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THE MEASUREMENT OF INFLATION:

A STOCHASTIC APPROACH

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I. INTRODUCTION

This paper considers the following familiar problem from index number theory. Given the prices of n goods in two periods t and t-1, $p_{1t}, \ldots, p_{nt}, p_{1,t-1}, \ldots, p_{n,t-1}$, how should we use this information to measure the overall rate of inflation, the proportionate change in the general price level? There are two basic approaches to this problem (Diewert, 1981, Frisch, 1936). The first is the functional index number approach, whereby the price index is related to the underlying utility or production functions.

The second method is what Frisch (1936) calls the "atomistic" or "stochastic" approach, associated with Jevons and Edgeworth (see Frisch for references). Here the proportionate price changes, $\Delta p_{1t}/p_{1,t-1}, \ldots, \Delta p_{nt}/p_{n,t-1}$, are viewed as independent observations on the underlying rate of inflation; that is, each observation is taken to be equal to this inflation rate plus an independent random component. Thus, the rate of inflation can be estimated by averaging over these n observations. This approach has been correctly criticized by Keynes (1930, pp. 85-88) on the basis that it requires the systematic component of each price change to be identical. In other words, all prices must change equi-proportionately, so that there can be no changes in relative prices.

The objective of this paper is to rehabilitate the stochastic approach by answering Keynes' criticism by allowing for systematic changes in relative prices. We do this by extending our previous work on this topic (Clements and Izan, 1981) and that of Theil, Suhm and Meisner (1981).

The approach of viewing the underlying rate of inflation as an unknown parameter to be estimated from the individual price changes leads to a link between index number theory and least squares theory. Accordingly, not only do we obtain a point estimate of the rate of inflation, but also its sampling variance. The source of the sampling error is that all prices do not move proportionately, so that the sampling variance of inflation is higher the more disproportionate are the individual price changes. In other words, the rate of inflation will be measured less precisely when there are large changes in relative prices. This attractive result provides a formal link between the measurement of inflation and changes in relative prices.

The sampling variance of inflation makes it possible to construct confidence intervals for the true rate of inflation. These confidence intervals could be of practical use for the measurement of inflation for wage negotiations, etc. The sampling variance may also be of use in econometric tests of rational expectations models regarding the effects of anticipated versus unanticipated shocks to the economy (see, e.g., Lucas, 1973). It could be easily argued that inflation would tend to be more fully anticipated when its sampling variance is lower; under rational expectations, such an inflation would be more neutral (have less real effects) than if it were unanticipated (i.e. have a large sampling variance).

Many of the results derived in this paper have applications in a number of other areas where index numbers are used. Although we work exclusively with prices, our methodology could equally well be applied to quantities and used for the measurement of real income, total factor productivity and so on. The framework could also be used to test the purchasing power parity hypothesis (see Miller, 1983) and to extend the analysis of Divisia monetary aggregates (Barnett, 1981, Sec. 7.11).

The paper is structured as follows. In Section II we provide a brief review of our previous results and then extend these to allow for systematic changes in relative prices. In that section we derive estimators of the rate of inflation and changes in relative prices, as well as their standard errors. All these expressions are simple and have clear economic interpretations. Moreover, they are easy to compute as they depend only on observed prices and budget shares, and not on unknown parameters. In Section III we apply our results to measuring inflation in Australia using data from the last 30 years.

II. INDEX NUMBERS AS MEANS

To motivate the analysis, in this section we first formulate the rate of inflation as an unweighted average of the individual price changes. In the following sub-sections we extend this idea by (i) introducing a budget-share-weighted average of prices and (ii) allowing for systematic changes in relative prices. This material extends the results of Clements and Izan (1981) and the weighted country-product-dummy method of Theil, Suhm and Meisner (1981, Section 2.3).

An Unweighted Average of Prices

Let p_{it} be the price of commodity i (i=1,...,n) in period t and $Dp_{it} = \log p_{it} - \log p_{i,t-1}$ be the price log-change. For each period, let each price log-change be made up of a systematic part α_t and a zero mean, random component ε_{it},

$$(1) \qquad Dp_{it} = \alpha_t + \varepsilon_{it}, \qquad\qquad i=1,...,n.$$

The term α_t has no i subscript and is interpreted as the common trend in all prices (due to, e.g., monetary expansion), while ε_{it} represents everything else. Since (1) specifies that, apart from random factors, all prices change equiproportionally, it can be viewed as a stochastic version of the Hicksian composite commodity aggregation condition.

Let us assume that the random terms are independent over commodities,

$$(2) \qquad \text{cov}(\varepsilon_{it}, \varepsilon_{jt}) = 0, \qquad\qquad i \neq j,$$

and that they have a common variance,

$$(3) \qquad \text{var } \varepsilon_{it} = \sigma_t^2.$$

Under these assumptions the best linear unbiassed estimator of α_t is just the unweighted average of the n price log-changes,

$$(4) \qquad \hat{\alpha}_t = \frac{1}{n} \sum_{i=1}^{n} Dp_{it},$$

which has sampling variance

$$(5) \qquad \text{var } \hat{\alpha}_t = \frac{\sigma_t^2}{n}.$$

The variance σ_t^2 can be estimated unbiassedly by

$$(6) \qquad \hat{\sigma}_t^2 = \frac{1}{n-1} \sum_{i=1}^{n} (Dp_{it} - \hat{\alpha}_t)^2.$$

To interpret these results, we note from equation (1) that $\varepsilon_{it} = Dp_{it} - \alpha_t$ is the change in the ith price deflated by the common trend in all prices; i.e. ε_{it} is the change in the ith relative price. Accordingly, when there is substantial variation in relative prices, the random errors in (1) will be large, so that, from (5) and (6), the sampling variance of the estimated rate of inflation will also be higher. This agrees with the intuitive notion that when there are substantial variations in relative prices, the overall rate of inflation is less well defined. In our framework inflation being less well defined means that our estimate of inflation has a higher sampling variance.

To illustrate, consider the situation with n = 3 commodities and the following two sets of price data:

	Dp_{1t}	Dp_{2t}	Dp_{3t}
Case 1	.10	0	.05
Case 2	.05	.05	.05

Using (4)-(6), we obtain $\hat{\alpha}_t = .05$, $\hat{\sigma}_t/\sqrt{3} = .029$ for case 1 and $\hat{\alpha}_t = .05$ and $\hat{\sigma}_t/\sqrt{3} = 0$ for case 2. In both cases the estimated rate of inflation is (approximately) 5 percent. In the first case relative prices change substantially and the standard error of $\hat{\alpha}_t$ is 2.9 percent, implying a 95 percent confidence interval of [17.4, -7.4] percent (using the t distribution with two degrees of freedom). In contrast, there is no change in relative prices in the second case and no estimation uncertainty about the rate of inflation as $\hat{\sigma}_t = 0$.

As ε_{it} is the change in the ith relative price, the interpretation of (2) is that relative prices are independent. The interpretation of (3) is that all relative price changes have the same variance. These conditions are obviously very stringent and in the following two subsections we extend the model to relax (2) and (3).

A Budget-Share-Weighted Average of Prices

Let q_{it} be the quantity consumed of commodity i, $M_t = \Sigma_{i=1}^n p_{it} q_{it}$ be total expenditure, $w_{it} = p_{it} q_{it}/M_t$ be the ith budget share and let $\bar{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1})$ be the arithmetic average over $t-1$ and t of this share. We continue to take the relative price changes as being independent, but we now replace (3) with the assumption that the variance of the change in the relative price of i is inversely proportional to $\bar{w}_{it}$,

$$(7) \qquad \text{var } \varepsilon_{it} = \frac{\lambda_t^2}{\bar{w}_{it}}.$$

This means that the variability of a relative price falls as the commodity becomes more important in the consumer's budget.

The error specification (7) is similar to that obtained when data are in the form of group means, such as some survey data. The effect of grouping is to make the error variance inversely proportional to the number of observations in each group (Kmenta, 1971, p. 323). In (7) $\bar{w}_{it}$ is approximately proportional to expenditure on commodity i, which can be viewed as a dollar measure of the number of items in or the size of the group.

We write (1) in vector form as

$$(8) \qquad Dp_t = \alpha_t \iota + \varepsilon_t,$$

where $Dp_t = [Dp_{it}]$, $\iota = [1,\ldots,1]'$ and $\varepsilon_t = [\varepsilon_{it}]$. Under (2) and (7) the $n \times n$ covariance matrix of ε_t is

$$(9) \qquad \text{var } \varepsilon_t = \lambda_t^2 \bar{W}_t^{-1},$$

where $\bar{W}_t = \text{diag}[\bar{w}_{1t},\ldots,\bar{w}_{nt}]$. Application of generalized least squares to (8) and (9) gives

$$\tilde{\alpha}_t = (\iota' \bar{W}_t \iota)^{-1} \iota' \bar{W}_t Dp_t.$$

Since $\iota'\bar{W}_t\iota = \Sigma_{i=1}^n \bar{w}_{it} = 1$ and $\iota'\bar{W}_t Dp_t = \Sigma_{i=1}^n \bar{w}_{it} Dp_{it}$, this simplifies to

$$(10) \qquad \tilde{\alpha}_t = \sum_{i=1}^n \bar{w}_{it} Dp_{it} .$$

The sampling variance of $\tilde{\alpha}_t$ is $\lambda_t^2 (\iota'\bar{W}_t\iota)^{-1} = \lambda_t^2$,

$$(11) \qquad \text{var } \tilde{\alpha}_t = \lambda_t^2 ,$$

which can be estimated unbiassedly by $(Dp_t - \tilde{\alpha}_t\iota)'\bar{W}_t(Dp_t - \tilde{\alpha}_t\iota)/(n-1) = \Sigma_{i=1}^n \bar{w}_{it}(Dp_{it} - \tilde{\alpha}_t)^2/(n-1)$,

$$(12) \qquad \tilde{\lambda}_t^2 = \frac{1}{n-1} \sum_{i=1}^n \bar{w}_{it}(Dp_{it} - \tilde{\alpha}_t)^2 .$$

According to equation (10), the estimate of inflation is now a weighted average of the n price log-changes, the weights being the arithmetic averages of the budget shares. This is identical to the well-known Divisia price index DP (see, e.g., Theil 1975/76). Thus, the Divisia index emerges as the generalized least squares estimator of inflation under (9).

The Divisia index can be viewed as a weighted first-order moment of the price log-changes, $Dp_{1t},\ldots,Dp_{nt}$. The corresponding second-order moment is the Divisia variance of relative price changes,

$$(13) \qquad \Pi_t = \sum_{i=1}^n \bar{w}_{it}(Dp_{it} - DP_t)^2 ,$$

which measures the degree to which prices move disproportionately; $\Pi_t = 0$ only if all prices increase at the same rate, i.e. if there are no changes in relative prices. As $\tilde{\alpha}_t = DP_t$, we see from (12) and (13) that the sampling variance of the Divisia index is a multiple $1/(n-1)$ of the Divisia price variance Π_t . Accordingly, again we see that the sampling variance of inflation will be higher the larger are the relative price movements. The only difference between the measures of this

subsection and those of the previous subsection is that now everything is budget-share-weighted, rather than unweighted.

Allowing for Systematic Changes in Relative Prices

We now extend the analysis to allow the changes in relative prices to be correlated across commodities. We do this by adding a commodity dummy to (1),

$$(14) \qquad Dp_{it} = \alpha_t + \beta_i + \zeta_{it} \, ,$$

where ζ_{it} is a zero mean, random error. As before, α_t is the common trend in all prices, which we identify as the underlying rate of inflation. The change in the ith relative price is now $Dp_{it} - \alpha_t = \beta_i + \zeta_{it}$, which has expectation β_i. Thus β_i is interpreted as the systematic change in this relative price.

We assume that the ζ_{it}'s are independent over commodities and time

$$(15) \qquad cov(\zeta_{it}, \zeta_{js}) = 0 \, , \qquad\qquad i \neq j, t \neq s \, ,$$

and that their variances are inversely proportional to the corresponding arithmetic averages of the budget shares,

$$(16) \qquad var \, \zeta_{it} = \frac{\eta_t^2}{\overline{w}_{it}} \, .$$

Equations (14)-(16) imply that relative prices are correlated over both commodities and time:

$$E(Dp_{it} - \alpha_t)(Dp_{js} - \alpha_s) = \beta_i \beta_j + \delta_{ij} \delta_{st} \frac{\eta_t^2}{\overline{w}_{it}} \, ,$$

where δ_{ij} is the Kronecker delta (=1 if $i=j$, 0 otherwise).

The estimation of (14) is somewhat more complicated than before because now we have to pool the data across time and commodities.

Previously we only used the prices of different commodities within one period. We proceed by initially ignoring that the variance (16) is time dependent. That is, we replace (16) with

$$(16') \qquad \text{var } \zeta_{it} = \frac{\eta^2}{\bar{w}_i} \text{ ,}$$

where $\bar{w}_i$ is the sample mean of $\bar{w}_{it}$. We shall subsequently adjust the estimation procedure to allow for the time dependence of this variance.

Model (14) is not identified. This can be seen by noting that an increase in α_t for each t by any number k and a lowering of β_i for each i by the same k does not affect the right-hand side of (14). To identify the model we impose the constraint

$$(17) \qquad \sum_{i=1}^{n} \bar{w}_i \beta_i = 0 \text{ .}$$

This has the simple interpretation that a budget-share-weighted average of the systematic components of the relative price changes is zero.

Multiplying both sides of (14) by $\sqrt{\bar{w}_i}$, we obtain

$$(18) \qquad y_{it} = \alpha_t x_i + \beta_i x_i + \zeta'_{it} \text{ ,}$$

where $y_{it} = \sqrt{\bar{w}_i} Dp_{it}$, $x_i = \sqrt{\bar{w}_i}$ and $\zeta'_{it} = \sqrt{\bar{w}_i} \zeta_{it}$. It follows from (16') that var $\zeta'_{it} = \eta^2$, a constant, so that least squares can be applied to (18). In the Appendix we show that the LS estimators, constrained by (17), are

$$(19) \qquad \alpha_t^* = \sum_{i=1}^{n} \bar{w}_i Dp_{it} \text{ ,} \qquad \beta_i^* = \frac{1}{T} \sum_{t=1}^{T} (Dp_{it} - \alpha_t^*) \text{ ,}$$

where T is the number of periods.

The only difference between α_t^* and the Divisia index DP_t $(=\tilde{\alpha}_t)$ is that the former uses constant weights $(\bar{w}_i)$, while the weights of the

latter vary with time ($\bar{w}_{it}$) . In fact, this difference will have little practical importance as budget shares tend to change only slowly over time. The estimator of the systematic component of the ith relative price change, β_i^* , is just the change in this relative price, averaged over all T periods.

The estimators defined in (19) are <u>first-round</u> in that they are based on (16'), rather than (16). To derive the <u>second-round</u> estimators we use (19) to estimate var ζ_{it} and then apply weighted least squares. As the budget shares are fairly stable over time, as an approximation we replace $\bar{w}_{it}$ in (16) with $\bar{w}_i$ (= sample mean of $\bar{w}_{it}$) ,

(16") $\qquad \text{var } \zeta_{it} = \dfrac{\eta_t^2}{\bar{w}_i} .$

Substituting (19) in (18), the residual from that model is

$$\zeta_{it}'^* = \sqrt{\bar{w}_i}\left[Dp_{it} - \alpha_t^* - \frac{1}{T}\sum_{t=1}^{T}(Dp_{it} - \alpha_t^*)\right] = \sqrt{\bar{w}_i}\left[(Dp_{it} - \alpha_t^*) - (D\bar{p}_i - \bar{\alpha}^*)\right] ,$$

where $D\bar{p}_i = (1/T)\sum_{t=1}^{T}Dp_{it}$ and $\bar{\alpha}^* = (1/T)\sum_{t=1}^{T}\alpha_t^*$. Thus the sum over $i=1,\ldots,n$ of squared residuals is

$$(20) \qquad \theta_t^2 = \sum_{i=1}^{n}(\zeta_{it}'^*)^2 = \sum_{i=1}^{n}\bar{w}_i(Dp_{it} - \alpha_t^*)^2$$

$$+ \sum_{i=1}^{n}\bar{w}_i(D\bar{p}_i - \bar{\alpha}^*)^2$$

$$- 2\sum_{i=1}^{n}\bar{w}_i(Dp_{it} - \alpha_t^*)(D\bar{p}_i - \bar{\alpha}^*) .$$

The first term on the far right of this equation is the Divisia price variance with $\bar{w}_{it}$ replaced by $\bar{w}_i$; this measures the variability of relative prices within the period. The second term measures the variability of relative prices over the whole period. The third term is minus twice a weighted covariance, which measures the degree to

which relative price changes this period coincide with price changes over the whole period; this covariance is positive when, on average, the relative prices of those goods that increase (decrease) this period also increase (decrease) over the whole period.

The term θ_t^2 rises with increased relative price variability both within the period and over the whole period, as long as these two measures are of the same order of magnitude. To see this, let a_t^2 and b_t^2 be the first and second terms on the far right of (20), so that

$$\theta_t^2 = a_t^2 + b_t^2 - 2\rho_t a_t b_t,$$

where $\rho_t = \Sigma_{i=1}^{n} \bar{w}_i (Dp_{it} - \alpha_t^*)(D\bar{p}_i - \bar{\alpha}^*)/a_t b_t$ is the Divisia correlation coefficient. For a fixed value of ρ_t it then follows that

$$\frac{\partial \theta_t^2}{\partial a_t} = 2a_t - 2\rho_t b_t, \qquad \frac{\partial \theta_t^2}{\partial b_t} = 2b_t - 2\rho_t a_t,$$

which are both positive for $\rho_t < 1$ as long as a_t is not too different from b_t.

As $\theta_t^2/(n-1)$ is an unbiassed estimator of η_t^2, we divide both sides of (18) by θ_t to give

(21) $\qquad \tilde{y}_{it} = \alpha_t \tilde{x}_{it} + \beta_i \tilde{x}_{it} + \tilde{\zeta}_{it},$

where $\tilde{y}_{it} = \sqrt{\bar{w}_i} Dp_{it}/\theta_t$, $\tilde{x}_{it} = \sqrt{\bar{w}_i}/\theta_t$ and $\tilde{\zeta}_{it} = \sqrt{\bar{w}_i}\zeta_{it}/\theta_t$. It follows from (16") that $\tilde{\zeta}_{it}$ has a constant variance so that least squares can be applied to (21). In the Appendix we show that the LS estimators, with constraint (17) imposed, are

(22) $\qquad \alpha_t^{**} = \sum_{i=1}^{n} \bar{w}_i Dp_{it}, \qquad \beta_i^{**} = \sum_{t=1}^{T} \phi_t (Dp_{it} - \alpha_t^{**}),$

where

$$(23) \qquad \phi_t \ = \ \frac{1/\theta_t^2}{\sum\limits_{t=1}^{T} (1/\theta_t^2)} \ .$$

As can be seen, the estimator α_t^{**} of inflation is identical to that in (19). The reason is that the new weighting factor in (21), $1/\theta_t$, is the same for all i within the period t and we measure inflation during t by using only $\bar{w}_1, \ldots, \bar{w}_n$ and $Dp_{1t}, \ldots, Dp_{nt}$. The estimator β_i^{**} of the systematic component of the ith relative price change is now a weighted average of this price change over all T periods. In contrast, β_i^{*}, defined in (19), is an unweighted average. The weights in (22), $\phi_1, \ldots, \phi_T$, are inversely proportional to θ_t^2, which in turn is proportional to the error variance in period t; thus, less weight is accorded to those observations with a higher error variance.

In the Appendix, we show that the sampling variances of the estimators defined in (22) are

$$(24) \qquad var \ \alpha_t^{**} \ = \ \frac{\theta_t^2}{n-1} \ , \quad var \ \beta_i^{**} \ = \ \frac{1}{(n-1)\Sigma_{t=1}^{T} (1/\theta_t^2)} \left(\frac{1}{\bar{w}_i} - 1 \right) \ .$$

As can be seen, the sampling variance of inflation increases with θ^2, which in turn rises with relative price variability. Therefore, the same general result as before emerges here; the sampling variance of inflation will be higher the larger are the relative price movements. The sampling variance of the change in the ith relative price is proportional to the difference between two terms, (i) $1/\bar{w}_i$ which increases as the good becomes less important in the consumer's budget; and (ii) a constant term. Accordingly, var β_i^{**} increases as $\bar{w}_i$ falls. We also show in the Appendix that $cov(\alpha_s^{**}, \alpha_t^{**}) = 0$ for $s \neq t$ and $cov(\beta_i^{**}, \beta_j^{**}) = -\left[(n-1)\Sigma_{t=1}^{T} (1/\theta_t^2) \right]^{-1}$ for $i \neq j$.

Finally, we define the estimate of the mean rate of inflation over all T periods as $\bar{\alpha}^{**} = (1/T)\Sigma_{t=1}^{T}\alpha_t^{**}$. In the Appendix we show that

(25) $\text{var } \bar{\alpha}^{**} = \dfrac{1}{T^2} \sum_{t=1}^{T} \text{var } \alpha_t^{**} = \dfrac{1}{(n-1)T^2} \sum_{t=1}^{T} \theta_t^2$.

III. APPLICATIONS

In this section we first describe the data used in the subsequent computations. We then give a simple summary measure of the degree of relative price variability. Finally, we present the estimates of inflation and relative price changes and their standard errors, using equations (22), (24) and (25).

The Data

Our data are from the quarterly, six state capitals combined, consumer price index (CPI) in Australia. The whole period is from 1952 to 1981, which is broken into four sub-periods:

(i)   June 1952 - December 1966, with n = 5 commodity groups

        and T = 58 quarters.

(ii)   March 1967 - September 1973, n = 33 and T = 26.

(iii)   December 1973 - June 1976, n = 36 and T = 10.

(iv)   September 1976 - September 1981, n = 41 and T = 20.

Tables 1-4 list the commodity groups in each sub-period, the mean price log-changes and budget shares. The budget shares are the CPI weights, adjusted for changes in prices within the period. In order to use the whole period also, we aggregate the commodities in the latter three sub-periods into the same n = 5 groups. Thus for the combined period June 1952 - September 1981, T = 114. The aggregation procedure involves taking weighted means of the price log-changes within each group, the weights being arithmetic averages of the budget shares within the group. Table 5 gives the means for the whole period. Full details of the data and their sources are given in Clements and Izan (1984), available on request.

TABLE 1

CPI COMMODITY GROUPS, PRICE LOG-CHANGES AND BUDGET SHARES:

AUSTRALIA, JUNE 1952 - DECEMBER 1966

Number	Commodity group	Sample mean	
		Quarterly price log-change $D\bar{p}_i \times 100$	Arithmetic average of budget share $\bar{w}_i \times 100$
1.	Food	.62	32.87
2.	Clothing and drapery	.35	19.17
3.	Housing	1.12	11.17
4.	Household supplies and equipment	.28	12.42
5.	Miscellaneous	.74	24.37

Mean quarterly CPI log-change × 100 = .61

TABLE 2

CPI COMMODITY GROUPS, PRICE LOG-CHANGES AND BUDGET SHARES:

AUSTRALIA, MARCH 1967 - SEPTEMBER 1973

Number	Commodity group	Sample mean	
		Quarterly price log-change $D\bar{p}_i \times 100$	Arithmetic average of budget share $\bar{w}_i \times 100$
1.	Cereal products	1.34	4.11
2.	Dairy produce	.75	6.06
3.	Preserved fruit and vegetables	.46	1.27
4.	Potatoes and onions	3.67	.95
5.	Soft drink, ice cream and confectionery	1.42	4.33
6.	Beef	1.75	4.83
7.	Mutton	2.46	1.39
8.	Lamb	1.87	1.78
9.	Pork	.76	.76
10.	Processed meat	.96	2.25
11.	Other food (excluding meat)	.56	3.35
12.	Predominantly summerwear (clothing and drapery)	1.01	1.99
13.	Predominantly winterwear (clothing and drapery)	1.19	3.83
14.	Predominantly non-seasonal (clothing and drapery)	.92	6.23
15.	Footwear	1.69	2.63
16.	Privately-owned dwellings (rental)	1.64	4.90
17.	Government-owned dwellings (rental)	1.13	.87
18.	Local government rates and charges	1.99	2.86
19.	House price, repairs and maintenance	1.32	5.69
20.	Fuel and light	.60	3.91
21.	Household appliances	.10	2.70
22.	Furniture and floorcoverings	1.25	2.02
23.	Household utensils, sundries and stationery	.88	1.82
24.	Personal requisites and proprietary medicines	1.12	2.08
25.	Postal and telephone services	1.32	1.09
26.	Fares	1.80	2.80
27.	Goods (motoring)	.70	5.97
28.	Services and charges (motoring)	1.88	2.62
29.	Cigarettes and tobacco	1.19	3.70
30.	Beer	1.27	3.80
31.	Newspapers and magazines	1.93	1.30
32.	Radio and TV operation	1.25	1.16
33.	Other services	1.83	4.98

Mean quarterly CPI log-change × 100 = 1.28

TABLE 3

CPI COMMODITY GROUPS, PRICE LOG-CHANGES AND BUDGET SHARES:

AUSTRALIA, DECEMBER 1973 - JUNE 1976

		Sample mean	
Number	Commodity group	Quarterly price log-change $D\bar{p}_i \times 100$	Arithmetic average of budget share $\bar{w}_i \times 100$
1.	Cereal products	4.23	3.09
2.	Dairy produce	3.20	4.63
3.	Preserved fruit and vegetables	3.83	1.16
4.	Potatoes and onions	-1.29	1.10
5.	Soft drink, ice cream and confectionery	3.92	4.09
6.	Beef	-1.25	3.77
7.	Mutton	-1.37	.41
8.	Lamb	.78	1.92
9.	Pork	3.78	.85
10.	Processed meat	2.44	2.90
11.	Other food (excluding meat)	3.12	2.46
12.	Snacks and take-aways	3.61	.91
13.	Predominantly summerwear (clothing and drapery)	4.33	2.78
14.	Predominantly winterwear (clothing and drapery)	5.44	2.25
15.	Predominantly non-seasonal (clothing and drapery)	3.58	5.12
16.	Footwear	4.34	2.31
17.	Privately-owned dwellings (rental)	3.23	6.33
18.	Government-owned dwellings (rental)	4.75	.60
19.	Local government rates and charges	4.86	2.45
20.	House price, repairs and maintenance	4.62	5.43
21.	Fuel and light	3.64	3.00
22.	Household appliances	1.69	2.08
23.	Furniture and floorcoverings	4.20	1.98
24.	Household utensils, sundries and stationery	3.41	1.96
25.	Personal requisites and proprietary medicines	3.27	2.29
26.	Postal and telephone services	5.42	1.49
27.	Fares	2.90	1.80
28.	Goods (motoring)	3.33	9.16
29.	Services and charges (motoring)	5.94	4.15
30.	Cigarettes and tobacco	4.35	3.65
31.	Beer	4.57	4.60
32.	Wines and spirits	4.10	1.25
33.	Newspapers and magazines	4.13	1.18
34.	Recreational goods and services	2.83	1.72
35.	Other services	5.96	1.59
36.	Health services	-3.32	3.57

Mean quarterly CPI log-change × 100   =   3.36

TABLE 4

CPI COMMODITY GROUPS, PRICE LOG-CHANGES AND BUDGET SHARES:

AUSTRALIA, SEPTEMBER 1976 - SEPTEMBER 1981

		Sample mean	
Number	Commodity group	Quarterly price log-change $D\bar{p}_i \times 100$	Arithmetic average of budget share $\bar{w}_i \times 100$
1.	Cereal products	2.21	2.26
2.	Dairy produce	2.37	2.07
3.	Processed fruit and vegetables	1.76	.81
4.	Fresh fruit and vegetables	3.50	2.08
5.	Soft drinks, ice cream and confectionery	2.67	2.09
6.	Beef and veal	4.11	2.56
7.	Lamb and mutton	3.22	1.03
8.	Pork	2.29	.25
9.	Other meat	5.30	1.36
10.	Fish	2.78	.47
11.	Poultry	2.10	.49
12.	Other food (excluding meat)	2.18	2.17
13.	Meals out and take-away	2.47	4.39
14.	Men's and boys' wear	2.03	2.85
15.	Women's and girls' wear	1.95	4.46
16.	Piecegoods	2.52	.55
17.	Footwear	2.07	1.59
18.	Clothing service	2.54	.53
19.	Privately-owned dwellings (rental)	1.84	4.48
20.	Government-owned dwellings (rental)	3.43	.54
21.	Local government rates and charges	2.64	1.80
22.	House price, repairs and maintenance	2.04	6.12
23.	Fuel and light	2.86	2.25
24.	Household appliances	1.23	1.66
25.	Furniture and floorcoverings	1.81	3.03
26.	Household utensils and tools	2.36	1.46
27.	Household supplies and services	2.44	3.22
28.	Drapery	2.45	1.10
29.	Postal and telephone services	.46	1.22
30.	Public transport fares	2.48	2.01
31.	Motor vehicle purchase	1.68	5.14
32.	Motor vehicle operation	2.57	10.78
33.	Cigarettes and tobacco	1.81	2.99
34.	Alcohol	2.14	6.66
35.	Books, newspapers and magazines	3.28	1.51
36.	Other recreational goods	.94	2.80
37.	Other recreational services	2.40	2.28
38.	Holiday accommodation	2.43	.92
39.	Personal care goods	2.30	1.73
40.	Personal care services	2.69	.70
41.	Health services	6.19	3.63

Mean quarterly CPI log-change × 100 = 2.36

TABLE 5

CPI COMMODITY GROUPS, PRICE LOG-CHANGES AND BUDGET SHARES:

AUSTRALIA, JUNE 1952 - SEPTEMBER 1981

Number	Commodity group	Sample mean	
		Quarterly price log-change $D\bar{p}_i \times 100$	Arithmetic average of budget share $\bar{w}_i \times 100$
1.	Food	1.32	30.07
2.	Clothing and drapery	1.19	16.14
3.	Housing	1.65	12.51
4.	Household supplies and equipment	.98	12.51
5.	Miscellaneous	1.42	28.77

Mean quarterly CPI log-change × 100   =   1.31

The Importance of Relative Price Changes

In this sub-section we introduce a simple summary measure of the extent to which relative price changes in one period are similar to those in another. Consider the n relative price changes in period s, $Dp_{1s} - DP_s, \ldots, Dp_{ns} - DP_s$, and those in period t, $Dp_{1t} - DP_t, \ldots, Dp_{nt} - DP_t$. The extent to which relative price changes coincide in these two periods can be measured by

$$(26) \qquad \Pi_{st} = \sum_{i=1}^{n} \bar{w}_{ist} (Dp_{is} - DP_s)(Dp_{it} - DP_t),$$

where $\bar{w}_{ist} = \frac{1}{2}(\bar{w}_{is} + \bar{w}_{it})$. This Π_{st} is positive when the relative price changes in period s are, on average, in the same direction as those in t. Thus $\Pi_{st} > 0$ when the changes in relative prices in s are sustained into period t. We shall refer to (26) as the Divisia price covariance between periods s and t. Note that (26) for s = t is the Divisia price variance, defined in (13). The correlation coefficient corresponding to (26) is

$$(27) \qquad \rho_{st} = \frac{\Pi_{st}}{\sqrt{\Pi_{ss}\Pi_{tt}}}.$$

We use our data to compute the Divisia correlations (27) for s,t=1,...,T and the results are summarized in Table 6 in the form of frequency distributions of the ρ_{st}'s, s=1,...,T, t=s+1,...,T. As can be seen, in the first sub-period 60 percent of the coefficients are positive, 51 percent in the second and third, 43 percent in the fourth and 55 percent in the whole period. Taken as a whole, there is a rather surprising lack of sustained changes in relative prices.

TABLE 6

FREQUENCY DISTRIBUTIONS OF DIVISIA PRICE CORRELATION COEFFICIENTS:

AUSTRALIA, JUNE 1952 - SEPTEMBER 1981

Range	June 1952 to December 1966 %	March 1967 to September 1973 %	December 1973 to June 1976 %	September 1976 to September 1981 %	June 1952 to September 1981 %
$-1.0 \leq \rho_{st} \leq -.5$	15.8	14.2	8.9	3.2	15.9
$-.5 < \rho_{st} \leq 0$	24.1	35.4	40.0	53.7	28.7
$0 < \rho_{st} \leq .5$	29.4	34.2	48.9	37.9	32.3
$.5 < \rho_{st} \leq 1.0$	30.7	16.3	2.2	5.3	23.1

Estimates of Inflation and Relative Price Changes

The results of applying equations (22), (24) and (25) to the
data of the four sub-periods, as well as the whole period are given
in Tables 7-16. In the first sub-period, the mean quarterly rate
of inflation is estimated to be about .61 percent, with a standard error
of .05 percent. This point estimate is exactly equal to the mean
CPI log-change. Looking at Table 8, the relative price of clothing
and drapery is estimated to have declined by .13 percent per quarter
during the first sub-period, a decline which is significantly different
from zero (the t-value is 2.6). The relative price of housing increased
significantly, while that of household supplies fell significantly.
The relative prices of the other two commodities, food and miscellaneous,
were constant.

As a way of summarizing the results for the four sub-periods,
we present in Figure 1 a scattergram of the estimate of inflation in
each quarter against the corresponding standard error. As can be seen,
the standard error increases along with inflation. In other words,
the higher is inflation the more difficult it is to measure it precisely;
it becomes more difficult in the absolute sense that the standard error
tends to rise. However, the increase in the standard error tends to
be less than the increase in inflation. Consequently, in a relative
sense higher inflation tends to be estimated more precisely. This is
illustrated in Figure 2, where the estimated inflation rates are plotted
against the corresponding t-values. There is a fairly distinct positive
relationship between these two variables, which confirms the tendency
for the standard error to rise less rapidly than inflation.

As a further way of summarizing the results for the four
sub-periods, in Figure 3 we plot against time estimated inflation and
the 95 percent confidence band constructed using the normal distribution,

TABLE 7

ESTIMATES OF INFLATION AND CPI LOG-CHANGES:

AUSTRALIA, JUNE 1952 - DECEMBER 1966

(Standard errors in parentheses)

Year and quarter	Estimate of inflation $\alpha_t^{**} \times 100$		CPI log-change $\times 100$	Year and quarter	Estimate of inflation $\alpha_t^{**} \times 100$		CPI log-change $\times 100$
1952 S	1.84	(.21)	1.73	1960 M	.85	(.26)	.84
D	.43	(.67)	.50	J	1.70	(.32)	1.75
1953 M	1.06	(.29)	1.00	S	1.20	(.55)	1.15
J	1.01	(.34)	.99	D	.67	(.17)	.65
S	.67	(.51)	.69	1961 M	.74	(.27)	.73
D	-.42	(.25)	-.39	J	.60	(.18)	.64
1954 M	.46	(.13)	.39	S	-.18	(.33)	-.16
J	-.01	(.17)	.00	D	-.40	(.64)	-.40
S	-.19	(.20)	-.29	1962 M	-.18	(.16)	-.16
D	.29	(.19)	.39	J	-.10	(.26)	-.08
1955 M	.68	(.34)	.68	S	.24	(.09)	.24
J	.69	(.26)	.68	D	.17	(.08)	.08
S	.90	(.28)	.86	1963 M	.01	(.08)	.08
D	1.33	(.58)	1.33	J	.32	(.09)	.32
1956 M	.96	(.37)	.94	S	.15	(.28)	.16
J	3.08	(1.14)	3.04	D	-.07	(.17)	-.08
S	2.40	(.82)	2.42	1964 M	.62	(.21)	.64
D	.06	(1.05)	.09	J	.98	(.38)	.95
1957 M	-.26	(.78)	-.35	S	1.15	(.41)	1.17
J	.87	(.22)	.97	D	1.19	(.36)	1.16
S	.21	(.24)	.18	1965 M	.65	(.18)	.69
D	-.12	(.40)	-.18	J	.94	(.43)	.91
1958 M	.47	(.40)	.53	S	1.05	(.50)	1.05
J	.44	(.31)	.44	D	1.16	(.64)	1.27
S	.11	(.17)	.09	1966 M	.19	(.40)	.15
D	.73	(.11)	.78	J	.80	(.25)	.81
1959 M	.54	(.34)	.43	S	.46	(.26)	.44
J	.38	(.12)	.43	D	.90	(.14)	.94
S	.46	(.13)	.43				
D	.58	(.13)	.59	Mean	.61	(.05)	.61

TABLE 8

ESTIMATES OF RELATIVE PRICE CHANGES:

AUSTRALIA, JUNE 1952 — DECEMBER 1966

(Standard errors in parentheses)

Commodity group	Estimate of relative price change $\beta_i^{**} \times 100$
1. Food	.00 (.04)
2. Clothing and drapery	−.13 (.05)
3. Housing	.48 (.07)
4. Household supplies and equipment	−.24 (.07)
5. Miscellaneous	.00 (.04)

TABLE 9

ESTIMATES OF INFLATION AND CPI LOG-CHANGES:

AUSTRALIA, MARCH 1967 - SEPTEMBER 1973

(Standard errors in parentheses)

Year and quarter	Estimate of inflation $\alpha_t^{**} \times 100$		CPI log-change $\times$ 100	Year and quarter	Estimate of inflation $\alpha_t^{**} \times 100$		CPI log-change $\times$ 100
1967 J	1.18	(.27)	1.19	1971 M	1.05	(.47)	1.05
S	1.32	(.36)	1.37	J	1.70	(.25)	1.72
D	.41	(.43)	.29	S	1.64	(.49)	1.69
1968 M	.45	(.15)	.39	D	2.31	(.54)	2.32
J	.76	(.19)	.77	1972 M	1.05	(.28)	1.14
S	.51	(.33)	.38	J	.87	(.36)	.89
D	.96	(.47)	1.05	S	1.35	(.38)	1.36
1969 M	.65	(.65)	.66	D	1.19	(.36)	1.18
J	.72	(.43)	.75	1973 M	2.06	(.53)	2.09
S	.52	(.28)	.56	J	3.17	(.59)	3.24
D	.85	(.37)	.83	S	3.28	(.71)	3.57
1970 M	.99	(.22)	1.01				
J	1.24	(.29)	1.27				
S	.66	(.53)	.63				
D	1.85	(.39)	1.86	Mean	1.26	(.08)	1.28

TABLE 10

ESTIMATES OF RELATIVE PRICE CHANGES:

AUSTRALIA, MARCH 1967 – SEPTEMBER 1973

(Standard errors in parentheses)

	Commodity group	Estimate of relative price change $\beta_i^{**} \times 100$	
1.	Cereal products	.33	(.30)
2.	Dairy produce	-.37	(.25)
3.	Preserved fruit and vegetables	-.52	(.55)
4.	Potatoes and onions	2.36	(.64)
5.	Soft drink, ice cream and confectionery	.22	(.29)
6.	Beef	.31	(.28)
7.	Mutton	.28	(.53)
8.	Lamb	.68	(.47)
9.	Pork	-.89	(.72)
10.	Processed meat	-.44	(.41)
11.	Other food (excluding meat)	-.47	(.34)
12.	Predominantly summerwear (clothing and drapery)	-.40	(.44)
13.	Predominantly winterwear (clothing and drapery)	.15	(.31)
14.	Predominantly non-seasonal (clothing and drapery)	-.30	(.24)
15.	Footwear	.19	(.38)
16.	Privately-owned dwellings (rental)	.51	(.28)
17.	Government-owned dwellings (rental)	-.17	(.67)
18.	Local government rates and charges	.77	(.37)
19.	House price, repairs and maintenance	.03	(.26)
20.	Fuel and light	-.47	(.31)
21.	Household appliances	-1.05	(.38)
22.	Furniture and floorcoverings	-.13	(.44)
23.	Household utensils, sundries and stationery	-.35	(.46)
24.	Personal requisites and proprietary medicines	.00	(.43)
25.	Postal and telephone services	-.33	(.60)
26.	Fares	-.11	(.37)
27.	Goods (motoring)	-.45	(.25)
28.	Services and charges (motoring)	.89	(.38)
29.	Cigarettes and tobacco	-.31	(.32)
30.	Beer	-.02	(.32)
31.	Newspapers and magazines	.77	(.55)
32.	Radio and TV operation	-.45	(.58)
33.	Other services	.53	(.28)

TABLE 11

ESTIMATES OF INFLATION AND CPI LOG-CHANGES:

AUSTRALIA, DECEMBER 1973 - JUNE 1976

(Standard errors in parentheses)

Year and quarter	Estimate of inflation $\alpha_t^{**} \times 100$		CPI log-change $\times$ 100
1974 M	2.37	(.64)	2.39
J	4.00	(.65)	3.97
S	5.03	(.77)	5.00
D	3.81	(.96)	3.70
1975 M	3.31	(1.07)	3.51
J	3.42	(.48)	3.44
S	.70	(1.63)	.77
D	5.07	(1.37)	5.41
1976 M	3.03	(.91)	2.93
J	2.47	(.64)	2.50
Mean	3.32	(.31)	3.36

TABLE 12

ESTIMATES OF RELATIVE PRICE CHANGES:

AUSTRALIA, DECEMBER 1973 - JUNE 1976

(Standard errors in parentheses)

	Commodity group	Estimate of relative price change $\beta_i^{**} \times 100$	
1.	Cereal products	.59	(1.35)
2.	Dairy produce	-.48	(1.09)
3.	Preserved fruit and vegetables	.52	(2.22)
4.	Potatoes and onions	-4.90	(2.28)
5.	Soft drink, ice cream and confectionery	.46	(1.16)
6.	Beef	-4.16	(1.21)
7.	Mutton	-3.26	(3.76)
8.	Lamb	-.25	(1.71)
9.	Pork	.55	(2.59)
10.	Processed meat	-.15	(1.39)
11.	Other food (excluding meat)	-.12	(1.51)
12.	Snacks and take-aways	.37	(2.50)
13.	Predominantly summerwear (clothing and drapery)	-1.34	(1.42)
14.	Predominantly winterwear (clothing and drapery)	5.69	(1.58)
15.	Predominantly non-seasonal (clothing and drapery)	.12	(1.03)
16.	Footwear	.96	(1.56)
17.	Privately-owned dwellings (rental)	-.16	(.92)
18.	Government-owned dwellings (rental)	-.17	(3.09)
19.	Local government rates and charges	2.56	(1.52)
20.	House price, repairs and maintenance	1.38	(1.00)
21.	Fuel and light	.48	(1.36)
22.	Household appliances	-1.69	(1.65)
23.	Furniture and floorcoverings	.76	(1.69)
24.	Household utensils, sundries and stationery	.00	(1.70)
25.	Personal requisites and proprietary medicines	-.01	(1.57)
26.	Postal and telephone services	-1.00	(1.95)
27.	Fares	-1.61	(1.77)
28.	Goods (motoring)	-.39	(.76)
29.	Services and charges (motoring)	1.83	(1.15)
30.	Cigarettes and tobacco	-.37	(1.23)
31.	Beer	-.09	(1.09)
32.	Wines and spirits	.49	(2.13)
33.	Newspapers and magazines	.56	(2.20)
34.	Recreational goods and services	-.21	(1.82)
35.	Other services	2.30	(1.89)
36.	Health services	-1.60	(1.25)

TABLE 13

ESTIMATES OF INFLATION AND CPI LOG-CHANGES:

AUSTRALIA, SEPTEMBER 1976 – SEPTEMBER 1981

(Standard errors in parentheses)

Year and quarter	Estimate of inflation $\alpha_t^{**} \times 100$		CPI log-change $\times$ 100
1976 D	7.64	(3.35)	5.82
1977 M	2.10	(.49)	2.25
J	2.36	(.33)	2.34
S	1.94	(.34)	1.94
D	2.37	(.31)	2.32
1978 M	1.27	(.39)	1.29
J	2.04	(.33)	2.04
S	1.94	(.22)	1.92
D	2.28	(1.52)	2.23
1979 M	1.78	(.48)	1.69
J	2.56	(.77)	2.63
S	2.27	(.30)	2.27
D	3.05	(.54)	2.97
1980 M	2.15	(.35)	2.16
J	2.74	(.33)	2.75
S	1.67	(.34)	1.86
D	2.05	(.39)	2.05
1981 M	2.33	(.29)	2.36
J	2.15	(.31)	2.18
S	2.05	(.43)	2.10
Mean	2.45	(.20)	2.36

TABLE 14

ESTIMATES OF RELATIVE PRICE CHANGES:

AUSTRALIA, SEPTEMBER 1976 - SEPTEMBER 1981

(Standard errors in parentheses)

	Commodity group	Estimate of relative price change $\beta_i^{**} \times 100$	
1.	Cereal products	.17	(.54)
2.	Dairy produce	.23	(.56)
3.	Processed fruit and vegetables	-.35	(.91)
4.	Fresh fruit and vegetables	1.46	(.56)
5.	Soft drinks, ice cream and confectionery	.66	(.56)
6.	Beef and veal	1.35	(.51)
7.	Lamb and mutton	.34	(.80)
8.	Pork	-.26	(1.63)
9.	Other meat	.38	(.70)
10.	Fish	-.02	(1.19)
11.	Poultry	-.28	(1.17)
12.	Other food (excluding meat)	.11	(.55)
13.	Meals out and take-away	.29	(.38)
14.	Men's and boys' wear	-.21	(.48)
15.	Women's and girls' wear	-.67	(.38)
16.	Piecegoods	.38	(1.10)
17.	Footwear	-.01	(.64)
18.	Clothing service	.54	(1.12)
19.	Privately-owned dwellings (rental)	-.39	(.38)
20.	Government-owned dwellings (rental)	1.97	(1.12)
21.	Local government rates and charges	-.07	(.61)
22.	House price, repairs and maintenance	-.14	(.32)
23.	Fuel and light	.65	(.54)
24.	Household appliances	-.81	(.63)
25.	Furniture and floorcoverings	-.30	(.46)
26.	Household utensils and tools	.38	(.67)
27.	Household supplies and services	.31	(.45)
28.	Drapery	.43	(.78)
29.	Postal and telephone services	-1.63	(.74)
30.	Public transport fares	.40	(.57)
31.	Motor vehicle purchase	-.11	(.35)
32.	Motor vehicle operation	.28	(.24)
33.	Cigarettes and tobacco	-1.01	(.47)
34.	Alcohol	-.65	(.31)
35.	Books, newspapers and magazines	.88	(.66)
36.	Other recreational goods	-1.22	(.48)
37.	Other recreational services	.17	(.54)
38.	Holiday accommodation	.07	(.85)
39.	Personal care goods	.27	(.62)
40.	Personal care services	.50	(.98)
41.	Health services	-.02	(.42)

TABLE 15

ESTIMATES OF INFLATION AND CPI LOG-CHANGES:

AUSTRALIA, JUNE 1952 – SEPTEMBER 1981

(Standard errors in parentheses)

Year and quarter	Estimate of inflation $\alpha_t^{**} \times 100$		CPI log-change $\times 100$	Year and quarter	Estimate of inflation $\alpha_t^{**} \times 100$		CPI log-change $\times 100$
1952 S	1.90	(.25)	1.73	1967 S	1.19	(.49)	1.37
D	.51	(.67)	.50	D	.40	(.56)	.29
1953 M	1.04	(.32)	1.00	1968 M	.40	(.12)	.39
J	.95	(.35)	.99	J	.72	(.19)	.77
S	.63	(.50)	.69	S	.48	(.31)	.38
D	-.42	(.28)	-.39	D	1.00	(.42)	1.05
1954 M	.48	(.12)	.39	1969 M	.59	(.26)	.66
J	.02	(.20)	.00	J	.74	(.17)	.75
S	-.18	(.19)	-.29	S	.57	(.25)	.56
D	.27	(.20)	.39	D	.81	(.19)	.83
1955 M	.67	(.33)	.68	1970 M	1.00	(.19)	1.01
J	.66	(.25)	.68	J	1.22	(.12)	1.27
S	.91	(.28)	.86	S	.62	(.38)	.63
D	1.48	(.62)	1.33	D	1.88	(.74)	1.86
1956 M	.97	(.41)	.94	1971 M	1.05	(.49)	1.05
J	3.30	(1.19)	3.04	J	1.72	(.42)	1.72
S	2.42	(.80)	2.42	S	1.67	(.59)	1.69
D	.21	(1.03)	.09	D	2.31	(.72)	2.32
1957 M	-.19	(.75)	-.35	1972 M	1.10	(.65)	1.14
J	.86	(.21)	.97	J	.90	(.40)	.89
S	.23	(.22)	.18	S	1.32	(.34)	1.36
D	-.09	(.38)	-.18	D	1.17	(.25)	1.18
1958 M	.43	(.39)	.53	1973 M	2.09	(.88)	2.09
J	.41	(.27)	.44	J	3.25	(1.02)	3.24
S	.13	(.16)	.09	S	3.53	(.66)	3.57
D	.76	(.12)	.78	D	-	-	-
1959 M	.52	(.33)	.43	1974 M	2.41	(.68)	2.39
J	.39	(.13)	.43	J	4.17	(2.19)	3.97
S	.45	(.11)	.43	S	4.84	(1.52)	5.00
D	.58	(.10)	.59	D	3.61	(1.47)	3.70
1960 M	.84	(.25)	.84	1975 M	3.19	(1.38)	3.51
J	1.76	(.35)	1.75	J	3.43	(.44)	3.44
S	1.14	(.55)	1.15	S	.92	(1.68)	.77
D	.68	(.19)	.65	D	5.33	(1.41)	5.41
1961 M	.72	(.28)	.73	1976 M	2.95	(1.26)	2.93
J	.60	(.19)	.64	J	2.45	(1.75)	2.50
S	-.14	(.32)	-.16	S	-	-	-
D	-.32	(.63)	-.40	D	6.38	(2.15)	5.82
1962 M	-.17	(.15)	-.16	1977 M	1.95	(.45)	2.25
J	-.05	(.27)	-.08	J	2.55	(1.33)	2.34
S	.24	(.07)	.24	S	2.17	(2.34)	1.94
D	.19	(.09)	.08	D	2.41	(.31)	2.32
1963 M	.03	(.06)	.08	1978 M	1.16	(.62)	1.29
J	.32	(.10)	.32	J	2.18	(.30)	2.04
S	.16	(.27)	.16	S	1.98	(.39)	1.92
D	-.06	(.17)	-.08	D	2.29	(.58)	2.23
1964 M	.61	(.20)	.64	1979 M	1.82	(.46)	1.69
J	.92	(.38)	.95	J	2.91	(.82)	2.63
S	1.20	(.42)	1.17	S	2.34	(.36)	2.27
D	1.28	(.38)	1.16	D	2.67	(.69)	2.97
1965 M	.65	(.17)	.69	1980 M	2.33	(.67)	2.16
J	.88	(.43)	.91	J	2.72	(.27)	2.75
S	1.01	(.49)	1.05	S	1.92	(.73)	1.86
D	1.33	(.69)	1.27	D	2.20	(.31)	2.05
1966 M	.28	(.40)	.15	1981 M	2.21	(.44)	2.36
J	.77	(.26)	.81	J	2.12	(.29)	2.18
S	.53	(.28)	.44	S	2.20	(.49)	2.10
D	.92	(.12)	.94				
1967 M	-	-	-				
J	1.14	(.60)	1.19	Mean	1.33	(.06)	1.31

TABLE 16

ESTIMATES OF RELATIVE PRICE CHANGES:

AUSTRALIA, JUNE 1952 - SEPTEMBER 1981

(Standard errors in parentheses)

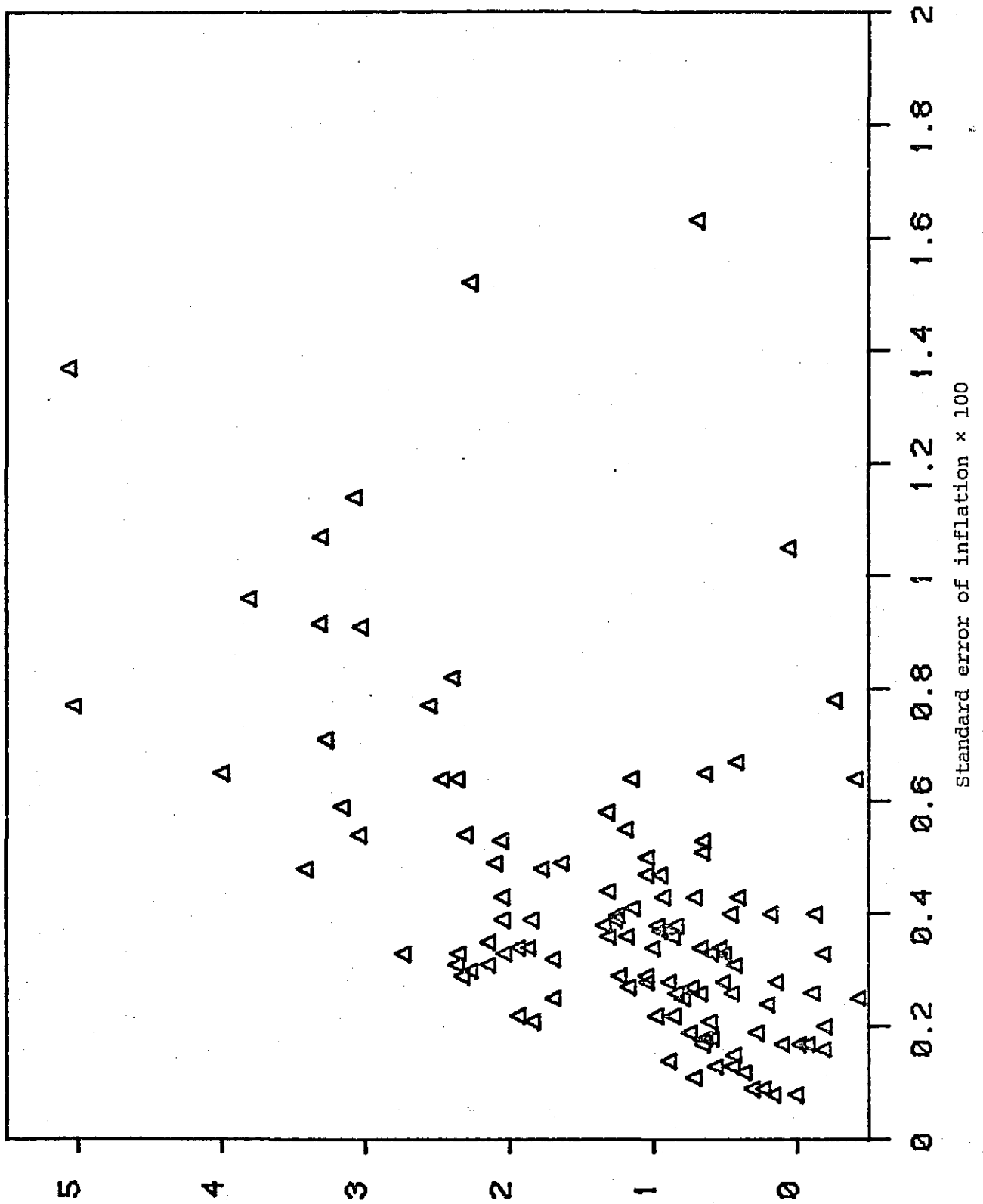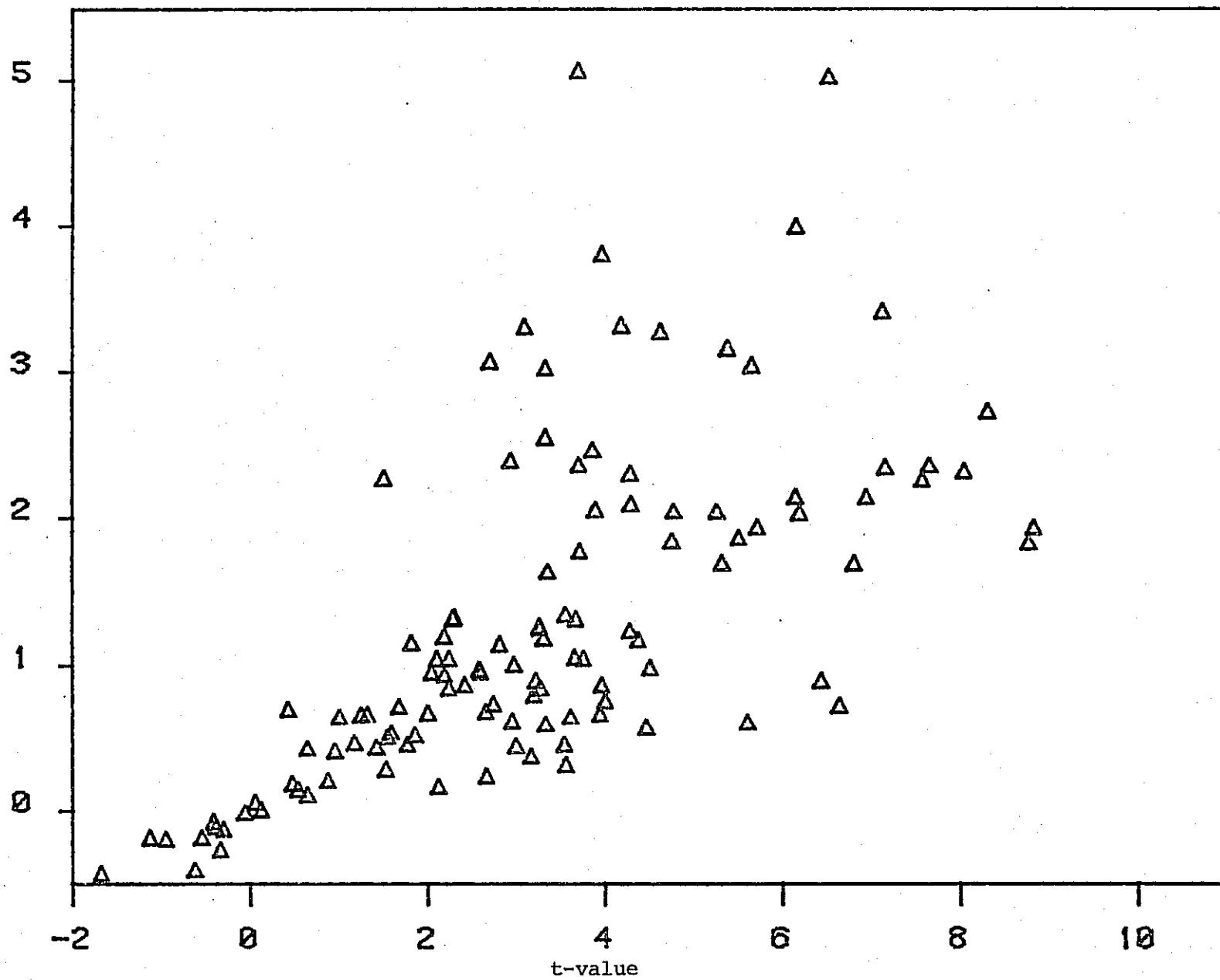| Commodity group	Estimate of relative price change $\beta_i^{**} \times 100$
1. Food	.01 (.03)
2. Clothing and drapery	-.11 (.05)
3. Housing	.40 (.06)
4. Household supplies and equipment	-.27 (.06)
5. Miscellaneous	.00 (.03)

Figure 1

Figure 2

YEAR
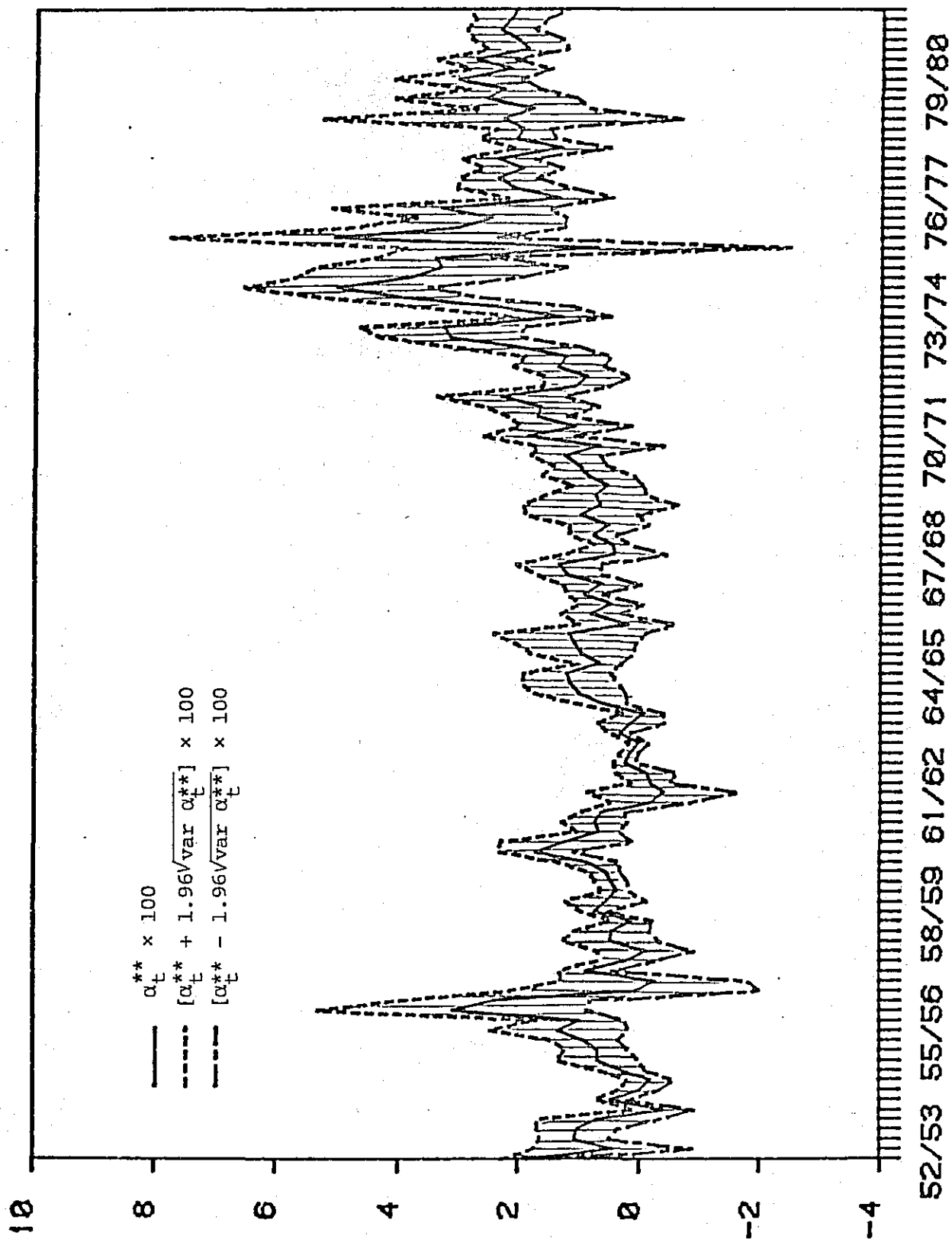
Figure 3

$\alpha_t^{**} \pm 1.96\sqrt{\text{var } \alpha_t^{**}}$. The jump in inflation and the increase in the

width of the confidence band in the mid 1950s and 70s is to be noted.

Compared to the first sub-period, the second, third and

fourth sub-periods have more commodity groups and fewer observations.

It is therefore not surprising that many of the relative price change

estimates for the latter three sub-periods are not significant.

This is also consistent with the results of Table 6 which show a

general absence of sustained changes in relative prices. Moreover,

there is a good deal of instability of the relative price changes,

from one sub-period to another. This can be seen in Figures 4-6, where

we plot the β_i^{**}'s from one sub-period against the corresponding

estimates from another. To construct these figures we have aggregated

the β_i^{**}'s to yield the same number of goods in the two sub-periods.

The aggregation procedure involves taking weighted means of the β_i^{**}'s,

the weights being proportional to the means of the arithmetic averages

of the budget shares.[1/]

The results for the whole period with 5 commodity groups

are summarized in Figures 7-9. The general picture which emerges is

the same as before, except that standard errors are somewhat higher,

t-values lower and the 95 percent confidence band wider, as is to be
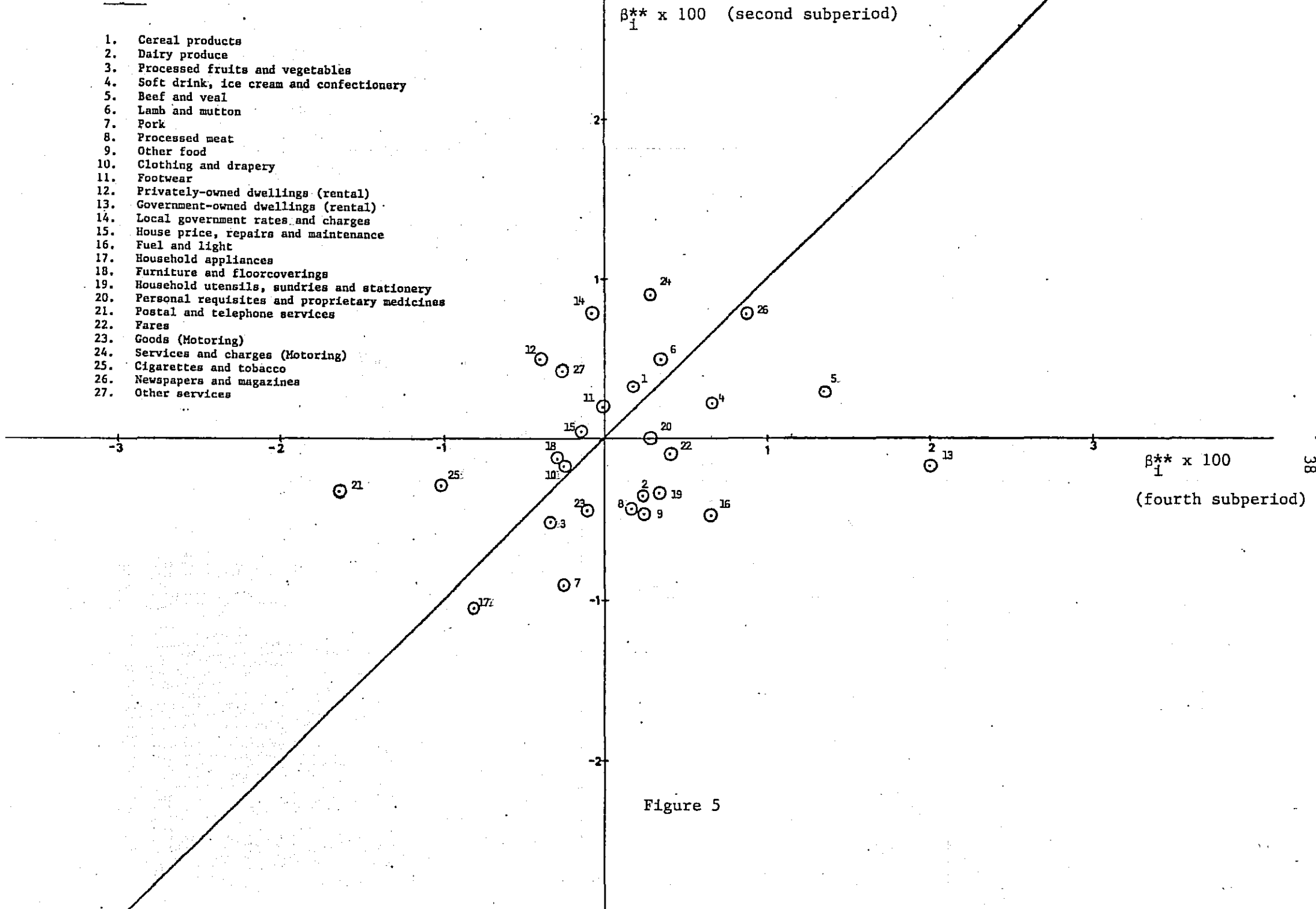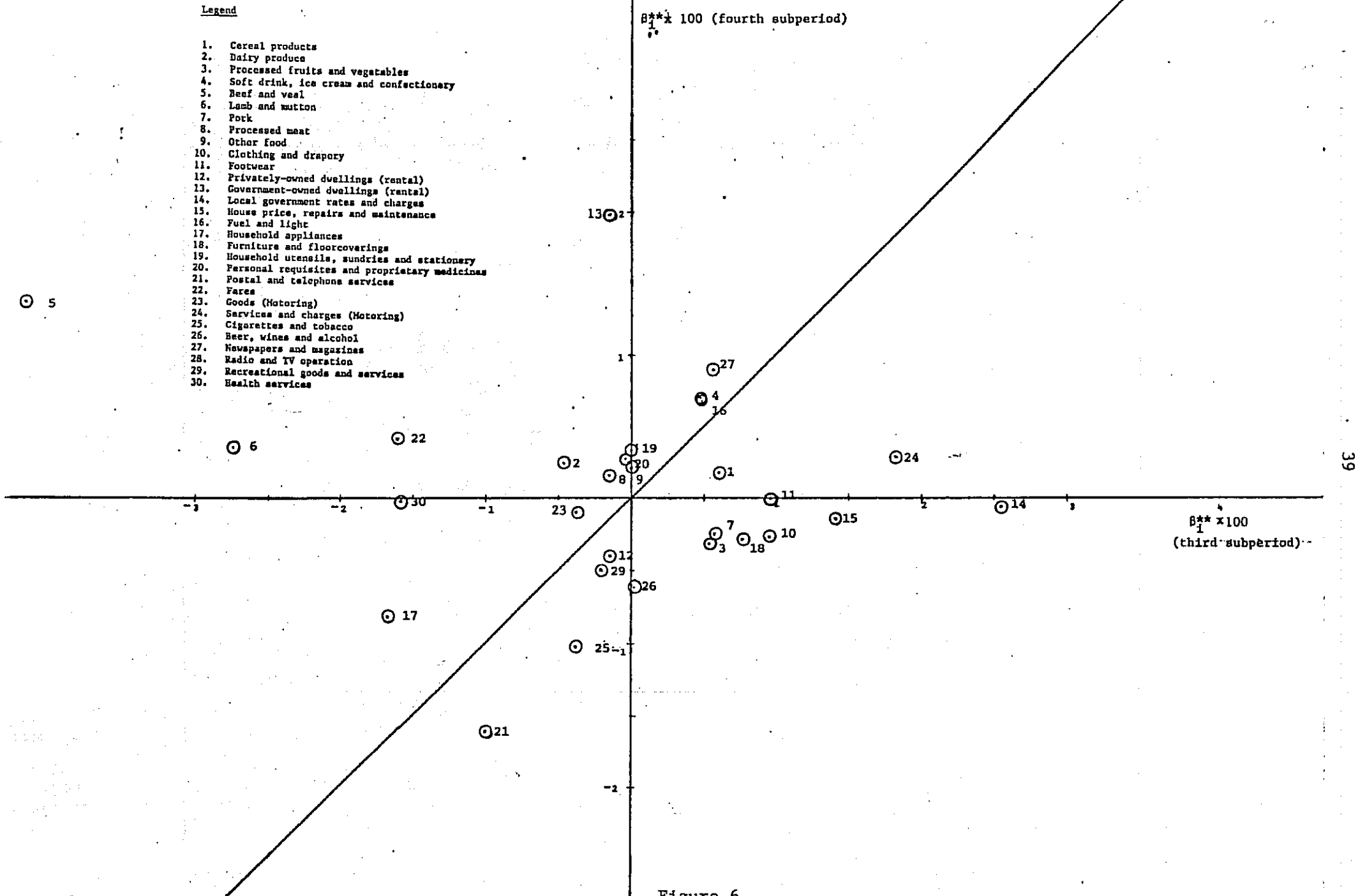
expected with fewer commodity groups.

Figure 4

Figure 5

1. Cereal products
2. Dairy produce
3. Processed fruits and vegetables
4. Soft drink, ice cream and confectionery
5. Beef and veal
6. Lamb and mutton
7. Pork
8. Processed meat
9. Other food
10. Clothing and drapery
11. Footwear
12. Privately-owned dwellings (rental)
13. Government-owned dwellings (rental)
14. Local government rates and charges
15. House price, repairs and maintenance
16. Fuel and light
17. Household appliances
18. Furniture and floorcoverings
19. Household utensils, sundries and stationery
20. Personal requisites and proprietary medicines
21. Postal and telephone services
22. Fares
23. Goods (Motoring)
24. Services and charges (Motoring)
25. Cigarettes and tobacco
26. Newspapers and magazines
27. Other services

Figure 6

Estimate of
inflation
$\alpha^{**}_t \times 100$
(5 commodity
groups)

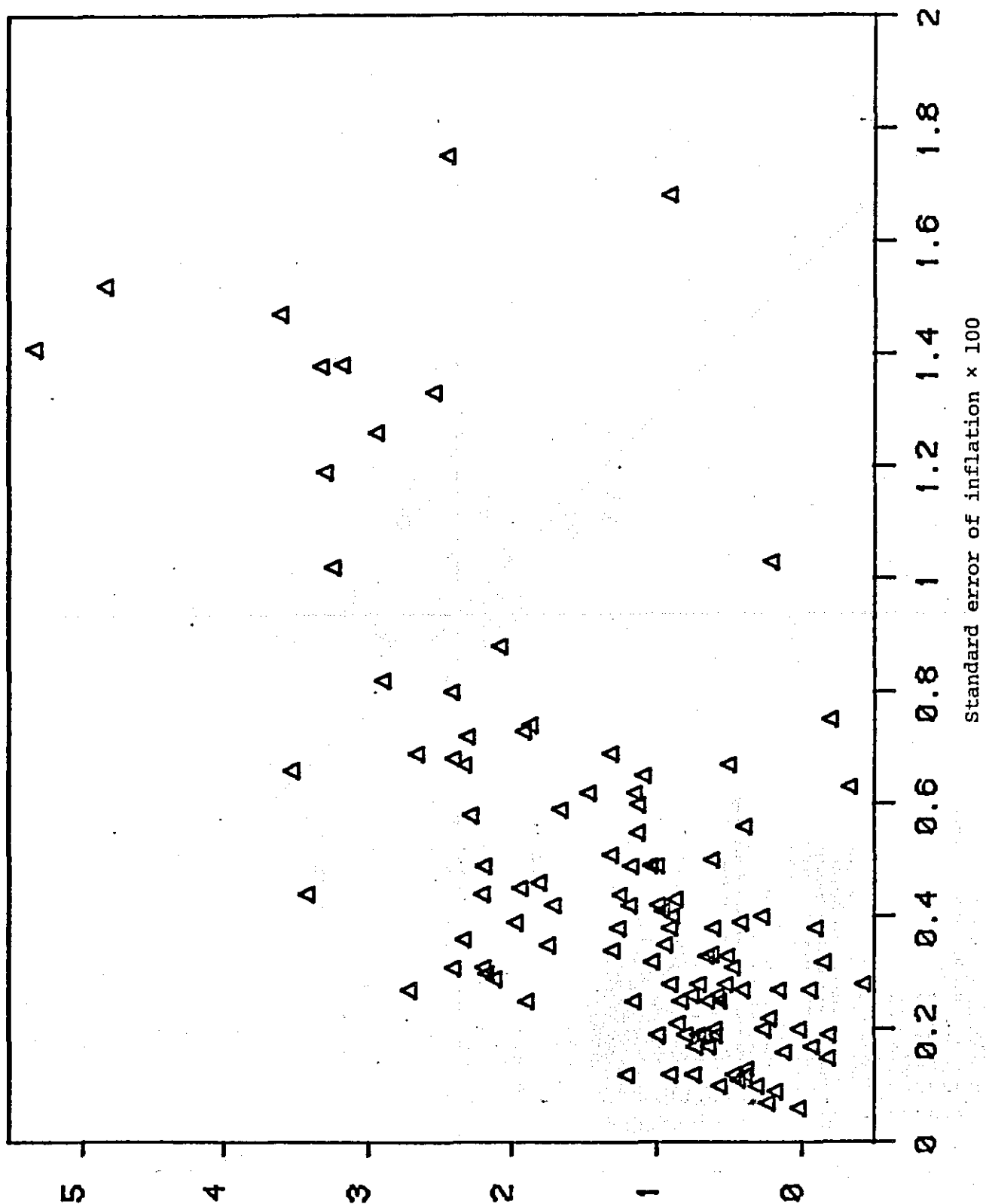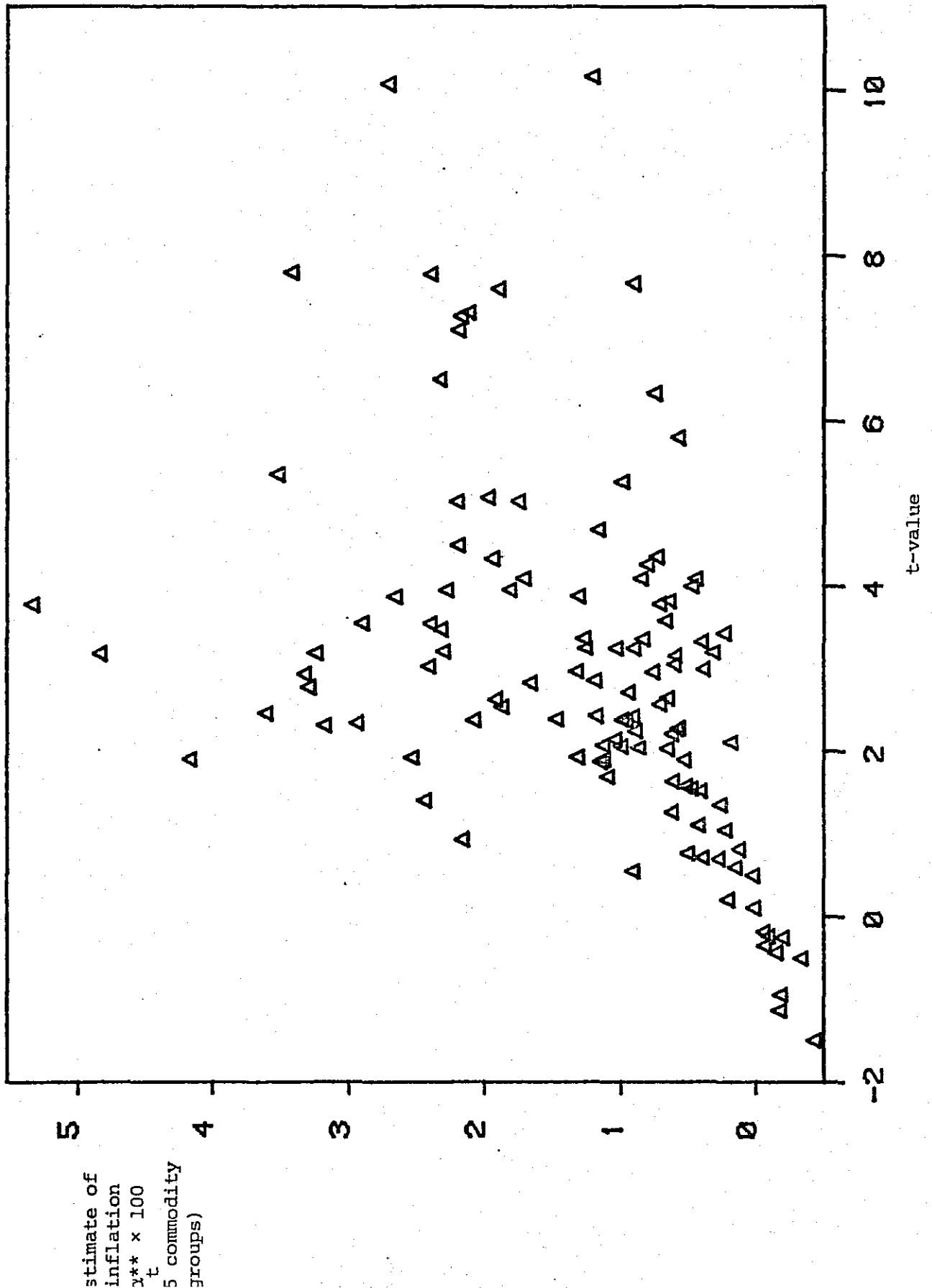Standard error of inflation × 100
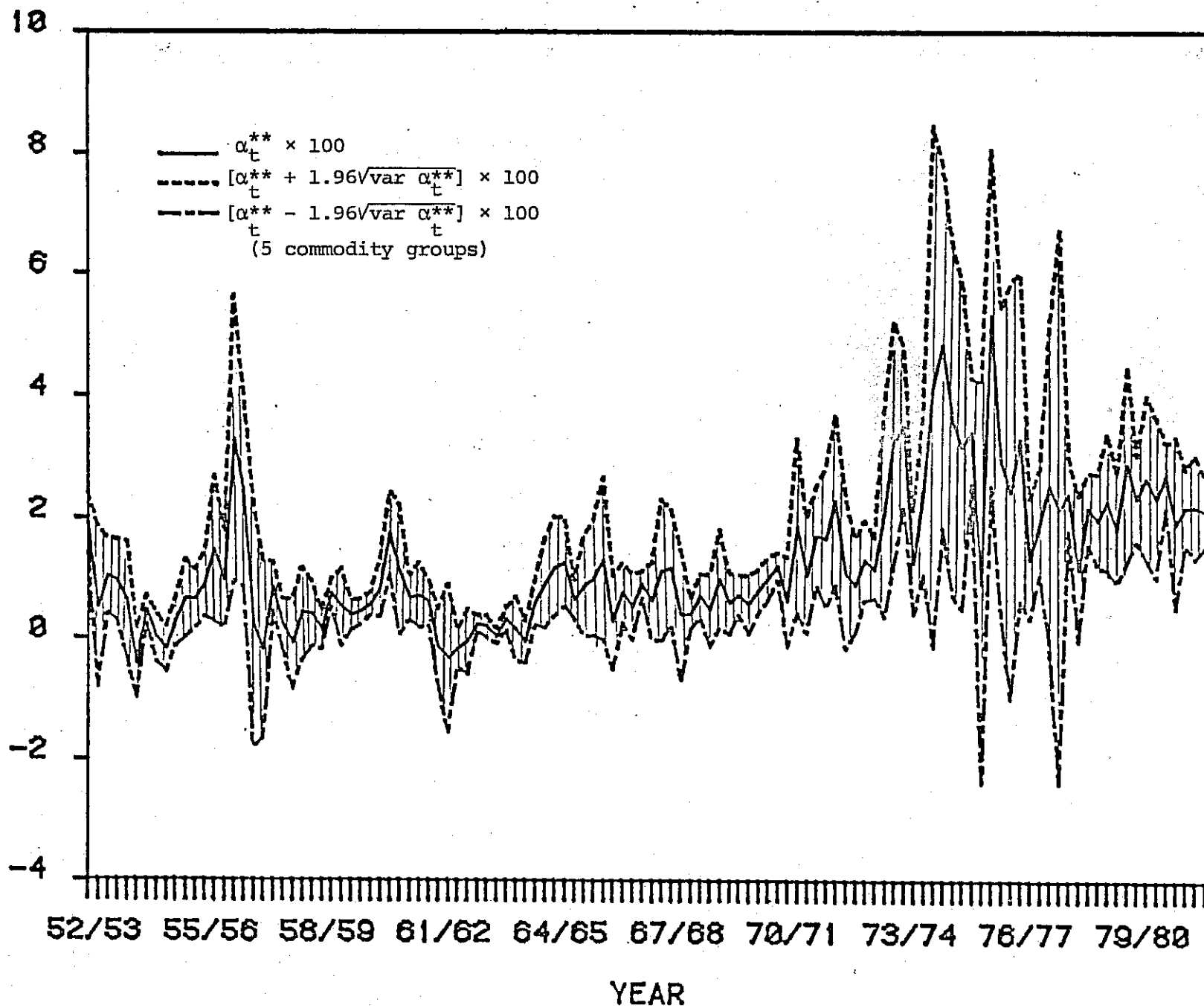
Figure 7

Figure 8

Figure 9

APPENDIX

Derivation of Equation (19)

To derive the least squares estimators of the α_t's and β_i's in (18) which satisfy the constraint (17), consider the Lagrangean function

(A1) $\quad \frac{1}{2} \sum_{i=1}^{n} \sum_{t=1}^{T} (y_{it} - \alpha_t x_i - \beta_i x_i)^2 - \mu \sum_{i=1}^{n} \bar{w}_i \beta_i$,

where μ is a Lagrangean multiplier. When we differentiate this function with respect to α_t and equate the derivate to zero, we obtain

$$-\sum_{i=1}^{n} x_i (y_{it} - \alpha_t x_i - \beta_i x_i) = 0 .$$

Using $x_i = \sqrt{\bar{w}_i}$, $y_{it} = \sqrt{\bar{w}_i} Dp_{it}$, $\sum_{i=1}^{n} \bar{w}_i = 1$ and $\sum_{i=1}^{n} \bar{w}_i \beta_i = 0$, this becomes $-\sum_{i=1}^{n} \bar{w}_i Dp_{it} + \alpha_t + 0 = 0$, from which the expression for α_t^* in (19) follows directly.

Next we differentiate (A1) with respect to β_i and equate the result to zero:

$$-x_i \sum_{t=1}^{T} (y_{it} - \alpha_t x_i - \beta_i x_i) - \mu \bar{w}_i = 0 ,$$

which is equivalent to

(A2) $\quad -\bar{w}_i \sum_{t=1}^{T} Dp_{it} + \bar{w}_i \sum_{t=1}^{T} \alpha_t + T\bar{w}_i \beta_i = \mu \bar{w}_i$.

Summing both sides of this over $i=1,\ldots,n$, using $\sum_{i=1}^{n} \bar{w}_i = 1$ and (17), we obtain

$$-\sum_{t=1}^{T} \sum_{i=1}^{n} \bar{w}_i Dp_{it} + \sum_{t=1}^{T} \alpha_t = \mu .$$

It follows from (19) that the first term on the left of this equation is $-\sum_{t=1}^{T} \alpha_t$, which means that $\mu = 0$. The zero value of the Lagrangean

multiplier is due to the fact that the constraint (17) is needed to identify model (18); i.e. the imposition of the constraint does not raise the residual sum of squares. Finally, substituting $\mu = 0$ in (A2) and rearranging gives the expression for β_i^* in (19).

Derivation of Equation (22)

If we replace y_{it} and x_i in (A1) with $\tilde{y}_{it}$ and $\tilde{x}_{it}$ respectively and follow exactly the same procedure as above for α_t, we obtain the expression for α_t^{**} given in (22).

We differentiate the (A1), with the above modifications, with respect to β_i and equate the result to zero:

$$-\sum_{t=1}^{T} \tilde{x}_{it}(\tilde{y}_{it} - \alpha_t \tilde{x}_{it} - \beta_i \tilde{x}_{it}) - \mu\bar{w}_i = 0.$$

This is equivalent to

(A3) $\qquad -\bar{w}_i \sum_{t=1}^{T} Dp_{it}/\theta_t^2 + \bar{w}_i \sum_{t=1}^{T} \alpha_t/\theta_t^2 + \bar{w}_i \beta_i \sum_{t=1}^{T} (1/\theta_t^2) = \mu\bar{w}_i.$

Summing both sides of this over $i=1,\ldots,n$, using $\Sigma_{i=1}^{n} \bar{w}_i = 1$ and (17), we obtain

$$-\sum_{t=1}^{T} \sum_{i=1}^{n} \bar{w}_i Dp_{it}/\theta_t^2 + \sum_{t=1}^{T} \alpha_t/\theta_t^2 = \mu,$$

from which it follows that $\mu = 0$.

Substituting $\mu = 0$ in (A3), dividing both sides by $\bar{w}_i$ and rearranging, we obtain

$$\beta_i^{**} = \frac{\displaystyle\sum_{t=1}^{T} (1/\theta_t^2)(Dp_{it} - \alpha_t^{**})}{\displaystyle\sum_{t=1}^{T} (1/\theta_t^2)}.$$

This is the expression given for β_i^{**} in (22) with ϕ_t as defined in (23).

Derivation of Equation (24)

To derive the sampling variances given in (24), we proceed in seven steps.

Substituting Out Constraint (17)

If we divide both sides of (17) by θ_t^2, this constraint can be expressed as

$$\sum_{i=1}^{n} \tilde{x}_{it}^2 \beta_i = 0 ,$$

since $x_{it} = \sqrt{\bar{w}_i}/\theta_t$. It then follows that

$$(A4) \qquad \beta_n = \sum_{i=1}^{n-1} \left(-\frac{\tilde{x}_{it}^2}{\tilde{x}_{nt}^2} \right) \beta_i .$$

Substituting the right side of (A4) for β_n in (21) for $i = n$, we obtain

$$(A5) \qquad \tilde{y}_{nt} = \alpha_t \tilde{x}_{nt} + \sum_{i=1}^{n-1} \left(-\frac{\tilde{x}_{it}^2}{\tilde{x}_{nt}} \right) \beta_i + \tilde{\zeta}_{nt} .$$

Matrix Formulation

Let

$$\underset{nT \times 1}{\tilde{y}} = \begin{bmatrix} \underset{T \times 1}{\tilde{y}_1} \\ \hline \underset{T \times 1}{\tilde{y}_2} \\ \hline \vdots \\ \hline \underset{T \times 1}{\tilde{y}_n} \end{bmatrix} = \begin{bmatrix} \tilde{y}_{11} \\ \tilde{y}_{12} \\ \vdots \\ \tilde{y}_{1T} \\ \hline \tilde{y}_{21} \\ \tilde{y}_{22} \\ \vdots \\ \tilde{y}_{2T} \\ \hline \vdots \\ \hline \tilde{y}_{n1} \\ \tilde{y}_{n2} \\ \vdots \\ \tilde{y}_{nT} \end{bmatrix} ;$$

$$\underset{T\times T}{\tilde{X}_i} = \begin{bmatrix} \tilde{x}_{i1} & 0 & \cdots & 0 \\ 0 & \tilde{x}_{i2} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \tilde{x}_{iT} \end{bmatrix} \quad , \quad i=1,\ldots,n;$$

$$\underset{T\times 1}{\tilde{X}_i\iota} = \begin{bmatrix} \tilde{x}_{i1} \\ \tilde{x}_{i2} \\ \vdots \\ \tilde{x}_{iT} \end{bmatrix} \quad , \quad \text{where } \iota = [1,1,\ldots,1]' \quad ;$$

$$\underset{T\times(n-1)}{A} = -\tilde{X}_n^{-1}(\tilde{X}_1\tilde{X}_1\iota \quad \tilde{X}_2\tilde{X}_2\iota \ldots \tilde{X}_{n-1}\tilde{X}_{n-1}\iota) = [a_{ti}] \ ,$$

where

$$a_{ti} = -\frac{\tilde{x}_{it}^2}{\tilde{x}_{nt}} = -\frac{\bar{w}_i}{\sqrt{\bar{w}_n}\,\theta_t} \quad ;$$

$$\underset{nT\times(T+n-1)}{Z} = \left[\begin{array}{c|cccc} \tilde{X}_1 & \tilde{X}_1\iota & 0 & \cdots & 0 \\ \tilde{X}_2 & 0 & \tilde{X}_2\iota & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \tilde{X}_{n-1} & 0 & 0 & \cdots & \tilde{X}_{n-1}\iota \\ \hline \tilde{X}_n & & & A & \end{array}\right] \quad ;$$

$$\underset{(T+n-1)\times 1}{\gamma} = [\alpha_1 \ \alpha_2 \ \cdots \ \alpha_T \ | \ \beta_1 \ \beta_2 \ \cdots \ \beta_{n-1}]' \quad ;$$

and

$$\underset{nT\times 1}{\tilde{\zeta}} = [\tilde{\zeta}_{11} \ \tilde{\zeta}_{12} \ \cdots \ \tilde{\zeta}_{1T} \ \tilde{\zeta}_{21} \ \tilde{\zeta}_{22} \ \cdots \ \tilde{\zeta}_{2T} \ \cdots \ \tilde{\zeta}_{n1} \ \tilde{\zeta}_{n2} \ \cdots \ \tilde{\zeta}_{nT}]' .$$

Using this notation, we write (21) for i=1,...,n-1, t=1,...,T and (A5) for t=1,...,T as

(A6) $\tilde{y} = Z\gamma + \tilde{\zeta}$.

As $\tilde{\zeta}_{it} = \sqrt{w_i}\,\zeta_{it}/\theta_t$, it follows from (16") that

(A7) $\text{var } \tilde{\zeta} = \dfrac{1}{n-1} I$,

which is a scalar matrix. Applying LS to (A6) and (A7), we obtain

(A8) $\gamma^{**} = (Z'Z)^{-1}Z'\tilde{y}$,

(A9) $\text{var } \gamma^{**} = \dfrac{1}{n-1}(Z'Z)^{-1}$.

Our objective is to obtain simple scalar expressions from (A9). We shall also show that (22) is a scalar version of (A8).

The Moment Matrix Z'Z

The moment matrix Z'Z is equal to

$$
\left[
\begin{array}{c|c}
\sum\limits_{i=1}^{n} \tilde{X}_i\tilde{X}_i & [\tilde{X}_1\tilde{X}_1\iota \quad \tilde{X}_2\tilde{X}_2\iota \ \ldots\ \tilde{X}_{n-1}\tilde{X}_{n-1}\iota] + \tilde{X}_n A \\
\hline
\begin{bmatrix} \iota'\tilde{X}_1\tilde{X}_1 \\ \iota'\tilde{X}_2\tilde{X}_2 \\ \vdots \\ \iota'\tilde{X}_{n-1}\tilde{X}_{n-1} \end{bmatrix} + A'\tilde{X}_n & \begin{bmatrix} \iota'\tilde{X}_1\tilde{X}_1\iota & 0 & \ldots & 0 \\ 0 & \iota'\tilde{X}_2\tilde{X}_2\iota & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & \iota'\tilde{X}_{n-1}\tilde{X}_{n-1}\iota \end{bmatrix} + A'A
\end{array}
\right] .
$$

It follows from the definition of A that

$$-\tilde{X}_n A = [\tilde{X}_1\tilde{X}_1\iota \quad \tilde{X}_2\tilde{X}_2\iota \ \ldots\ \tilde{X}_{n-1}\tilde{X}_{n-1}\iota] ,$$

so that the off-diagonal blocks of Z'Z are zero matrices. Thus Z'Z is

block-diagonal. The leading block is

$$
\sum_{i=1}^{n} \tilde{X}_i \tilde{X}_i \;=\;
\begin{bmatrix}
\sum\limits_{i=1}^{n} \tilde{x}_{i1}^{2} & 0 & \cdots & 0 \\
0 & \sum\limits_{i=1}^{n} \tilde{x}_{i2}^{2} & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & \sum\limits_{i=1}^{n} \tilde{x}_{iT}^{2}
\end{bmatrix}
$$

$$
\;=\;
\begin{bmatrix}
\dfrac{1}{\theta_1^2} & 0 & \cdots & 0 \\
0 & \dfrac{1}{\theta_2^2} & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & \dfrac{1}{\theta_t^2}
\end{bmatrix}
$$

$$
\;=\; B \, ,
$$

say, where the second step follows from $\Sigma_{i=1}^{n} \bar{w}_i = 1$.

The second diagonal block of $Z'Z$ is the sum of two matrices.
The first matrix is

$$
\begin{bmatrix}
\iota' \tilde{X}_1 \tilde{X}_1 \iota & 0 & \cdots & 0 \\
0 & \iota' \tilde{X}_2 \tilde{X}_2 \iota & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & \iota' \tilde{X}_{n-1} \tilde{X}_{n-1} \iota
\end{bmatrix}
$$

$$
\;=\;
\begin{bmatrix}
\sum\limits_{t=1}^{T} \tilde{x}_{1t}^{2} & 0 & \cdots & 0 \\
0 & \sum\limits_{t=1}^{T} \tilde{x}_{2t}^{2} & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & \sum\limits_{t=1}^{T} \tilde{x}_{n-1,t}^{2}
\end{bmatrix}
$$

$$
\;=\; \Theta \bar{w} \, ,
$$

where $\Theta = \Sigma_{t=1}^{T} (1/\theta_t^2)$ and $\bar{\bar{w}} = \text{diag}[\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_{n-1}]$. The second matrix in this sum is $A'A$, the $(i,j)^{th}$ element of which is

$$\sum_{t=1}^{T} a_{it} a_{tj} = \sum_{t=1}^{T} \frac{\bar{w}_i}{\sqrt{\bar{w}_n} \, \theta_t} \frac{\bar{w}_j}{\sqrt{\bar{w}_n} \, \theta_t} = \frac{\bar{w}_i \bar{w}_j}{\bar{w}_n} \Theta \ .$$

Thus

$$A'A = \frac{\Theta}{\bar{w}_n} \bar{w} \bar{w}' \ ,$$

where $\bar{w} = [\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_{n-1}]'$.

Combining these results we obtain

$$Z'Z = \begin{bmatrix} B & 0 \\ 0 & \Theta\left[\bar{\bar{w}} + \dfrac{1}{\bar{w}_n} \bar{w}\bar{w}'\right] \end{bmatrix} \ .$$

Partitioned Inversion of Z'Z

It can be easily verified that

$$\text{(A10)} \qquad (Z'Z)^{-1} = \begin{bmatrix} B^{-1} & 0 \\ 0 & \Theta^{-1}(\bar{\bar{w}}^{-1} - \iota\iota') \end{bmatrix} \ .$$

As B is diagonal, so is B^{-1}; and the $(t,t)^{th}$ element of B^{-1} is θ_t^2. The matrix $\bar{\bar{w}}^{-1}$ is also diagonal with $1/\bar{w}_i$ as the $(i,i)^{th}$ element.

Partitioned Multiplication of Z' and $\tilde{y}$

It follows from the definitions of Z and $\tilde{y}$ that

$$Z'\tilde{y} \;=\; \begin{bmatrix} \tilde{X}_1 & \tilde{X}_2 \;\ldots\; \tilde{X}_{n-1} & \vdots & \tilde{X}_n \\ \hline \iota'\tilde{X}_1 & 0 \;\ldots\; 0 & \vdots & a_1' \\ 0 & \iota'\tilde{X}_2 \;\ldots\; 0 & \vdots & a_2' \\ \vdots & \vdots \qquad \vdots & \vdots & \vdots \\ 0 & 0 \;\ldots\; \iota'X_{n-1} & \vdots & a_{n-1}' \end{bmatrix} \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_{n-1} \\ \hline \tilde{y}_n \end{bmatrix}$$

(A11)
$$= \begin{bmatrix} \sum_{i=1}^{n} \tilde{X}_i \tilde{y}_i \\ \hline \iota'\tilde{X}_1 \tilde{y}_1 + a_1' \tilde{y}_n \\ \iota'\tilde{X}_2 \tilde{y}_2 + a_2' \tilde{y}_n \\ \vdots \\ \iota'\tilde{X}_{n-1}\tilde{y}_{n-1} + a_{n-1}' \tilde{y}_n \end{bmatrix} ,$$

where a_i is the ith column of A.

The LS Estimators

Using (A10) and (A11) in (A8), we obtain

$$\gamma^{**} = (Z'Z)^{-1}Z'\tilde{y} \;=\; \begin{bmatrix} B^{-1} & \vdots & 0 \\ \hline & \vdots & \\ 0 & \vdots & \Theta^{-1}(\bar{W}^{-1}-\iota\iota') \\ & \vdots & \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{n} \tilde{X}_i \tilde{y}_i \\ \hline \iota'\tilde{X}_1 \tilde{y}_1 + a_1' \tilde{y}_n \\ \iota'\tilde{X}_2 \tilde{y}_2 + a_2' \tilde{y}_n \\ \vdots \\ \iota'\tilde{X}_{n-1}\tilde{y}_{n-1} + a_{n-1}' \tilde{y}_n \end{bmatrix}$$

It follows from the definition of γ that

$$
\begin{bmatrix} \alpha_1^{**} \\ \alpha_2^{**} \\ \vdots \\ \alpha_T^{**} \end{bmatrix} = B^{-1} \sum_{i=1}^{n} \tilde{X}_i \tilde{y}_i
$$

$$
= \begin{bmatrix} \theta_1^2 & 0 & \cdots & 0 \\ 0 & \theta_2^2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \theta_T^2 \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{n} \tilde{x}_{i1}\tilde{y}_{i1} \\ \sum_{i=1}^{n} \tilde{x}_{i2}\tilde{y}_{i2} \\ \vdots \\ \sum_{i=1}^{n} \tilde{x}_{iT}\tilde{y}_{iT} \end{bmatrix} ,
$$

or

$$
(A12) \qquad \alpha_t^{**} = \theta_t^2 \sum_{i=1}^{n} \tilde{x}_{it}\tilde{y}_{it} = \sum_{i=1}^{n} \bar{w}_i D p_{it} .
$$

It also follows that

$$
\begin{bmatrix} \beta_1^{**} \\ \beta_2^{**} \\ \vdots \\ \beta_{n-1}^{**} \end{bmatrix} = \Theta^{-1}(\bar{W}^{-1} - \mathbf{1}\mathbf{1}') \begin{bmatrix} \mathbf{1}'\tilde{X}_1\tilde{y}_1 + a_1'\tilde{y}_n \\ \mathbf{1}'\tilde{X}_2\tilde{y}_2 + a_2'\tilde{y}_n \\ \vdots \\ \mathbf{1}'\tilde{X}_{n-1}\tilde{y}_{n-1} + a_{n-1}'\tilde{y}_n \end{bmatrix} .
$$

As

$$
\bar{W}^{-1} - \mathbf{1}\mathbf{1}' = \begin{bmatrix} \left(\dfrac{1}{\bar{w}_1} - 1\right) & -1 & \cdots & -1 \\ -1 & \left(\dfrac{1}{\bar{w}_2} - 1\right) & \cdots & -1 \\ \vdots & \vdots & & \\ -1 & -1 & \cdots & \left(\dfrac{1}{\bar{w}_{n-1}} - 1\right) \end{bmatrix} ,
$$

it follows that

$$\beta_i^{**} = \Theta^{-1}\left[\left(\frac{1}{\bar{w}_i} - 1\right)(\imath'\tilde{X}_i\tilde{y}_i + a_i'\tilde{y}_n) - \sum_{\substack{j=1 \\ j\neq i}}^{n-1} (\imath'\tilde{X}_j\tilde{y}_j + a_j'\tilde{y}_n)\right]$$

$$= \Theta^{-1}\left[\frac{1}{\bar{w}_i}(\imath'\tilde{X}_i\tilde{y}_i + a_i'\tilde{y}_n) - \sum_{j=1}^{n-1} (\imath'\tilde{X}_j\tilde{y}_j + a_j'\tilde{y}_n)\right]$$

$$= \Theta^{-1}\left[\frac{1}{\bar{w}_i}\left(\sum_{t=1}^{T} \tilde{x}_{it}\tilde{y}_{it} - \frac{\bar{w}_i}{\sqrt{\bar{w}_n}}\sum_{t=1}^{T} \frac{\tilde{y}_{nt}}{\theta_t}\right) - \sum_{j=1}^{n-1}\left(\sum_{t=1}^{T} \tilde{x}_{jt}\tilde{y}_{jt} - \frac{\bar{w}_j}{\sqrt{\bar{w}_n}}\sum_{t=1}^{T} \frac{\tilde{y}_{nt}}{\theta_t}\right)\right]$$

$$= \Theta^{-1}\left[\sum_{t=1}^{T} \frac{Dp_{it}}{\theta_t^2} - \sum_{t=1}^{T} \frac{Dp_{nt}}{\theta_t^2} - \sum_{j=1}^{n-1}\left(\bar{w}_j\sum_{t=1}^{T} \frac{Dp_{jt}}{\theta_t^2} - \bar{w}_j\sum_{t=1}^{T} \frac{Dp_{nt}}{\theta_t^2}\right)\right]$$

$$= \Theta^{-1}\left[\sum_{t=1}^{T} \frac{1}{\theta_t^2}\left(Dp_{it} - Dp_{nt} - \sum_{j=1}^{n-1} \bar{w}_j Dp_{jt} + Dp_{nt}\sum_{j=1}^{n-1} \bar{w}_j\right)\right]$$

$$= \Theta^{-1}\left[\sum_{t=1}^{T} \frac{1}{\theta_t^2}\left(Dp_{it} - \sum_{j=1}^{n} \bar{w}_j Dp_{jt}\right)\right] \quad \text{as} \quad \sum_{j=1}^{n-1} \bar{w}_j = 1 - \bar{w}_n$$

$$(A13) \qquad = \sum_{t=1}^{T} \phi_t(Dp_{it} - \alpha_t^{**}) \ ,$$

where $\phi_t = \Theta^{-1}/\theta_t^2 = \dfrac{1/\theta_t^2}{\Sigma_{t=1}^{T}(1/\theta_t^2)}$.

Equations (A12) and (A13) are identical to (22) of the text.

The Sampling Variance of the Estimators

Using (A10) in (A9) we obtain

$$\text{var } \gamma^{**} = \frac{1}{n-1} \begin{bmatrix} B^{-1} & 0 \\ \\ 0 & \Theta^{-1}(\bar{W}^{-1} - \iota\iota') \end{bmatrix}.$$

Thus

$$\text{var} \begin{bmatrix} \alpha_1^{**} \\ \alpha_2^{**} \\ \vdots \\ \alpha_T^{**} \end{bmatrix} = \frac{1}{n-1} \begin{bmatrix} \theta_1^2 & 0 & \cdots & 0 \\ 0 & \theta_2^2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \theta_T^2 \end{bmatrix},$$

that is

(A14) $\quad \text{var } \alpha_t^{**} = \dfrac{\theta_t^2}{n-1}, \quad \text{cov}(\alpha_t^{**}, \alpha_{t'}^{**}) = 0, \qquad t \neq t'.$

It also follows that

$$\text{var} \begin{bmatrix} \beta_1^{**} \\ \beta_2^{**} \\ \vdots \\ \beta_{n-1}^{**} \end{bmatrix} = \frac{\Theta^{-1}}{n-1}(\bar{W}^{-1} - \iota\iota')$$

$$= \frac{\Theta^{-1}}{n-1} \begin{bmatrix} \left(\dfrac{1}{\bar{w}_1} - 1\right) & -1 & \cdots & -1 \\ -1 & \left(\dfrac{1}{\bar{w}_2} - 1\right) & \cdots & -1 \\ \vdots & & \vdots & \\ -1 & -1 & \cdots & \left(\dfrac{1}{\bar{w}_{n-1}} - 1\right) \end{bmatrix},$$

so that

$$(A15) \qquad \operatorname{var} \beta_i^{**} = \frac{1}{(n-1)\Sigma_{t=1}^{T}(1/\theta_t^2)}\left(\frac{1}{\bar{w}_i} - 1\right), \qquad i=1,\ldots,n-1;$$

$$(A16) \qquad \operatorname{cov}(\beta_i^{**}, \beta_j^{**}) = \frac{-1}{(n-1)\Sigma_{t=1}^{T}(1/\theta_t^2)}, \qquad i \neq j.$$

The variance of β_n^{**}, the estimate of the change in the relative price of good n (which has been substituted out), can be obtained from (A4):

$$\operatorname{var} \beta_n^{**} = \sum_{i=1}^{n-1} \frac{\bar{w}_i^2}{\bar{w}_n^2} \operatorname{var} \beta_i^{**} + \sum_{i=1}^{n-1}\sum_{\substack{j=1 \\ i \neq j}}^{n-1} \frac{\bar{w}_i\bar{w}_j}{\bar{w}_n^2} \operatorname{cov}(\beta_i^{**}, \beta_j^{**}).$$

Defining $k = \left[(n-1)\Sigma_{t=1}^{T}(1/\theta_t^2)\right]^{-1}$ and using (A15) and (A16) in the above, we obtain

$$\operatorname{var} \beta_n^{**} = \frac{k}{\bar{w}_n^2}\left[\sum_{i=1}^{n-1} \bar{w}_i^2\left(\frac{1}{\bar{w}_i} - 1\right) - \sum_{i=1}^{n-1}\sum_{\substack{j=1 \\ i \neq j}}^{n-1} \bar{w}_i\bar{w}_j\right]$$

$$= \frac{k}{\bar{w}_n^2}\left[\sum_{i=1}^{n-1} \bar{w}_i(1 - \bar{w}_i) - \sum_{i=1}^{n-1}\sum_{\substack{j=1 \\ i \neq j}}^{n-1} \bar{w}_i\bar{w}_j\right]$$

$$= \frac{k}{\bar{w}_n^2}\left[\sum_{i=1}^{n-1} \bar{w}_i - \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \bar{w}_i\bar{w}_j\right]$$

$$= \frac{k}{\bar{w}_n^2}\sum_{i=1}^{n-1} \bar{w}_i\left[1 - \sum_{j=1}^{n-1} \bar{w}_j\right]$$

$$= \frac{k}{\bar{w}_n^2}(1 - \bar{w}_n)\bar{w}_n \qquad \text{as } \sum_{i=1}^{n} \bar{w}_i = 1$$

$$= k\left(\frac{1 - \bar{w}_n}{\bar{w}_n}\right).$$

$$(A17) \qquad = \frac{1}{(n-1)\Sigma_{t=1}^{T}(1/\theta_t^2)}\left(\frac{1}{\bar{w}_n} - 1\right).$$

Thus the variance of β_n^{**} has exactly the same form as does (A15).

Equations (A14), (A15) and (A17) are equation (24) of the text.

Derivation of Equation (25)

The estimate of the mean rate of inflation over all T periods is

$$\bar{\alpha}^{**} = \frac{1}{T}\sum_{t=1}^{T}\alpha_t^{**}.$$

It follows from (A14) that

$$\text{var } \bar{\alpha}^{**} = \frac{1}{T^2}\sum_{t=1}^{T}\text{var }\alpha_t^{**}, = \frac{1}{(n-1)T^2}\sum_{t=1}^{T}\theta_t^2,$$

which is equation (25).

FOOTNOTE

1. Let there be $G < n$ groups of goods with n_g goods in group g.

Then the estimate of the relative price change for group g is

$$\beta_g^{**} = \sum_{i=1}^{n_g} \frac{\bar{w}_i}{\bar{w}_g} \beta_i^{**} ,$$

where $\bar{w}_g = \sum_{i=1}^{n_g} \bar{w}_i$ is the mean budget share of the group.

Note that a budget-share-weighted average of the group relative

price changes is zero:

$$\sum_{g=1}^{G} \bar{w}_g \beta_g^{**} = \sum_{g=1}^{G} \bar{w}_g \sum_{i=1}^{n_g} \frac{\bar{w}_i}{\bar{w}_g} \beta_i^{**} = \sum_{i=1}^{n} \bar{w}_i \beta_i^{**} = 0 ,$$

where the last step follows from (17).

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