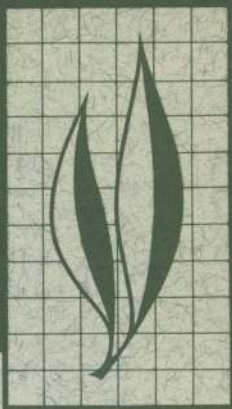


# HILGARDIA

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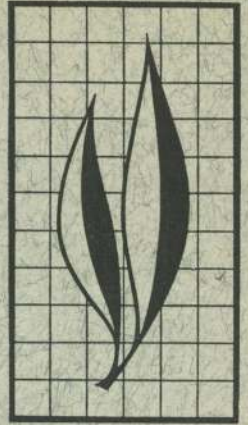


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## The Measurement and Description of Water Flow Through Columbia Silt Loam and Hesperia Sandy Loam

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Experimental data are presented for water entering air-dry soils in the horizontal and vertical position. All soils were wet with water below atmospheric pressure. The ability of a frequently used mathematical equation to describe the soil water movement is examined for several boundary conditions. The mathematical assumptions and their physical reality are discussed. A method of measuring the unsaturated capillary conductivity is presented.

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# The Measurement and Description of Water Flow Through Columbia Silt Loam and Hesperia Sandy Loam<sup>1</sup>

## INTRODUCTION

THE UNDERSTANDING and description of fluid moving within porous media is of vital importance to science and to mankind in general. In agriculture, it is necessary to know changes in soil water content under the influence of rainfall or irrigation, evapotranspiration, and drainage. To predict water content changes, a mathematical description of the physical processes involved should be obtained. This paper presents experimental data for water moving through soils and examines how well an existing mathematical equation describes the water movement.

The mathematical equations used herein are commonly employed in present day research in soil-water movement, but because the derivation of these equations is generally understood by many investigators it is usually not necessary to redevelop them. However, it is convenient to have all derivations closely at hand, as the primary purpose of this study is to scrutinize and compare the theoretical and experimental behavior of a soil-water system. With the exception of two nonagricultural porous materials [Youngs, 1957],<sup>2</sup> no work has been reported involving the description of water moving vertically

downward through soils under constant laboratory controls.

The simplest type of fluid flow exists when porous media are saturated or all pores are filled with the same fluid. For saturated sand, Darcy (1856) observed a linear relation between the volume flux of water and the gradient of the hydraulic head. A generalization made from that observation is Darcy's Law:

$$\bar{v} = -K\bar{\nabla}\Phi \quad [1]$$

where  $\Phi$  is the hydraulic head ( $L$ ),  $\bar{v}$  is the volume flux of water ( $LT^{-1}$ ) and  $K$  the proportionality constant ( $LT^{-1}$ ) commonly called hydraulic conductivity. This relation has received general acceptance for hydraulic gradients when laminar flow exists under steady-state conditions.

The more complex but most common type of fluid flow in agriculture is that which takes place through soil partially filled with water. Soil pores contain not only water but also air and other gases and vapors, and water movement is complicated by their presence. The air phase may be at pressures above or below atmospheric pressure, regardless of its continuity of distribution within the

<sup>1</sup> Submitted for publication November 12, 1962.

<sup>2</sup> See "Literature Cited" for citations referred to in text by author and date.

soil. The pressure of the soil water is related to the surface tension and the radii of curvature present at the air-water interfaces within the partially filled pores of the soil. This relation is not analytic owing, among other things, to the presence of dissolved constituents in the soil water. Nevertheless, progress has been made by assuming that a definite relation exists between the soil water content and the soil water pressure. By definition, soil water pressure is equal to that gauge pressure to which water must be subjected in order to be in hydraulic equilibrium, through a porous permeable wall, with the water in the soil.

Childs and Collis-George (1950) performed an experiment to test the validity of Darcy's Law for unsaturated flow. By measuring the flux of water passing through partially saturated columns oriented to several positions between the vertical and horizontal, they concluded equation [1] was valid for steady-state conditions. For unsaturated flow, the hydraulic conductivity is commonly called the capillary conductivity (Richards, 1952).

Richards (1931) used equation [1] in the equation of continuity:

$$\frac{\partial(\rho\theta)}{\partial t} = -\bar{\nabla} \cdot \rho\bar{v} \quad [2]$$

where  $\theta$  is the water content ( $L^3 L^{-3}$ ), and  $\rho$  the fluid density ( $ML^{-3}$ ) and  $t$  the time. It was assumed that changes in water content and pressure would take place slowly enough that a steady state relation used in equation [2] could describe the soil water system. The use of equation [1] in [2] yields:

$$\frac{\partial(\rho\theta)}{\partial t} = \bar{\nabla} \cdot (K\rho\bar{\nabla}\Phi) \quad [3]$$

For soils, the hydraulic head  $\Phi$  is generally considered the sum of two terms,  $\psi + x$ , where  $\psi$  is the soil water pressure head and  $x$  the gravitational head. It has been mathematically con-

venient to use a constant fluid density  $\rho$ , and to consider that a single-valued relation exists between water content and soil water pressure. This consideration allows the water content  $\theta$  to be the dependent variable of equation [3]. Recognizing hysteresis is most evident in the water content-pressure relation between wetting and drying processes, Childs and Collis-George (1948) introduced the following mathematics for a wetting process or a drying process, but not for both processes together:

$$K(\theta) \frac{\partial\psi(\theta)}{\partial x} = K(\theta) \frac{d\psi(\theta)}{d\theta} \frac{\partial\theta}{\partial x} = D(\theta) \frac{\partial\theta}{\partial x} \quad [4]$$

where  $D(\theta)$  is called the soil water diffusivity. The capillary conductivity  $K(\theta)$  is assumed to be a single-valued function of  $\theta$ . Some evidence to support the use of this assumption has been found by Nielsen and Biggar (1961). Substituting the soil water diffusivity in equation [3] for flow in the downward  $x$ -direction we have:

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x} \left[ D(\theta) \frac{\partial\theta}{\partial x} \right] - \frac{\partial K}{\partial x} \quad [5]$$

For horizontal movement the last term on the right-hand side of equation [5] is omitted, as it represents the external body force gravity. Without gravity, equation [5] takes the identical form of the well-known diffusion equation, where the diffusivity  $D(\theta)$  is concentration dependent. This does not, however, imply that the mechanism of fluid movement is diffusion in the same sense as diffusion in gases, liquids, or solids due to random molecular motion. The diffusion equation is commonly used to describe soil water problems because of the ease of measuring water contents, and its solutions are analogous to ordinary diffusion or heat flow equations.

This publication gives data for water at below atmospheric pressure entering air-dry soils. Measured soil water pro-

files for vertical and horizontal columns are compared with those calculated from the solution of equation [5]. A

method of measuring capillary conductivity for different water contents is also presented.

## THEORETICAL PROCEDURE

### Horizontal and Vertical Soil Water Profiles

Philip (1955, 1957a) has presented a numerical solution of equation [5] for infiltration into a semi-infinite homogeneous soil column either in the vertical or horizontal position. The initial soil water content is assumed constant with depth, and during the wetting process it is assumed that a greater constant water content exists at the soil surface. Thus, we have

$$\begin{aligned} \theta &= \theta_n, & t &= 0, & x &> 0 \\ \theta &= \theta_0, & t &> 0, & x &= 0 \end{aligned} \tag{6}$$

where  $\theta_0 > \theta_n$ . For these boundary conditions, the solution of equation [5] for the vertical case is

$$\begin{aligned} x &= \lambda(\theta)t^{1/2} + \chi(\theta)t + \psi(\theta)t^{3/2} \\ &+ \omega(\theta)t^2 + \dots \end{aligned} \tag{7}$$

where  $\lambda, \chi, \psi, \omega$ , etc. are single-valued functions of  $\theta$  to be determined by the numerical analysis of Philip.

For the horizontal case, the first term of the right-hand side of equation [7] is the only one used, leaving the solution

$$x = \lambda(\theta)t^{1/2} . \tag{8}$$

Equation [5] with its solution [7] or [8] may describe soil water movement provided certain assumptions are fulfilled, in addition to those made previously: (1) There must be no rearrangement of soil particles upon wetting. (2) The air movement does not influence the water movement. This condition requires water movement to be analogous to heat flow where consideration is given to only a single phase. (3) The proper-

ties of the water are uniform regardless of the position occupied by water. (4) An isothermal condition exists.

If all the assumptions and boundary conditions are fulfilled, a  $\lambda$  single-valued in  $\theta$  satisfying equation [8] can be found experimentally. If a constant water content may be visually observed at the wetting front of a horizontal column, the distance to the wetting front divided by the square root of time should yield the constant value  $\lambda$ . Still, a better means of ascertaining the existence of a unique  $\lambda$  versus  $\theta$  relation is to measure the water content distribution in a horizontal column at different times. If  $\lambda(\theta)$  exists, plots of  $x/t^{1/2}$  versus  $\theta$  will be identical for all times.

Equation [7] provides a theoretical formula for obtaining a curve of  $x$  versus  $\theta$  for comparing calculated and experimental values of water content in vertical columns. Another expression is needed for the net amount of water that infiltrates into the soil surface. Upon integrating with respect to  $\theta$  and differentiating with respect to  $t$ , equation [7] becomes

$$\begin{aligned} v_0 &= 1/2t^{-1/2} \int_{\lambda} + \int_x + K_n \\ &+ 3/2t^{1/2} \int_{\psi} + \dots \end{aligned} \tag{9}$$

where

$$\begin{aligned} \int_{\lambda} &= \int_{\theta_n}^{\theta_0} \lambda d\theta, \\ \int_x &= \int_{\theta_n}^{\theta_0} \chi d\theta, \\ \int_{\psi} &= \int_{\theta_n}^{\theta_0} \psi d\theta, \text{ etc.} \end{aligned} \tag{10}$$

The term  $K_n$  is the capillary conductivity for the water content  $\theta_n$ . For boundary condition [6] to be maintained ( $\theta = \theta_n$  to great depth), a flux equal to  $K_n$  must be supplied at the soil surface in addition to that derived from equation [7].

Knowing the infiltration velocity  $v_0$ , the volume of water per unit area which has infiltrated into the profile at time  $t$  is

$$i = \int_0^t v_0 dt \quad [11]$$

or, in view of equation [9], it is

$$i = t^{1/2} \int_{\lambda} + t \left[ \int_x + K_n \right] + t^{3/2} \int_{\psi} + \dots \quad [12]$$

The right-hand side of equation [12] gives the water stored in the profile plus that which has leaked out the bottom of the profile at great depth.

Equations [7], [9], [11] and [12] are asymptotic infinite series, that is, they fail to converge for large values of  $t$ . For these values a decreasing exponential curve is matched to that obtained by equation [9]. The exponential curve assumed for times greater than  $t_1$  minutes is

$$v_0 = K_0 + (V_1 - K_0) \exp[-\alpha(t - t_1)] \quad [13]$$

where  $\alpha$  is a constant to be obtained and  $V_1$  is the value of  $v_0$  at  $t = t_1$  minutes. To obtain  $\alpha$  the derivative of equation [13] at  $t = t_1$  minutes yields the value of the slope  $-\alpha(V_1 - K_0)$  which is equal to the derivative with respect to  $t$  of equation [11]. The value of the derivative of [11] is known and hence,  $\alpha$  may be calculated. Thus, equations [9] and [13] supply a single theoretical curve of  $v_0$  versus  $t$  for all times. Integration of  $v_0$  with respect to  $t$  yields the cumula-

tive infiltration. For  $t < t_1$ , equation [12] applies. For  $t > t_1$ , integration of equation [13] yields the following cumulative infiltration expression:

$$\int_{t_1}^t v_0 dt = K_0(t - t_1) + \frac{1}{\alpha} (V_1 - K_0) \{1 - \exp[-\alpha(t - t_1)]\} \quad [14]$$

A theoretical expression for  $x$  versus  $\theta$  for times greater than  $t_1$  minutes corresponding to equation [7] is obtained by assuming that the shape of the profile for these times is the same as that when time is infinite, Philip (1957b). If the profile between  $\theta_0$  and  $\theta_1$  is assumed to be linear,  $x_1$  (associated with  $\theta_1$ ) may be computed from

$$i = K_n t + \int_{\theta_n}^{\theta_1} x'_{\infty} d\theta + x_1(n - 1/2)\delta\theta \quad [15]$$

where  $x'_{\infty}$  is the distance defined by  $x = x_1 + x'_{\infty}$ . The value of  $i$  is known theoretically for any time from equation [12] and [14]. Once  $x_1$  is found, the entire soil water profile is known for any time and it is the shape and position of these calculated profiles that will be compared to the experimental profiles measured in the laboratory for the vertical cases.

### Capillary Conductivity

For large times and boundary conditions [6] for the vertical case the value of the infiltration velocity approaches that of  $K_0$  as seen in equation [13]. Physically, for these large times, a constant water content  $\theta_0$  establishes itself over that portion of the profile near  $x = 0$ . For infiltration, where  $\theta_0 > \theta_n$ ,  $\theta_0$  may take on any value of an unsaturated to saturated condition. For example, if  $\theta_0$  were  $0.35 \text{ cm}^3/\text{cm}^3$ , the infiltra-

tion velocity would approach  $K_0$ , the capillary conductivity at a water content of  $0.35 \text{ cm}^3/\text{cm}^3$ .

To obtain  $K_0$  by this procedure it is only necessary to assume that equation

[1] is valid (not equation [5]). This method of obtaining the capillary conductivity is similar to the method presented by Childs and Collis-George (1950), but easier to perform.

### EXPERIMENTAL PROCEDURE

The porous materials were Columbia silt loam, a recent alluvial soil bordering the Sacramento River; and Hesperia sandy loam, derived mainly from granitic alluvial sediments and occupying evenly sloping alluvial fans in the Bakersfield area.

Air-dried soils sieved to pass a 1 mm screen were packed into clear plastic cylinders 3.2 cm in diameter, composed of 1 cm wide sections. The sections were supported in a V-shaped container made from a 4 x 6 inch piece of lumber. To uniformly pack the column, soil was added in small amounts through a 1.5 cm powder funnel connected to a 1 cm diameter rubber tube. The rubber tube rested on the top of the previously added soil and as more soil was added,

the funnel and tube was raised and rotated simultaneously. After each addition of soil, the wood container that held the plastic column was systematically tapped with a rubber mallet.

The water used in all experiments was 0.01 N  $\text{CaSO}_4$  made from deaerated distilled water. The pressure of water entering the soils was controlled by a fritted glass bead plate described by Nielsen and Phillips (1958). The plate was filled with water and the desired pressure applied prior to placing the plate in contact with the porous material. Using the constant head buret shown in figure 1, the pressure at  $x = 0$

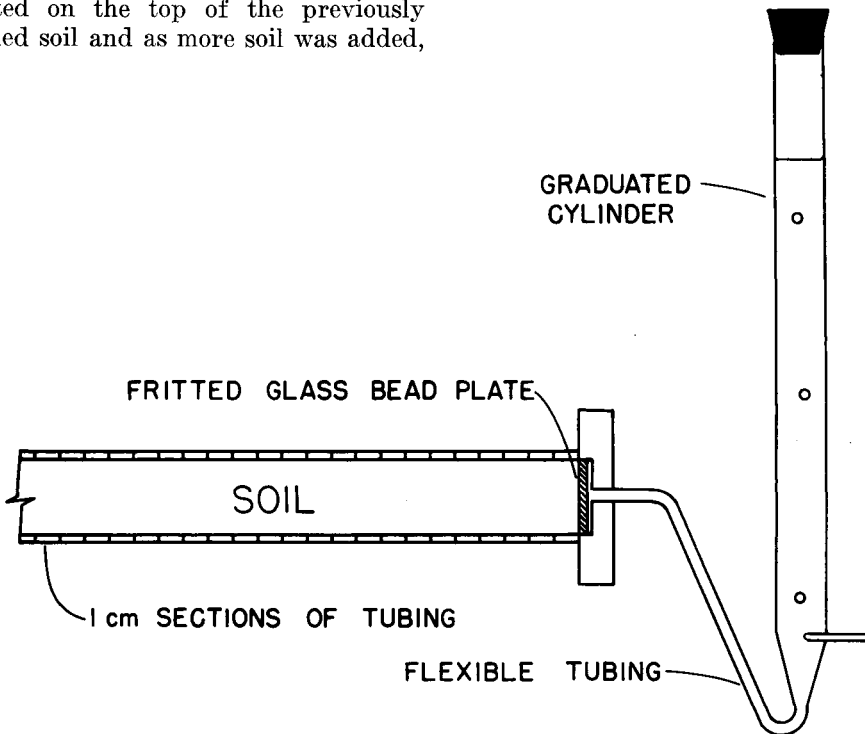


Fig. 1. Experimental apparatus used for both horizontal and vertical water movement.

was precisely controlled. Measurement of time in all experiments commenced the instant contact was established between the wetted plate and the soil. For each run the pressure was held constant at the value existing when the initial contact was made. The pressure drop across the plate and the soil-plate interface was initially in the order of 0.1 millibars and diminished to lesser values for longer times. This small pressure drop was made possible by using different size fritted beads and always using the fritted bead plate that remained just saturated at the desired pressure. It will later be shown that the water content at  $x=0$  remained constant for all observed times. This condition is required for the solution of the equation [5] using boundary conditions [6].

Water entering the columns was measured volumetrically in the constant head buret. Measurements of distance to the wetted front were visually observed. When flow had proceeded for a desired time the fritted plate was removed from the soil, the column was segmented, and the water content of each 1 cm section gravimetrically determined. These gravimetric values were converted to  $\theta$  using the average bulk density of the entire column. The ability to pack columns to equal average bulk density values has been discussed previously by Nielsen *et al.* (1962).

Boundary conditions [6] were imposed on both horizontal and vertical soil columns. A complete discussion of the experimental boundary conditions for horizontal flow through the soils of this study and other porous materials has been presented by Nielsen *et al.* (1962). Some of these data for horizontal flow through Columbia and Hesperia will be given here to make calculations of and comparisons with vertical profiles. The initial water contents  $\theta_n$  of Columbia and Hesperia soils were 0.031 and 0.026  $\text{cm}^3/\text{cm}^3$ , respectively. Values of  $\theta_0$  are given in *Results*, page 607. For these values, the soil water pressure at  $x=0$  ranged between -100 to -2 mb.

The length of time,  $t_0$ , during which boundary condition [6] was maintained before gravimetrically determining the soil water distribution ranged from approximately 60 minutes to about 3 weeks.

Capillary conductivity measurements were made on Columbia soil by the method outlined in *Theoretical Procedure*, page 604. For the vertical columns the values of  $\theta_0$  maintained by pressures of -100, -75, -50 and -2 mb at  $x=0$  were 0.325, 0.35, 0.425 and 0.45  $\text{cm}^3/\text{cm}^3$ , respectively. For these conditions, all columns were allowed to wet until the wetting front reached a depth of 75 cm. For all cases except those samples wetting at -100 mb, the infiltration velocity approached a constant value  $K_0$ . These values of capillary conductivity will be compared to those obtained for steady-state conditions using the more common two-plate method (Richards, 1931).

Soil water diffusivity (defined in equation [4]) versus water content relations were obtained by the method of Bruce and Klute (1956). This method is based upon the assumption that equation [8] exists for horizontal flow under conditions [6]. Upon integration of equation [5] without its right-hand term, the soil water diffusivity  $D$  is calculated from the soil water profiles using the following equation:

$$D(\theta) = -\frac{1}{2t_0} \frac{dx}{d\theta} \int_{\theta_n}^{\theta} x d\theta \quad [16]$$

Soil water diffusivity values were obtained for both Columbia and Hesperia soils for values of  $\theta_0$  corresponding to -2 mb. Diffusivity relations calculated for other values of  $\theta_0$  have been reported elsewhere (Nielsen *et al.*, 1962).

Values of capillary conductivity for Columbia soil at water contents less than 0.30  $\text{cm}^3/\text{cm}^3$  were calculated using the method of Childs and Collis-George (1950). These values together with those determined with equation



[13] for large times provided a complete  $K$  versus  $\theta$  relation. This relation together with the diffusivity data above was used to obtain the solution of equation [5] for the vertical case. For the

Hesperia soil, the necessary  $K$  versus  $\theta$  relation was determined from the above diffusivity relation and the soil water profile developed under conditions [6] for  $t_0 = 97$  minutes.

## RESULTS

### Horizontal Infiltration

If equation [5] is capable of describing soil water movement for conditions [6], a  $\lambda$  single-valued in  $\theta$  will exist (equation [8]). Thus, any water content between  $\theta_n$  and  $\theta_0$  would proceed along the horizontal proportionally to the square root of time. In this investigation, two means are available to ascertain the existence of a unique  $\lambda$  versus  $\theta$  relation. The first is to divide the distance to the wetting front by the square root of time. If these ratios are constant during the experiment, they define the value of  $\lambda$  corresponding to the water content immediately ahead of the wetting front. The second means of determining the uniqueness of  $\lambda$  versus  $\theta$  is from the water content distributions for different times. If the distances are divided by the square root of the time each sample was allowed to wet, a common  $\lambda$  versus  $\theta$  relation should exist. For figures 2 and 3 it has been assumed

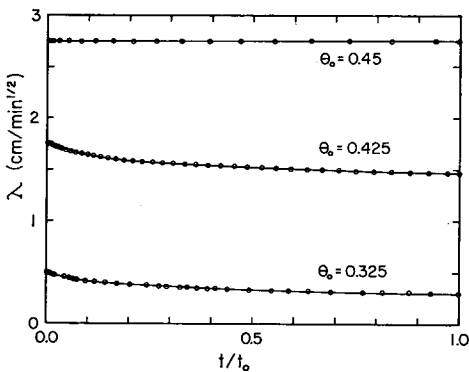


Fig. 2. Values of  $\lambda$  determined by visual distance to the wetting front divided by the square root of time for water infiltrating air-dry Columbia silt loam.  $\theta_0$  is the soil water content at  $x = 0$ , the inflow end of the column.

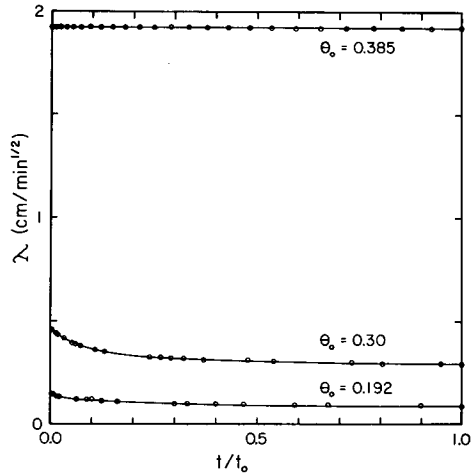


Fig. 3. Values of  $\lambda$  determined by visual distance to the wetting front divided by the square root of time for water infiltrating air-dry Hesperia sandy loam.

that the water content immediately in front of the visually observed wetting front is constant and that the distances to the wetting front divided by the square root of time are values of  $\lambda$  for that water content. For Columbia soil where  $\theta_0$  at  $x = 0$  equals  $0.45 \text{ cm}^3/\text{cm}^3$ ,  $\lambda$  does exist at a value of  $2.75 \text{ cm min}^{-1/2}$  as shown in figure 2. However, for  $\theta_0 = 0.425$  and  $0.325 \text{ cm}^3/\text{cm}^3$ , a constant relation does not exist. Similar results were found for Hesperia soil. For  $\theta_0 = 0.385$  corresponding to an applied soil water pressure of  $-2 \text{ mb}$ ,  $\lambda$  was  $1.92 \text{ cm min}^{-1/2}$ . For smaller values of  $\theta_0$ , the values of distance to the wetting front divided by square root of time decreased during the experiment.

After water had entered the Columbia soil with  $\theta_0 = 0.45$  for three time-periods, the soil water content distribution with distance was measured. These distances, when divided by the square root

of each corresponding time-period  $t_0$ , are the values of  $\lambda$  with their corresponding water contents  $\theta$  described by equation [8]. Equation [8] is apparently a physical reality for the Columbia soil water system values given in figure 4, because plots of the experimental data for all three time-periods yield the same  $\lambda$  versus  $\theta$  relation. In figure 5 for  $\theta_0 = 0.325$ , the  $\lambda$  versus  $\theta$  relation is not unique for three time-periods. Although not presented, the same was found for  $\theta_0 = 0.425$ . A comparison of the values of  $\lambda$  in figure 2 and those near the wetting front in figures 4 and 5 reveals that the assumption regarding the observation of a constant water content in front of the wetting front is reasonable.

Only for  $\theta_0 = 0.385 \text{ cm}^3/\text{cm}^3$  (applied soil water pressure equal to  $-2 \text{ mb}$ ) was there a unique  $\lambda$  versus  $\theta$  relation for different time-periods of wetting Hesperia soil (figure 6). When the soil was allowed to wet at a smaller pressure,

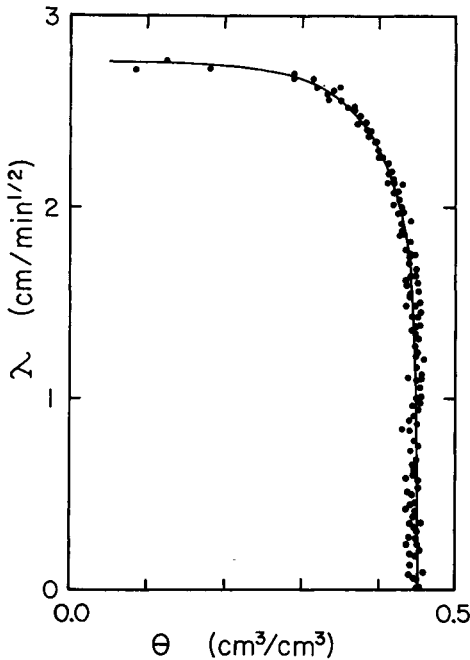


Fig. 4. Values of  $\lambda$  for Columbia soil determined from water content distributions measured for three time-periods of infiltration with  $\theta_0 = 0.45 \text{ cm}^3/\text{cm}^3$ .

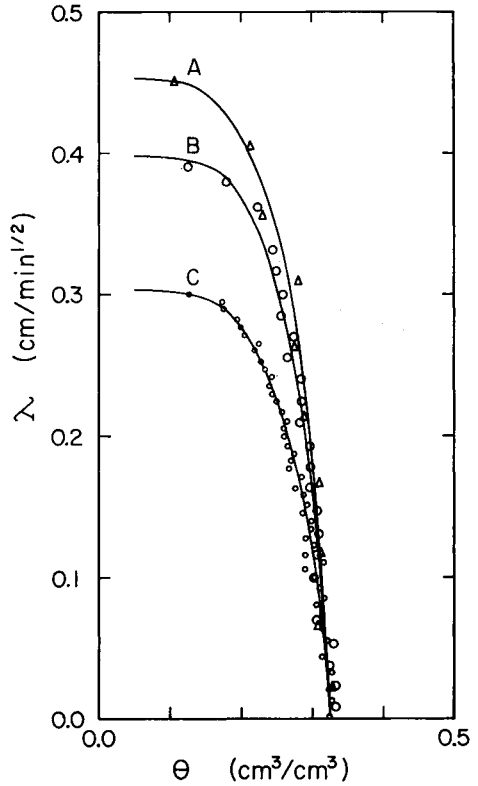


Fig. 5. Values of  $\lambda$  for Columbia soil determined from water content distributions measured for three time-periods of infiltration with  $\theta_0 = 0.325 \text{ cm}^3/\text{cm}^3$ . Curves A, B, and C correspond to time  $t_0$  equal to 441, 4182 and 28224 minutes, respectively.

producing a value of  $\theta_0$  equal to  $0.30 \text{ cm}^3/\text{cm}^3$ , results similar to those of Columbia were measured—that is, a unique  $\lambda$  versus  $\theta$  relation did not exist (figure 7).

Values of soil water diffusivity were calculated (only when  $\lambda$  existed) from the measured soil water distribution curves using equation [16]. Diffusivity values for Columbia soil allowed to wet at  $\theta_0 = 0.45 \text{ cm}^3/\text{cm}^3$  are plotted against water content in figure 8. Values for Hesperia soil which was wet at  $\theta_0 = 0.385$  are given in figure 9. The successful prediction of horizontal soil water movement using these exact relations has been reported previously (Nielsen et al., 1962).

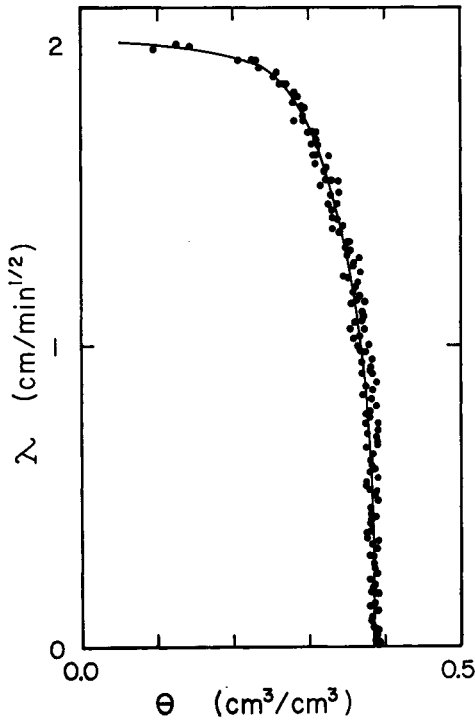


Fig. 6. Values of  $\lambda$  for Hesperia soil determined from water content distributions measured for three time-periods of infiltration with  $\theta_0 = 0.385 \text{ cm}^3/\text{cm}^3$ .

### Vertical Infiltration

Figures 10 and 11 show observations of the wetting front advance into Columbia silt loam and Hesperia sandy loam for both horizontal and vertical movement. For the Columbia soil data shown in figure 10, the values of  $\theta_0$  were 0.45, 0.425, 0.35 and 0.325  $\text{cm}^3/\text{cm}^3$ . The distance the wetting front advanced in the vertical direction is always equal to or greater than that in the horizontal direction. For a given  $\theta_0$ , the initial rates of advance are identical for both directions. Similar results for Hesperia soil wet at  $\theta_0$  equal to 0.385 and 0.30  $\text{cm}^3/\text{cm}^3$  are given in figure 11. It is of interest to observe that the effect of the gravitational field for both soils is more obvious as  $\theta_0$  is decreased. For example, when water entered Columbia soil at 0.45  $\text{cm}^3/\text{cm}^3$ , the time required for the wetting front to advance 50 cm hori-

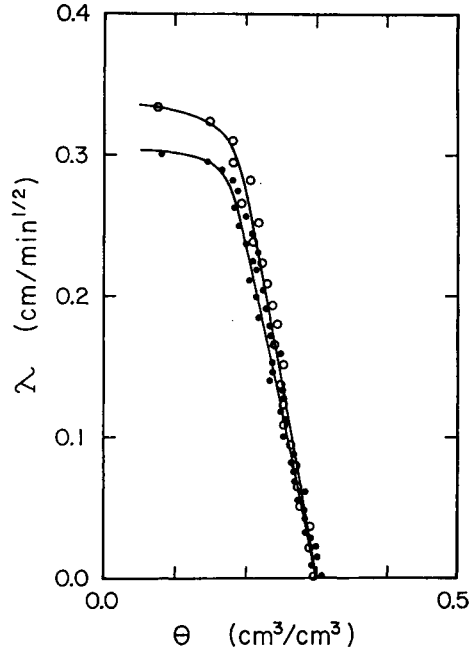


Fig. 7. Values of  $\lambda$  for Hesperia soil determined from water content distributions measured for two time-periods of infiltration with  $\theta_0 = 0.30 \text{ cm}^3/\text{cm}^3$ . Solid points and open circles represent data corresponding to times  $t_0$  equal to 4820 and 23677 minutes respectively.

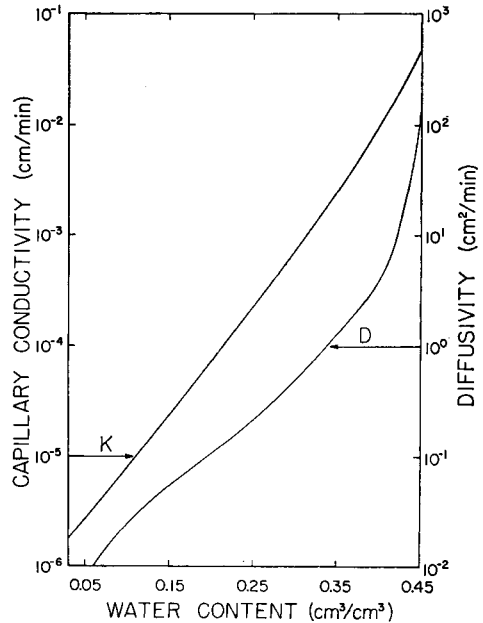


Fig. 8. Experimental values of capillary conductivity K and soil water diffusivity D for Columbia silt loam used to calculate vertical soil water movement.

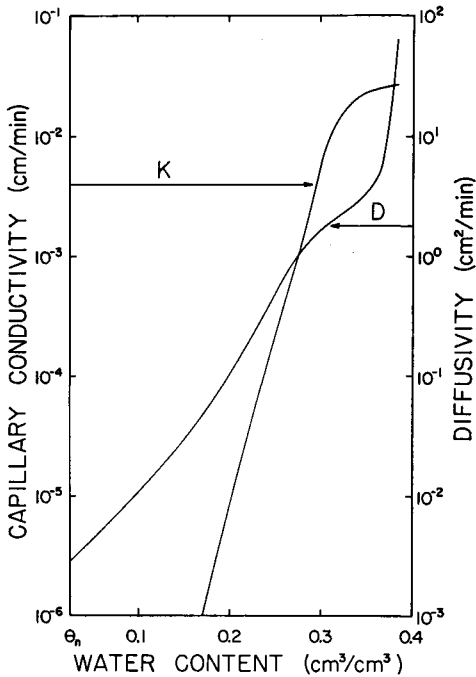


Fig. 9. Experimental values of capillary conductivity K and soil water diffusivity D for Hesperia sandy loam used to calculate vertical soil water movement.

zontally was 1.4 times greater than that vertically. But when water entered at 0.425 and 0.325 cm<sup>3</sup>/cm<sup>3</sup>, the times required to advance 50 cm horizontally as

compared to 50 cm vertically was 1.9 and 2.1, respectively.

Columbia soil water profiles developed during time periods of 64, 226, and 467 minutes where  $\theta_0$  was 0.45 cm<sup>3</sup>/cm<sup>3</sup> are presented in figure 12. In approximately 225 minutes a constant water content over the first 30 cm depth (approximately) is established. Profiles for  $\theta_0$  equal to 0.425 cm<sup>3</sup>/cm<sup>3</sup> are similar except that the times involved are greater. A time greater than 500 minutes is required to develop a 'θ-straight' (Philip 1957b) at 0.425. For water entering at the least water content of 0.325 cm<sup>3</sup>/cm<sup>3</sup>, a 'θ-straight' is

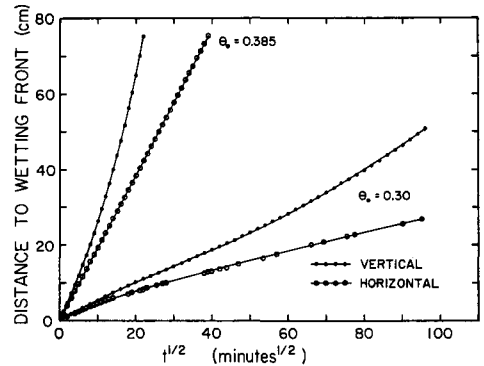


Fig. 11. Distance to the wetting front of air-dry Hesperia sandy loam versus square root of time for horizontal and vertical movement.

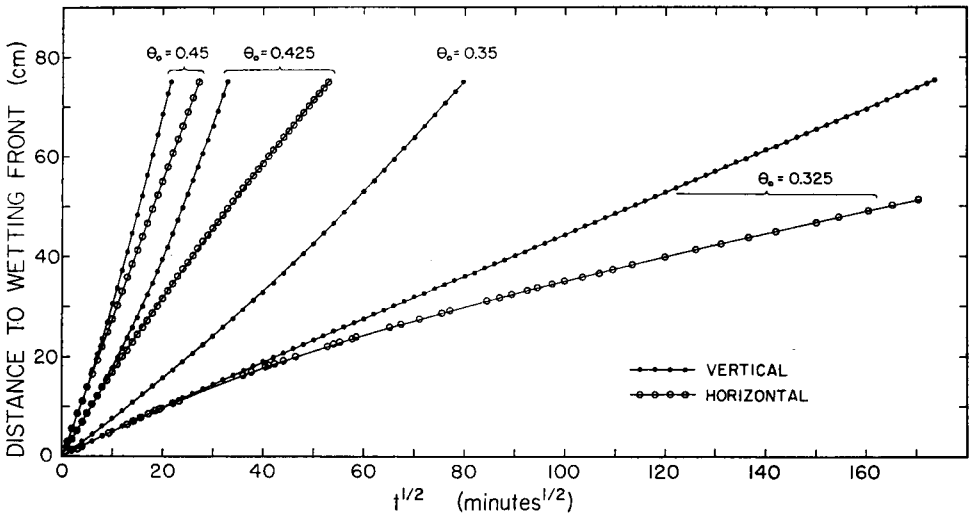


Fig. 10. Distance to the wetting front of air-dry Columbia silt loam versus square root of time for horizontal and vertical movement.

being approached but is not established after 30,200 minutes or nearly 3 weeks. Figures 15 and 16 give Hesperia soil water profiles developed for  $\theta_0$  equal to 0.385 and 0.30  $\text{cm}^3/\text{cm}^3$ . For the greater water content a ' $\theta$ -straight' exists after 300 minutes, while for the smaller water

content such a condition failed to completely establish for the deepest profile.

The soil water profiles shown in figures 12 through 16 differ from those reported by Bodman and Colman (1943) which have been the subject of considerable discussion (Baver, 1956; Philip,

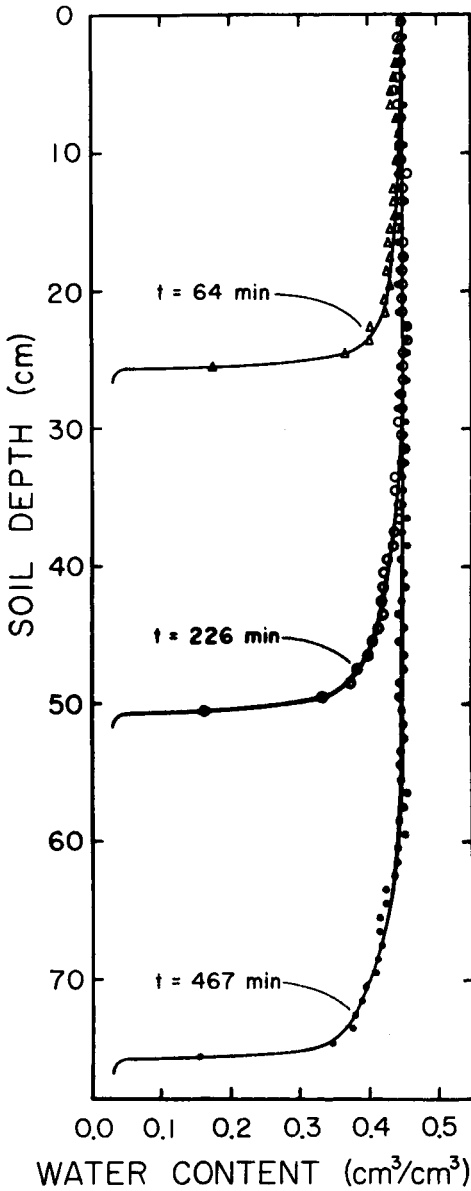


Fig. 12. Columbia soil water content distributions for vertical profiles developed in air-dry soil with  $\theta_0 = 0.45 \text{ cm}^3/\text{cm}^3$ .

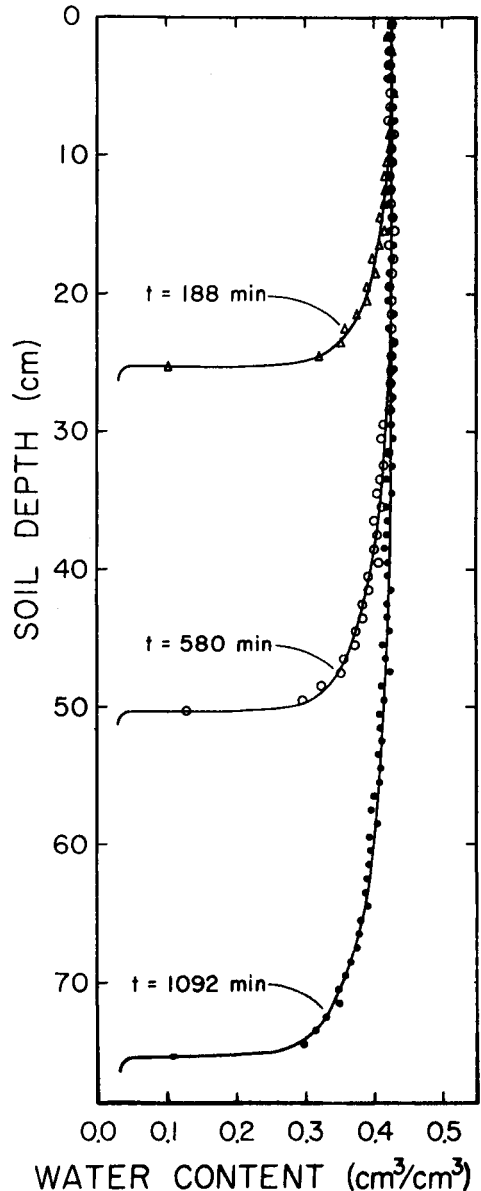


Fig. 13. Columbia soil water content distributions for vertical profiles developed in air-dry soil with  $\theta_0 = 0.425 \text{ cm}^3/\text{cm}^3$ .



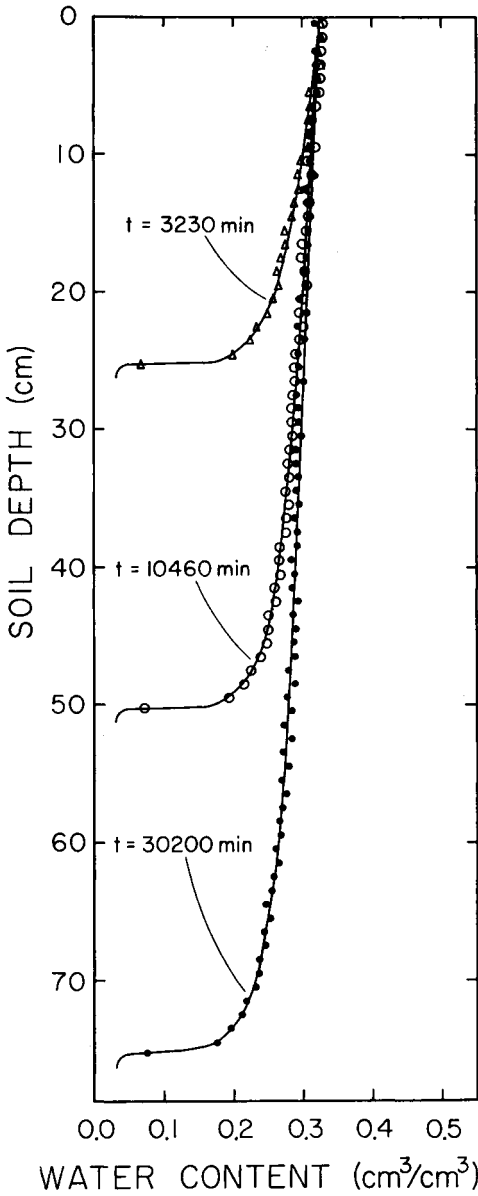


Fig. 14. Columbia soil water content distributions for vertical profiles developed in air-dry soil with  $\theta_0 = 0.325 \text{ cm}^3/\text{cm}^3$ .

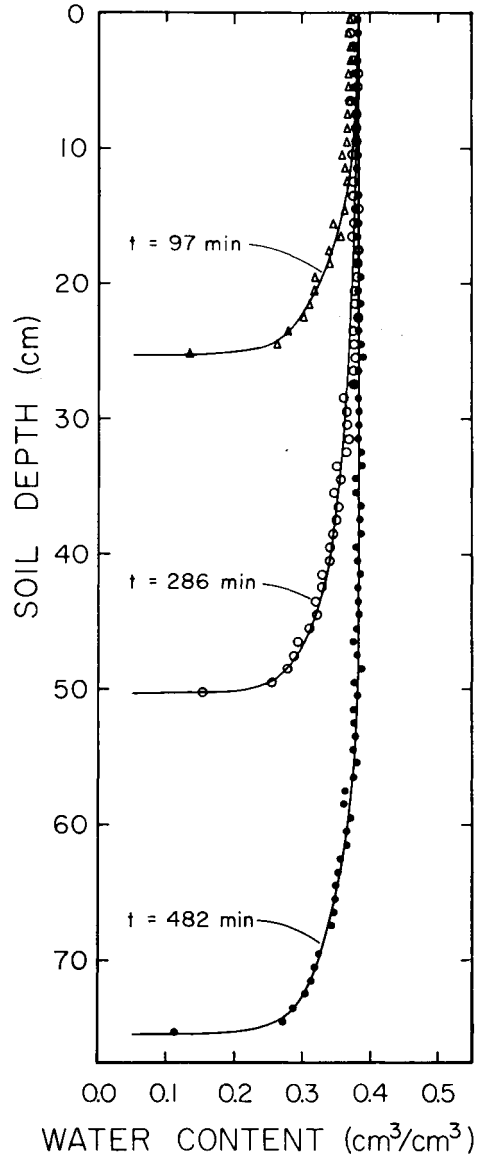


Fig. 15. Hesperia soil water content distributions for vertical profiles developed in air-dry soil with  $\theta_0 = 0.385 \text{ cm}^3/\text{cm}^3$ .

1957d; Youngs, 1957). Their profiles were S-shaped, having a sharp reduction in soil water content a few centimeters from  $x=0$ . The profiles given in this paper are not S-shaped and without exception tend to develop a ' $\theta$ -straight' with time.

Table 1 gives the capillary conductivity values for Columbia soil determined by the method described in this paper and by the two-plate method (Nielsen and Biggar, 1961). Values of

TABLE 1  
MEASURED VALUES OF CAPILLARY CONDUCTIVITY OF COLUMBIA SILT LOAM

Water content $\theta$ (cm <sup>3</sup> /cm <sup>3</sup> )	Capillary conductivity	
	$K^*$ (cm/min)	$K^\dagger$ (cm/min)
0.45.....	0.0464	.....
0.44.....	.....	0.0215
0.425.....	0.0193	0.0150
0.35.....	0.00294	0.00260
0.30.....	0.000623	.....

\*  $K$  from method described in text, page 604.  
 †  $K$  from method of Nielsen and Biggar, 1961.

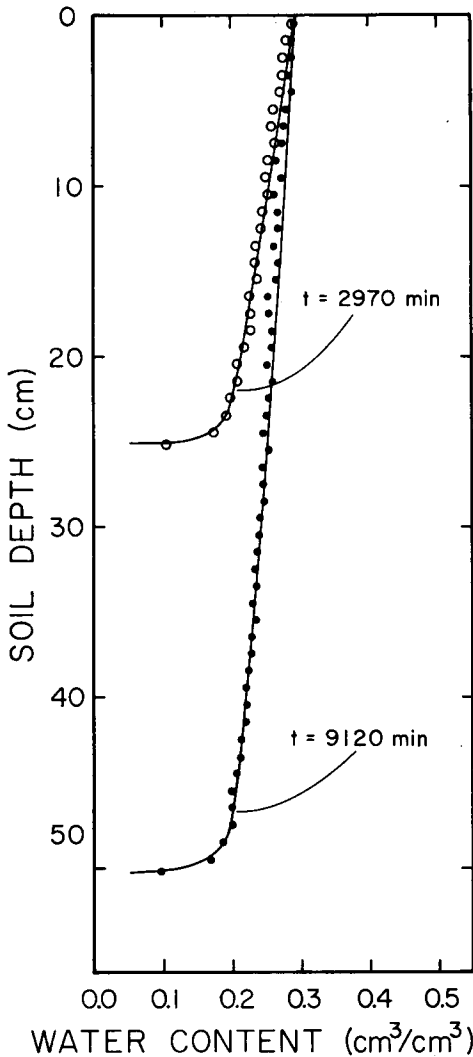


Fig. 16. Hesperia soil water content distributions for vertical profiles developed in air-dry soil with  $\theta_0 = 0.30$  cm<sup>3</sup>/cm<sup>3</sup>.

capillary conductivity taken equal to the infiltration velocities measured for soil columns wet to 75 cm depth compare favorably with those obtained using the two-plate steady-state method. The method provides a simple means of obtaining capillary conductivities for high soil water contents for the imbibing process—heretofore a difficult measurement to make.

Figure 8 gives the capillary conductivity relation for Columbia soil measured for water contents between 0.45 to 0.30 cm<sup>3</sup>/cm<sup>3</sup> by the above method, and calculated by the method of Childs and Collis-George (1950) for water contents less than 0.30 cm<sup>3</sup>/cm<sup>3</sup>. The capillary conductivity relation of Hesperia sandy loam given in figure 9 was calculated from the soil water profile for  $t_0 = 97$  minutes presented in figure 15 by the method of Philip (1957c) outlined in *Theoretical Procedure*.

Figures 17 and 18 show the parameters  $\lambda$ ,  $\chi$ , and  $\psi$  of equation [7]. The  $\lambda$  and diffusivity relations are those obtained from the horizontal flow studies. The values of  $\chi$  and  $\psi$  were calculated using the iterative procedure of Philip (1955, 1957a). It should be noted for both soils that the ordinate scales are not the same for each parameter and that  $\chi$  and  $\psi$  are much smaller than  $\lambda$ .

Figure 19 presents Columbia soil water profiles calculated from the above parameters for infiltration times of 64, 226 and 467 minutes for  $\theta_0$  equal to 0.45 cm<sup>3</sup>/cm<sup>3</sup>. Agreement exists between the

measured and theoretical plots for all three infiltration times. For Hesperia soil having  $\theta_0$  equal to 0.385, the soil water profiles are successfully predicted for infiltration times of 286 and 482 minutes using the capillary conductivity relations calculated from the profile measured at 97 minutes.

Soil water profiles given in figures 13,

14 and 16 could not be calculated using the solution of equation [5]. This solution depends upon the calculation of a unique  $\lambda$  versus  $\theta$  relation based upon measurements made on the horizontal samples. These relations were not unique, as is shown in figures 5 and 7.

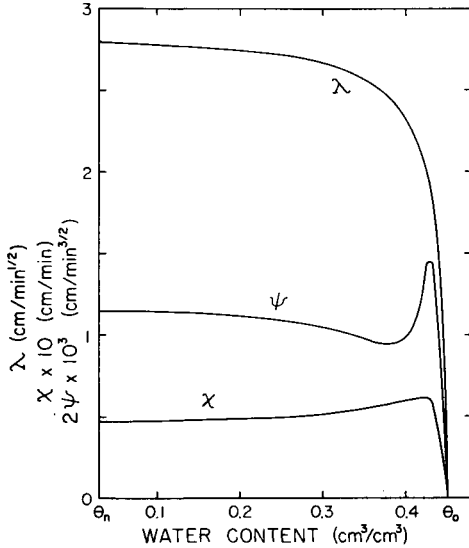


Fig. 17. Calculated values of  $\lambda$ ,  $\chi$ , and  $\psi$  defined in equation [7] for Columbia silt loam.

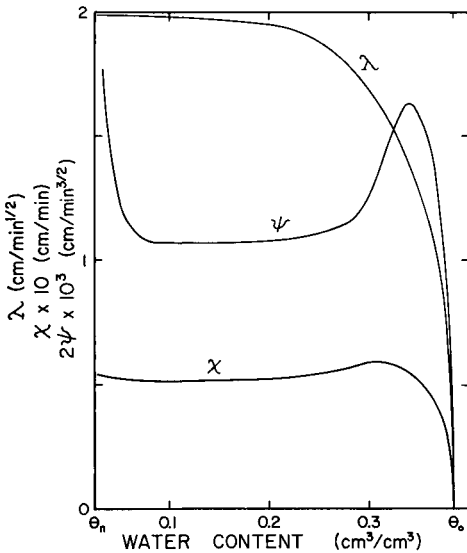


Fig. 18. Calculated values of  $\lambda$ ,  $\chi$ , and  $\psi$  defined in equation [7] for Hesperia sandy loam.

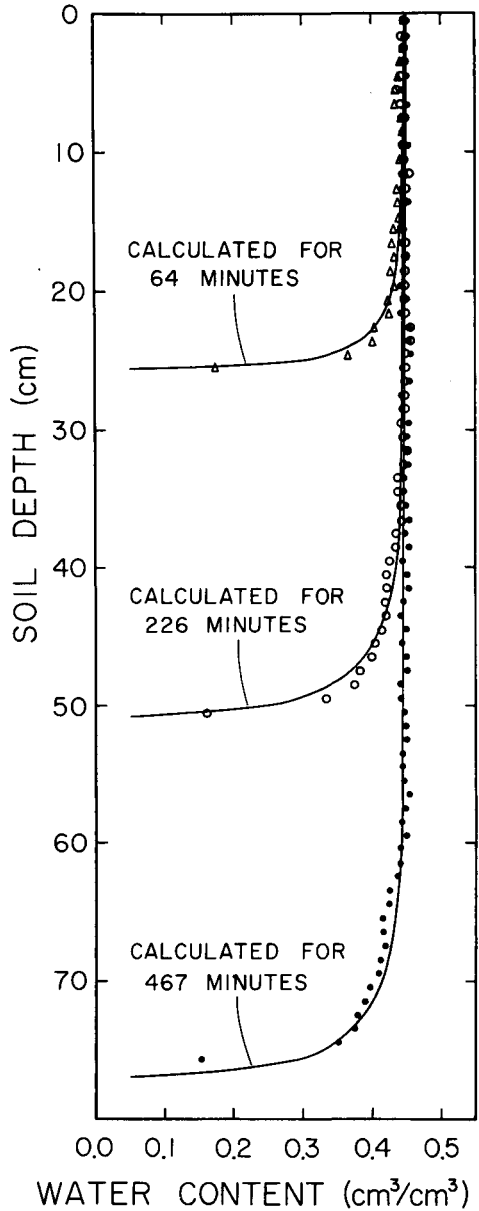


Fig. 19. Calculated and measured soil water profiles for air-dry Columbia soil allowed to wet at  $\theta_0 = 0.45 \text{ cm}^3/\text{cm}^3$ .

## DISCUSSION

When water under near-atmospheric pressure entered Columbia or Hesperia soil, soil water profiles were described by equation [5] subject to [6] for both the horizontal and vertical cases. When the soil water pressure was reduced, causing a reduction in water content at

$x = 0$ , neither horizontal nor vertical soil water profiles could be predicted. The failure of the equation to describe horizontal flow for these soils and sandstone wet with not only water but oil has already been partially discussed (Nielsen *et al.*, 1962). From experimental evidence it was concluded that rearrangement of soil particles or clay migration or swelling could not account for the lack of agreement between the measured and calculated profiles, and other evidence showed that bacterial activity was not responsible. This would suggest that assumption (1) given in *Theoretical Procedure*, page 603, be satisfied.

Let us consider assumption (2), which requires water movement to be analogous to heat flow where only a single phase is studied. Experimentally, the cylinder that supported the soil was composed of 1 cm segments which allowed air to be displaced between these segments and also out the open end of the column. Because of the large differences in viscosity between that of water and air it would seem logical that this assumption might be satisfied. Miller and Miller (1955) have discussed the wetting and drying of soils with particular emphasis given to hysteresis occurring in a single pore or sequence. The heterogeneous nature of the size, shape, composition and arrangement of soil particles complicates the task of physically describing the addition or removal of soil water at different rates. The contact angles between the water and the various surfaces would vary and would also depend upon rate of movement (Biggar and Taylor, 1960).

The discontinuity of air and the possibility of its displacement is recognized at water contents near saturation. Once continuous air passages exist within the soil, further consideration of air movement has been generally neglected. With visual observation of a Christianson filter (Davidson *et al.*, 1962) having

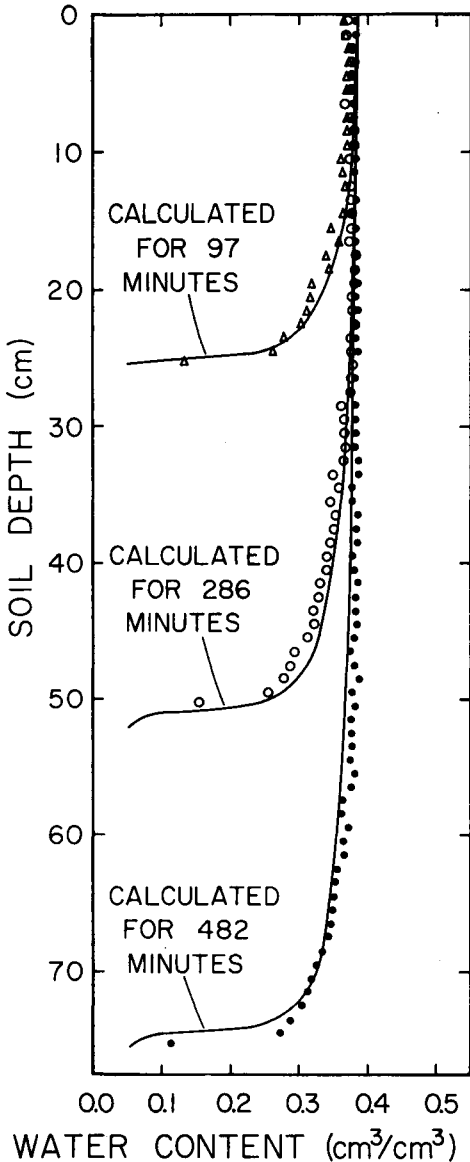


Fig. 20. Calculated and measured soil water profiles for air-dry Hesperia soil allowed to wet at  $\theta_0 = 0.385 \text{ cm}^3/\text{cm}^3$ .

air as one of its fluids, it is easily recognized that the water distribution within a porous material is not unique for a given fluid content. Depending upon the position of the source of air and the rate at which the air is allowed to displace the original fluid, different fluid distributions occur. The particular distribution of air within the mass will again influence subsequent displacements. In the transient condition of soil water movement, a sudden emptying of a relatively large pore sequence connected to the bulk soil mass by only smaller necks or openings will produce a disturbance that persists long enough to influence the draining of other pores in close proximity. A comparison made by Elrick (1963) of transient and steady-state water flow in unsaturated sand also suggests the same description. Concerning the movement of the second phase, a suitable experiment to perform would be the following: Subject samples of equal initial water content, but of unequal lengths, to identical increments of applied soil water pressure and observe rate of water content change. In addition, for equal soil lengths, a study of the water content relations for unequal pressure increments applied over the same pressure range would yield further insight into the problem. It is also possible to study water movement with the total air pressure reduced below normal atmospheric pressure, although the use of gases other than air would probably be more convenient.

The third assumption states that the properties of the fluid or water do not vary. On the basis of work such as that of Anderson and Low (1958) it is reasonable to conclude that the physical state of water in films of considerable thickness differs markedly from that in bulk quantities. The presence of ions in the soil solutions also works against the validity of this assumption. The surface properties of the soil colloids, together with their residual charge, are responsible for a non-uniform ion distribution

within the liquid phase. Moreover, these distributions depend upon the pore diameter or the liquid film thickness. Experimental evidence of the behavior of water and aqueous solutions flowing through small capillaries of great length would be helpful in ascertaining the limits of applicability of this assumption.

Anderson and Linville (1962) have measured substantial temperature fluctuations in initially dry porous materials during water infiltration. For  $35\mu$  diameter glass beads, the accompanying temperature change was greater than  $0.1^\circ\text{C}$ , while for bentonites changes have been measured as high as  $40^\circ\text{C}$ . Temperature increases of 2 to  $5^\circ\text{C}$  are commonly measured on air-dry agricultural soils. Because the Columbia and Hesperia soils were initially air-dry, it would be expected that significant temperature fluctuations occurred during infiltration. These fluctuations would influence the water movement to a greater degree as values of  $\theta_0$  became substantially less than saturation.

It is also worthwhile to compare the experimental data with previously published data. Vertical soil water profiles reported by Bodman and Colman (1943) and horizontal profiles reported by Bruce and Klute (1956) have sharp increases in water content near  $x=0$ . This increase is not peculiar to vertical or horizontal water movement, and is not found in figures 12 through 16. It is possible to produce such a water content distribution in Columbia and Hesperia soils with the apparatus shown in figure 1. By merely initiating flow with a slight instantaneous positive pressure or by pinching off with flexible tubing of the porous plate to cease flow at time  $t_0$ , the water content near  $x=0$  is increased. Such experimental consequences demonstrate the necessity of additional carefully planned and executed experiments. With such information at hand, the physical processes revealed could be described.



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