the older material more strongly and raises the value of $\bar{t}_{s,3}$ above $\bar{t}_{M,3}$ (curve A). The curve $\bar{t}_{s,3}$ for model II has a more subdued behavior compared with model I for more recent additions. Similarly, if a long-lived parent is much more strongly enriched than the daughter in the crust, then the factor $\bar{t}_{r/s}$ shows a behavior like that of curve B. Subsequent to $\tau = t$ no mass is added, so $(d\bar{t}/d\tau) = 1$ for both curves for the time interval $\tau = t$ to $\tau = T_0$. The difference between the two curves is given by (66).

values of $\epsilon_{d,j}{}^*$ are the average values in each reservoir. Insofar as each new added segment was isolated, the added segment would follow its own evolutionary path governed by its initial state. This model has the property that new crustal and new mantle material would at the time of addition have the isotopic values of the undifferentiated reservoir 1. Magmas derived from subsequent melting of the crust or the depleted mantle would have distinctive characteristics at a given time. For a constant rate of crustal growth, (20a) takes the following form:

$$\epsilon_{d,j}{}^*(\tau) = \frac{Q_d{}^*(\tau)f_j{}^{r/s}}{\lambda}\left[\frac{e^{\lambda \tau} - 1}{\lambda \tau} - 1\right] \qquad (21a)$$

and if $\lambda \tau \ll 1$, this equation reduces to

$$\epsilon_{d,j}{}^*(\tau) = Q_d{}^*f_j{}^{r/s}(\tau/2) \qquad (21b)$$

The average time of residence $(\bar{t})$ of a stable atom s, which is not produced by radioactive decay, in reservoir j ($j = 2$ or 3) at a time τ is, using (11) and (12),

$$\bar{t}_{s,j} \equiv \langle(\tau - \xi)\delta N_{s,j}(\xi)/N_{s,j}(\tau)\rangle = \frac{1}{N_{s,j}(\tau)}\int_0^\tau (\tau - \xi)\left(\frac{\delta N_{s,j}}{\delta \xi}\right)d\xi$$

$$= \frac{1}{M_j(\tau)}\int_0^\tau (\tau - \xi)M_j'(\xi)\,d\xi = \frac{1}{M_j(\tau)}\int_0^\tau M_j(\xi)\,d\xi \qquad (22)$$

We have here assumed that there was initially no crust or depleted mantle, so that $N_{s,j}(0) = M_j(0) = 0$. Expression (22) is equivalent to the average age as measured today of continental crustal material if $\tau = T_0$, the age of the earth, otherwise it is the average age as measured $T = T_0 - \tau$ years ago. Note that for this model, $\bar{t}_{s,j}$ is the same as the average age of the crust or depleted mantle and is independent of the species or of the reservoir. We may therefore drop the subscripts on $\bar{t}$ for this model.

Defining

$$\langle t^n \rangle \equiv \frac{1}{M_j(\tau)}\int_0^\tau (\tau - \xi)^n \left(\frac{dM_j}{d\xi}\right)d\xi$$

$$= \frac{n}{M_j(\tau)}\int_0^\tau (\tau - \xi)^{n-1} M_j(\xi)\,d\xi \qquad (23)$$

where n is an integer, then the integral in (20) can be written in the following form:

$$I_j(\tau) = \frac{1}{M_j(\tau)}\int_0^\tau M_j(\xi)\exp[\lambda(\tau - \xi)]\,d\xi$$

$$= \frac{1}{M_j(\tau)}\int_0^\tau \sum_{n=0}^\infty \frac{\lambda^n}{n!}(\tau - \xi)^n M_j(\xi)\,d\xi = \sum_{n=1}^\infty \frac{\lambda^{n-1}}{n!}\langle t^n \rangle \qquad (24)$$

Then we have from (20)

$$\epsilon_{d,j}{}^*(\tau) = Q_d{}^*(\tau)f_j{}^{r/s}\sum_{n=1}^\infty \frac{\lambda^{n-1}}{n!}\langle t^n \rangle$$

$$= Q_d{}^*(\tau)f_j{}^{r/s}\langle t^1 \rangle\left[1 + \frac{\lambda\langle t^2 \rangle}{2\langle t^1 \rangle} + \frac{\lambda^2\langle t^3 \rangle}{6\langle t^1 \rangle} + \cdots\right] \qquad (25)$$

We note that for long-lived isotopes with $\lambda \tau \ll 1$ we get, to a good approximation,

$$\epsilon_{d,j}{}^*(\tau) = Q_d{}^*(\tau)f_j{}^{r/s}\bar{t} \qquad (26)$$

or

$$\epsilon_{d,j}(T) = Q_d(T)f_j{}^{r/s}\bar{t}$$

as $\bar{t} = \langle t^1 \rangle$.

If we can measure $\epsilon_{d,j}{}^*(\tau)$ and $f_j{}^{r/s}$ for a very long lived isotope at $T = T_0 - \tau$ years ago, then the only information we can obtain is the mean age of reservoir j as measured $T = T_0 - \tau$ years ago. The resulting mean age is independent of which very long lived species is used. This is independent of the rate of growth of the mass of reservoir j or the rate of transport of species i. This general treatment gives an integral which resembles a Laplace transform of $M_j(\tau)$ with a transform variable λ. This would permit a unique inversion for $M_j(\tau)$ if the transform was known for all values of λ. Insofar as

TABLE 1. Values of $I_j(\tau)$ at $\tau = 4.55$ Aeons for Various Parent-Daughter Systems for a Constant Rate of Crustal Growth With a Mean Age of $\bar{t} = 2.275$ Aeons

Radioactive Parent	Decay Constant, aeons^{-1}	$I_j(\tau)$, aeons
...	0	2.275
^{147}Sm	0.00654	2.30
^{87}Rb	0.0142	2.32
^{187}Re	0.0161	2.33
^{176}Lu	0.0198	2.34
^{232}Th	0.049475	2.45
^{238}U	0.155125	2.92
^{40}K	0.5543	6.39
^{235}U	0.98485	18.77
^{244}Pu	8.48	1.44×10^{15}

there is only a limited number of decay schemes a unique inversion is not possible, although a substantial amount of information may be deduced. We have calculated $I_j(\tau)$ for a constant rate of crustal growth for various parent-daughter systems (Table 1) for $\tau = 4.55$ aeons and thus a mean age of the mass of j of 2.275 aeons. The isotopes ^{147}Sm, ^{87}Rb, ^{187}Re, ^{176}Lu, and ^{232}Th all have mean lives that are sufficiently long so that $I_j(\tau) \approx \bar{t}$ as shown in Table 1. The concordant results obtained by *Jacobsen and Wasserburg* [1979] for Rb-Sr and Sm-Nd single-stage ages for the oceanic mantle clearly do not represent a unique time event but an average age and the concordance for model I is a consequence of the long mean lives of both ^{147}Sm and ^{87}Rb. Inspection of Table 1 shows that ^{238}U has a significant deviation from the case with $\lambda = 0$, and ^{40}K and ^{235}U are greatly different from this case. These species have sufficiently short mean lives that the higher-order terms become important, and the equations are no longer redundant. From this result it is clear that the detailed history of the early earth must be derived from the latter two decay schemes. These different decay schemes when taken together yield information on the detailed time evolution by measurement at a single point in time.

The mean age $\bar{t}$ shows the relationship

$$\frac{d\bar{t}}{d\tau} = 1 - \frac{\bar{t}}{M}\left(\frac{dM}{d\tau}\right) = 1 - \bar{t}\left(\frac{d \ln M}{d\tau}\right) \qquad (27)$$

so that for $(dM/d\tau) > 0$, $(d\bar{t}/d\tau) < 1$. When $(dM/d\tau) = 0$, it follows that $(d\bar{t}/d\tau) = 1$. We also note that $(d \ln \bar{t}/d\tau) \geq -(d \ln M/d\tau)$ and that $(d\bar{t}/d\tau) \leq 0$ if $(d \ln M/d\tau) \geq (1/\bar{t})$. This is shown schematically in Figure 2. It follows from (27) that a similar relationship must hold for the ϵ value for all systems involving very long lived isotopes. If we know $\bar{t}$ as a function of time and the mass of the crust today, then we may calculate the mass of the continents as a function of time from the following equation:

$$M_3(\tau) = M_3(T_0)\frac{\bar{t}(T_0)}{\bar{t}(\tau)}\exp\left[-\int_\tau^{T_0}\frac{d\xi}{\bar{t}(\xi)}\right] \qquad (28)$$

where T_0 is the age of the earth as measured today.

Geometrical Constraints for Model I

The mechanism for crustal growth in this model is by deriving melts from undepleted mantle. The mass transfer from a deep-seated undepleted reservoir may occur through rising blobs. At shallow levels in the mantle the blobs may intersect the solidus and be partially melted. However, only a fraction β of the blobs may rise underneath the continental crust and

contribute to crustal growth. The remaining fraction of the blobs $(1 - \beta)$ rise underneath the oceanic crust and may contribute to oceanic volcanism. However, these may be mixed back into the mantle with a short time scale (<0.2 eon) owing to the rapid turnover in the oceanic mantle implied by sea floor spreading and not become attached to the continents. This would correspond to adding undifferentiated mantle to the depleted mantle. The possiblity that undifferentiated mantle in this way gets mixed with depleted mantle may easily be incorporated in the equations for model I. We assume that a differential mass δM_1 is derived from reservoir 1 and adds a differential mass δM_0^* to blobs underneath the oceans and δM_C^* to blobs rising underneath the continental crust. Then we have that $\delta M_C^*/\delta M_1 = \beta$ and $\delta M_0^*/\delta M_1 = 1 - \beta$, where β is the probability that a mass taken from reservoir 1 is directed toward a continental segment. The blobs underneath the continental crust are partially melted to a degree F and thus the differential mass added to the crust is $\delta M_3 = F\delta M_C^*$, and the corresponding residue R of $\delta M_R^* = (1 - F)\delta M_C^*$ is left in the mantle and remixed. The blobs δM_0^* which rise in the oceanic regions will also differentiate but are not isolated and are taken to be remixed consistent with the general scheme of model I. The contributions of δM_0^* to reservoir 2 are thus not depleted. In this process a mass $\delta M_2 = \delta M_0^* + \delta M_R^*$ is added to reservoir 2. Conservation of mass and species implies that the concentration of i in the crust $c_{i,3}$ is given by $C_{i,1} = c_{i,3}[F + D_i(1 - F)]$ as before (equation (3)). However, the relationship between the rates of growth of the crust and depleted mantle and the composition of the depleted mantle will be changed. The relationship between the masses of reservoirs 2 and 3 is given by

$$M_3(\tau) = M_2(\tau)\left[\frac{\beta F}{1 - \beta F}\right] \qquad (29)$$

In this case we have from mass and species conservation that $C_{i,1} = c_{i,3}\beta F + \bar{c}_{i,2}(1 - \beta F)$, where $\bar{c}_{i,2}$ is the effective concentration in the total material added to layer 2 in masses δM_0^* and δM_R^*. The concentration of species i in reservoir 2 is given by

$$\frac{\bar{c}_{i,2}}{C_{i,1}} = \frac{F(1 - \beta) + D_i(1 - F)}{(1 - F\beta)[F + D_i(1 - F)]} \qquad (30)$$

where $(\bar{c}_{i,2}/C_{i,1}) > D_i/[F + D_i(1 - F)]$. This version of model I changes (3), (4), and (6) for reservoir 2 and gives $(\delta N_{i,2}/\delta\tau) = \bar{c}_{i,2}(\delta M_2/\delta\tau)$ in (5). All the equations for the continental crust will be the same as those derived previously. For reservoir 2 the transport equation corresponding to (6) is

$$\frac{dN_{i,2}}{d\tau} = \frac{F(1 - \beta) + D_i(1 - F)}{(1 - F\beta)[F + D_i(1 - F)]}C_{i,1}(\tau)M_2'(\tau) - \lambda_i N_{i,2}(\tau) \qquad (31)$$

The solution of (31) for a stable isotope is

$$N_{s,2}(\tau) = \frac{F(1 - \beta) + D_s(1 - F)}{(1 - \beta F)[\mu + D_s(1 - F)]}C_{s,1}(0)M_2(\tau) \qquad (32)$$

and corresponding changes for radioactive and daughter isotopes have to be made in (13) and (15) given previously. The

enrichment factor as defined in (17) is then given for reservoir 2 by

$$f_2^{r/s} = \left[\frac{F(1-\beta)+D_r(1-F)}{F(1-\beta)+D_s(1-F)}\right]\left[\frac{F+D_s(1-F)}{F+D_r(1-F)}\right] - 1 \quad (33)$$

and this value should be used in (19) and (20). From the mass and species conservation equations given previously for this case written for radioactive and stable species it may be shown that

$$D_s = \left\{\frac{(\beta-1)f_2^{r/s}+\beta[1-(C_{r,3}/C_{r,1})F]}{(C_{r,3}/C_{r,1})F\beta+f_2^{r/s}}\right\}\frac{F}{1-F} > 0 \quad (34)$$

Since $0 < F(C_{r,3}/C_{r,1}) < 1$, it follows from (34) that for $f_2^{r/s} < 0$ and $0 < D_r < D_s < 1$ that

$$\beta > (C_{r,3}/C_{r,1})F\beta > -f_2^{r/s} \quad (35)$$

and for $f_2^{r/s} > 0$ and $0 < D_s < D_r < 1$ we have that

$$\beta > \frac{f_2^{r/s}}{f_2^{r/s}+1} \quad (36)$$

Thus given a value for $f_2^{r/s}$, this may put serious constraints on the value of β. If, for example, the continents always occupied 1/3 of the surface area and if the continents only grew from rising material by this geometrical consideration, then β would be 1/3. Were it possible to show that β is very different from this, then this would indicate that the geometrical considerations were not controlling the rate of continental growth and some sweeping or disaggregation process was required. We note that the case with $\beta < 1$ has many similarities with models involving refluxing of material from the crust to the depleted mantle. If a fraction $1 - \beta$ of new additions to the crust is always immediately subducted back to the depleted mantle, then the exact solution of this problem is also given by (29)–(36) above. Similarly, these equations also describe the case where a fraction $1 - \beta$ of the melt is always trapped in the residual solid. The conceptual pictures in Figure 1 and the descriptions of model I in the text do not fully describe the variety of physical circumstances which are represented by the equations derived for model I.

Model II

We now consider a model where the continental crust 3 grows from an initially undifferentiated mantle reservoir 2 which is depleted at all times subsequent to $\tau = 0$ as a result of crustal growth. This reservoir 2 may make up all or part of the mantle. We define a reference reservoir 1 which corresponds to the total of reservoirs 2 and 3. Initially, the whole mantle is supposed to be homogeneous and undifferentiated. A remaining portion of the mantle which is not involved in crust formation may remain isolated and undifferentiated. This reservoir will follow the isotopic evolution of the reference reservoir 1 but otherwise has nothing to do with the model. The model is shown in Figure 1. Let us assume that a differential mass is removed from reservoir 2 as a partial melt to form new crust and increases the mass of the crust by δM_3 and decreases the mass of reservoir 2 by $\delta M_2 = -\delta M_3$. The concentration of an element i in the partial melt at time τ is $c_{i,3}(\tau)$.

There are two different cases of this model that merit discussion. For case A, assume that the differential melt δM_3 equilibrates with all of reservoir 2. Then the mass of the residue is all of reservoir 2, and the degree of melting $F = \delta M_3/M_2$ must be a differential. This means that for an element i

with bulk distribution coefficient $D_i = 0$ all of the element would be transported into the crust as soon as any differential melt formed from reservoir 2. This does not seem physically realistic. For case B, assume that a differential mass δM^* of reservoir 2 is differentiated into a melt δM_3 and a residue δM_R^* with the degree of melting F being finite. Case B seems physically more realistic than case A. For case B we have that $\delta M_3/\delta M^* = F$ and $\delta M_R^*/\delta M^* = 1 - F$. The residue after removal of a melt fraction from reservoir 2 is assumed to be homogenized instantaneously with the remaining part of the depleted mantle. Conservation of species and mass gives $C_{i,2}(\tau)\delta M^* = c_{i,3}(\tau)\delta M_3 + c_{i,R}(\tau)\delta M_R^*$, and since $c_{i,R}(\tau)/c_{i,3}(\tau) = D_i$ it follows that $c_{i,3}(\tau) = d_i C_{i,2}(\tau)$, where

$$d_i \equiv 1/[F+D_i(1-F)] \quad (37)$$

This case thus corresponds to the melting law in model I. The difference is that in model II the residue always has to be homogenized with the remaining part of the mantle reservoir it is derived from, and this reservoir is continuously depleted. Case B will be used in the following discussion.

The number of species i in the differential melt (δM_3) that is added to the crust is given by

$$\delta N_{i,3}(\tau) = c_{i,3}(\tau)\delta M_3 = d_i \frac{\delta M_3}{M_2(\tau)} N_{i,2}(\tau) \quad (38)$$

If $\delta\tau$ is the time over which the mass δM_3 is added to the continental crust, then the transport equations for reservoirs $j = 2, 3$ are

$$\frac{dN_{i,j}(\tau)}{d\tau} = (-1)^{j+1}\left(\frac{\delta N_{i,3}}{\delta\tau}\right) - \lambda_i N_{r,j}(\tau) \quad (39)$$

$$\begin{aligned}\lambda_i &= 0 & i &= s\\ \lambda_i &= \lambda_r & i &= r\\ \lambda_i &= -\lambda_r & i &= d\end{aligned}$$

For simplicity we again ignore mass transport from 3 to 2. Using the expression for $\delta N_{i,3}$ and calling $\delta M_3/\delta\tau \equiv M_3'(\tau)$, we obtain the following basic transport equations for model II:

$$\frac{dN_{i,j}(\tau)}{d\tau} = (-1)^{j+1}d_i\frac{M_3'(\tau)}{M_2(\tau)}N_{i,2}(\tau) - \lambda_i N_{r,j}(\tau) \quad (40)$$

The masses of the total reservoirs 2 and 3 must satisfy

$$M_2(\tau) + M_3(\tau) = M_2(0) \qquad M_2'(\tau) = -M_3'(\tau) \quad (41)$$

The integrals of the transport equations for a stable species s in reservoirs 2 and 3 are

$$N_{s,2}(\tau) = N_{s,2}(0)\left[\frac{M_2(\tau)}{M_2(0)}\right]^{d_s} \quad (42)$$

$$N_{s,3}(\tau) = N_{s,2}(0) - N_{s,2}(\tau) \quad (43)$$

Equation (42) for a stable isotope in reservoir 2 may be written in the following form:

$$C_{s,2}(\tau) = C_{s,2}(0)\left[1 - \frac{M_3(\tau)}{M_2(0)}\right]^{(d_s-1)}$$

This expression has the same general form as the equation of *Shaw* [1970] for the solid residue after modal fractional (Rayleigh) melting, however, d_s takes the place of the inverse bulk distribution coefficient and the degree of crust formation $M_3(\tau)/M_2(0)$ takes the place of the degree of melting.

The solutions of (40) for radioactive and daughter isotopes are

$$N_{r,2}(\tau) = N_{r,2}(0)e^{-\lambda\tau}\left[\frac{M_2(\tau)}{M_2(0)}\right]^{d_r} \tag{44}$$

$$N_{r,3}(\tau) = N_{r,2}(0)e^{-\lambda\tau} - N_{r,2}(\tau) \tag{45}$$

$$N_{d,2}(\tau) = \left\{N_{d,2}(0) + N_{r,2}(0)\lambda \int_0^\tau \left[\frac{M_2(\xi)}{M_2(0)}\right]^{(d_r-d_s)}\right.$$
$$\left. \cdot \exp\left[-\lambda\xi\right] d\xi\right\}\left[\frac{M_2(\tau)}{M_2(0)}\right]^{d_s} \tag{46}$$

$$N_{d,3}(\tau) = N_{d,2}(0) + N_{r,2}(0)[1 - e^{-\lambda\tau}] - N_{d,2}(\tau) \tag{47}$$

where $d_s = d_d$, since d and s are isotopes of the same element. The ratios of a radiogenic isotope to a stable isotope for the two reservoirs are

$$\frac{N_{d,2}(\tau)}{N_{s,2}(\tau)} = \frac{N_{d,1}(\tau)}{N_{s,1}(\tau)} - \lambda\frac{N_{r,1}(\tau)}{N_{s,1}(\tau)}\int_0^\tau\left\{1 - \left[\frac{M_2(\xi)}{M_2(0)}\right]^{(d_r-d_s)}\right\}$$
$$\cdot \exp\left[\lambda(\tau - \xi)\right] d\xi \tag{48}$$

$$\frac{N_{d,3}(\tau)}{N_{s,3}(\tau)} = \frac{N_{d,1}(\tau)}{N_{s,1}(\tau)} - \lambda\frac{N_{r,1}(\tau)}{N_{s,1}(\tau)}$$

$$\left\{\int_0^\tau\left[1 - \left(\frac{M_2(\xi)}{M_2(0)}\right)^{(d_r-d_s)}\right]\exp\left[\lambda(\tau-\xi)\right]d\xi\right\}\left[1 - \left(\frac{M_2(\xi)(-d_s)}{M_2(0)}\right)^{-1}\right] \tag{49}$$

From (17) and (42)–(45) the enrichment factors of the ratio of a radioactive isotope to a stable isotope in the total reservoirs 2 and 3 to that in reservoir 1 are given by

$$f_2^{r/s}(\tau) = \left[\frac{M_2(\tau)}{M_2(0)}\right]^{(d_r-d_s)} - 1 \approx \exp\left[-(d_r-d_s)\frac{M_3(\tau)}{M_2(0)}\right] - 1 \tag{50}$$

$$\frac{f_2^{r/s}(\tau)}{f_3^{r/s}(\tau)} = 1 - \left[\frac{M_2(\tau)}{M_2(0)}\right]^{(-d_s)} \approx 1 - \exp\left[d_s\frac{M_3(\tau)}{M_2(0)}\right] \tag{51}$$

where the approximations hold for the mass of the crust much smaller than that of the depleted mantle $[(M_3(\tau)/M_2(0)) \ll 1]$. Equations (48) and (49) may be written in terms of deviations in parts in 10^4 from the reference reservoir 1:

$$\epsilon_{d,2}^*(\tau) = Q_d^*(\tau)\int_0^\tau f_2^{r/s}(\xi)\exp\left[\lambda(\tau - \xi)\right]d\xi \tag{52}$$

$$\epsilon_{d,3}^*(\tau) = \epsilon_{d,2}^*(\tau)\frac{f_3^{r/s}(\tau)}{f_2^{r/s}(\tau)} \tag{53}$$

where $Q_d^*(\tau)$ is defined by (20). Equations (50)–(53) in conjunction with (37) are the basic equations for calculations with this model for arbitrary rates of crustal growth. In this class of models the average enrichment factors are a function of the time and correspond to a time dependent chemical evolution. For $\lambda\tau \ll 1$ the deviation $\epsilon_{d,j}^*$ are then proportional to the weighted time average of the enrichment factors $\langle f_j^{r/s}\rangle$ and given by

$$\epsilon_{d,2}^*(\tau) = Q_d^*(\tau)\langle f_2^{r/s}(\tau)\rangle\tau \tag{54}$$

$$\epsilon_{d,3}^*(\tau) = Q_d^*(\tau)\frac{f_3^{r/s}(\tau)}{f_2^{r/s}(\tau)}\langle f_2^{r/s}(\tau)\rangle\tau \tag{55}$$

This means that the simple species independent time averages which govern the ϵ function for models of type I no longer apply, and an average age is not directly applicable to reservoir 2.

In this model the depleted mantle is assumed to always be homogeneous. For the crust the equations give the average values. If each new addition to the crust is isolated, then the new individual segments follow their own evolutionary path. The initial states of new additions to 3 at time τ' are given by

$$f_{new}^{r/s}(\tau') = \frac{d_r}{d_s}[f_2^{r/s}(\tau') + 1] - 1 \tag{56}$$

$$\epsilon_{d,new}^*(\tau') = \epsilon_{d,2}^*(\tau') \tag{57}$$

and subsequent evolution will follow:

$$\epsilon_{d,new}^*(\tau) = \epsilon_{d,3}^*(\tau') + \frac{Q_d^*(\tau')}{\lambda}f_{new}^{r/s}(\tau')\{\exp[\lambda(\tau - \tau')] - 1\} \tag{58}$$

This model has the property that new crust would at the time of addition have the isotopic values of depleted mantle 2, and the interpretation of T_{CHUR}^{Nd} model ages [McCulloch and Wasserburg, 1978] of new crust would depend on the detailed history of reservoir 2.

For this model we may again define the mean age of species s in the crust 3 by

$$\bar{t}_{s,3} = \frac{1}{N_{s,3}(\tau)}\int_0^\tau(\tau - \xi)\frac{dN_{s,3}}{d\xi}d\xi = \frac{1}{N_{s,3}(\tau)}\int_0^\tau N_{s,3}(\xi)d\xi \tag{59}$$

and the mean age of the mass M_3 by

$$\bar{t}_{M,3} = \frac{1}{M_3(\tau)}\int_0^\tau(\tau - \xi)M_3'(\xi)d\xi = \frac{1}{M_3(\tau)}\int_0^\tau M_3(\xi)d\xi \tag{60}$$

The mean age of the depleted mantle reservoir 2 may not be defined in the same manner. Note that for model II, $N_{s,3}(\tau)/M_3(\tau)$ is not a constant independent of τ, so that $\bar{t}_{s,3} \neq \bar{t}_{M,3}$. The result of this is distinct from model I and is shown schematically in Figure 2. Using (43) and (59), we may write

$$\bar{t}_{s,3} = \left\{1\middle/\left[1 - \left(1 - \frac{M_3(\tau)}{M_2(0)}\right)^{d_s}\right]\right\}\int_0^\tau\left\{1 - \left[1 - \frac{M_3(\xi)}{M_2(0)}\right]^{d_s}\right\}d\xi$$

$$\approx \left\{1\middle/\left[1 - \exp\left(-\frac{d_sM_3(\tau)}{M_2(0)}\right)\right]\right\}\int_0^\tau\left\{1 - \exp\left[-d_s\frac{M_3(\xi)}{M_2(0)}\right]\right\}d\xi \tag{61}$$

assuming that $(M_3(\tau)/M_2(0)) \ll 1$ in the second expression. In the case that $[d_sM_3(\tau)/M_2(0)] \ll 1$, then $\bar{t}_{s,3} \approx \bar{t}_{M,3}$. If $[d_sM_3(\tau)/M_2(0)] \gg 1$ for $\tau \geq \tau^*$, then $\bar{t}_{s,3} \approx$ const $+ (\tau - \tau^*)$ for $\tau \geq \tau^*$, and $(d\bar{t}_{s,3}/d\tau) = 1$. This corresponds to the bulk of the species s being transported into the crust by time τ^* after the formation of the earth. The mass transfer is then subsequently decoupled from addition of the trace element s, and the average age of the mass $\bar{t}_{M,3}$ may be substantially younger than $\bar{t}_{s,3}$ as the different trace element abundances of the crust change with time. The difference between $\bar{t}_{s,3}$ and $\bar{t}_{M,3}$ is of interest and may be evaluated quantitatively. We have

$$\bar{t}_{s,3} - \bar{t}_{M,3} = \left\{\int_0^\tau\left[M_3(\tau) - M_3(\xi) + M_3(\xi)\exp\left(-d_s\frac{M_3(\tau)}{M_2(0)}\right)\right.\right.$$
$$\left.\left. - M_3(\tau)\exp\left(-d_s\frac{M_3(\xi)}{M_2(0)}\right)\right]d\xi\right\}\left\{\left[1 - \exp\left(-d_s\frac{M_3(\tau)}{M_2(0)}\right)\right]M_3(\tau)\right\}^{-1}$$
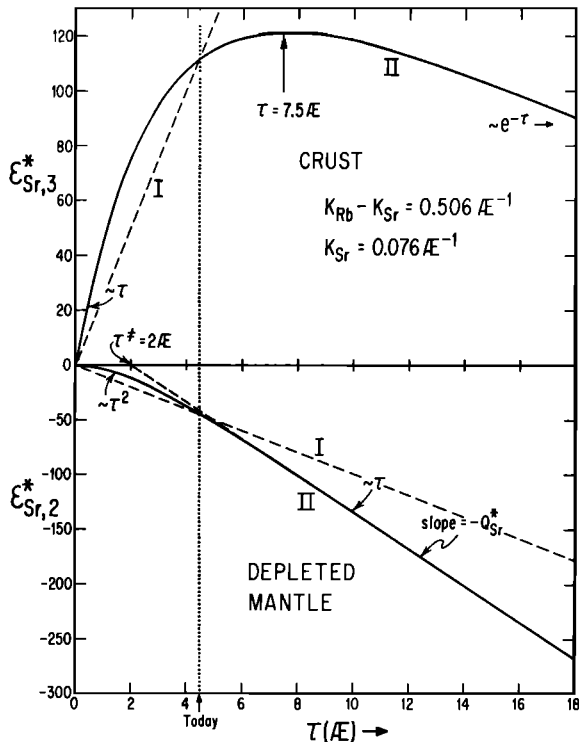
Fig. 3. Shown are $\epsilon_{Sr,j}^*$ curves for the bulk depleted mantle (2) and the bulk crust (3) as a function of time τ for a constant rate of growth of the crust. Curves that give the same present day ϵ_{Sr}^* values are shown for both model I and model II and are identified by the symbols I and II, respectively. Note the marked difference in behavior for ϵ_{Sr}^* as a function of time τ for the two models. Model I is a straight line for both reservoirs, and model II starts out as nearly a straight line for the crust but with a significantly different slope. The curves for model II are constrained to give $f_2^{Rb/Sr} = -0.9$ and $f_3^{Rb/Sr} = +2.18$ today. The implied values of $K_{Rb} - K_{Sr}$ and K_{Sr} are given. The systematics shown is valid for decay systems with $\lambda \tau \ll 1$ and $(d_r - d_s) > 0$.

$$= \left\{ \left(\frac{d_s}{M_2(0)} \right)^2 \int_0^\tau M_3(\xi) \left[\sum_{n=2}^\infty \frac{(-1)^n}{n!} \left(M_3(\tau) \left(\frac{d_s M_3(\tau)}{M_2(0)} \right)^{n-2} \right. \right. \right.$$

$$\left. \left. \left. - M_3(\xi) \left(\frac{d_s M_3(\xi)}{M_2(0)} \right)^{n-2} \right) \right] d\xi \right\} \left\{ \left[1 - \exp\left(-d_s \frac{M_3(\tau)}{M_2(0)} \right) \right] \right\}^{-1} \tag{62a}$$

If $d_s < 0$, it may be seen that $\bar{i}_{s,3} - \bar{i}_{M,3} < 0$. This expression converges rather well even for large values of $d_s[M_3(\tau)/M_2(0)]$ (≤ 2). In the limit for small $d_s/M_2(0)$ we have rather precisely

$$\frac{\bar{i}_{s,3} - \bar{i}_{M,3}}{(d_s/M_2(0))} \approx \frac{1}{2M_s(\tau)} \int_0^\tau M_3(\xi)[M_3(\tau) - M_3(\xi)] \, d\xi \tag{62b}$$

so in this approximation, $\bar{i}_{s,3} - \bar{i}_{M,3} > 0$ for $d_s > 0$. For a model of a constant rate of crustal growth we obtain

$$\bar{i}_{s,3} - \bar{i}_{M,3} \approx \frac{d_s M_3(\tau)}{12 M_2(0)} \tau \tag{62c}$$

which gives a difference of 0.65 eon for $d_s = 100$ with $M_3(\tau)/M_2(0) = 0.0174$. It follows that for $d_s \leq 10$ there are only small differences in the mean ages.

We note that the expressions for ϵ^* ((52) and (53)) may be written·in a form analogous to that for model I. Inspection of (61) and (52) shows that $\epsilon_{d,2}^*$ may be written as

$$\epsilon_{d,2}^*(\tau) = Q_d^*(\tau) f_2^{r/s}(\tau) \bar{i}_{r/s} \tag{63}$$

where

$$\bar{i}_{r/s} \equiv \frac{1}{f_2^{r/s}(\tau)} \int_0^\tau f_2^{r/s}(\xi) \, d\xi = \frac{\tau \langle f_2^{r/s}(\tau) \rangle}{f_2^{r/s}(\tau)} \tag{64}$$

The factor $\bar{i}_{r/s}$ is the time weighted average of $f_2^{r/s}$ for reservoir 2. If $d_r - d_s > 0$, this is equivalent to the mean age of an element r/s with a distribution coefficient $d_{r/s} = d_r - d_s$ and an enrichment factor $f_2^{r/s}$. Note that $\bar{i}_{r/s}$ is explicitly dependent on the particular parent-daughter system as distinct from the case for model I. If the differentiation took place in only a single event, then $t_{r/s} = t_{M,3}$. More generally, there will be a difference between $\bar{i}_{r/s}$ and $\bar{i}_{M,3}$, which may be calculated from

$$\bar{i}_{r/s} - \bar{i}_{M,3} \approx \frac{d_r - d_s}{2 M_3(\tau) M_2(0)} \int_0^\tau M_3(\xi)[M_3(\tau) - M_3(\xi)] \, d\xi \tag{65}$$

The derivation of expression (65) follows that given previously for $\bar{i}_{s,3} - \bar{i}_{M,3}$. For a model of constant rate of crustal growth we have

$$\bar{i}_{r/s} - \bar{i}_{M,3} = \frac{(d_r - d_s) M_3(\tau)}{12 M_2(0)} \tau \tag{66}$$

If $|(d_r - d_s) M_3(\tau)/M_2(0)| \ll 1$, then (50) and (52) may for very long lived isotopes be approximated by

$$\epsilon_{d,2}^*(\tau) \approx -(d_r - d_s) \frac{Q_d^*(\tau) M_3(\tau)}{M_2(0)} \bar{i}_{M,3} \tag{67}$$
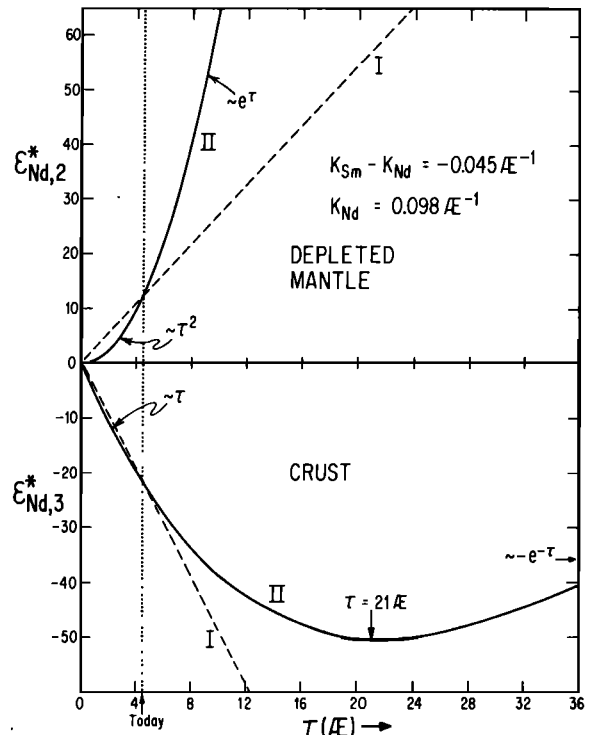
and

Fig. 4. Shown are $\epsilon_{Nd,j}^*$ curves for bulk depleted mantle (2) and bulk crust (3) as a function of time τ for a constant rate of growth of the crust. The notation is the same as in Figure 3. The curves for model II are constrained to give $f_2^{Sm/Nd} = +0.225$ and $f_3^{Sm/Nd} = -0.4$ today. The implied values of $K_{Sm} - K_{Nd}$ and K_{Nd} are shown. Note that the curves are reversed from those in Figure 3 owing to the change in sign of $K_r - K_s$. The systematics shown is valid for decay systems with $\lambda \tau \ll 1$ and $(d_r - d_s) < 0$.
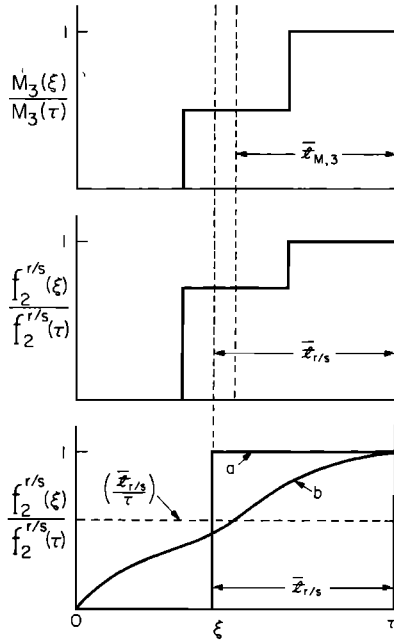
Fig. 5. Cartoon showing the relationship between the mean age of the mass of the crust $\bar{t}_{M,3}$, the mass growth curve of the crust M_3, the enrichment factor of the depleted mantle $f_2^{r/s}$, and the factor $\bar{t}_{r/s}$. The upper diagram shows a mass growth curve where the crust is formed in two stages with a mean age $\bar{t}_{M,3}$ at time τ. The middle diagram shows the corresponding change in $f_2^{r/s}$ with time if $(d_r - d_s) = 100$ and $[M_3(\tau)/M_2(0)] = 0.0174$. The difference between $\bar{t}_{r/s}$ and $\bar{t}_{M,3}$ is also indicated. If we know $\bar{t}_{r/s}, f_2^{r/s}$, and M_3 today, there is insufficient information to determine a unique solution for $M_3(\xi)$ and $f_2^{r/s}(\xi)$. In the lower part of the diagram some other possible $f_2^{r/s}$ curves that give the same value for $\bar{t}_{r/s}$ are shown. For a given curve, $f_2^{r/s}$ with knowledge of $(d_r - d_s)$ determines a corresponding M_3 curve from (50). The area underneath the $[f_2^{r/s}(\xi)/f_2^{r/s}(\tau)]$ curve must be the same for all M_3 curves with the same value for $\bar{t}_{r/s}$; $\bar{t}_{r/s}$ is the shortest time in which differentiation could have occurred as a single-stage process, and curve a shows such an evolution. Curve b shows a possible continuous curve. The dashed horizontal curve gives $[(f_2^{r/s})/f_2^{r/s}] = [\bar{t}_{r/s}/\tau]$. All of these curves in the lower figure are compatible with the $\bar{t}_{r/s}$ fixed by the assumed model with $M_3(\xi)$ as given in the top and the $f_2^{r/s}(\tau)$ value.

$$f_2^{r/s}(\tau) \approx -(d_r - d_s) \frac{M_3(\tau)}{M_2(0)} \tag{68}$$

If it also holds that $[d_s M_3(\tau)/M_2(0)] \ll 1$, then

$$f_3^{r/s}(\tau) \approx (d_r/d_s) - 1 \tag{69}$$

and

$$\epsilon_{d,3}^*(\tau) \approx Q_d^*(\tau)[(d_r/d_s) - 1]\bar{t}_{M,3} \tag{70}$$

An important special case is when $(dM_3/d\tau)$ is a constant. In this case, $M_3(\tau) = [M_3(T_0)/T_0]\tau$, and defining $K_i = [(d_i M_3(T_0))/(T_0 M_2(0))]$ then for very long lived isotopes,

$$\epsilon_{d,2}^*(\tau) = -Q_d^*(\tau) \left[\tau - \frac{1 - \exp[-(K_r - K_s)\tau]}{(K_r - K_s)} \right] \tag{71}$$

$$f_2^{r/s}(\tau) = \exp[-(K_r - K_s)\tau] - 1 \tag{72}$$

$$\frac{f_2^{r/s}(\tau)}{f_3^{r/s}(\tau)} = 1 - \exp(K_s\tau) \tag{73}$$

$$\epsilon_{d,3}^*(\tau) = \frac{\epsilon_{d,2}^*(\tau)}{1 - \exp(K_s\tau)} \tag{74}$$

There are two different classes of solutions depending on whether $(d_r - d_s)$ is negative (as for Sm-Nd) or positive (as for Rb-Sr). Examples are shown for Rb-Sr and Sm-Nd in Figures 3 and 4, respectively. For short times we have in both cases that $\epsilon_{d,2}^* \propto \tau^2$ and $\epsilon_{d,3}^* \propto \tau$. For long times we get $\epsilon_{Sr,2}^* \approx -Q_{Sr}^*(\tau - \tau^\dagger)$, where $\tau^\dagger = 1/(K_{Rb} - K_{Sr}) = 2$ aeons and $\epsilon_{Nd,2}^* \propto \exp[-(K_{Sm} - K_{Nd})\tau]$ for the depleted mantle. The curves for the crust go to an extremum and then asymptotically approach zero. The inclusion of the term $\exp[\lambda(\tau - \xi)]$ for shorter-lived isotopes in the exact integral of (52) will, of course, quantitatively alter the nature of these results. The curve for $\epsilon_{Sr,3}^*$ has a maximum at $\tau = 7.5$ aeons, and for much longer times $\epsilon_{Sr,3}^* \propto \exp[-K_{Sr}\tau]$. Similarly, the $\epsilon_{Nd,3}^*$ curve for the crust has a minimum at $\tau = 21$ aeons, and $\epsilon_{Nd,3}^* \propto -\exp[-K_{Nd}\tau]$ for much longer times. These solutions enable us to qualitatively understand the nature of the results for more general cases of crustal growth with time. From these results we may infer that for any continuous or multiepisodic mass growth of the crust the basic isotopic systematics for long-lived isotopes must follow those outlined in Figures 3 and 4. We note that for the case of episodic mass growth, $\epsilon_{d,3}^*$ will be discontinuous owing to the factor $f_3^{r/s}/f_2^{r/s}$ outside of the integral in (53). The value of $\epsilon_{d,2}^*$ for the depleted mantle will still be continuous but will show changes in slope corresponding to the discontinuities in the mass growth curve. The general case for arbitrary mass transport is of course included in the exact basic equations (50)–(53).

Calculations with model II are more complex than with model I, where the average age $\bar{t}$ is independent of species. For the mantle we have previously obtained the exact results:

$$\epsilon_{d,2}^* = Q_d^*(\tau)\langle f_2^{r/s}\rangle\tau = Q_d^*(\tau)f_2^{r/s}(\tau)\bar{t}_{r/s} \tag{75}$$

It follows that if $\epsilon_{d,2}^*(\tau)$ is known at τ, then $\langle f_2^{r/s}\rangle$ is determined. If $\epsilon_{d,2}^*$ and $f_2^{r/s}$ are known, then $(d_r - d_s)$ is known from (50), and $\bar{t}_{r/s}$ is determined; $\bar{t}_{r/s}$ is the time required to generate $\epsilon_{d,2}^*$ with the present value of $f_2^{r/s}$. As $f_2^{r/s}$ is monotonic in this model, $\bar{t}_{r/s}$ is the shortest time in which differentiation could have occurred (as a single-stage process). All possible crustal mass growth laws $M_3(\tau)$ must satisfy the relationship between $f_2^{r/s}$ and M_3. The relationships are shown schematically in Figure 5. If decay schemes with very long lived parents have $|(d_r - d_s)M_3/M_2(0)| \ll 1$, then they will all yield the same values of $\bar{t}_{r/s}$, and no further information may be derived about the change of M_3 with time from the different systems at one time τ. If two systems have distinctive parameters (such as Rb-Sr and Sm-Nd), then they will have different values of $\bar{t}_{r/s}$ which will give more information about $M_3(\tau)$. For N independent values of $\bar{t}_{r/s}$ from N different decay schemes with $\lambda\tau \ll 1$, it is, in principle, possible to calculate models of multiple-stage mass evolution with time. For example, for an n stage evolution we write

$$\bar{t}_{r/s} = \sum_{i=1}^{n} \frac{\exp[-\alpha_{r/s}X_{i-1}] - 1}{\exp[-\alpha_{r/s}] - 1}(\tau_i - \tau_{i-1}) \tag{76}$$

where $\tau_0 = 0$, $\tau_n = \tau$, $X_{n-1} = 1$, and $X_0 = 0$. The solutions for X_i may in general not be a unique description of earth evolution. Here $X_i - X_{i-1}$ is the fraction of $M_3(\tau)$ made at time τ_i, and $\alpha_{r/s} = (d_r - d_s)M_3(\tau)/M_2(0)$. Such equations may be solved numerically for the X_i and τ_i which must provide simultaneous solutions for different $\bar{t}_{r/s}$ values. In the degenerate case when $\alpha_{r/s} \ll 1$,

$$\bar{t}_{r/s} = \sum_{i=1}^{n} X_{i-1}(\tau_i - \tau_{i-1}) \tag{77}$$

which is independent of r/s.

TABLE 2. Masses of Various Reservoirs in the Earth

Reservoir	Mass, 10^{25} grams	Fraction of Total Earth Mass
Continental crust	1.807	0.003023
Subcontinental crust	0.430	0.000719
Oceanic sediment (layer 1)*	0.019	0.000032
Total continental crust plus subcontinental crust plus oceanic sediment	2.256	0.003774
Oceanic crust (layers 2 and 3)†	0.590	0.000987
Mantle	400.5	0.6701
Mantle plus crust	403.4	0.6749
Core	194.3	0.3251
Total earth	597.7	1

Masses of core and mantle are from Smith [1971], and masses of the crustal reservoirs are from Renov and Yaroshevsky [1976].

*Considered to be mainly derived by werathering of continental crust.

†Basaltic part of the oceanic crust.

BOUNDARY CONDITIONS FOR THE MODELS

The parameters involved for both models described above are the initial and final concentrations and isotopic compositions for each reservoir j, the solid/melt distribution coefficients, and the mass of reservoir j as a function of time. All these parameters need not be specified, and in the following we will choose the parameters we believe are the best known and use them to make estimates of others. We will limit the discussion to the Rb-Sr and Sm-Nd decay systems, but their relation to the concentrations of K, Ba, and U will also be discussed. The masses of the various reservoirs in the earth are given in Table 2. In both models the depleted mantle is taken to include the basaltic part of the oceanic crust, since it is derived from the depleted mantle and is subducted back to the depleted mantle at a short time scale (<0.2 aeon). The total mass of the continental crust (2.256 × 10^{25} grams) is taken as the total amount of continental materials including layer 1 of the oceanic crust, which is considered to be mainly sediments derived from weathering of continental crust. The bulk earth values (reservoir 1) used for Rb-Sr and Sm-Nd are given in the appendix. By definition, $\epsilon_{4,1} = f_1''^{/s} = 0$ for reservoir 1 (undepleted mantle). Estimates of ϵ and f values for the depleted mantle and the continental crust are given in Table 3, and partition coefficients for garnet peridotite minerals in Table 4. The values for the depleted mantle in Table 3 are the estimates for the MORB source given by Jacobsen and Wasserburg [1979], where the estimates for 0.5 aeon ago are based on data from the Bay of Islands complex.

The 'silicate portion of the earth is assumed to be initially chemically homogeneous, and the core is assumed to have insignificant amounts of the lithophile elements considered here. The concentrations of these elements in the silicate portion of the earth are thus 1.48 times their concentrations in the

total earth. It is known that the earth is depleted in volatile elements relative to chondrites [Gast, 1960]. However, the ratios of refractory elements in the earth appear to be equal to those in chondrites from both isotopic studies [DePaolo and Wasserburg, 1976a] and chemical studies [Sun and Nesbitt, 1977]. This gives a minimum of 1.48 times chondritic abundances for these elements in the silicate portion of the earth. If a steady state relationship between heat production from radioactive sources and heat flow exist, then the U content of the crust plus mantle is 35 ppb [Wasserburg et al., 1964; Tera et al., 1974] or 46 ppb if the new global heat flow estimate of Williams and Von Herzen [1975] is used [O'Nions and Pankhurst, 1977]. Ordinary chondrites have 13 ppb U, so the heat flow data give an upper limit of 2.7 or 3.5 times chondrites for the concentration of U and other refractory elements for the silicate portion of the earth. It has been concluded elsewhere that the concentrations of Ca, Al, U, Th, Ba, Sr, and REE in the mantle are about 2.0 ± 0.5 those in ordinary chondrites [Frey and Green, 1974; Loubet et al., 1975; Sun and Nesbitt, 1977]. We have thus used a value 2 times ordinary chondrites for the refractory elements in the crust plus mantle. Estimates for the Rb and K concentrations in the silicate portion of the earth can then be estimated from the bulk earth Rb/Sr ratio of 0.029 [DePaolo and Wasserburg, 1976b] and the bulk earth K/U ratio. Previous estimates of the K/U ratio of the earth [Heier and Rogers, 1963; Wasserburg et al., 1964] were based mainly on upper crustal rocks and are very close to 10^4. This is most likely a lower limit, since lower crustal granulites have (K/U) > 10^4 [Lambert and Heier, 1968b] and since it seems plausible that U is more enriched in the crust than K. Our estimates of the concentrations of K, U, Ba, Rb, Sr, Sm, and Nd in the crust plus mantle are compared with other recent estimates in Table 5. Most estimates agree to within a factor of 2. The largest disagreement for element ratios is with the Sm/Nd ratio of Smith [1977] and the Sr/Nd ratio of O'Nions et al. [1978].

Extensive work has established that the upper crustal composition is close to that of granodiorite [Poldervaart, 1955; Shaw et al., 1967; Eade and Fahrig, 1971]. The most recent estimates for the elements of interest here [Shaw et al., 1976] are given in Table 6. It is well known from the heatflow data that the upper crustal concentrations of K, U, and Th cannot extend to a depth of more than 10–15 km, so that the lower crust must be depleted in these elements [Heier, 1965, 1973]. This had led to the hypothesis that the lower crust is made up of dry granulite facies rocks and related igneous rocks, since they show the necessary depletions in U and Th [Heier, 1965], and they also show Rb depletions and generally low Rb/Sr ratios [Heier, 1964]. Extensive geochemical studies of granulite facies rocks with particular emphasis on the distribution of heat-producing elements have been carried out in Australia and Norway [Heier and Adams, 1965; Lambert and Heier, 1967, 1968a, b; Heier and Thoresen, 1971]. The estimates of

TABLE 3. Ranges of Parameters Based on Directly Observed Average Values

	T, aeons	$f^{Rb/Sr}$	ϵ_{Sr}	$f^{Sm/Nd}$	ϵ_{Nd}
Depleted mantle	0	$-0.9 \, ^{+0.1}_{-0.05}$	-27 ± 5	$+0.22 \, ^{+0.08}_{-0.07}$	$+10 \pm 2$
	0.5	-0.9	-19.3	$\sim+0.2$	$+7.6$
Continental crust	0	$+1$ to $+10$	$+60$ to $+250$	-0.4 ± 0.1	-10 to -25

Sources are McCulloch and Wasserburg [1978], DePaolo and Wasserburg [1979a], and Jacobsen and Wasserburg [1979].

TABLE 4. Mineral/Melt and Bulk Partition Coefficients

	K	Rb	Sr	Ba	Nd	Sm	U
Clinopyroxene	0.003	0.003	0.12	0.002	0.17	0.26	0.003
Garnet	0.006	0.01	0.08	0.003	0.087	0.217	0.003
Orthopyroxene	0.003	0.003	0.02	0.002	0.013	0.022	0.0001
Olivine	0.0002	0.0002	0.0002	0.0001	0.007	0.010	0.0001
Initial bulk distribution coefficient*							
$D_{0,K}$	0.002						
$D_{0,Rb}$		0.0025					
$D_{0,Sr}$			0.033				
$D_{0,Ba}$				0.0014			
$D_{0,Nd}$					0.044		
$D_{0,Sm}$						0.080	
$D_{0,U}$							0.001

Sources: *Griffin and Murthy* [1969], *Philpotts and Schnetzler* [1970], *Schnetzler and Philpotts* [1970], *Hart and Brooks* [1974], *Shimizu* [1974], *Grutzeck et al.* [1975], *Shimizu and Kushiro* [1975], *Weill and McKay* [1975], *McKay and Weill* [1976], *Benjamin et al.* [1978].
$D_U^{cpx/ol} \approx D_U^{cpx/opx} \approx 30$, *Tatsumoto* [1978].
$D_U^{cpx/ga} \approx 1$, *Morgan and Lovering* [1971].
*Calculated for a garnet peridotite mineralogy with the following initial mass fractions of the various phases: olivine = 0.55, orthopyroxene = 0.15, clinopyroxene = 0.15, garnet = 0.15.

the concentrations in the lower crust from *Heier and Thoresen* [1971] are given in Table 6. *Lambert and Heier* [1968b] used estimates of the heat-producing elements in upper and lower crustal materials together with constraints from heatflow to calculate concentration profiles of K, U, and Th through the continental crust. They estimated total crust concentrations of K = 1.5 wt % and U = 0.7 ppm for Precambrian shields. Similar reasoning gives K/Rb $\approx$ 450 for the total crust on the basis of upper and lower crustal K and Rb concentrations. To arrive at the estimates we give for the total crust in Table 6, we have used the estimates of *Lambert and Heier* [1968b] for K and U together with K/Rb $\approx$ 450 and K/Ba $\approx$ 30 [*Philpotts and Schnetzler*, 1970]. Most estimates of the average Sr concentration in crustal materials lie in the range 320–400 ppm [*Taylor*, 1964; *Hurley*, 1968a, b; *Larimer*, 1971; *Taylor*, 1977] typically with an average value of ~370 ppm which we have used.

Average Sm and Nd concentrations seem to be approximately the same in both upper and lower crustal rocks. We have thus used Nd = 26 ppm from *Shaw et al.* [1976]. Average continental crustal materials typically have $f_{Sm/Nd} \approx -0.4$ [*McCulloch and Wasserburg*, 1978], and this gives Sm = 5 ppm in the total crust.

RESULTS FOR MODEL I

We now will attempt to evaluate the parameters of this model for both Rb-Sr and Sm-Nd by first only considering estimates from observed ϵ values and f values (Table 3) using (26). These parameters are more directly known than the concentrations in the various reservoirs. The enrichment factor for Rb-Sr in the MORB source is estimated to be $sf_2^{Rb/Sr} = -0.9$. As $f_2^{Rb/Sr}$ is close to the limit of -1, it is therefore not subject to substantial error. The mean ϵ_{Sr} value of MORB is -27. These values give a reliable estimate of the mean age of the MORB source today ($\bar{t} = 1.8$ aeons). If the rate of production of crust were constant over 4.5 aeons, then $\bar{t} = T_0/2 = 2.3$ aeons. As $\bar{t}$ is much less than 2.3 aeons, this shows that if model I is valid, the rate of crustal and mantle formation was much greater in the time subsequent to 1.8 aeons in comparison with the rate between 1.8 and 4.55 aeons. It is of interest to note that for a uniform rate of crustal formation, we calculate $T_0 = 3.6$ aeons the age of the oldest known terrestrial

rocks. This model suggests a low initial rate of crustal formation. The MORB show a range of $f_2^{Sm/Nd}$ from ~0 to +0.4, but their average $\epsilon_{Nd,2} = +10$ is well defined. As the model has the same value of $\bar{t}$ for different species, we therefore use the mean age of 1.8 aeons from the Rb-Sr data to arrive at a self-consistent estimate of $f_2^{Sm/Nd} = +0.225$. This compares well with the estimate given in Table 3 of $f_2^{Sm/Nd} = +0.22$. For the continental crust the only parameter for which we have a reliable estimate is $f_3^{Sm/Nd} = -0.4$. With our estimate of the mean age $\bar{t} = 1.8$ aeons this gives $\epsilon_{Nd,3}(0) = -17.8$, which is in the middle of the range of values reported for Canadian shield composite samples and sedimentary rocks by *McCulloch and Wasserburg* [1978]. Reliable estimates of $\epsilon_{Sr,3}$ and $f_3^{Rb/Sr}$ for the continental crust are much harder to obtain directly, owing to the strong upward concentration of Rb in the crust as discussed previously. Estimates of $f^{Rb/Sr}$ for the upper crust give $f^{Rb/Sr} \sim +10$ [*McCulloch and Wasserburg*, 1978]. However, average continental runoff shows $\epsilon_{Sr}(0) = +163$ [*Veizer and Compston*, 1974], which would yield $f_3^{Rb/Sr} = +5.4$ if $\bar{t} = 1.8$ aeons. Strontium has a short residence time in sea water (4 × 10^6 yr), and if we assume that the sea water ($\epsilon_{Sr}(0) = +65$) [*Papanastassiou et al.*, 1970; *Hildreth and Henderson*, 1971] represents average crust, then we get $f_3^{Rb/Sr} = +2.16$. This is identical to our estimate in Table 6 based on a crustal model with a granulite facies lower crust. However, the similarity may be fortuitous, as Sr in seawater may be affected by isotope exchange with oceanic crust during hydrothermal convection within oceanic ridges [*Spooner*, 1976]. The self-consistent values of the model parameters discussed above are given in Table 7.

Assuming that the Bay of Islands data are a proper representation of the MORB source of 0.5 aeon ago (Table 3), we may calculate the various model parameters at that time. For Rb-Sr using $f_2^{Rb/Sr} = -0.9$ from Table 7 and the observed $\epsilon_{Sr,2} = -19.3$, we obtain $\bar{t} = 1.3$ aeons at 0.5 aeon ago. Similarly, for Sm-Nd using the observed $\epsilon_{Nd,2} = +7.6$ and the parameters in Table 7, we get $\bar{t} = 1.4$ aeons. Equation (27) gives the variation of the mean age with time. Since $d\bar{t}/d\tau = 1 - \bar{t}(d \ln M/d\tau) \leq 1$, it follows that $\bar{t}$ (today) $- \bar{t}$ (0.5 aeon ago) ≤ 0.5. For Rb/Sr this difference is 0.5, implying that $dM/d\tau \cong 0$ for the last 500 m.y. For Sm/Nd this difference is 0.4, implying that $(d \ln M/d\tau) = 0.11$ aeon^{-1} for the last 500 m.y., which is half of the

TABLE 5. Recent Estimates of Average Lithophile Element Concentrations, in Parts per Million, in the Silicate Portion of the Earth (Crust Plus Mantle)

Author	K	Rb	Sr	Ba	Sm	Nd	U	$f^{Rb/Sr}$	$f^{Sm/Nd}$	Sr/Nd	Sr/U	K/Rb	K/U
Hurley [1968a, b]	207	0.59	17.8	...	...	...	0.0148	+0.16	...	...	1187	331	1.4×10^4
Larimer [1971]	192	0.62	17.8	7.8	...	...	0.0178	+0.22	...	...	1000	310	1.1×10^4
Shaw [1972]	160	0.59	27	5.5	...	...	0.027	−0.24	...	...	1000	271	0.6×10^4
Ganapathy and Anders [1974]*	252	0.86	26.9	7.5	0.38	1.27	0.0266	+0.12	−0.07	21.2	1011	293	0.944×10^4
Tera et al. [1974]	350	1.2	31	...	...	...	0.035	+0.35	...	...	886	292	10^4
Ringwood and Kesson [1977]†	286	0.85	24.2	8.4	0.42	1.39	0.0286	+0.23	≡0	17.5	846	341	10^4
Smith [1977]	192	0.73	29.6	7.3	0.33	1.78	0.0385	−0.14	−0.42	16.6	769	263	0.5×10^4
Sun and Nesbitt [1977]†	237	0.67	22	7.6	0.406	1.26	0.026	+0.06	≡0	17.5	846	353	0.9×10^4
O'Nions et al. [1978]	462	1.31	41	...	0.30	0.95	0.046	+0.12	≡0	43.2	891	350	10^4
This work	≥260	0.63	22	7.6	0.406	1.26	0.026	≡0	≡0	17.5	846	≥413	$≥10^4$
Chondrites‡	805	2.3	11	3.8	0.203	0.63	0.013	+6.2	...	17.5	846	350	6.2×10^4

We assume that these elements are totally excluded from the core, and all bulk earth concentrations from the literature have therefore been recalculated to the average value for the crust-mantle system (total earth mass = 1.48 × (crust + mantle)).

*Refractory elements = 2.2 × ordinary chondrites, (K/U) = 10^4 and Rb/U = 29.2.

†Refractory elements = 2.0 × chondrites.

‡Hubbard and Gast [1971], Nakamura [1974], and Mason [1971].

average growth rate of the crust [$(1/T_0) = 0.22$ aeon^{-1}]. However, if the mean age measured 0.5 aeon ago was 1.5 aeons instead of 1.3 or 1.4 aeons, then $(dM/d\tau)$ for the last 0.5 aeon would be equal to the average growth rate of the crust. Thus the value for $(dM/d\tau)$ during the last 0.5 aeon is very sensitive to small differences in the mean ages. No firm conclusion can be drawn, but the data suggest that the average growth rate of the continents and the rate of differentiation of the mantle for the last 0.5 aeon is less than half of the average value over the history of the earth.

We now turn to the problem of evaluating the concentrations of Rb, Sr, Nd, and Sm and the masses of the various reservoirs. The concentrations used for the undepleted mantle and the crust are those from Tables 5 and 6, respectively, except for the Rb concentration in the crust, which will be calculated in the following. These parameters are given in Table 7. From conservation of mass and species we have as before that $C_{i,1} = C_{i,2}(1 - x) + C_{i,3}x$, where $x = M_2/(M_2 + M_3)$ for both stable and radioactive isotopes. This together with the definition of $f_i^{r/s}$ (equation (17)) leads to the following equation that relates the masses of reservoirs 2 and 3 to the f values of 2 and 3 and the concentrations of the stable isotope in 1 and 3:

$$\frac{M_3(\tau)}{M_2(\tau) + M_3(\tau)} = \frac{f_2^{r/s}(\tau)C_{s,1}(\tau)}{[f_2^{r/s}(\tau) - f_3^{r/s}(\tau)]C_{s,3}(\tau)} \qquad (78)$$

This equation holds for any model as long as reservoirs 2 and 3 are derived from the reference reservoir 1. For model I the left-hand side of (78) must be equal to the degree of melting $F = M_3(\tau)/[M_2(\tau) + M_3(\tau)]$. The degree of melting must be the same for all species, so (78) gives us a relation between the f values for the Rb/Sr and the Sm/Nd system. From the values of $f_2^{Sm/Nd}$, $f_3^{Sm/Nd}$, $C_{Nd,1}$, and $C_{Nd,3}$ in Table 7, it follows that $F = 0.0174$. Using this value and the mass of the total continental crust given in Table 2, we get that the mass of the depleted mantle today is 127×10^{25} grams. This corresponds to approximately the upper 650 km of the mantle or ~1/3 of the mass of the mantle. As was noted previously, the values of $f_3^{Rb/Sr}$ and $C_{Rb,3}$ are not well known. We therefore use (78), the values of $f_2^{Rb/Sr}$, $C_{Sr,1}$, and $C_{Sr,2}$ from Table 7, and $F = 0.0174$ as estimated above to calculate $f_3^{Rb/Sr} = 2.18$ and $C_{Rb,3} = 33.7$. This is identical to estimates made previously of these parameters, and it gives $\epsilon_{Sr,3}(0) = +65.5$ if $\bar{\imath} = 1.8$ aeons. From this information we can calculate the concentrations of all the elements in reservoir 2 and the bulk distribution coefficients using (11) and (12). The complete set of self-consistent parameters for Rb, Sr, Sm, and Nd are given in Table 7 for model I. The calculated bulk distribution coefficients agree to better than a factor of 2 with garnet peridotite bulk distribution coefficients estimated from the mineral melt distribution coefficients from Table 4 ($D_{Rb} \approx 0.0025$, $D_{Sr} \approx 0.033$, $D_{Nd} \approx 0.044$, and $D_{Sm} \approx 0.080$ for small degrees of melting of garnet peridotite).

This approach can also be extended to include other elements by using (11) and (12). We briefly investigate the consequences of this model for K, U, and Ba, since these are strongly incompatible elements with $D_i \approx 0$. If $D_i = 0$, then $C_{i,3}/C_{i,1} = 57$ for $F = 0.0174$, which is the maximum enrichment we may have for this model. Bulk distribution coefficients estimated in Table 4 for these elements are $D_K = 0.002$, $D_{Ba} = 0.0014$, and $D_U = 0.001$, and these values will be used for the calculations. The concentrations for reservoirs 1 and 3 are obtained from Tables 5 and 6. The concentration of K in the total crust (1.5 wt.%) is used with the above mentioned pa-

TABLE 6. Estimates of Lithophile Element Concentrations in the Continental Crust

	K	Rb	Sr	Ba	Sm	Nd	U	$f^{Rb/Sr}$	$f^{Sm/Nd}$	Sr/Nd	Sr/U	K/Rb	K/U
Upper crust, *Shaw et al.* [1976]	2.57	110	316	1070	4.5	26	2.45	+11	-0.46	12.2	129	234	10^4
Lower crust, *Heier and Thoresen* [1971]	2.49	56	572	1162	(4.8)*	(28)*	0.5	+2.4	(-0.47)*	20.4	1144	629	5×10^4
Total crust, *Lambert and Heier* [1968]	1.50	...	...	...	...	...	0.7	...	...	...	...	...	2×10^4
This work	1.50	33	370	500	5	26	0.7	+2.1	-0.40	14.2	528	450	2×10^4

*Estimated from REE patterns of *Green et al.* [1972] on some of the samples used by *Heier and Thoresen* [1971].

rameters to get $C_{K,1}$ = 290 ppm and $C_{K,2}$ = 30 ppm. It follows then that the bulk earth K/Rb ratio is 460 and that this ratio is 680 in the depleted mantle. This ratio for the depleted mantle still seems somewhat low, as K/Rb ~ 1000 for MORB, but this difference may be accounted for by only increasing D_K from 0.002 to 0.003. For Ba and U the concentrations in the undepleted mantle are considered to be better established than for K (Table 5). Using (11), we get $C_{Ba,3}$ = 405 ppm and $C_{U,3}$ = 1.4 ppm, which compare reasonably well with the estimates given for the average total crust in Table 6. The calculated concentrations in the depleted reservoir are $C_{Ba,2}$ = 0.57 ppm and $C_{U,2}$ = 1.4 ppb. Existing data for K, Rb, and Ba are thus reasonably consistent with model I.

We will now consider the possibility that undifferentiated material is being mixed with the depleted mantle by the mechanism described by (29)–(33). In this case, $\beta F = M_3(\tau)/(M_2(\tau) + M_3(\tau))$. From the value of $f_2^{Rb/Sr}$ = -0.9 from Table 7 we get from (35) that β > 0.9. This means that the amount of undifferentiated material that may be mixed with the depleted mantle is less than 10%. Considering only the surface area covered by continents and oceans today, we would expect that β = 0.42 using the surface areas given by *Ronov and Yaroshevsky* [1976]. This suggests that if model I is valid, there must be some mechanism for efficient addition to continents of melt fractions from undifferentiated material that differentiates beneath oceans. As was discussed previously, we may also consider the case $\beta < 1$ to be analogous to refluxing of crust to depleted mantle in a generalized version of model I. This shows that only 10% of the total amount of material added to the crust may have been subducted back to the depleted mantle.

RESULTS FOR MODEL II

We now evaluate the present day parameters for this model. As for model I, we again start out with the most reliable values given in Table 3. Substituting $\epsilon_{Sr,2}$ = -27 and $f_2^{Rb/Sr}$ = -0.9 into (63) gives $\bar{i}_{Rb/Sr}$ = 1.8 aeons. It follows for this model that the mean mass age of the continents is substantially less than $(T_0/2)$ = 2.3 aeons. This clearly implies that the rate of growth of the continental crust yields a mass growth curve which accelerates with time; i.e., the crustal growth rate for the past 2 aeons being much higher than for the period from 4.5 to 3.6 aeons. This result can be modified only if the bulk earth parameters for Rb-Sr given in the Appendix were substantially altered. To obtain self-consistent values for the Sm-Nd system, we note that the possible fractional range of $f^{Sm/Nd}$ values for the depleted mantle is substantially larger than the corresponding fractional range of $f^{Rb/Sr}$ and the range in possible crustal ϵ_{Nd} values is also large, thus placing only rather broad bounds on the value of $\bar{i}_{Sm/Nd}$. Stricter bounds of $f^{Sm/Nd}$ are placed by using $\bar{i}_{Rb/Sr}$, since $\bar{i}_{M,3} \approx \bar{i}_{Sm/Nd} \leq \bar{i}_{Rb/Sr}$ = 1.8 aeons. So from (63) we get $f_2^{Sm/Nd} \geq$ 0.225, and the direct observational upper bound from the Appendix is $f_2^{Sm/Nd} \leq$ 0.3. Note that for model II, knowledge of the present day f_2 values explicitly determines the quantity $a_{r/s}(T_0)$ = $[(d_r - d_s)M_3(T_0)/M_2(0)]$ using (50). Using $f_2^{Rb/Sr}(T_0)$ = -0.9 for the present value, we obtain $a_{Rb/Sr}(T_0)$ = +2.30. This shows that $f_2^{Rb/Sr}$ = exp $[-a_{Rb/Sr}(\tau)] - 1$ is in the strongly nonlinear range. In contrast, $0.225 \leq f_2^{Sm/Nd} \leq 0.3$, which implies that $-0.264 \leq a_{Sm/Nd}(T_0) \leq -0.203$, so that $f_2^{Sm/Nd}$ is determined to within 10% by the first-order expansion over the history of the earth. This contrast between Rb-Sr and Sm-Nd is different from that of model I and reflects the fundamental

TABLE 7. A Self-Consistent Set of Parameters for Model I

Reservoir j	$C_{Rb,j}$	$C_{Sr,j}$	$f_j^{Rb/Sr}$	$\epsilon_{Sr,j}(0)$	$C_{Sm,j}$	$C_{Nd,j}$	$f_j^{Sm/Nd}$	$\epsilon_{Nd,j}(0)$	$M_j^{(today)}$, g
Continental crust $j = 3$	33.7	370	2.18	65.5	5.0	26	−0.4	−17.8	2.256×10^{25}
Depleted mantle $j = 2$	0.044	15.8	−0.9	−27	0.32	0.82	0.225	10	127.4×10^{25}
Undepleted mantle $j = 1$	0.63	22	≡0	≡0	0.406	1.26	≡0	≡0	273.7×10^{25}

$\bar{T} = 1.8$ aeons; $F = 0.0174$; $\beta = 1$; $D_{Rb} = 0.0013$; $D_{Sr} = 0.043$; $D_{Nd} = 0.032$; $D_{Sm} = 0.065$.
Concentrations are given in parts per million.

differences between the two models. The values of $\alpha_{Rb/Sr}$ and $\alpha_{Sm/Nd}$ may be used to find $M_2(0)$ and $(d_r - d_s)$ using the value of $M_3(T_0)$ from Table 2. If we assume that the whole mantle is involved in crust formation, then $[M_3(T_0)/M_2(0)] = 0.00448$, which implies that $d_{Rb} - d_{Sr} = 513$ and $d_{Sm} - d_{Nd} = -46$. We now assume that reasonable values for the degree of melting F would be in the range from 0.5 to 5% by mass. Using the bulk distribution coefficients from Table 3, we find a range of $(d_{Rb} - d_{Sr})$ from 7 to 108 for Rb-Sr (equation (50)). This implies that $[M_3(T_0)/M_2(0)]$ should be in the range from 0.021 to 0.33 using $\alpha_{Rb/Sr} = 2.3$. Similarly, we get that $(d_{Sm} - d_{Nd})$ must be in the range from −8.7 to −3, and using the range of $\alpha_{Sm/Nd}$ given previously implies that $0.023 \le [M_3(T_0)/M_2(0)] \le 0.081$. We may also estimate $M_3(T_0)/M_2(0)$ from (78) using $C_{Nd,1} = 1.26$ ppm and $C_{Nd,3} = 26$ ppm. Since $f_3^{Sm/Nd} = -0.4$ and $0.225 \le f_2^{Sm/Nd} \le 0.3$ as discussed previously, it follows from (78) that $0.0174 \le [M_3(T_0)/M_2(0)] \le 0.0208$. From these estimates the most reasonable choice seems to be $M_3(T_0)/M_2(0) = 0.020 \pm 0.003$, and this gives $M_2(T_0) = 110.5 \times 10^{25}$ grams, which is 28% of the mantle. Using this value for $M_3(T_0)/M_2(0)$ in (78) gives $f_2^{Sm/Nd} = +0.281$ and thus $\alpha_{Sm/Nd} = -0.248$. From (63) it now follows that $\bar{t}_{Sm/Nd} = 1.44$ aeons, which together with $f_3^{Sm/Nd} = -0.4$ gives $\epsilon_{Nd,3} = 14.3$. Similarly, using (78) for Rb-Sr with $f_2^{Rb/Sr} = -0.9$, $[M_3(T_0)/M_2(0)] = 0.02$, $C_{Sr,1} = 22$ ppm, and $C_{Sr,3} = 370$ ppm, we get $f_3^{Rb/Sr} = +1.78$, $C_{Rb,3} = 29.4$ ppm, and since $\bar{t}_{Rb/Sr} = 1.8$ aeons it also follows that $\epsilon_{Sr,3} = +53.4$.

We have now determined a self-consistent set of f and ϵ values for model II, which are given in Table 8. From the values $f_2^{r/s}$ and $f_3^{r/s}$ for Sm/Nd and Rb/Sr together with $[M_3(T_0)/M_2(0)] = 0.020$ we can, by using (50) and (51), determine all the d values which are given in Table 8. Since $D_{Rb} > 0$, it follows from the value we calculated for $d_{Rb} = 135.5$ that $F < 0.0076$. Choosing $D_{Rb} = 0.0025$ from Table 4 gives $F = 0.005$. Using this value of F, we may calculate D_{Sr}, D_{Nd}, and D_{Sm} (Table 8). From the d values listed in Table 8 and the concentrations of the undepleted reservoir we may now calculate the concentrations of these elements in the depleted reservoir. A complete set of self-consistent parameters for Rb, Sr, Sm, and Nd are given in Table 8 for model II. The parameters for Rb imply that the concentration of Rb in new additions to the crust has changed from 86 ppm initially to 6 ppm today giving an average concentration today of ~30 ppm. This large change of Rb concentration in the crust over geologic time does not appear supportable from the existing data. The magnitude of this time dependence is not changed by any choice of parameters so long as $f_2^{Rb/Sr}$ is close to its lower limit of −1.

We now turn our attention to see what the data can tell us about the time evolution of the continental crust. The simplest growth model would be $(dM_3/d\tau) = $ const over the history of the earth. Using this assumption, we get $\bar{t}_{Sm/Nd} - \bar{t}_{M,3} = -0.09$ aeon from (66) with $\tau = 4.5$ aeons and $\alpha_{Sm/Nd} = -0.248$, so since $\bar{t}_{Sm/Nd} = 1.44$ aeons, it follows that $\bar{t}_{M,3} \approx 1.5$ aeons. However, since $\bar{t}_{M,3} \le \bar{t}_{Rb/Sr} = 1.8$ aeons $< (T_0/2)$, this would require that the crust started to form late. The longest time

compatible with a uniform growth may be determined from (71) and (72). For Rb-Sr we get 2.7 aeons and for Sm-Nd we get 3.0 aeons for the longest time compatible with a constant growth rate of the crust using the f and ϵ values for reservoir 2 given in Table 3. These are both unrealistically low values, so we must conclude that a uniform growth rate can hardly explain the data. It is, however, clear that this model also requires the dominant parts of the continents to have formed late in the earth's history.

From the Bay of Islands data [*Jacobsen and Wasserburg*, 1979], similar estimates can be made for the depleted mantle 0.5 aeon ago using the values given in Table 3. Our estimates of the enrichment factors for the Bay of Islands source are $f_2^{Rb/Sr} = -0.9$ and $f_2^{Sm/Nd} \approx +0.2$ (Table 3). For Rb-Sr using $\epsilon_{Sr,2} = -19.3$, we obtain $\bar{t}_{Rb/Sr} = 1.3$ aeons at 0.5 aeon ago, which indicates that $(dM_3/d\tau) \approx 0$ for the last 500 m.y. as in model I. For Sm-Nd using $\epsilon_{Nd,2} = +7.6$, we obtain by using the estimate of the present day $f_2^{Sm/Nd}$ of +0.28 (Table 8) that $\bar{t}_{Sm/Nd} \approx 1.1$ aeons. From (66) we get $\bar{t}_{Sm/Nd} - \bar{t}_{M,3} = -0.08$ at 0.5 aeon ago, so it follows that $\bar{t}_{M,3} \approx 1.2$ aeons at that time for this model. This is only slightly lower than the value obtained for model I.

DISCUSSION OF THE MODELS AND CONCLUSIONS

The basic transport equations for the earth differentiation models shown in Figure 1 give simple results when solved for arbitrary mass growth curves $M_j(\tau)$ for the continental crust $(j = 3)$ and the depleted mantle $(j = 2)$. The solution for model I is

$$\epsilon_{d,j}^*(\tau) = \frac{Q_d^*(\tau) f_j^{r/s}}{M_j(\tau)} \int_0^\tau M_j(\xi) \exp[\lambda(\tau - \xi)] \, d\xi$$

where

$$Q_d^*(\tau) = 10^4 \lambda \left[\frac{N_{r,1}(\tau)}{N_{d,1}(\tau)} \right]$$

and $f_j^{r/s}$ is the enrichment factor, which is constant for all time. The mean age $\bar{t}_{s,j}$ of stable isotopes is equal to the mean age of the mass of j, $\bar{t}_{M,j} = \int_0^\tau [M_j(\xi)/M_j(\tau)] \, d\xi$. For long-lived isotopes $(\lambda^{-1} \gg 4.5 \text{ aeons})$, $\epsilon_{d,j}^* = Q_d^* f_j^{r/s} \bar{t}_{M,j}$, and $Q_d^* \simeq$ const. Knowledge of $\epsilon_{d,j}^*$ and $f_j^{r/s}$ at a single time fixes $\bar{t}_{M,j}$. The mean age shows the relationship $(d\bar{t}/d\tau) = 1 - \bar{t}(d \ln M_j/d\tau)$, so if $M_j(\tau)$ and $f_j^{r/s}$ are known, then $\bar{t}_{M,j}$ and $\epsilon_{d,j}^*(\tau)$ are known.

For model II which has continuous fractionation in the source reservoir the concentrations are functions of time, and we get

$$\epsilon_{d,2}^*(\tau) = Q_d^*(\tau) \int_0^\tau f_2^{r/s}(\xi) \exp[\lambda(\tau - \xi)] \, d\xi$$

$$\epsilon_{d,3}^*(\tau) = \epsilon_{d,2}^*(\tau) \frac{f_3^{r/s}(\tau)}{f_2^{r/s}(\tau)}$$

TABLE 8. A Self-Consistent Set of Parameters for Model II at $\tau = T_0$ (Today)

Reservoir j	$C_{Rb,j}$	$C_{Sr,j}$	$f_j^{Rb/Sr}$	$\epsilon_{Sr,j}*$	$C_{Sm,j}$	$C_{Nd,j}$	$f_j^{Sm/Nd}$	$\epsilon_{Nd,j}*$	M_j, g
Continental crust $j = 3$	29.4	370	1.78	53.4	5.0	26	−0.4	−14.3	2.256×10^{25}
Depleted mantle $j = 2$	0.041	14.5	−0.9	−27	0.29	0.74	0.281	10	110.5×10^{25}
Undepleted mantle $j = 1$	0.63	22	≈0	≈0	0.406	1.26	≈0	≈0	290.6×10^{25}

$T_{Rb/Sr} = 1.8$ aeons; $T_{Sm/Nd} = 1.44$ aeons; $T_{M,3} = 1.5$ aeons; $M_3(T_0)/M_2(0) = 0.020$; $d_{Rb} = 135.5$; $d_{Sr} = 20.5$; $d_{Nd} = 26.6$; $s_{Sm} = 16.2$; $\alpha_{Rb/Sr}(T_0) = +2.30$; $\alpha_{Sm/Nd}(T_0) = -0.248$; $F = 0.005$; $D_{Rb} = 0.0025$; $D_{Sr} = 0.044$; $D_{Nd} = 0.033$; $D_{Sm} = 0.057$.
Concentrations are given in parts per million.

$$f_2^{r/s}(\tau) = \left[\frac{M_2(\tau)}{M_2(0)}\right]^{(d_r - d_s)}$$

$$\frac{f_2^{r/s}(\tau)}{f_3^{r/s}(\tau)} = 1 - \left[\frac{M_2(\tau)}{M_2(0)}\right]^{(-d_s)}$$

Define $\bar{i}_{r/s} = \int_0^\tau [f_2^{r/s}(\xi)/f_2^{r/s}(\tau)]\, d\xi$. Then for long-lived isotopes we have $\epsilon_{d,j}*(\tau) = Q_d* f_j^{r/s}(\tau)\bar{i}_{r/s}$, where $\bar{i}_{r/s}$ is the time required to generate $\epsilon_{d,j}*$ with the present value of $f_j^{r/s}$. Note that $\bar{i}_{r/s}$ depends on the parent-daughter system and is in general different from $\bar{i}_{M,3}$. The difference is given by

$$\bar{i}_{r/s} - \bar{i}_{M,3} \approx \frac{(d_r - d_s)}{2M_3(\tau)M_2(0)} \int_0^\tau M_3(\xi)[M_3(\tau) - M_3(\xi)]\, d\xi$$

for small $[(d_r - d_s)/M_2(0)]$. Decay systems with small $(d_r - d_s)$ may be used to estimate $\bar{i}_{M,3}$. These simple expressions may be used to calculate earth models with great facility without requiring a computer calculation.

We first compare the results from models I and II (Tables 7 and 8). The mean age of the crust is 1.8 aeons for model I and 1.5 aeons for model II, so a young mean age of the crust is confirmed by both models. Similarly, both models indicate that the rate of growth over the last 0.5 aeon is much less than the average rate of growth. Therefore most of the crust probably formed in the time interval from ~0.5 to ~3 aeons ago. Both models also require the depleted mantle complementary to the continental crust to make up about 1/3 of the mantle, which corresponds approximately to the upper 650 km of the mantle. This does not seem to be compatible with whole mantle convection as discussed by *Hager and O'Connell* [1979]. In future studies it is of importance to arrive at mantle structure models which satisfy both the boundary conditions on mantle convection and isotope geochemistry. The present day ϵ and f values and the concentration in each reservoir are fairly similar for both models as shown in Tables 7 and 8. Insofar as the mantle consists of an undepleted reservoir and a depleted reservoir that is complementary to the crust, the above conclusions are basically model independent. Model I indicates a larger degree of melting (1.7%) than model II (0.5%). At the present time both of these values seem to be plausible estimates of the average degree of melting through time. The major difference in the results for the two models is in the composition of new additions to continental crust. Model I predicts that $\epsilon_{Nd} = \epsilon_{Sr} = 0$ for new additions to the continental crust, and model II predicts that the ϵ_{Sr} and ϵ_{Nd} of new additions are those of the source of MORB. The fact that island arcs have bulk composition and REE abundances similar to average continental crust and that they usually are found at or near continental margins has led to the hypothesis that the continents grow by accreting island arcs along their margins [*Taylor and White*, 1965; *Ringwood*, 1975]. If this is the case, then the Nd isotopic studies reported by *DePaolo and Wasserburg* [1977] and *Hawkesworth et al.* [1977] show that new continental material should have ϵ_{Nd} values similar

to MORB and are thus derived from depleted mantle. This mechanism for continental growth is thus described by model II. However, this model cannot explain why typically most continental crustal materials have $\epsilon_{Nd} \approx 0$ [*DePaolo and Wasserburg*, 1976a, b; *McCulloch and Wasserburg*, 1978; *Hamilton et al.*, 1977, 1978, 1979] throughout the earth's history. While values of $\epsilon \approx 0$ may be obtained at early times for both models, it is not possible to continue production of such materials in more recent geologic time for model II.

In model I the concentrations of stable isotopes in new additions to the crust are the same for all time, however, in model II the concentrations may decrease by over a factor of 10 for highly incompatible elements as K, Rb, Ba, and U. There is no evidence for such drastic changes with time. We therefore conclude that model I best describes crust and mantle evolution. Some additions to the continents during the last 1.5 aeons have $\epsilon_{Nd} > 0$ [*DePaolo and Wasserburg*, 1976a, b], so some new additions to the crust may follow model II. A modified version of model I where it is allowed for mixing of undepleted mantle with depleted mantle shows that this contribution must be less than 10%. This model also shows that less than 10% of the total continental crust has been refluxed back to the depleted mantle.

Recent isotopic studies of young volcanic rocks have shown that ϵ_{Nd} and ϵ_{Sr} are strongly correlated in ocean floor and ocean island basalts and selected continental basalts [*DePaolo and Wasserburg*, 1976b; *O'Nions et al.*, 1977]. This correlation line is assumed to be due to mantle characteristics and has been called the 'mantle array' by *DePaolo and Wasserburg* [1979a]. Magmas in model I may be mixtures derived from depleted and undepleted mantle and may thus explain the mantle array. Model II does not explain the mantle array unless a range of source reservoirs individually following this model are involved. This would, however, require that some of them be close to undepleted mantle material, but it is not clear that this would satisfy a strict correlation between initial Nd and initial Sr.

Considering the young mean ages derived for the continents, we are left with very little evidence of crust forming very early in the earth's history. Search for old crustal rocks by direct dating has established a low abundance of ~3.6-aeon-old crustal rocks and the apparent absence of much older rocks at the surface of the earth. An alternative way of looking for old crust is to look at the residue in the mantle after crust formation. If the pre-3.6-aeon-old crust were buried in the lower crust, then we would still expect to see it in the isotopic systematics of the depleted mantle. The depleted mantle gives a young mean age of ~1.8 aeons, and this implies either that very little early crust ever formed or that the early crust was destroyed by rapid refluxing into the mantle in the period from 4.5 to 3.6 aeons ago. If the latter is the case, then this would have caused early degassing of the earth's Ar as suggested by $^{40}Ar/^{36}Ar$ ratios in the mantle [*Ozima*, 1975]. The detailed history of this time period must be derived from

data for ^{40}K and ^{235}U and perhaps also ^{244}Pu and the extinct ^{129}I.

APPENDIX: PRESENT DAY BULK EARTH VALUES

The following values are from DePaolo and Wasserburg [1976a, b] and O'Nions et al. [1977]:

$$\frac{^{87}Sr(T_0)_1}{^{86}Sr(T_0)_1} = 0.7045 \qquad \frac{^{87}Rb(T_0)_1}{^{86}Sr(T_0)_1} = 0.0827$$

$$\frac{^{143}Nd(T_0)_1}{^{144}Nd(T_0)_1} = 0.511836 \qquad \frac{^{147}Sm(T_0)_1}{^{144}Nd(T_0)_1} = 0.1936$$

$$Q_{Nd}{}^*(T_0) = 24.74 \text{ aeons}^{-1} \qquad Q_{Sr}{}^*(T_0) = 16.70 \text{ aeons}^{-1}$$

The decay constants used are $\lambda(^{87}Rb) = 0.0142$ aeons^{-1} and $\lambda(^{147}Sm) = 0.00654$ aeons^{-1}. $T_0 = 4.55$ aeons.

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