Chapter 66

THE MEASUREMENT OF PRODUCTIVITY FOR NATIONS*

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Abstract

This chapter covers the theory and methods for productivity measurement for nations. Labor, multifactor and total factor productivity measures are defined and are related to each other and to gross domestic product (GDP) per capita. Their growth over time and relative counterparts are defined as well.

Different conceptual meanings have been proposed for a total factor productivity growth (TFPG) index. These are easiest to understand for the case in which the index number problem is absent: a production process that involves one input and one output (a 1–1 process). It is easily seen that four common concepts of TFPG all lead to the same result in the 1–1 case. Moving on to a general N input, M output production scenario, we demonstrate that a Paasche, Laspeyres or Fisher index number formula provides a measure for all four of the concepts of TFPG introduced for the 1–1 case. This is an advantage of the Paasche–Laspeyres–Fisher family of formulas.

When multiple inputs or outputs are involved, there is the problem of choosing among alternative functional forms. The axiomatic and economic approaches to index formula choice are reviewed.

In addition, we briefly cover the Divisia index number approach and growth accounting, including the KLEMS (capital, labor, energy, materials and services) approach. The gross output measures of the KLEMS approach are contrasted with value added output measures such as GDP. Also, an alternative family of revenue function based productivity growth indexes proposed by Diewert, Kohli and Morrison (DKM) is outlined. The DKM approach facilitates the decomposition of productivity growth into economically meaningful components. This approach is useful, for example, for examining the effects of changes in the terms of trade on productivity growth.

Keywords

total factor productivity growth, labor productivity, living standards, exact index numbers, capital deepening, real income growth, gross versus net output, growth accounting, KLEMS, terms of trade, aggregation of capital, embodiment of technical progress, depreciation, deterioration, obsolescence, index number theory

JEL classification: O4, O4.7, C43, C82, D24, D31, F14, I3

1. Introduction

> "The two main sources of economic growth in output are increases in the factors of production (the labor and capital devoted to production) and efficiency or productivity gains that enable an economy to produce more for the same amount of inputs."
>
> [Baldwin, Harchaoui, Hosein and Maynard (2001), "Productivity: Concepts and Trends", Statistics Canada]

> "Productivity is commonly defined as a ratio of a volume measure of output to a volume measure of input use. While there is no disagreement on this general notion, a look at the productivity literature and its various applications reveals very quickly that there is neither a unique purpose for nor a single measure of productivity."
>
> [Paul Schreyer (2001), OECD Productivity Manual]

Productivity for nations is like love. Much is said about the benefits of having more of it, but consensus is elusive on what "it" really is. As Schreyer (2001) writes, "productivity is commonly defined as a ratio of a volume measure of output to a volume measure of input use." But how can the output and input volumes be defined and measured for a nation? This paper deals with the methods used for measuring aggregate productivity, by which we mean the productivity of unique entities such as nations or entire industries.

The best of all times for reviewing a subject area is when the reported findings are impacting important decision processes, so the research matters; when there is a large volume of recent research to be digested and integrated with previous findings; when important data developments have taken place or are in progress; and when there is informed and truly interactive debate on how best to proceed in areas where researchers disagree on the appropriate directions. This is the current state of affairs for the subject of this paper: the measurement of productivity for nations.

Multiple types of productivity measures are produced for nations. Official statistics agencies in countries, including the United States and Canada, produce three sorts of labor productivity measures. In this paper we refer to these using the designations of per worker labor productivity (LP), per hour labor productivity (HLP), and per weighted hour labor productivity (WHLP).

Many official statistics agencies also produce a multifactor productivity measure (MFP) that takes account of machinery and equipment and other capital inputs as well as labor, and sometimes energy and materials inputs as well. Even though it is probably not possible to measure *all* inputs at a national level, economists define and consider and estimate approximations to total factor productivity (TFP) measures.

In the rest of this paper we focus mostly on the TFP and TFPG measures, where we use TFPG to denote both TFP growth and relative TFP. (Note that others sometimes make this same distinction by using TFP for both the growth and relative total factor productivity measures with the qualifier of "levels" in referring to what we denote as simply TFP.)

National productivity estimates are of special importance because they are an input into many aspects of policy making.[1] Although useful analogies can be drawn and there are methodological commonalities, the measurement of productivity for nations is a fundamentally different undertaking from the sorts of productivity measurement dealt with by engineers for specific machines and production lines, and by accountants and business analysts and economists working with micro level data for individual production units. At this level of aggregation, the data available are limited to fairly short time series, putting bounds on the scope for econometric estimation. Also feedback effects among the measured inputs and outputs cannot be ruled out *a priori*. Index number methods (including growth accounting) are the mainstay methodology.

Estimates of relative productivity or productivity growth do not, by themselves, provide causal insights. However, many aspects of federal government and other economic planning are affected by reported productivity measures. Also, causal research on productivity depends as well on having measures of productivity.[2]

Many economists seem not to look on index number theory and applied research as belonging within the discipline of economics. And yet, there is scarcely an empirical paper published in economics that does not utilize price or volume, if not productivity, index numbers. Certainly index numbers are ubiquitous in empirical macroeconomics.

Also, economic theory and empirical findings provide the basis for a wide array of choices made in defining and evaluating national price, volume and productivity indexes. For example, the exact index number method for choosing among alternative index number formulas involves showing that specific ones can be derived from optimizing models for firms or households where these models include production, revenue,

[1] A sense of the range of relevant public policy issues can be acquired from studies including Aschauer (1989), Atrostic and Nguyen (2006), Baily (1981), Baldwin, Jarmin and Tang (2004), Balk (1998, 2003), Basu and Fernald (1997), Basu et al. (2004), Berndt and Wood (1975), Black and Lynch (1996), Boskin (1997), Bresnahan and Gordon (1997), Denison (1979), Diewert (1993a, 1995, 1998a, 1998b, 2001a, 2001c, 2002a, 2005e, 2005f, 2006c, 2007b), Diewert and Fox (1999), Diewert and Lawrence (2005), Diewert, Lawrence and Fox (2006), Duguay (1994, 2006), Ellerman, Stoker and Berndt (2001), Feenstra and Hanson (2005), Fortin (1996), Griliches (1997), Ho, Rao and Tang (2004, 2007), Hulten (1986, 2001), Jog and Tang (2001), Jorgenson (2001, 2004), Jorgenson, Ho and Stiroh (2005), Jorgenson and Landefeld (2006), Jorgenson and Lee (2001), Jorgenson and Motohashi (2005), Jorgenson and Nomura (2005), Jorgenson and Yun (1986, 1990, 1991), Kuroda and Nomura (2003), Lee and Tang (2001a, 2001b), Lipsey and Carlaw (2004), Lipsey, Carlaw and Bekar (2006), Nakamura and Lipsey (2006), Maddison (1987), Mankiw (2001), Morrison (1992), Muellbauer (1986), Nadiri (1980), Nakamura and Diewert (2000), Nordhaus (1982), Power (1998), Prescott (1998), Smith (2005), Stiroh (2002) Tang and Wang (2004, 2005), Triplett and Bosworth (2004), van Ark, Inklaar and McGuckin (2003), and Wolff (1996).

[2] For gaining a causal understanding of the determinants of national productivity, data at lower levels of aggregation are of obvious value, as are suitable econometric methods for analyzing panel and other sorts of micro data files. See, for example, Bartelsman and Doms (2000), Foster, Haltiwanger and Krizan (2001), Levinsohn and Petrin (1999), Olley and Pakes (1996), and Pavcnik (2002).

cost, expenditure, transformation, or other aggregators with specific functional forms such as the translog and the generalized quadratic with properties that have been explored by economists. We feel that index number theory and practice should (once more) be a core subject within economics.

The traditional index number measures of TFPG are defined as ratios of output and input volume indexes. As is appropriate, statistics agencies collect mostly value and price, rather than volume and price, information, and then create the needed volume data by deflating value data using price indexes. We show the relationships among price, volume and productivity indexes, and how productivity indexes relate to real revenue/cost ratios.

Several different conceptual meanings have been proposed for a TFPG index. The alternative concepts are easiest to understand for a one period production process that uses a single input factor to make a single output product (a 1–1 process). In Section 2 we show that four common concepts of TFPG all lead to the same measure in the 1–1 case. Of course, the aggregation challenges that must be confronted in the construction of national productivity measures do not arise in a 1–1 case context. To introduce these issues, we use a hypothetical two input, one output production scenario (a 2–1 process). We then move on to the general *N* input, *M* output case that is relevant for national level productivity measurement.

In the final subsection of Section 2 we introduce three different labor productivity indexes in common use, and relate these to the multifactor productivity (MFP) and total factor productivity (TFP) measures that are our main focus in this paper. The Törnqvist, and implicit Törnqvist volume and price indexes[3] and the corresponding TFPG indexes are also introduced and discussed.

In Section 3 we define Laspeyres, Paasche and Fisher measures for the general *N* input, *M* output case for the four concepts of TFPG introduced in Section 2 for the 1–1 case.

With multiple inputs and outputs, different formula choices lead to different TFPG findings. This raises the issue of choice among alternative TFPG formulas. The two main approaches to choosing among the different index number functional forms are the axiomatic (or test) approach and the economic approach.[4]

The axiomatic approach is taken up in Section 4. It was used extensively by the founding contributors to index number theory, including Fisher (1911, 1922). This approach makes use of lists of desired properties referred to as axioms or tests. They are either formalizations of common sense properties of good index numbers or generalizations of properties that hold for virtually all proposed index number formulas in the simplistic 1–1 case.

[3] Perhaps the best source for learning about or checking details of price indexes are the new international Consumer Price Index Manual [T.P. Hill (2004)] and Producer Price Index Manual [Armknecht (2004)]. The Diewert chapters in the new International CPI and PPI Manuals are Chapters 15–20 and 22–23 of the CPI Manual and Chapters 15–22 of the PPI Manual. See also Diewert (2002b).

[4] A third approach – the statistical approach – is not discussed here. See Diewert (1981a, 1987, 2002b, 2004c, 2007c) on this parallel approach to the index number formula choice problem.

The axiomatic approach to index number choice focuses on properties of the index number formula itself. In contrast, the economic approach seeks to use principles and implications of economic theory as a basis for choosing among proposed index formulas.

Exact index number theory is one important stream within the economic approach to index numbers: the stream on which we focus in Section 5. The exact approach transforms the index number choice problem into a problem of choosing the correct functional form for a behavioral aggregator function of some sort. In order to use the exact approach to derive the functional form for a TFPG index, it is first necessary to decide on the perspective for the productivity analysis. When a producer perspective is adopted, then the aggregator function for the economic approach can be the production function, or it can be the corresponding cost, profit, or other dual representation of the production process. Once the form of the aggregator has been determined, then the exact index number approach can be applied in order to determine the corresponding functional form for the TFPG index, as shown in Section 5.

When it can be established that some particular index number formula corresponds, by the "exact" index number approach, to a linearly homogeneous producer behavioral relationship that is "flexible", meaning that it provides a second order approximation to an arbitrary twice continuously differentiable linearly homogeneous function, then the index number is said to be "superlative". Diewert established that, under ordinary conditions, all of the commonly used superlative index number formulas (including the Fisher, Törnqvist, and implicit Törnqvist formulas introduced in Section 3) approximate each other to the second order when evaluated at an equal price and volume point. Diewert established as well that the two most commonly used index number formulas that are not superlative – the Laspeyres and the Paasche indexes – approximate the superlative indexes to the first order at an equal price and volume point.

The exact index number approach, together with Diewert's numerical analysis approximation results for superlative index numbers, reduces the *a priori* information requirements for choosing an index number formula to a list of general characteristics of the production scenario. So long as there is agreement on those characteristics, under ordinary conditions, any one of the commonly used superlative TFPG index number formulas should provide a reasonable estimate to the theoretical Malmquist TFPG index introduced in Section 6.

The exact and the axiomatic approaches single out some of the same index number formulas as especially desirable. The exact approach can be viewed as a methodology for exploring the meaning of the proposed measures of TFPG and also of the intuitions on which the axiomatic approach is based. This approach helps us interpret TFPG indexes in the language of neoclassical theory. That the index number formulas which have been in use since the early 1900s have interpretations in the language of modern microeconomic theory suggests that the intuitions which guided the axiomatic approach to index number theory and the axioms of microeconomic theory may have more in common than is readily apparent.

The data used in evaluating measures of productivity are discrete. Nevertheless, various properties of national productivity measures have been worked out utilizing the convergence of continuous approximations. The Divisia method reviewed in Section 8 treats time as continuous. The Divisia method has been used extensively in growth accounting studies for nations, which is the subject of Section 9. Section 9 also briefly takes up the KLEMS (capital, labor, energy, materials and services) approach, and the World KLEMS data development and analysis initiatives.

In Section 10, further consideration is also given to the choice of the measure of output incorporated into productivity analyses and we review efforts to relax the assumption of constant returns to scale.

In Section 11, an alternative family of theoretical productivity growth indexes proposed by Diewert and Morrison (1986) and Kohli (1990) is introduced.[5] This approach has special advantages for examining the components of TFP growth.

Section 12 concludes.

2. Alternative productivity measurement concepts

> "**Productivity**
> A ratio of output to input."
>
> [Atkinson, Kaplan and Young (1995, p. 514)]

> "While, for example, we look at the cost of power as a number of 'analysed' items such as coal, water-rate, ash removal, drivers' and stokers' wages, etc., it will probably be a long time before it dawns upon us that all this expenditure can be reduced to a horse-power-hour rate, and that such a factor, once known, may turn out to be a standing reproach. The burning of 200 tons of coal per week may mean anything or nothing, but the cost of a horse-power hour can be compared at once with standard data ... the publication of figures based on them would reveal amazing inefficiencies that under present conditions are unsuspected and unknown because no means of comparison exists."
>
> [A. Hamilton Church (1909, p. 190)]

The basic definition of total factor productivity (TFP) is the rate of transformation of total input into total output. The output-over-input index approach to the measurement of total factor productivity has early origins.[6] In his Simon Kuznets Memorial Lecture, Griliches remarked that "the first mention of what might be called an output-over-input index that I can find appears in Copeland (1937)". However, in an endnote to the written version of the lecture Griliches (1997) writes:

[5] This approach has been used lately in a growing number of other studies such as Feenstra et al. (2005).

[6] Output over input measures are sometimes referred to as productivity levels measures.

> "Nothing is really new. Kuznets (1930) used the 'cost of capital and labor per pound of cotton yarn,' the inverse of what would later become a total factor productivity index (if the cost is computed in constant prices) ... as a '(reflection of) the economic effects of technical improvement' and a few sentences later as a measure of 'the effect of technical progress' (p. 14). More thorough research is likely to unearth even earlier references".

Indeed, the early engineering and cost accounting literature contains numerous references to unit costs used as efficiency measures (e.g., [Church (1909)]). For a one output production process, the unit cost is the reciprocal of the TFP index.

Virtually all real production processes make use of multiple inputs and most yield multiple outputs. Nevertheless, it is convenient to introduce basic concepts, terms and notation in the simplified context of a production process with a single homogeneous input factor and a single homogeneous output product. In a 1–1 context, the concepts of total factor productivity and total factor productivity growth (TFPG) are easy to think about because the measures are not complicated by choices about how different types of inputs and different types of outputs should be aggregated. By the same token, of course, the aggregation difficulties that arise when there are multiple inputs or outputs cannot be introduced in a 1–1 context because they do not arise. Thus in Subsection 2.2 we also briefly consider a two input, one output process, a 2–1 case before moving on in Subsection 2.3 to a general N input, M output setting. Labor, multifactor, and total factor productivity measures are introduced in Subsection 2.3.

2.1. The 1–1 case[7]

For each time period (or scenario), suppose we know the volume of the one input used, given by x_1^t, its unit price w_1^t, and the volume of the one output produced, given by y_1^t, and its unit price p_1^t. TFP can be defined conceptually as the rate of transformation of total input into total output. For the 1–1 case, the ratio of output produced to input used is the measure for TFP for period t:

$$\text{TFP} \equiv \left(y_1^t/x_1^t\right) \equiv a^t. \tag{2.1-1}$$

The parameter a^t that is defined as well in (2.1-1) is a conventional output–input coefficient.[8]

Total factor productivity growth, or TFPG, can be defined in several ways, four of which are considered here.[9] Our first concept of TFPG is the rate of transformation of

[7] This section and some of what follows draws on Diewert (2000).

[8] An output–input coefficient always involves just one output and one input. However, these coefficients can be defined and used in multiple input, multiple output situations too, as is done in Diewert and Nakamura (1999).

[9] Some authors also use TFP to refer to total factor productivity growth. In line with Bernstein (1999), we use TFPG rather than TFP for total factor productivity growth so as to avoid the inevitable confusion that otherwise results.

input into output for production period t versus s, where s comes before t here and elsewhere in this paper if these are time periods. This concept of TFPG, denoted here by TFPG(1), can be measured in the 1–1 case as[10]:

$$\mathrm{TFPG}(1) \equiv \left(\frac{y_1^t}{x_1^t}\right) \Big/ \left(\frac{y_1^s}{x_1^s}\right) = a^t/a^s. \tag{2.1-2}$$

Three other concepts of total factor productivity growth are also in common use:

- the ratio of the output and the input growth rates, denoted by TFPG(2);
- the rate of growth in the real revenue/cost ratio; i.e., the rate of growth in the revenue/cost ratio controlling for price change, denoted by TFPG(3); and
- the rate of growth in the margin after controlling for price change, denoted by TFPG(4).

For a 1–1 production process, the obvious measure for the second concept of TFPG is:

$$\mathrm{TFPG}(2) \equiv \left(\frac{y_1^t}{y_1^s}\right) \Big/ \left(\frac{x_1^t}{x_1^s}\right). \tag{2.1-3}$$

The third and fourth concepts of TFPG are financial in nature. Expressions for actual revenue and cost are needed to form measures for these. For the 1–1 case, total revenue and total cost are given by

$$R^t \equiv p_1^t y_1^t \quad \text{and} \quad C^t \equiv w_1^t x_1^t. \tag{2.1-4}$$

Thus, the third concept of TFPG can be measured by

$$\mathrm{TFPG}(3) \equiv \left[\frac{R^t/R^s}{p_1^t/p_1^s}\right] \Big/ \left[\frac{C^t/C^s}{w_1^t/w_1^s}\right] = \left(\frac{y_1^t}{y_1^s}\right) \Big/ \left(\frac{x_1^t}{x_1^s}\right), \tag{2.1-5}$$

where

$$\left(R^t/R^s\right) \Big/ \left(p^t/p^s\right) = \left(p_1^t y_1^t / p_1^s y_1^s\right) / \left(p_1^t/p_1^s\right) = y_1^t/y_1^s, \quad \text{and} \tag{2.1-6}$$

$$\left(C^t/C^s\right) / \left(w^t/w^s\right) = \left(w_1^t x_1^t / w_1^s x_1^s\right) / \left(w_1^t/w_1^s\right) = x_1^t/x_1^s. \tag{2.1-7}$$

Business managers are usually interested in ensuring that revenues exceed costs, and this leads to an interest in margins. The period t margin, m^t, can be defined by

$$1 + m^t \equiv R^t/C^t. \tag{2.1-8}$$

Using this definition, in the 1–1 case TFPG(4) can be measured by

$$\mathrm{TFPG}(4) \equiv \left[\left(1+m^t\right)/\left(1+m^s\right)\right]\left[\left(w_1^t/w_1^s\right)/\left(p_1^t/p_1^s\right)\right]. \tag{2.1-9}$$

[10] Here we refer to t and s as time periods. However, the 'period s' comparison situation could be for some other unit of production in the same time period.

If we interpret the margin as a reward for managerial or entrepreneurial input, then TFPG(4) can be interpreted as the rate of growth of input prices, broadly defined so as to include managerial and entrepreneurial input, divided by the rate of growth of output prices. Note that if the margins are zero, then TFPG(4) reduces to $(w_1^t/w_1^s)/(p_1^t/p_1^s)$.[11]

Using (2.1-8) to eliminate the margin growth rate on the right-hand side of (2.1-9), and comparing the resulting expression and those in (2.1-2), (2.1-3) and (2.1-5), it can readily be seen that the four concepts of total factor productivity growth introduced here all lead to the same pure volume measure. That is, for the 1–1 case the measures for all four of the concepts for TFPG reduce to

$$\text{TFPG} \equiv \left(\frac{y_1^t}{y_1^s}\right) \Big/ \left(\frac{x_1^t}{x_1^s}\right). \tag{2.1-10}$$

2.2. The 2–1 case

We next use a slightly more complex production process as the context for introducing key choices that must be faced in order to specify multiple input, multiple output measures of TFP and TFPG. This hypothetical 2–1 production process uses the labor hours of one man and logs as inputs and yields firewood as the output. The man buys the loads of logs, splits them with an axe, and then sells the split logs as firewood. The axe was inherited and has no resale or rental value. The man's time, in hours, is denoted by x_1^t, and the number of truckloads of logs purchased is denoted by x_2^t. The firewood output is measured in kilograms and denoted by y_1^t.

The labor productivity in each period is given by (y_1^t/x_1^t). The materials utilization productivity can also be defined as (y_1^t/x_2^t). These are the two output–input coefficient measures that can be specified for this production scenario, and their values will tend to move in opposite directions from period to period. When the man splits logs at a faster pace, unless he pays extra attention, he uses the raw resource input more wastefully. The fact that the single factor productivity measures do not necessarily move together closely (or even in the same direction) is a key reason why TFP and TFPG measures are needed instead of just labor productivity measures.

In order to measure TFP for our log splitting process, a measure for total input is needed. That is, we need a way of adding hours of labor and truckloads of logs. Different perspectives can be adopted for forming this aggregate.[12]

In the economic approach to index number theory, the goal of producer revenue or cost optimization dictates that unit revenues or costs should be used as weights in aggregating the volumes of the different inputs and outputs.

[11] One set of conditions under which the margins will be zero is perfect competition and constant returns to scale.

[12] This issue of perspective is taken up, for example, in Schultze and Mackie (2002).

In our firewood production example, if the unit cost for an hour of labor is w_1^t and the unit cost of a load of logs is w_2^t, then the input volume aggregate could be defined as the following price weighted sum:

$$w_1^t x_1^t + w_2^t x_2^t. \tag{2.2-1}$$

If the total input is measured as in (2.2-1), then total factor productivity, defined as the rate of transformation of total input into total output, can be measured as

$$\text{TFP} = y_1^t \big/ \left(w_1^t x_1^t + w_2^t x_2^t\right). \tag{2.2-2}$$

Now, suppose we want to measure TFPG. That is, suppose we want to compare the ratio of output to input in period (or scenario) t with the ratio of output to input for some earlier period (or some different production scenario) s. Should period t price weights be used in forming both the period t and period s aggregates? Or, should period s price weights be used in forming both of the aggregates? Or, should some sort of combination of the period s and t prices be used as weights? Also, are there other functional forms besides the linear one that might be preferable for combining the volumes of the different inputs? These are the sorts of issues that are faced in the theory of index numbers when it comes to choosing among alternative functional forms that have been proposed for the indexes.

2.3. *Different types of measures of productivity*

For nations, a general N input, M output production setting applies. In the next sections, we introduce formulas. Here, however, we first show how TFP and TFPG measures fit with other general types of productivity measures that are commonly used at a national level and with per capita gross domestic product (GDP). We do this here using words rather than mathematical expressions for the relevant component parts.

GDP per capita equals the product of GDP per hour of work, the average hours of work per worker, the employment rate, and the proportion of the population (denoted by POP) that is old enough to work and hence in the potential labor force:

$$\frac{\text{GDP}}{\text{POP}} \equiv \frac{\text{GDP}}{\begin{bmatrix}\text{Total}\\ \text{work}\\ \text{hours}\end{bmatrix}} \times \frac{\begin{bmatrix}\text{Total}\\ \text{work}\\ \text{hours}\end{bmatrix}}{\begin{bmatrix}\text{Number}\\ \text{of}\\ \text{workers}\end{bmatrix}} \times \frac{\begin{bmatrix}\text{Number}\\ \text{of}\\ \text{workers}\end{bmatrix}}{\begin{bmatrix}\text{Potential}\\ \text{labor}\\ \text{force}\end{bmatrix}} \times \frac{\begin{bmatrix}\text{Potential}\\ \text{labor}\\ \text{force}\end{bmatrix}}{\text{POP}}. \tag{2.3-1}$$

Variants of the above identity have been used in many published studies. For understanding the commonly used measures of productivity, it is useful to expand this expression as follows:

$$\frac{\text{GDP}}{\text{POP}} \equiv \frac{\text{GDP}}{\begin{bmatrix}\text{Total}\\ \text{input}\end{bmatrix}} \times \frac{\begin{bmatrix}\text{Total}\\ \text{input}\end{bmatrix}}{\begin{bmatrix}\text{Total}\\ \text{measured}\\ \text{input}\end{bmatrix}} \times \frac{\begin{bmatrix}\text{Total}\\ \text{measured}\\ \text{input}\end{bmatrix}}{\begin{bmatrix}\text{Total}\\ \text{labor}\\ \text{input}\end{bmatrix}} \times \frac{\begin{bmatrix}\text{Total}\\ \text{labor}\\ \text{input}\end{bmatrix}}{\begin{bmatrix}\text{Total}\\ \text{work}\\ \text{hours}\end{bmatrix}} \times \frac{\begin{bmatrix}\text{Total}\\ \text{work}\\ \text{hours}\end{bmatrix}}{\begin{bmatrix}\text{Number}\\ \text{of}\\ \text{workers}\end{bmatrix}}$$
$$\times \frac{\begin{bmatrix}\text{Number}\\ \text{of}\\ \text{workers}\end{bmatrix}}{\begin{bmatrix}\text{Potential}\\ \text{labour}\\ \text{force}\end{bmatrix}} \times \frac{\begin{bmatrix}\text{Potential}\\ \text{labour}\\ \text{force}\end{bmatrix}}{\text{POP}}$$
$$= (\text{A}) \times (\text{B}) \times (\text{C}) \times (\text{D}) \times (\text{E}) \times (\text{F}) \times (\text{G}). \tag{2.3-2}$$

For expositional convenience, we denote the terms on the right-hand side by A–G, respectively.

All of the productivity measures we consider have as their numerator some measure of total output. We follow common practice here in using GDP as the measure of national output.[13] On a conceptual level, productivity is just output over input – that is, it is the rate of conversion of input into output. These various productivity measures differ in terms of the categories of included input.[14]

Productivity measures in common use and our designations for these are total factor productivity (TFP), multi factor productivity (MFP), labor productivity with wage weighted hours of work used as the measure of labour input (WHLP), labor productivity with hours of work used as the measure of labor input (what we denote here as HLP), and labor productivity with the number of workers used as the measure of labor input (LP).

To be meaningfully interpreted, productivity measures must usually be placed in a comparative context. The two most common contexts are comparisons of productivity for two different time periods for the same productive unit – e.g., for the same nation –

[13] Arguments for using other measures of total national output can be found, for example, in Kohli (1978, 1991, 2004, 2005, 2007). Diewert (2006d, 2007a) argues for the use of measures that are net of anticipated depreciation and obsolescence of capital assets. See also Diewert, Nakamura and Schreyer (2007).

[14] There are large literatures on measuring the various input volumes. On the labor input, see for example Ahmad et al. (2003), Baldwin, Maynard and Wong (2005), Baldwin et al. (2005), Breshnahan, Brynjolfsson and Hitt (2002), Nakamura (1995), Jorgenson and Fraumeni (1992), Jorgenson, Gollop and Fraumeni (1987), Tang and MacLeod (2005), and Triplett (1990, 1991). On the capital inputs see, for example, Diewert (1977, 1980a, 1983, 2001b, 2004a, 2004b, 2005a, 2005b, 2005c), Diewert and Lawrence (2000, 2005), Diewert, Mizobuchi and Nomura (2007), Diewert and Schreyer (2006), Diewert and Wykoff (2007), Hicks (1961), T.P. Hill (1999, 2000), Hulten (1986, 1990, 1992, 1996), Jorgenson (1963, 1980, 1989, 1995a, 1995b, 1996), and Schreyer (2001, 2005). See also Baldwin and Tanguay (2006), de Haan et al. (2005), Gu and Tang (2004), Harper (2004), Harper, Berndt and Wood (1989), Hayashi and Nomura (2005), R.J. Hill and T.P. Hill (2003), Inklaar, O'Mahony and Timmer (2005), Kuroda and Nomura (2004), Morrison (1988, 1999), Nomura (2004, 2005), Schreyer (2001, 2005), Timmer and van Ark (2005) and Triplett (1996).

or a contemporaneous comparison for two different productive units such as for Canada and the United States. Comparative productivity measures are sometimes referred to more specifically as productivity growth, or as relative productivity, measures, depending on the nature of the comparison.

Economists have tended to prefer the most comprehensive of possible productivity statistics: total factor productivity, designated commonly as TFP and defined as output divided by a measure of total input – i.e., a price weighted aggregate of the volumes of all of the inputs used in producing the designated output. In terms of the components of (2.3-2) above, we can represent TFP as follows:

$$\text{TFP} \equiv \frac{\text{GDP}}{\begin{bmatrix}\text{Total}\\ \text{input}\end{bmatrix}} = (\text{A}). \tag{2.3-3}$$

Statistical agencies charged with producing productivity figures for nations are painfully aware that they do not manage to take account of *all* of the inputs used in producing the output of a nation. Thus official statistics agencies usually refer to the measures they compile, which are intended and used as approximations to TFP indexes, as multifactor productivity measures. These MFP measures can also be represented in terms of the components of the decomposition of GDP; i.e., we have

$$\text{MFP} \equiv \frac{\text{GDP}}{\begin{bmatrix}\text{Measured}\\ \text{input}\end{bmatrix}} = \text{TFP} \times (\text{B}). \tag{2.3-4}$$

Labor productivity measures are far easier to compile than TFP and MFP type measures because the only input information needed is for the volume of labor used in producing the designated output. Labor productivity measures also have an especially transparent relationship to per capita GDP, which has given these productivity measures special public policy appeal.

One way of measuring the labor input is as an average wage weighted aggregate of the hours of work for different types of workers. The resulting weighted hours productivity measure can be specified as follows in terms of the components of per capita GDP given in (2.3-2):

$$\text{WHLP} \equiv \frac{\text{GDP}}{\begin{bmatrix}\text{Total}\\ \text{labor}\\ \text{input}\end{bmatrix}} = \text{TFP} \times (\text{B}) \times (\text{C}) = \text{MFP} \times (\text{C}). \tag{2.3-5}$$

A simpler and more common way of measuring the labor input is as total hours of work (i.e., as the *un*weighted sum). The resulting hours labor productivity measure can be specified as:

$$\begin{aligned}\text{HLP} &\equiv \frac{\text{GDP}}{\begin{bmatrix}\text{Total}\\ \text{work}\\ \text{hours}\end{bmatrix}} = \text{TFP} \times (\text{B}) \times (\text{C}) \times (\text{D}) = \text{MFP} \times (\text{C}) \times (\text{D})\\ &= \text{WHLP} \times (\text{D}).\end{aligned} \tag{2.3-6}$$

An even simpler way of measuring the labor input is as the number of workers. The resulting worker labor productivity measure is:

$$\mathrm{LP} \equiv \frac{\mathrm{GDP}}{\begin{bmatrix}\text{Number}\\ \text{of}\\ \text{workers}\end{bmatrix}} = \mathrm{TFP} \times (\mathrm{B}) \times (\mathrm{C}) \times (\mathrm{D}) \times (\mathrm{E}) = \mathrm{MFP} \times (\mathrm{C}) \times (\mathrm{D}) \times (\mathrm{E})$$
$$= \mathrm{WHLP} \times (\mathrm{D}) \times (\mathrm{E}) = \mathrm{HLP} \times (\mathrm{E}). \qquad (2.3\text{-}7)$$

As noted above, to be meaningfully interpreted, productivity measures must usually be placed in a comparative context. Productivity growth (or relative productivity) is evaluated by the ratio of the labor productivity, MFP or TFP measures for period (or production scenario) t versus s.

3. Four TFPG concepts in the N–M case

> "But even if we confine our attention to what is ordinarily called a commodity, such as 'wheat,' we find ourselves dealing with a composite commodity made up of winter wheat, spring wheat, of varying grades."
>
> [Paul A. Samuelson (1983, p. 130), Foundations of Economic Analysis]

Obviously, nations produce multiple outputs using multiple inputs. How can we measure the four concepts of TFPG introduced in Subsection 2.1 in general multiple input, multiple output production situations? This is the question explored in this section.

We begin by defining volume aggregates that are components of the Paasche, Laspeyres, and Fisher Ideal (referred to hereafter simply as Fisher) volume, price and TFPG indexes, and then give the formulas for these indexes. Törnqvist and implicit Törnqvist index numbers are also defined.

3.1. Price weighted volume aggregates

For a general N-input, M-output production process, the period t input and output price vectors are denoted by $w^t \equiv [w_1^t, \ldots, w_N^t]$ and $p^t \equiv [p_1^t, p_2^t, \ldots, p_M^t]$, while $x^t \equiv [x_1^t, \ldots, x_N^t]$ and $y^t \equiv [y_1^t, \ldots, y_M^t]$ denote the period t input and output volume vectors.

Nominal total cost C^t and revenue R^t can be viewed as price weighted volume aggregates of the micro data for the transactions, and are defined as follows for period s and t:

$$C^t \equiv \sum_{n=1}^{N} w_n^t x_n^t, \qquad R^t \equiv \sum_{m=1}^{M} p_m^t y_m^t, \qquad (3.1\text{-}1)$$

$$C^s \equiv \sum_{n=1}^{N} w_n^s x_n^s \quad \text{and} \quad R^s \equiv \sum_{m=1}^{M} p_m^s y_m^s. \qquad (3.1\text{-}2)$$

We also define four hypothetical volume aggregates.[15] The first two result from evaluating period t volumes using period s price weights:

$$\sum_{n=1}^{N} w_n^s x_n^t \quad \text{and} \quad \sum_{m=1}^{M} p_m^s y_m^t. \tag{3.1-3}$$

These aggregates are what the cost and revenue would have been if the period t inputs and outputs had been transacted at period s prices. In contrast, the third and fourth aggregates are sums of period s volumes evaluated using period t prices:

$$\sum_{n=1}^{N} w_n^t x_n^s \quad \text{and} \quad \sum_{m=1}^{M} p_m^t y_m^s. \tag{3.1-4}$$

These are what the cost and revenue would have been if the period s inputs had been purchased and the period s outputs had been sold at period t prices. No assumptions are involved in defining the hypothetical volume aggregates.[16]

3.2. The Paasche, Laspeyres and Fisher volume and price indexes

The Paasche (1874), Laspeyres (1871), and Fisher (1922, p. 234) output volume indexes can be defined, respectively, as follows using the volume aggregates given in (3.1-1)–(3.1-4):

$$Q_P \equiv \sum_{m=1}^{M} p_m^t y_m^t \Big/ \sum_{m=1}^{M} p_m^t y_m^s, \tag{3.2-1}$$

$$Q_L \equiv \sum_{m=1}^{M} p_m^s y_m^t \Big/ \sum_{m=1}^{M} p_m^s y_m^s, \quad \text{and} \tag{3.2-2}$$

$$Q_F \equiv (Q_P Q_L)^{(1/2)}. \tag{3.2-3}$$

[15] Formally, the first two of these can be obtained by deflating the period t nominal cost and revenue by a Paasche price index. The second two result from deflating the period t nominal cost and revenue by a Laspeyres price index. See Horngren and Foster (1987, Chapter 24, Part One) or Kaplan and Atkinson (1989, Chapter 9) for examples of this common accounting practice of controlling for price level change without mention of price indexes. See also Armitage and Atkinson (1990).

[16] Traditionally these aggregates were defined as weighted averages of volume and price relatives. A volume (price) relative for a good is the ratio of the volume (price) for that good in a specified period t to the volume (price) for that good in some comparison period s. One advantage of defining a volume (or price) index as a weighted average of relatives is that the relatives are unit free, making it clear that this is an acceptable way of incorporating even goods (prices) for which there is no generally accepted unit of measure. The equivalent definitions presented here are more convenient for establishing that each of these TFPG indexes is a measure of all four of the different concepts of TFPG introduced in Subsection 2.1.

Similarly, the Paasche, Laspeyres, and Fisher input volume indexes can be defined as:

$$Q_P^* \equiv \sum_{n=1}^{N} w_n^t x_n^t \Big/ \sum_{n=1}^{N} w_n^t x_n^s, \tag{3.2-4}$$

$$Q_L^* \equiv \sum_{n=1}^{N} w_n^s x_n^t \Big/ \sum_{n=1}^{N} w_n^s x_n^s, \quad \text{and} \tag{3.2-5}$$

$$Q_F^* \equiv \left(Q_P^* Q_L^*\right)^{(1/2)}. \tag{3.2-6}$$

Output and input volume indexes are all that are needed to define measures of the first and second concepts of TFPG. However, in order to specify measures of the third and fourth concepts for the multiple input, multiple output case, price indexes are needed too.

Price indexes can be constructed using any of the functional forms given for volume indexes simply by reversing the roles of the prices and volumes. Thus output and input price indexes for the Paasche, Laspeyres and Fisher formulas are given by:

$$P_P \equiv \sum_{m=1}^{M} p_m^t y_m^t \Big/ \sum_{m=1}^{M} p_m^s y_m^t, \tag{3.2-7}$$

$$P_P^* \equiv \sum_{n=1}^{N} w_n^t x_n^t \Big/ \sum_{n=1}^{N} w_n^s x_n^t, \tag{3.2-8}$$

$$P_L \equiv \sum_{m=1}^{M} p_m^t y_m^s \Big/ \sum_{m=1}^{M} p_m^s y_m^s, \tag{3.2-9}$$

$$P_L^* \equiv \sum_{n=1}^{N} w_n^t x_n^s \Big/ \sum_{n=1}^{N} w_n^s x_n^s, \tag{3.2-10}$$

$$P_F \equiv (P_P P_L)^{(1/2)}, \quad \text{and} \tag{3.2-11}$$

$$P_F^* \equiv \left(P_P^* P_L^*\right)^{(1/2)}. \tag{3.2-12}$$

A price index is defined to be the implicit counterpart of a volume index if the product rule (also called the product test or axiom) is satisfied.[17] This rule requires that the product of the volume and price indexes must equal the total cost ratio for input side indexes or the total revenue ratio for output side indexes.[18] Usually the implicit price index will not have the same functional form as the volume index it is associated with. For example, the Paasche price index is the implicit counterpart of a Laspeyres volume

[17] For more on the properties of direct versus implicit indexes, see Allen and Diewert (1981).

[18] The implicit price (volume) index corresponding to a given volume (price) index can always be derived by imposing the product test and solving for the price (volume) index that satisfies this rule. The product test is part of the axiomatic approach to the choice of an index number functional form that is reviewed in Section 4.

index, and the Laspeyres price index is the implicit counterpart of a Paasche volume index. The Fisher indexes are unusual in that the Fisher price index satisfies the product test rule when paired with a Fisher volume index.[19]

In defining and proving equalities for the measures of the four concepts of TFPG for a general multiple input, multiple output production situation, we use the following implications of the product rule. In particular, for the Paasche, Laspeyres and Fisher indexes, on the input side we have

$$Q_P^* \times P_L^* = Q_L^* \times P_P^* = Q_F^* \times P_F^* = C^t/C^s, \tag{3.2-13a}$$

and on the output side we have

$$Q_P \times P_L = Q_L \times P_P = Q_F \times P_F = R^t/R^s. \tag{3.2-13b}$$

3.3. *TFPG measures for the N–M case*

The traditional definition of a total factor productivity growth index in the index number literature is as a ratio of output and input volume indexes:

$$\text{TFPG} \equiv Q/Q^*. \tag{3.3-1}$$

Thus the Paasche, Laspeyres, and Fisher TFPG indexes can be defined using the Paasche, Laspeyres, and Fisher volume indexes. Given a choice of *any one* of these three functional forms, we prove here that the corresponding multiple input, multiple output case measures are all equal for the four concepts of TFPG introduced in Subsection 2.1.

We proceed as follows to establish these equalities for the measures of the TFPG(1), TFPG(2) and TFPG(3) concepts. We first use the product rule results to define Paasche, Laspeyres and Fisher TFPG(3) measures. We substitute in the definitions of the components of the TFPG(3) measures and rearrange terms to establish the equalities with the TFPG(2) and TFPG(1) measures. Then we take up the TFPG(4) case.

For a Paasche TFPG index we have:

$$\begin{aligned}\text{TFPG}_P &= \frac{Q_P}{Q_P^*} = \frac{(R^t/R^s)/P_L}{(C^t/C^s)/P_L^*} \equiv \text{TFPG}(3)_P \quad \text{using (3.3-1) and (3.2-13)}\\ &= \frac{\sum_{m=1}^M p_m^t y_m^t / \sum_{m=1}^M p_m^t y_m^s}{\sum_{n=1}^N w_n^t x_n^t / \sum_{n=1}^N w_n^t x_n^s} \equiv \text{TFPG}(2)_P\\ &\qquad \text{using (3.1-1), (3.1-2) and also (3.2-9) and (3.2-10)}\end{aligned}$$

[19] When the product of a price and a volume index that both have the same formula equals the value ratio (i.e., the revenue ratio in the case of output indexes, or the cost ratio in the case of input indexes), then the formula satisfies the factor reversal test. The Fisher formula is unusual, but not unique, in satisfying this test. See Diewert (1987) on the factor reversal test.

$$= \frac{\sum_{m=1}^{M} p_m^t y_m^t / \sum_{n=1}^{N} w_n^t x_n^t}{\sum_{m=1}^{M} p_m^t y_m^s / \sum_{n=1}^{N} w_n^t x_n^s} \equiv \text{TFPG}(1)_P. \tag{3.3-2}$$

For a Laspeyres TFPG index we have:

$$\text{TFPG}_L = \frac{Q_L}{Q_L^*} = \frac{(R^t/R^s)/P_P}{(C^t/C^s)/P_P^*} \equiv \text{TFPG}(3)_L \quad \text{using (3.3-1) and (3.2-13)}$$

$$= \frac{\sum_{m=1}^{M} p_m^s y_m^t / \sum_{m=1}^{M} p_m^s y_m^s}{\sum_{n=1}^{N} w_n^s x_n^t / \sum_{n=1}^{N} w_n^s x_n^s} \equiv \text{TFPG}(2)_L$$

using (3.1-1), (3.1-2) and also (3.2-7) and (3.2-8)

$$= \frac{\sum_{m=1}^{M} p_m^s y_m^t / \sum_{n=1}^{N} w_n^s x_n^t}{\sum_{m=1}^{M} p_m^s y_m^s / \sum_{n=1}^{N} w_n^s x_n^s} \equiv \text{TFPG}(1)_L. \tag{3.3-3}$$

And for a Fisher TFPG index we have:

$$\text{TFPG}_F = \frac{Q_F}{Q_F^*} = \frac{(R^t/R^s)/P_F}{(C^t/C^s)/P_F^*} \equiv \text{TFPG}(3)_F \quad \text{using (3.3-1) and (3.2-13)}$$

$$= \frac{[(\frac{R^t}{R^s})P_L]^{1/2}[(\frac{R^t}{R^s})P_P]^{1/2}}{[(\frac{C^t}{C^s})P_L^*]^{1/2}[(\frac{C^t}{C^s})P_P^*]^{1/2}} = \frac{\left[\frac{\sum_{m=1}^{M} p_m^t y_m^t}{\sum_{m=1}^{M} p_m^t y_m^s}\right]^{1/2}\left[\frac{\sum_{m=1}^{M} p_m^s y_m^t}{\sum_{m=1}^{M} p_m^s y_m^s}\right]^{1/2}}{\left[\frac{\sum_{n=1}^{N} w_n^t x_n^t}{\sum_{n=1}^{N} w_n^t x_n^s}\right]^{1/2}\left[\frac{\sum_{n=1}^{N} w_n^s x_n^t}{\sum_{n=1}^{N} w_n^s x_n^s}\right]^{1/2}}$$

$$\equiv \text{TFPG}(2)_F$$

using (3.2-3), (3.2-13), (3.1-1), (3.1-2), and (3.2-7)–(3.2-10)

$$= \frac{\left[\frac{\sum_{m=1}^{M} p_m^t y_m^t}{\sum_{n=1}^{N} w_n^t x_n^t}\right]^{1/2}\left[\frac{\sum_{m=1}^{M} p_m^s y_m^t}{\sum_{n=1}^{N} w_n^s x_n^t}\right]^{1/2}}{\left[\frac{\sum_{m=1}^{M} p_m^t y_m^s}{\sum_{n=1}^{N} w_n^t x_n^s}\right]^{1/2}\left[\frac{\sum_{m=1}^{M} p_m^s y_m^s}{\sum_{n=1}^{N} w_n^t x_n^s}\right]^{1/2}} \equiv \text{TFPG}(1)_F. \tag{3.3-4}$$

The TFPG(4) concept is the rate of growth in the margin after controlling for price change. In the N–M case, just as in the 1–1 one, the margin m^t is given by

$$1 + m^t \equiv R^t/C^t. \tag{3.3-5}$$

Depending on whether Laspeyres, Paasche or Fisher price indexes are used to deflate the cost and revenue components of the margin, the respective expressions for TFPG(3) given in (3.3-2), (3.3-3) and (3.3-4) can be rewritten as:

$$\text{TFPG}(4)_P \equiv \left[\left(1 + m^t\right)/\left(1 + m^s\right)\right]\left[P_L^*/P_L\right], \tag{3.3-6}$$

$$\text{TFPG}(4)_L \equiv \left[\left(1 + m^t\right)/\left(1 + m^s\right)\right]\left[P_P^*/P_P\right], \quad \text{and} \tag{3.3-7}$$

$$\text{TFPG}(4)_F \equiv \left[\left(1 + m^t\right)/\left(1 + m^s\right)\right]\left[P_F^*/P_F\right]. \tag{3.3-8}$$

Notice that if the margins are zero, regardless of the reasons, then each of these expressions for TFPG(4) reduces to the ratio of the input price index to the output price index.[20]

3.4. Other index number formulas

Many other index number formulas have been proposed besides the Paasche, Laspeyres and Fisher.[21] Here we will use Q_G and P_G and Q_G^* and P_G^* to denote any two pairs of direct and implicit output and input volume and price indexes. These are any output side and input side pairs of volume and price indexes that satisfy the product rule so that $Q_G P_G = (R^t/R^s)$ and $Q_G^* P_G^* = (C^t/C^s)$. From these product rule results and (3.3-5), it is easily seen that the following measures of concepts (3.3-2), (3.3-3) and (3.3-4) of TFPG are all equal:

$$\begin{aligned}\frac{(R^t/R^s)/P_G}{(C^t/C^s)/P_G^*} &\equiv \text{TFPG}(3)_G \\ &= Q_G/Q_G^* \equiv \text{TFPG}(2)_G \\ &= \big[(1+m^t)\big/(1+m^s)\big]\big[P^*/P\big] \equiv \text{TPFG}(4).\end{aligned} \tag{3.4-1}$$

This is a general result that nests the results given in Subsection 3.3.

But what about $\text{TFPG}(1)_G$? A measure of the growth in the rate of transformation of total input into total output ideally should be defined using measures of input and output that are comparable for period s and t in the sense that the micro level volumes for both periods are aggregated using the same price weights. This is a desirable property if levels comparisons are to be made for pairs of nations. The volume aggregates that are the components of the Paasche, Laspeyres and Fisher TFPG(1) measures defined in the first line of (3.3-2), (3.3-3) and (3.3-4) satisfy what we refer to as this *comparability over time ideal*.[22] There are many other index number formulas for which it is not possible to define this sort of a measure for the TFPG(1) concept that also equals the corresponding measures for the other three concepts of TFPG. For those that are nevertheless superlative, an approximate equality of $\text{TFPG}(1)_G$ with the expressions for the other three concepts of TFPG is established as follows.

[20] One set of conditions under which the margins will be zero is perfect competition and a constant returns to scale technology.

[21] See Diewert (1993b, 1993c) and Fisher (1911, 1922).

[22] The period t cost and revenue and the hypothetical aggregates of period s output and input volumes defined in expressions (3.1-1) and (3.1-4) are comparable in this sense because the volumes for period s and t are evaluated using the same period t price vectors. Similarly, the period s cost and revenue and the hypothetical aggregates of period t output and input volumes defined in expressions (3.1-2) and (3.1-3) are comparable in this sense because the volumes of the output and input goods are evaluated using the same period s price vectors. These aggregates are what are used to define the Paasche, Laspeyres and Fisher measures given in (3.3-2), (3.3-3) and (3.3-4).

For any pair of volume and price indexes satisfying the product test, from (3.4-1) and the product rule implications we see that the following expressions equal, respectively, those given in (3.4-1) for TFPG$(2)_G$, TFPG$(3)_G$ and TFPG$(4)_G$:

$$\frac{Q_G}{Q_G^*} = \frac{(R^t/R^s)/P}{(C^t/C^s)/P^*} = \frac{\sum_{m=1}^{M}(p_m^t/P_G)y_m^t / \sum_{m=1}^{M} p_m^s y_m^s}{\sum_{n=1}^{N}(w_n^t/P_G^*)x_n^t / \sum_{n=1}^{N} w_n^s x_n^s}. \tag{3.4-2}$$

In the last of these expressions, the price vectors (p^t/P_G) and (w^t/P_G^*) appearing in the period t output and input volume aggregates are the period t prices expressed in period s dollars. If we choose this expression as the measure of TFPG$(1)_G$, then with the choice of a Paasche, Laspeyres or Fisher formula, this measure will be ideal in the sense of using the same price weights to compare the period t and s volumes. When some other formula is used, there is an approximate solution to this problem for indexes that satisfy the product rule and are also “superlative”. This approximate solution makes use of the Fisher functional form with the TFPG(1) measure, defined as in the last line of (3.3-4).

Diewert coined the term superlative for an index number functional form that is “exact” in that it can be derived algebraically from a producer or consumer behavioral equation that satisfies the Diewert flexibility criterion. According to this criterion, a functional form is flexible if it can provide a second order approximation to an arbitrary twice continuously differentiable linearly homogeneous function. Diewert (1976, 1978b) established that under usual conditions, all of the commonly used superlative index number formulas (including the Fisher, and also the Törnqvist and implicit Törnqvist functional forms introduced below) approximate each other to the second order when evaluated at an equal price and volume point. This is a numerical analysis approximation result that does not rely on any further assumptions.[23]

Because the Fisher volume and price indexes satisfy the product rule, we have

$$Q_G P_G = \left(R^t/R^s\right) = Q_F P_F \quad \text{and} \quad Q_G^* P_G^* = \left(C^t/C^s\right) = Q_F^* P_F^*,$$

and dividing through by P_G and P_G^*, respectively, yields

$$\frac{Q_G}{Q_G^*} = \left[\frac{Q_F}{Q_F^*}\right]\left[\frac{P_F/P_G}{P_F^*/P_G^*}\right]. \tag{3.4-3}$$

From (3.4-3), (3.4-1) and (3.3-4) we see that if we define the measure for the first concept of TFPG as

$$\text{TPFG}(1)_G \equiv \text{TPFG}(1)_F\left[\frac{P_F/P_G}{P_F^*/P_G^*}\right], \tag{3.4-4}$$

this measure will equal TFPG$(2)_G$, TFPG$(3)_G$ and TFPG$(4)_G$ as defined in (3.4-1). In this TFPG$(1)_G$ measure, the period t price vectors, p^t and w^t, of the TFPG$(1)_F$ component are replaced by $(p^t/(P_F/P_G))$ and $(w^t/(P_F^*/P_G^*))$. As a consequence, unless

[23] R.J. Hill (2006) shows, however, that being superlative does not, by itself, ensure an index is desirable.

the given price indexes are Laspeyres or Paasche or Fisher ones, the period t and s volumes compared by the measure will not be aggregated using the same price weights when there have been changes in relative prices. Nevertheless, for superlative index numbers, it follows that when the chosen volume and price indexes are any of the commonly used ones such as the Törnqvist or implicit Törnqvist, then we can use the result that, under usual conditions, all of the superlative indexes in common use approximate each other to the second order at an equal price and volume point. That is, we have $\mathrm{TFPG}(1)_G \cong \mathrm{TFPG}(1)_F$.

3.5. The Törnqvist (or Translog) indexes[24]

Törnqvist (1936) indexes are weighted geometric averages of growth rates for the volume or price relatives for the different products. These indexes have been widely used by statistical agencies and in the economics literature. The formula for the natural logarithm of a Törnqvist index is usually shown as the definition for this index. For the output volume index, this is

$$\ln Q_T = (1/2) \sum_{m=1}^{M} \left[\left(p_m^s y_m^s \Big/ \sum_{i=1}^{M} p_i^s y_i^s \right) + \left(p_m^t y_m^t \Big/ \sum_{j=1}^{M} p_j^t y_j^t \right) \right] \ln\left(y_m^t / y_m^s\right). \tag{3.5-1}$$

The Törnqvist input volume index Q_T^* is defined analogously, with input volumes and prices substituted for the output volumes and prices in (3.5-1).

Reversing the role of the prices and volumes in the formula for the Törnqvist output volume index yields the Törnqvist output price index, P_T, defined by

$$\ln P_T = (1/2) \sum_{m=1}^{M} \left[\left(p_m^s y_m^s \Big/ \sum_{i=1}^{M} p_i^s y_i^s \right) + \left(p_m^t y_m^t \Big/ \sum_{j=1}^{M} p_j^t y_j^t \right) \right] \ln\left(p_m^t / p_m^s\right). \tag{3.5-2}$$

The input price index P_T^* is defined in a similar manner.

The implicit Törnqvist output volume index, denoted by $Q_{\widetilde{T}}$, is defined implicitly by[25] $(R^t/R^s)/P_T \equiv Q_{\widetilde{T}}$, and the implicit Törnqvist input volume index, $Q_{\widetilde{T}}^*$, is defined analogously using the cost ratio and P_T^*. The implicit Törnqvist output price index, $P_{\widetilde{T}}$, is given by $(R^t/R^s)/Q_T \equiv P_{\widetilde{T}}$, and the implicit Törnqvist input price index, $P_{\widetilde{T}}^*$, is defined analogously.

Using the Törnqvist volume and the implicit Törnqvist price indexes, or the implicit Törnqvist volume and the Törnqvist price indexes, measurement formulas for the second, third and fourth concepts of TFPG can be specified as in (3.4-1) above. Moreover,

[24] Törnqvist indexes are also known as translog indexes following Jorgenson and Nishimizu (1978) who introduced this terminology because Diewert (1976, p. 120) related Q_T^* to a translog production function. The exact index number approach used for relating specific volume indexes to specific production functions is the topic of Section 5.

[25] See Diewert (1992a, p. 181).

these are superlative indexes for which Section 3.4 approximation result applies; that is, we have $\text{TFPG}(1)_T \cong \text{TFPG}(1)_F$ and $\text{TFPG}(1)_{\widetilde{T}} \cong \text{TFPG}(1)_F$.

4. The axiomatic (or test) approach to index formula choice

Multiple TFPG index number formulas can all be viewed as measures of total factor productivity growth. This was demonstrated in Section 3 for the commonly used Laspeyres, Paasche, Fisher and Törnqvist indexes, and this result could be established for other proposed index number formulas as well. Since different formulas will yield different estimates for TFPG, which one should be used, and why? Historically, index number theorists have relied on what is called the axiomatic or test approach to address this functional form choice problem. An overview of this approach is provided here.

As before, Q denotes an output volume index and P denotes an output price index. The corresponding input volume and price indexes are denoted by the same symbols with a star superscript added. The axiomatic approach to the determination of the functional forms for Q and P on the output side, or for Q^* and P^* on the input side, works as follows. The starting point is a list of mathematical properties that *a priori* reasoning suggests a price index should satisfy. These are the index number theory 'tests' or 'axioms'. Mathematical reasoning is applied to determine whether the *a priori* tests are mutually consistent and whether they uniquely determine, or usefully narrow, the choice of the functional form for the price index.[26] Once the form of the price index has been decided on, imposition of the product test rule determines the functional form of the volume index as well.

The *product test* was already introduced in Subsection 3.2.[27] On the output side, this rule states that the product of the output price and output volume indexes, P and Q, should equal the nominal revenue ratio for periods t and s:

$$PQ = R^t/R^s. \tag{4-1}$$

If the functional form for the output price index P is given, then imposing the product rule means that the functional form for the volume index must be given by the expression[28]

$$Q = \left(R^t/R^s\right)/P. \tag{4-2}$$

[26] Contributors to this approach include Walsh (1901, 1921), Fisher (1911, 1922), Eichhorn (1976), Eichhorn and Voeller (1976), Funke and Voeller (1978, 1979), Diewert (1976, 1987, 1988, 1992b, 1999), Balk (1995) and Armstrong (2003).

[27] The product test was proposed by Irving Fisher (1911, p. 388) and named by Frisch (1930, p. 399).

[28] Volume or price indexes derived by imposing the product rule and specifying the form of the price or volume index are sometimes referred to as implicit indexes. The $\sim$ symbol is sometimes added on top of the symbol for the index number when it is desired to call attention to the implicit nature of the index. Any test that satisfies the factor reversal test would also satisfy the product test.

Thus, unlike the other tests introduced below that are applied to the alternative price indexes of interest and that may be passed or failed by each of the index number formulas tested, the product test is often imposed at the outset as part of the formula choice process.[29]

We conclude this overview of the axiomatic approach by listing four tests that can be applied for choosing among alternative functional forms for the price index. Only the output side price indexes are considered here, but the tests are applied in the same manner on the input side.

The *identity* or *constant prices test* is[30]

$$P\left(p, p, y^s, y^t\right) = 1. \tag{4-3}$$

What this means is that if all prices stay the same over the current and comparison time periods so that $p^s = p^t = p = (p_1, \ldots, p_M)$, then the price index should be one regardless of the volume values for period s and t.

The *constant basket test*, also called the *constant volumes test*, is[31]

$$P\left(p^s, p^t, y, y\right) = \sum_{i=1}^{N} p_i^t y_i \Big/ \sum_{j=1}^{N} p_j^s y_j. \tag{4-4}$$

This test states that if the volumes produced for all output goods stay the same for period s and t so that $y^s = y^t = y \equiv (y_1, \ldots, y_M)$, then the level of prices in period t compared to s should equal the value of the constant basket of volumes evaluated at the period t prices divided by the value of this same basket evaluated at the period s prices.

The *proportionality in period t prices test* is[32]

$$P\left(p^s, \lambda p^t, y^s, y^t\right) = \lambda P\left(p^s, p^t, y^s, y^t\right) \quad \text{for } \lambda > 0. \tag{4-5}$$

According to this test, if each of the elements of p^t is multiplied by the positive constant λ, then the level of prices in period t relative to s should differ by the same multiplicative factor λ.

Our final example of a price index test is the *time reversal test*[33]:

$$P\left(p^t, p^s, y^t, y^s\right) = 1/P\left(p^s, p^t, y^s, y^t\right). \tag{4-6}$$

[29] Note that the product test is not the same as the factor reversal test, although any formula that satisfies the factor reversal test will satisfy the product test. As pointed out to us by Andy Baldwin in private correspondance, in imposing the product test on a price index, one normally has already chosen the volume index and the price index is chosen by default to satisfy the product index. Thus the Paasche formula is chosen for the price index because one would like to have a Laspeyres volume index.

[30] This test was proposed by Laspeyres (1871, p. 308), Walsh (1901, p. 308) and Eichhorn and Voeller (1976, p. 24).

[31] This test was proposed by many researchers including Walsh (1901, p. 540).

[32] This test was proposed by Walsh (1901, p. 385) and Eichhorn and Voeller (1976, p. 24).

[33] This test was first informally proposed by Pierson (1896, p. 128) and was formalized by Walsh (1901, p. 368, 1921, p. 541) and Fisher (1922, p. 64).

If this test is satisfied, then when the prices and volumes for period s and t are interchanged, the resulting price index will be the reciprocal of the original price index.

The Paasche and Laspeyres indexes, P_P and P_L, fail the time reversal test (4-6). The Törnqvist index, P_T, fails the constant basket test (4-4), and the implicit Törnqvist index, $\widetilde{P}_T$, fails the constant prices test (4-3). On the other hand, the Fisher price index P_F satisfies all four of these tests. When a more extensive list of tests is compiled, the Fisher price index continues to satisfy more tests than other leading candidates.[34] These results favor the Fisher TFPG index. However, the Paasche, Laspeyres, Törnqvist, and implicit Törnqvist indexes all rate reasonably well according to the axiomatic approach.

5. The exact approach and superlative index numbers

> "Tinbergen (1942, pp. 190–195) interprets the geometric volume index of total factor productivity as a Cobb–Douglas production function. As further examples of index-number formulas that have been interpreted as production functions, a fixed-weight Laspeyres volume index of total factor productivity may be interpreted as a 'linear' production function, that is, as a production function with infinite elasticity of substitution, as Solow (1957, p. 317) and Clemhout (1963, pp. 358–360) have pointed out. In a sense, output-capital or output-labor ratios correspond to Leontief-type production functions, that is, to production functions with zero elasticity of substitution, as Domar (1961, pp. 712–713) points out."
>
> [Dale W. Jorgenson (1995a, p. 48), Productivity Vol. 1]

An alternative approach to the determination of the functional form for a measure of total factor productivity growth is to derive the TFPG index from a producer behavioral model. Diewert's (1976) exact index number approach is a paradigm for doing this. This approach places the index number formula choice problem on familiar territory for economists, allowing the choice to be based on axioms of economic behavior or empirical evidence about producer behavior rather than, or in addition to, the traditional tests of the axiomatic approach.

The exact index number approach is perhaps most easily explained by outlining the main steps in an actual application. In this section we sketch the steps involved in deriving a TFPG index that is exact for a translog cost function for which certain stated restrictions hold.

The technology of a firm can be summarized by its period t production function f^t. If we focus on the production of output 1, then the period t production function can be represented as

$$y_1 = f^t(y_2, y_3, \ldots, y_M, x_1, x_2, \ldots, x_N). \tag{5-1}$$

[34] See Diewert (1976, p. 131, 1992b) and also Funke and Voeller (1978, p. 180).

This function gives the amount of output 1 the firm can produce using the technology available in any given period t if it also produces y_m units of each of the outputs $m = 2, \ldots, M$ using x_n units for each of the inputs $n = 1, \ldots, N$.

The production function f^t can be used to define the period t cost function:

$$c^t(y_1, y_2, \ldots, y_M, w_1, w_2, \ldots, w_N). \tag{5-2}$$

This function is postulated to give the minimum cost of producing the output volumes $y_1, \ldots, y_M$ using the period t technology and with the given input prices w_n^t, $n = 1, 2, \ldots, N$. Under the assumption of cost minimizing behavior, the observed period t cost of production, denoted by C^t, is the minimum possible cost, and we have

$$C^t \equiv \sum_{n=1}^{N} w_n^t x_n^t = c^t\left(y_1^t, \ldots, y_M^t, w_1^t, \ldots, w_N^t\right). \tag{5-3}$$

We need some way of relating the cost functions for different time periods (or scenarios) to each other. One way is to assume the cost function for each period is a period specific multiple of an atemporal cost function. As a simplest (and much used case), we might assume that

$$c^t(y_1, \ldots, y_M, w_1, \ldots, w_N) = \left(1/a^t\right)c(y_1, \ldots, y_M, w_1, \ldots, w_N), \quad t = 0, 1, \ldots, T, \tag{5-4}$$

where $a^t > 0$ denotes a period t relative efficiency parameter and c denotes an atemporal cost function which does not depend on time. We have assumed in (5-4) that technological change is Hicks neutral. The normalization $a^0 \equiv 1$ is usually imposed. Given (5-4), a natural measure of productivity change (or relative productivity) for a productive unit for period t versus s is the ratio

$$a^t/a^s. \tag{5-5}$$

If this ratio is greater than 1, efficiency is said to have improved.

Taking the natural logarithm of both sides of (5-4), we have

$$\ln c^t\left(y_1^t, \ldots, y_M^t, w_1^t, \ldots, w_N^t\right) = -\ln a^t + \ln c\left(y_1^t, \ldots, y_M^t, w_1^t, \ldots, w_N^t\right). \tag{5-6}$$

Suppose that *a priori* information is available indicating that a translog functional form is appropriate for $\ln c$. In this case, the atemporal cost function c on the right-hand side of (5-6) can be represented by

$$\begin{aligned} &\ln c\left(y_1^t, \ldots, y_M^t, w_1^t, \ldots, w_N^t\right) \\ &\quad = b_0 + \sum_{m=1}^{M} b_m \ln y_m^t + \sum_{n=1}^{N} c_n \ln w_n^t + (1/2)\sum_{i=1}^{M}\sum_{j=1}^{M} d_{ij} \ln y_i^t \ln y_j^t \\ &\qquad + (1/2)\sum_{n=1}^{N}\sum_{j=1}^{N} f_{nj} \ln w_n^t \ln w_j^t + \sum_{m=1}^{M}\sum_{n=1}^{N} g_{mn} \ln y_m^t \ln w_n^t. \end{aligned} \tag{5-7}$$

An advantage of the choice of the translog functional form for the atemporal cost function part of (5-6) is that it does not impose *a priori* restrictions on the admissible patterns of substitution between inputs and outputs, but this flexibility results from a large number of free parameters.[35] There are $M+1$ of the b_m parameters, N of the c_n parameters, MN of the g_{mn} parameters, $M(M+1)/2$ independent d_{ij} parameters and $N(N+1)/2$ independent f_{nj} parameters even when it is deemed reasonable to impose the symmetry conditions that $d_{ij} = d_{ji}$ for $1 \leqslant i < j \leqslant M$ and $f_{nj} = f_{jn}$ for $1 \leqslant n < j \leqslant N$. If homogeneity of degree one in the input prices is also assumed, then the following additional restrictions hold for the parameters of (5-7):

$$\sum_{n=1}^{N} c_n = 1, \quad \sum_{j=1}^{N} f_{nj} = 0 \quad \text{for } n = 1, \ldots, N, \quad \text{and}$$
$$\sum_{n=1}^{N} g_{mn} = 0 \quad \text{for } m = 1, \ldots, M. \tag{5-8}$$

With all of the above restrictions, the number of independent parameters in (5-6) and in (5-7) is still $T + M(M+1)/2 + N(N+1)/2 + MN$. The number of parameters can easily end up being larger than the number of available observations.[36] Thus, without imposing more restrictions, it may not be possible to reliably estimate the parameters of (5-6) or to derive a productivity index from this sort of an estimated relationship.

One way of proceeding is to assume the producer is minimizing costs so that the following demand relationships hold[37]:

$$x_n^t = \partial c^t\big(y_1^t, \ldots, y_M^t, w_1^t, \ldots, w_N^t\big)/\partial w_n$$
$$\text{for } n = 1, \ldots, N \text{ and } t = 0, 1, \ldots, T. \tag{5-9}$$

Since $\ln c^t$ can also be regarded as a quadratic function in the variables

$$\ln y_1, \ln y_2, \ldots, \ln y_M, \ln w_1, \ln w_2, \ldots, \ln w_N,$$

[35] The translog functional form for a single output technology was introduced by Christensen, Jorgenson and Lau (1971, 1973). See also Christensen and Jorgenson (1973). The multiple output case was defined by Burgess (1974) and Diewert (1974a, p. 139).

[36] On the econometric estimation of cost and related aggregator functions using more flexible functional forms that permit theoretically plausible types of substitution, see for example Berndt (1991), Berndt and Khaled (1979) and also Diewert (1969, 1971, 1973, 1974b, 1978a, 1981a, 1982) and Diewert and Wales (1992, 1995).

[37] This follows by applying a theoretical result due initially to Hotelling (1925) and Shephard (1953, p. 11).

Diewert's (1976, p. 119) logarithmic quadratic identity can be applied. Accordingly, we have[38]:

$$\begin{aligned}
\ln c^t - \ln c^s &= (1/2)\sum_{m=1}^{M}\left[y_m^t \frac{\partial \ln c^t}{\partial y_m}(y^t, w^t) + y_m^s \frac{\partial \ln c^s}{\partial y_m}(y^s, w^s)\right]\ln(y_m^t/y_m^s) \\
&\quad + (1/2)\sum_{n=1}^{N}\left[w_n^t \frac{\partial \ln c^t}{\partial w_n}(y^t, w^t) + w_n^s \frac{\partial \ln c^s}{\partial w_n}(y^s, w^s)\right]\ln(w_n^t/w_n^s) \\
&\quad + (1/2)\left[\frac{\partial \ln c^t}{\partial a}(y^t, w^t) + \frac{\partial \ln c^s}{\partial a}(y^s, w^s)\right]\ln(a^t/a^s) \qquad (5\text{-}10) \\
&= (1/2)\sum_{m=1}^{M}\left[y_m^t \frac{\partial \ln c^t}{\partial y_m}(y^t, w^t) + y_m^s \frac{\partial \ln c^s}{\partial y_m}(y^s, w^s)\right]\ln(y_m^t/y_m^s) \\
&\quad + (1/2)\sum_{n=1}^{N}\left[(w_n^t x_n^t/C^t) + (w_n^s x_n^s/C^s)\right]\ln(w_n^t/w_n^s) \\
&\quad + (1/2)\left[-1 + (-1)\right]\ln(a^t/a^s). \qquad (5\text{-}11)
\end{aligned}$$

If it is acceptable to impose the additional assumption of competitive profit maximizing behavior, we can simplify (5-11) even further. More specifically, suppose we can assume that the output volumes $y_1^t, \dots, y_M^t$ solve the following profit maximization problem for $t = 0, 1, \dots, T$:

$$\max_{y_1,\dots,y_M}\left[\sum_{m=1}^{M} p_m^t y_m - c^t(y_1, \dots, y_M, w_1^t, \dots, w_N^t)\right]. \qquad (5\text{-}12)$$

This leads to the usual price equals marginal cost relationships that result when competitive price taking behavior is assumed; i.e., we now have

$$p_m^t = \partial c^t(y_1^t, \dots, y_M^t, w_1^t, \dots, w_N^t)/\partial y_m, \quad m = 1, \dots, M. \qquad (5\text{-}13)$$

This key step permits the use of observed prices as weights for aggregating the observed volume data for the different outputs and inputs. Making use of the definition of total costs in (5-3), expression (5-11) can now be rewritten as:

$$\begin{aligned}
\ln(C^t/C^s) &= (1/2)\sum_{m=1}^{M}\left[(p_m^t y_m^t/C^t) + (p_m^s y_m^s/C^s)\right]\ln(y_m^t/y_m^s) \\
&\quad + (1/2)\sum_{n=1}^{N}\left[(w_n^t x_n^t/C^t) + (w_n^s x_n^s/C^s)\right]\ln(w_n^t/w_n^s) - \ln(a^t/a^s).
\end{aligned} \qquad (5\text{-}14)$$

[38] Expression (5-11) follows from (5-10) by applying the Hotelling–Shephard relations (5-9) for period t and s.

Total costs in period s and t presumably can be observed, as can output and input prices and volumes. Thus the only unknown in Equation (5-14) is the productivity change measure going from period s to t. Solving (5-14) for this measure yields

$$a^t/a^s = \left\{ \prod_{m=1}^{M} \left(y_m^t/y_m^s\right)^{(1/2)[(p_m^t y_m^t/C^t)+(p_m^s y_m^s/C^s)]} \right\} / \widetilde{Q}_T^*, \tag{5-15}$$

where $\widetilde{Q}_T^*$ is the implicit Törnqvist input volume index that is defined analogously to the implicit Törnqvist output volume index introduced in Subsection 3.5.

Formula (5-15) can be simplified still further if it is appropriate to assume that the underlying technology exhibits constant returns to scale. If costs grow proportionally with output, then it can be shown [e.g., see Diewert (1974a, pp. 134–137)] that the cost function must be linearly homogeneous in the output volumes. In that case, with competitive profit maximizing behavior, revenues must equal costs in each period. In other words, under the additional hypothesis of constant returns to scale, for each time period $t = 0, 1, \ldots, T$ we have the equality:

$$c^t\left(y^t, w^t\right) = C^t = R^t. \tag{5-16}$$

Using (5-16), we can replace C^t and C^s in (5-15) by R^t and R^s, and (5-15) becomes

$$a^t/a^s = Q_T/\widetilde{Q}_T^*, \tag{5-17}$$

where Q_T is the Törnqvist output volume index and $\widetilde{Q}_T^*$ is the implicit Törnqvist input volume index. This means that if we can justify the choice of a translog cost function and if the assumptions underlying the above derivations are true, then we have a basis for choosing $(Q_T/\widetilde{Q}_T^*)$ as the appropriate functional form of the TFPG index.

The hypothesis of constant returns to scale that must be invoked in moving from expression (5-15) to (5-17) is very restrictive. However, if the underlying technology is subject to diminishing returns to scale, we can convert the technology into an artificial one still subject to constant returns to scale by introducing an extra fixed input, x_{N+1} say, and setting this extra fixed input equal to one (that is, $x_{N=1}^t = 1$ for each period t). The corresponding period t price for this input, w_{N+1}^t, is set equal to the firm's period t profits, $R^t - C^t$. With this extra factor, the firm's period t cost is redefined to be the adjusted cost given by

$$C_A^t = C^t + w_{N+1}^t x_{N+1}^t = \sum_{n=1}^{N+1} w_n^t x_n^t = R^t. \tag{5-18}$$

The derivation can now be repeated using the adjusted cost C_A^t rather than the actual cost C^t. This results in the same productivity change formula except that $\widetilde{Q}_T^*$ is now the implicit translog volume index for $N + 1$ instead of N inputs. Thus, in the diminishing returns to scale case, we could use formula (5-15) as our measure of productivity change between period s and t, or we could use formula (5-17) with the understanding that the extra fixed input would then be added into the list of inputs and incorporated into the adjusted costs.

Formulas (5-15) and (5-17) illustrate the exact index number approach to the derivation of productivity change measures. The method may be summarized as follows: (1) *a priori* or empirical evidence is used as a basis for choosing a specific functional form for the firm's cost function,[39] (2) competitive profit maximizing behavior is assumed (or else cost minimizing plus competitive revenue maximizing behavior), and (3) various identities are manipulated and a productivity change measure emerges that depends only on observable prices and volumes.

In this section, the use of the exact index number method has been demonstrated for a situation where the functional form for the cost function was assumed to be adequately approximated by a translog with parameters satisfying symmetry, homogeneity, cost minimization, profit maximization, and possibly also constant returns to scale conditions. The resulting productivity change term a^t/a^s given by the formula on the right-hand side of (5-15) or (5-17) can be directly evaluated even with thousands of outputs and inputs.

It is important to bear in mind, however, that all of the index number TFPG measures defined in Section 3 can be evaluated numerically for each time period given suitable volume and price data regardless of whether assumptions such as those made above are true. The assumptions are used only to show that particular TFPG index number formulas can be derived from certain optimizing models of producer behavior. Such a model might then be used in interpreting the TFPG value. For instance, the model might be used as a basis for breaking up the TFPG value into returns to scale and technical progress components. Decompositions of this sort are taken up in Sections 6.1, 10.2 and 11.

6. Production function based measures of TFPG

When a TFPG index can be related to a producer behavioral relationship that is derived from an optimizing model of producer behavior, this knowledge provides a potential theoretical basis for defining various decompositions of TFPG and interpreting component parts. This is the approach adopted here.

We begin in Subsection 6.1 by considering some production function based alternatives for factoring TFPG into technical progress (TP) and returns to scale (RS) components in the simplified one input, one output case. Even in the general multiple input, multiple output case, a TP and RS decomposition of TFPG has no direct implications for the choice of a measurement formula for TFPG since the new parameters introduced

[39] In place of step (1) where a specific functional form is assumed for the firm's cost function, some researchers have specified functional forms for the firm's production function [e.g., Diewert (1976, p. 127)] or the firm's revenue or profit function [e.g., Diewert (1988)] or for the firm's distance function [e.g., Caves, Christensen and Diewert (1982a and 1982b)].

in making these decompositions cancel out in the representation of TFPG as a product of the TP and RS components. However, the decomposition makes us more aware that *an index number TFPG measure typically includes the effects of both technical progress (a shift in the production function) and nonconstant returns to scale if present (a movement along a nonconstant returns to scale production function).*[40]

After defining TP and RS components for the 1–1 case in Subsection 6.1, in Subsection 6.2 theoretical Malmquist output growth, input growth and TFPG indexes are defined for a general multiple input, multiple output production situation.

6.1. Technical progress (TP) and returns to scale (RS) in the simple 1–1 case

The amount of output obtained from the same input volumes could differ in period t versus s for two different sorts of reasons: (1) the same technology might be used, but with a different scale of operation, or (2) the technology might differ. The purpose of the decompositions introduced here is to provide a conceptual framework for thinking about returns to scale versus technological shift changes in TFPG.

In the 1–1 case, TFPG can be measured as the ratio of the period t and s output–input coefficients, as in (2.1-2). Suppose we know the period s and t volumes for the single input and the single output, as well as the true period s and t production functions given by:

$$y_1^s = f^s\left(x_1^s\right) \quad \text{and} \tag{6.1-1}$$

$$y_1^t = f^t\left(x_1^t\right). \tag{6.1-2}$$

Technical progress can be conceptualized as a shift in a production function due to a switch to a new technology for some given scale of operation for the productive process. Four of the possible measures of shift for a production function are considered here. For the first two, the scale is hypothetically held constant by fixing the input level. For the second two, the scale is hypothetically held constant by fixing the output level.

Some hypothetical volumes are needed for defining the four shift measures given here: two on the output side and two on the input side. The output side hypothetical volumes are

$$y_1^{s*} \equiv f^t\left(x_1^s\right) \quad \text{and} \tag{6.1-3}$$

$$y_1^{t*} \equiv f^s\left(x_1^t\right). \tag{6.1-4}$$

The first of these is the output that hypothetically *could be* produced with the scale fixed by the period s input volume x_1^s but using the newer period t technology embodied in f^t. Given technical progress rather than regress, y_1^{s*} should be larger than y_1^s. The second

[40] Favorable or adverse changes in environmental factors facing the firm going from period s to t are regarded as shifts in the production function. We are assuming here that producers are on their production frontier each period; i.e., that they are technically efficient. In a more complete analysis, we could allow for technical inefficiency.

volume, y_1^{t*}, is the output that hypothetically *could be* produced with the scale fixed by the period t input volume x_1^t but using the older period s technology. Given technical progress rather than regress, y_1^{t*} should be smaller than y_1^t.

Turning to the input side now, x_1^{s*} and x_1^{t*} are defined implicitly by

$$y_1^s = f^t(x_1^{s*}) \quad \text{and} \tag{6.1-5}$$

$$y_1^t = f^s(x_1^{t*}). \tag{6.1-6}$$

The first of these is the hypothetical amount of the single input factor required to produce the actual period s output, y_1^s, using the more recent period t technology. Given technical progress, x_1^{s*} should be less than x_1^s. The second volume x_1^{t*} is the hypothetical amount of the single input factor required to produce the period t output y_1^t using the older period s technology, so we would usually expect x_1^{t*} to be larger than x_1^t.

The first two of the four technical progress indexes to be defined here are the output based measures given by[41]

$$\text{TP}(1) \equiv y_1^{s*}/y_1^s = f^t(x_1^s)/f^s(x_1^s) \quad \text{and} \tag{6.1-7}$$

$$\text{TP}(2) \equiv y_1^t/y_1^{t*} = f^t(x_1^t)/f^s(x_1^t). \tag{6.1-8}$$

Each of these describes the percentage increase in output resulting solely from switching from the period s to the period t production technology with the scale of operation fixed by the actual period s or the period t input level for TP(1) and TP(2), respectively.

The other two indexes of technical progress defined here are input based[42]:

$$\text{TP}(3) \equiv x_1^s/x_1^{s*} \quad \text{and} \tag{6.1-9}$$

$$\text{TP}(4) \equiv x_1^{t*}/x_1^t. \tag{6.1-10}$$

Each of these gives the reciprocal of the percentage decrease in input usage resulting solely from switching from the period s to the period t production technology with the scale of operation fixed by the actual period s or the period t output level for TP(3) and TP(4), respectively. That is, for TP(3), technical progress is measured with the output level fixed at y_1^s whereas for TP(4) the output level is fixed at y_1^t.

Each of the technical progress measures defined above is related to TFPG as follows:

$$\text{TFPG} = \text{TP}(i)\,\text{RS}(i) \quad \text{for } i = 1, 2, 3, 4, \tag{6.1-11}$$

where, depending on the selected technical progress measure, the corresponding returns to scale measure is given by

$$\text{RS}(1) \equiv \left[y_1^t/x_1^t\right]\big/\left[y_1^{s*}/x_1^s\right], \tag{6.1-12}$$

[41] TP(1) and TP(2) are the output based 'productivity' indexes proposed by Caves, Christensen, and Diewert (1982b, p. 1402) for the simplistic case of one input and one output.

[42] TP(3) and TP(4) are the input based 'productivity' indexes proposed by Caves, Christensen, and Diewert (1982b, p. 1407) for the simplistic case of one input and one output.

$$\mathrm{RS}(2) \equiv \left[y_1^{t*}/x_1^t\right]/\left[y_1^s/x_1^s\right], \tag{6.1-13}$$

$$\mathrm{RS}(3) \equiv \left[y_1^t/x_1^t\right]/\left[y_1^s/x_1^{s*}\right], \quad \text{or} \tag{6.1-14}$$

$$\mathrm{RS}(4) \equiv \left[y_1^t/x_1^{t*}\right]/\left[y_1^s/x_1^s\right]. \tag{6.1-15}$$

In the TFPG decompositions given by (6.1-11), the technical progress term, $\mathrm{TP}(i)$, can be viewed as a production function *shift*[43] caused by a change in technology, and the returns to scale term, $\mathrm{RS}(i)$, can be viewed as a *movement along* a production function with the technology held fixed. Each returns to scale measure will be greater than one if output divided by input increases as we move along the production surface. Obviously, if $\mathrm{TP}(1) = \mathrm{TP}(2) = \mathrm{TP}(3) = \mathrm{TP}(4) = 1$, then RS = TFPG and increases in TFPG are due solely to changes of scale.

For two periods, say $s = 0$ and $t = 1$, and with just one input factor and one output good, the four measures of TP defined in (6.1-7)–(6.1-10) and the four measures of returns to scale defined in (6.1-12)–(6.1-15) can be illustrated graphically, as in Figure 1. (Here the subscript 1 is dropped for both the single input and the single output.)

The lower curved line is the graph of the period 0 production function; i.e., it is the set of points (x, y) such that $x \geqslant 0$ and $y = f^0(x)$. The higher curved line is the graph of the period 1 production function; i.e., it is the set of points (x, y) such that $x \geqslant 0$ and $y = f^1(x)$. The observed data points are A with coordinates (x^0, y^0)

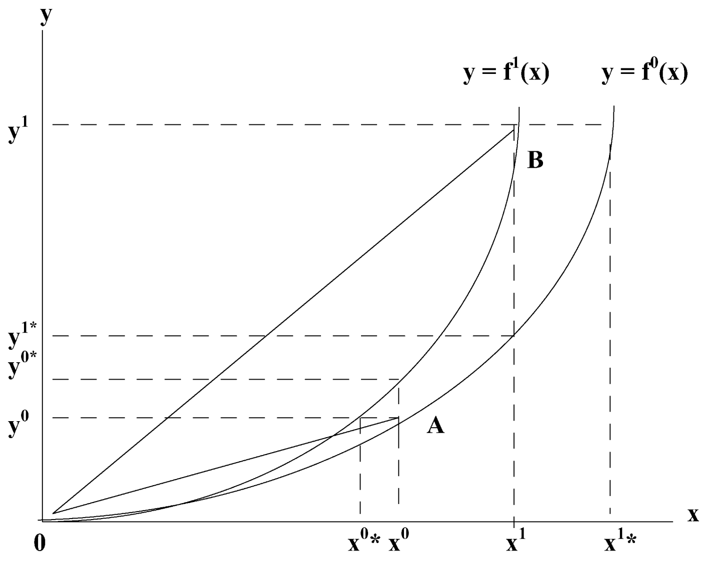

Figure 1. Production function based measures of technical progress.

43 This shift can be conceptualized as either a move from one production function to another, or equivalently as a change in the location and perhaps the shape of the original production function.

for period 0, and B with coordinates (x^1, y^1) for period 1.[44] Applying formula (2.1-2) from Section 2, for this example we have TFPG $= [y^1/x^1]/[y^0/x^0]$. In Figure 1, this is the slope of the straight line OB divided by the slope of the straight line OA. The reader can use Figure 1 and the definitions provided above to verify that each of the four decompositions of TFPG given by (6.1-11) corresponds to a different combination of shifts of, and movements along, a production function that take us from observed point A to observed point B.[45] Of course, there would be no way of distinguishing among the different possible mechanisms that could yield a move from A to B if nothing were known but the values of the points.

Geometrically, each of the specified measures for the returns to scale is the ratio of two output–input coefficients, say $[y^j/x^j]$ divided by $[y^k/x^k]$ for points (y^j, x^j) and (x^k, y^k) on the *same* fixed production function with $x^j > x^k$. For the ith measure, if the returns to scale component $\mathrm{RS}(i) = [y^j/x^j]/[y^k/x^k]$ is greater than 1, the production function exhibits increasing returns to scale, while if $\mathrm{RS}(i) = 1$ we have constant returns to scale, and if $\mathrm{RS}(i) < 1$ we have decreasing returns. If the returns to scale are constant, then $\mathrm{RS}(i) = 1$ and TP = TFPG.[46] *Note, however, that it is unnecessary to assume constant returns to scale in order to evaluate the index number TFPG measures presented here or in previous sections.*

6.2. Malmquist indexes

If the technology for a multiple input, multiple output production process can be represented in each time period by some known production function, this function can be used as a basis for defining theoretical Malmquist volume and Malmquist TFPG indexes. Malmquist indexes are introduced here, and then in the following subsection we show conditions under which these theoretical Malmquist indexes can be evaluated using the same information needed in order to evaluate the TFPG index numbers introduced in Section 3.

Here as previously, we let y_1^t denote the amount of output 1 produced in period t for $t = 0, 1, \ldots, T$. Here we also let $\tilde{y}^t \equiv [y_2^t, y_3^t, \ldots, y_M^t]$ denote the vector of other outputs jointly produced in each period t along with output 1 using the vector of input volumes $x^t \equiv [x_1^t, x_2^t, \ldots, x_N^t]$. Using these notational conventions, the production

44 In Figure 1, note that if the production function shifts were measured in absolute terms as differences in the direction of the y axis, then these shifts would be given by $y^{0*} - y^0$ (at point A) and $y^1 - y^{1*}$ (at point B). If the shifts were measured in absolute terms as differences in the direction of the x axis, then the shifts would be given by $x^0 - x^{0*}$ (at point A) and $x^{1*} - x^1$ (at point B). An advantage of measuring TP (and TFPG) using ratios is that the relative measures are invariant to changes in the units of measurement whereas the differences are not.

45 In a regulated industry, increasing returns to scale is often the reason for the regulation. See Diewert (1981b).

46 Solow's (1957, p. 313) Chart I is similar, but his figure is for the simpler case of constant returns to scale.

functions for output 1 in period s and t can be represented compactly as:

$$y_1^s = f^s(\tilde{y}^s, x^s) \quad \text{and} \quad y_1^t = f^t(\tilde{y}^t, x^t). \tag{6.2-1}$$

Three alternative Malmquist output volume indexes will be defined.[47]

The first Malmquist output index, α^s, is the number which satisfies

$$y_1^t/\alpha^s = f^s(\tilde{y}^t/\alpha^s, x^s). \tag{6.2-2}$$

This index is the number which just deflates the period t vector of outputs, $y^t \equiv [y_1^t, y_2^t, \ldots, y_M^t]$, into an output vector y^t/α^s that can be produced with the period s vector of inputs, x^s, using the period s technology. Due to substitution, when the number of output goods, M, is greater than 1, then the hypothetical output volume vector y^t/α^s will not usually be equal to the actual period s output vector, y^s. However, with only one output good, we have $y_1^t/\alpha^s = f^s(x^s) = y_1^s$ and this Malmquist output index reduces to $\alpha^s = y_1^t/y_1^s$.

A second Malmquist output index, α^t, is defined as the number which satisfies

$$\alpha^t y_1^s = f^t(\alpha^t \tilde{y}^s, x^t). \tag{6.2-3}$$

This index is the number that inflates the period s vector of outputs y^s into $\alpha^t y^s$, an output vector that can be produced with the period t vector of inputs x^t using the period t technology. The index $\alpha^t y^s$ will not usually be equal to y^t when there are multiple outputs. However, when $M = 1$, then $\alpha^t y_1^s = f^t(x^t) = y_1^t$ and $\alpha^t = y_1^t/y_1^s$.

When there is no reason to prefer either the index α^s or α^t, we recommend taking the geometric mean of these indexes. This is the third Malmquist index of output growth, defined as

$$\alpha \equiv [\alpha^s \alpha^t]^{1/2}. \tag{6.2-4}$$

When there are only two output goods, the Malmquist output indexes α^s and α^t can be illustrated as in Figure 2 for $t = 1$ and $s = 0$. The lower curved line represents the set of outputs $\{(y_1, y_2,): y_1 = f^0(y_2, x^0)\}$ that can be produced with period 0 technology and inputs. The higher curved line represents the set of outputs $\{(y_1, y_2,): y_1 = f^1(y_2, x^1)\}$ that can be produced with period 1 technology and inputs. The period 1 output possibilities set will generally be higher than the period 0 one for two reasons: (i) technical progress and (ii) input growth.[48] In Figure 2, the point $\alpha^1 y^0$ is the straight line projection of the period 0 output vector $y^0 = [y_1^0, y_2^0]$ onto the period 1 output possibilities

[47] These indexes correspond to the two output indexes defined in Caves, Christensen, and Diewert (1982b, p. 1400) and referred to by them as Malmquist indexes because Malmquist (1953) proposed indexes similar to these in concept, though his were for the consumer context. Indexes of this sort were subsequently defined as well by Moorsteen (1961) and Hicks (1961, 1981, pp. 192 and 256) for the producer context. See also Balk (1998, Chapter 4).

[48] However, with technical regress, production would become less efficient in period 1 compared to period 0. Also, if the utilization of inputs declined, then the period 1 output production possibilities set could lie below the period 0 one.

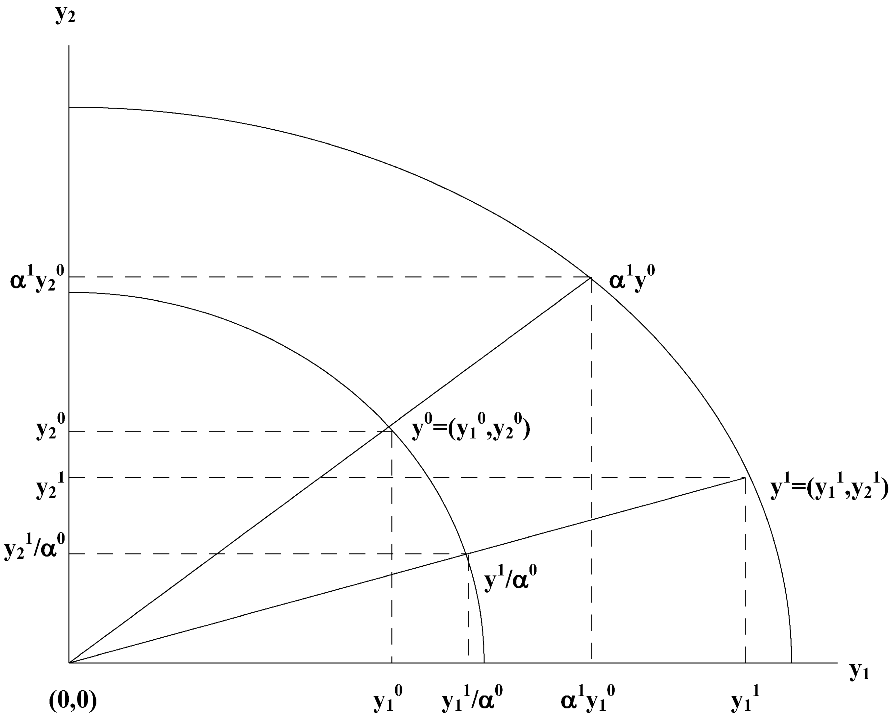

Figure 2. Alternative economic output indexes illustrated.

set, and $y^1/\alpha^0 = [y_1^1/\alpha^0, y_2^1/\alpha^0]$ is the straight line contraction of the output vector $y^1 = [y_1^1, y_2^1]$ onto the period 0 output possibilities set.

We now turn to the input side. A first Malmquist input index, β^s, is defined as follows:

$$y_1^s = f^s\left(\tilde{y}^s, x^t/\beta^s\right) \equiv f^s\left(y_2^s, \ldots, y_M^s, x_1^t/\beta^s, \ldots, x_N^t/\beta^s\right). \tag{6.2-5}$$

This index measures input growth holding fixed the period s technology and output vector. A second Malmquist input index, denoted by β^t, is the solution to the following equation

$$y_1^t = f^t\left(\tilde{y}^t, \beta^t x^s\right) \equiv f^t\left(y_2^t, \ldots, y_M^t, \beta^t x_1^s, \ldots, \beta^t x_N^s\right). \tag{6.2-6}$$

This index measures input growth holding fixed the period t technology and output vector.

When there is no reason to prefer β^s to β^t, we recommend a third Malmquist input index:

$$\beta \equiv \left[\beta^s \beta^t\right]^{1/2}. \tag{6.2-7}$$

Figure 3 illustrates the Malmquist indexes β^s and β^t for the case where there are just two input goods and for $t = 1$ and $s = 0$.

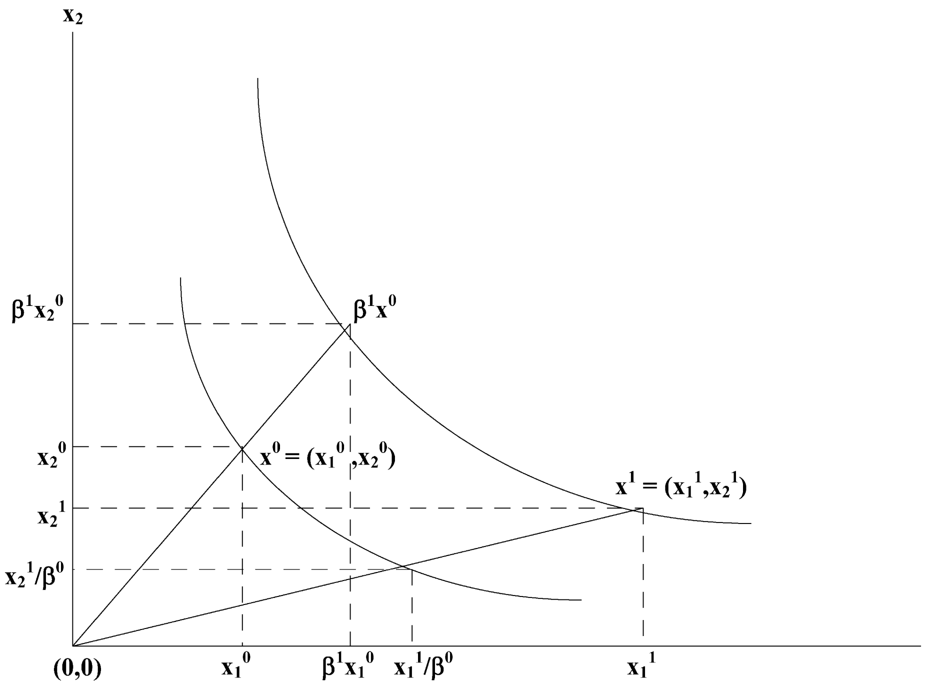

Figure 3. Alternative Malmquist input indexes illustrated.

The lower curved line in Figure 3 represents the set of inputs that are needed to produce the vector of outputs y^0 using period 0 technology. This is the set $\{(x_1, x_2)\colon\ y_1^0 = f^0(\tilde{y}^0, x_1, x_2)\}$. The higher curved line represents the set of inputs that are needed to produce the period 1 vector of outputs y^1 using period 1 technology. This is the set $\{(x_1, x_2)\colon\ y_1^1 = f^1(\tilde{y}^1, x_1, x_2)\}$.[49] The point $\beta^1 x^0 = [\beta^1 x_1^0, \beta^1 x_2^0]$ is the straight line projection of the input vector $x^0 \equiv [x_1^0, x_2^0]$ onto the period 1 input requirements set. The point $x^1/\beta^0 \equiv [x_1^1/\beta^0, x_2^1/\beta^0]$ is the straight line contraction of the input vector $x^1 \equiv [x_1^1, x_2^1]$ onto the period 0 input requirements set.

Once theoretical Malmquist volume indexes have been defined that measure the growth of total output and the growth of total input, then a Malmquist TFPG index for the general N–M case can be defined too. The definition we recommend for the Malmquist TFPG index is

$$\mathrm{TFPG}_M \equiv \alpha/\beta. \tag{6.2-8}$$

[49] If technical progress were sufficiently positive or if output growth between the two periods were sufficiently negative, then the period 1 input requirements set could lie *below* the period 0 input requirements set.

In the 1–1 case, expression (6.2-8) reduces to TFPG(2) as defined in expression (2.1-3), which equals the single measure for TFPG for the 1–1 case.

6.3. Direct evaluation of Malmquist indexes for the N–M case

Using the exact index number approach, Caves, Christensen, and Diewert (1982b, pp. 1395–1401) give conditions under which the Malmquist output and input volume indexes $\alpha \equiv [\alpha^s \alpha^t]^{1/2}$ and $\beta \equiv [\beta^s \beta^t]^{1/2}$ defined in (6.2-4) and (6.2-7) equal Törnqvist indexes. More specifically, Caves, Christensen, and Diewert give conditions under which

$$\alpha = Q_T \quad \text{and} \tag{6.3-1}$$

$$\beta = Q_T^*, \tag{6.3-2}$$

where Q_T is the Törnqvist output volume index and Q_T^* is the Törnqvist input volume index. The assumptions required to derive (6.3-1) and (6.3-2) are, roughly speaking: (i) price taking, revenue maximizing behavior, (ii) price taking, cost minimizing behavior, and (iii) a translog technology. Under these assumptions, we can evaluate the theoretical Malmquist measure TFPG_M by taking the ratio of the Törnqvist output and input volume indexes since we have

$$\text{TFPG}_M = \alpha/\beta = Q_T/Q_T^* \equiv \text{TFPG}_T\,. \tag{6.3-3}$$

The practical importance of (6.3-3) is that the Malmquist TFPG index can be evaluated directly from observable prices and volumes without knowing the parameter values for the true period specific production functions. This sort of result can be established as well for other representations of the technology, as we show now.

An intuitive explanation for the remarkable equalities in (6.3-1) and (6.3-2) rests on the following fact: if $f(z)$ is a quadratic function, then we have $f(z^t) - f(z^s) = (1/2)[\nabla f(z^t) + \nabla f(z^s)]^T[z^t - z^s]$. This result follows from applying Diewert's (1976, p. 118) Quadratic Approximation Lemma. Under the assumption of optimizing behavior on the part of the producer, the vectors of first order partial derivatives, $\nabla f(z^t)$ and $\nabla f(z^s)$, will be equal to or proportional to the observed prices. Thus the right-hand side of the above identity becomes observable without econometric estimation.

Recall that the "best" productivity index from the axiomatic point of view is the Fisher productivity index defined in (3.3-4) as

$$\text{TFPG}_F \equiv Q_F/Q_F^*,$$

with the Fisher output volume index Q_F defined by (3.2-3) and input volume index Q_F^* defined by (3.2-6). Diewert (1992b, pp. 240–243) shows these Fisher indexes equal Malmquist indexes when the firm's output distance function over the relevant time span has the functional form

$$\text{d}^t(y,x) = \sigma^t\big[y^T Ay\big(x^T Cx\big)^{-1} + \alpha^t \cdot y\beta^t \cdot x^{-1} y^T B^t x^{-1}\big]^{1/2}.$$

Here superscript T denotes a transpose, the parameter matrices A and C are symmetric and independent of time t, and the parameter vectors α^t and β^t and also the parameter matrix B^t can depend on time. The vector x^{-1} is defined as consisting of components that are the reciprocals of the components of the vector x of input volumes. The parameter matrices and vectors must also satisfy some additional restrictions that are listed in Diewert (1992b, p. 241).

It should be noted that the above results do *not* rely on the assumption of constant returns to scale in production. *These results extend the concept of superlative index numbers, which were originally defined under the assumption of constant returns to scale*. Also, the assumption of revenue maximizing behavior can be dropped if we know the marginal costs in the two periods under consideration, in which case we could directly evaluate the Malmquist indexes. However, usually we do not know these marginal costs.

In many respects, the Fisher TFPG index is the most attractive index formula.[50] Nevertheless, both the Fisher and the Törnqvist indexes should yield similar results.[51] Both are superlative index numbers. Diewert (1976, 1978b) established that all of the commonly used superlative index number formulas approximate each other to the second order when each index is evaluated at an equal price and volume point.[52] These approximation results, and also Diewert's (1978b) result for the Paasche and Laspeyres indexes, hold *without* the assumption of optimizing behavior and regardless of whether the assumptions about the technology are true. These are findings of numerical rather than economic analysis.

[50] Recall that the Fisher TFPG index satisfies what we have termed the comparability over time ideal, as shown in Subsections 3.3 and 3.4. For an index that satisfies this property, the aggregates that make up the components are comparable for period s and t in the sense that the micro level volumes are aggregated using the same price weights. Diewert (1992b) also shows that the Fisher index satisfies more of the traditional index number axioms than any other formula considered.

[51] See Diewert (1978b, p. 894).

[52] The term superlative means that an index is exact for a flexible functional form. Since the Fisher and the Törnqvist indexes are both superlative, they will both have the same first and second order partial derivatives with respect to all arguments when the derivatives are evaluated at a point where the price and volume vectors take on the same value for both period t and s. T.P. Hill (1993, p. 384) explains current accepted practice as follows: "Thus economic theory suggests that, in general, a symmetric index that assigns equal weight to the two situations being compared is to be preferred to either the Laspeyres or Paasche indices on their own. The precise choice of superlative index – whether Fisher, Törnqvist or other superlative index – may be of only secondary importance as all the symmetric indices are likely to approximate each other, and the underlying theoretic index fairly closely, at least when the index number spread between the Laspeyres and Paasche is not very great". R.J. Hill (2006) showed that whereas the approximation result of Diewert (1978b) which the remarks of T.P. Hill (1993) quoted above are based on and which have found their way into the manuals of statistical agencies around the world do indeed apply to all of the commonly used superlative indexes including the Fisher, Törnqvist, and implicit Törnqvist, the approximation can be poor for some other superlative indexes.

7. Cost function based measures

In this section, we define another set of theoretical output and input growth rate and TFPG measures based on the true underlying cost function instead of the production function as in Section 6. We give conditions under which these indexes equal the Laspeyres and the Paasche indexes. For the two output case, we also show how the Laspeyres and Paasche indexes relate to the Malmquist indexes defined in the previous section.

The period t cost function given by $c^t(y_1, y_2, \ldots, y_M, w_1, w_2, \ldots, w_N)$ in (5-2) is the minimum cost of producing the given volumes $y_1, y_2, \ldots, y_M$ of the M output goods using the input volumes $x_1, x_2, \ldots, x_N$ purchased at the unit prices $w_1, w_2, \ldots, w_N$ and using the period t technology summarized by the production function constraint $y_1 = f^t(y_2, \ldots, y_M, x_1, x_2, \ldots, x_N)$. In this section, we assume that the period s and t cost functions, c^s and c^t, are known and we examine theoretical output, input and productivity indexes that can be defined using these cost functions.

Under the assumptions of perfect information and cost minimizing behavior on the part of the production unit, the actual period t total cost equals the period t cost function evaluated at the period t output volumes and input prices. Thus we have

$$c^t\left(y^t, w^t\right) = \sum_{n=1}^{N} w_n^t x_n^t \equiv w^t \cdot x^t \equiv C^t. \tag{7-1}$$

(As in the above expression, weighted sums will sometimes be represented as inner products of vectors in addition to, or as an alternative to, the summation sign representation.) The cost function in (7-1) is assumed to be differentiable with respect to the components of the vector y at the point (y^t, w^t). Under the assumed conditions, the ith marginal cost for period t, denoted by mc_i^t, is given by

$$\mathrm{mc}_i^t \equiv \partial c^t\left(y^t, w^t\right)/\partial y_i, \quad i = 1, 2, \ldots, M. \tag{7-2}$$

Marginal costs for period s are defined analogously.

Just as the output unit prices were used as weights for the period s and period t volumes in the formulas for the Laspeyres and Paasche volume indexes given in Section 3, here the marginal cost vectors, mc^s and mc^t, are used to define theoretical Laspeyres and Paasche type output and input volume indexes. These indexes are given by

$$\gamma_L \equiv \mathrm{mc}^s \cdot y^t/\mathrm{mc}^s \cdot y^s \quad \text{and} \tag{7-3}$$

$$\gamma_P \equiv \mathrm{mc}^t \cdot y^t/\mathrm{mc}^t \cdot y^s. \tag{7-4}$$

When we have no reason to prefer γ_L over γ_P, we recommend using as a theoretical measure of the output growth rate the geometric mean of γ_L and γ_P; that is, we recommend

$$\gamma \equiv [\gamma_L \gamma_P]^{1/2}. \tag{7-5}$$

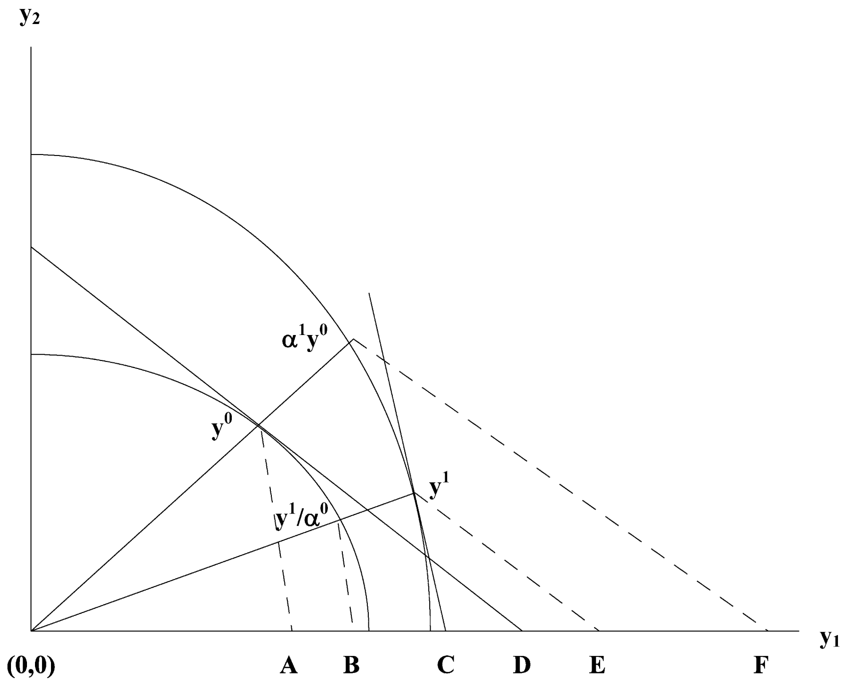

Figure 4. Alternative price based theoretical output indexes.

With price taking, profit maximizing behavior, the observed output volume vector y^t is determined as the solution to the first order necessary conditions for the period t profit maximization problem and economic theory implies that $p^t = \mathrm{mc}^t$. If this is the case, then γ_L defined in (7-3) equals the usual Laspeyres output index, Q_L, defined in (3.2-2), and γ_P defined in (7-4) equals the usual Paasche output index, Q_P, defined in (3.2-1). Moreover, in this case, γ defined in (7-5) equals the Fisher output index, Q_F, defined in (3.2-3).

With just two outputs and under the assumptions of price taking, profit maximizing behavior, the differences between the new theoretical output indexes γ_P and γ_L and the Malmquist output indexes α^0 and α^1 can be illustrated using Figure 4.

The lower curved line in Figure 4 is the period $s = 0$ output possibilities set, $\{(y_1, y_2)\colon\ y_1 = f^0(y_2, x^0)\}$. The higher curved line is the period $t = 1$ output possibilities set, $\{(y_1, y_2)\colon\ y_1 = f^1(y_2, x^1)\}$. The straight line ending in D is tangent to the period 0 output possibilities set at the observed period 0 output vector $y^0 \equiv [y_1^0, y_2^0]$, and the straight line ending in C is tangent to the period 1 output possibilities set at the observed period 1 output vector $y^1 \equiv [y_1^1, y_2^1]$. The marginal costs for period 0 and period 1 are denoted by mc_i^0 and mc_i^1 for outputs $i = 1, 2$. The tangent line through y^0, the output volume vector for period 0, has the slope $-(\mathrm{mc}_1^0/\mathrm{mc}_2^0)$ and the tangent line through y^1, the period 1 output volume vector, has the slope $-(\mathrm{mc}_1^1/\mathrm{mc}_2^1)$. The straight line ending in E passes through y^1, and the straight line ending in F passes through $\alpha^1 y^0$. Both of these lines are parallel to the line ending in D, which is the tangent to the period 0 output possibility set at the point (y_1^0, y_2^0). Similarly, the straight line ending in A passes through y^0, and is parallel to the straight line ending in B passes through y^1/α^0, and both are parallel to the line ending in C,[53] which is the tangent to the period 1 output possibility set at the point (y_1^1, y_2^1).

For the theoretical output indexes defined above, we will always have $\gamma_L = \mathrm{OE}/\mathrm{OD} < \mathrm{OF}/\mathrm{OD} = \alpha^1$ and $\gamma_P = \mathrm{OC}/\mathrm{OA} > \mathrm{OC}/\mathrm{OB} = \alpha^0$. Although the four output indexes can be quite different in magnitude as illustrated in Figure 4, the geometric average of γ_L and γ_P should be reasonably close to the geometric average of α^0 and α^1. Moving to the input side, the theoretical input volume indexes are given by[54]

$$\delta_L \equiv c^t\left(y^t, w^s\right)/c^s\left(y^s, w^s\right) \quad \text{and} \tag{7-6}$$

$$\delta_P \equiv c^t\left(y^t, w^t\right)/c^s\left(y^s, w^t\right). \tag{7-7}$$

In the case of two inputs and under the assumptions of price taking, profit maximizing behavior, the differences between δ_L and δ_P on the one hand and the Malmquist indexes β^s and β^t on the other hand can be illustrated as in Figure 5. The lower curved line is the period $s = 0$ set of combinations of the two input factors that can be used to produce y^0 under f^0. The upper curved line is the period $t = 1$ set of input combinations that can be used to produce y^1 under f^1.

The straight line ending at the point E in Figure 5 is tangent to the input possibilities curve for period 1 at the observed input vector $x^1 \equiv [x_1^1, x_2^1]$. This tangent line has slope $-(w_1^1/w_2^1)$ and, by construction, the lines ending in A, B, and C have this same slope. The line ending at point C passes through the period 0 observed input vector $x^0 \equiv [x_1^0, x_2^0]$. The line ending at B passes through $x^1/\beta^0 \equiv [x_1^1/\beta^0, x_2^1/\beta^0]$. Finally, the line ending at A is tangent to the period 0 input possibilities set.

Similarly, the straight line ending at the point D in Figure 5 is tangent to the period 0 input possibilities set at the point x^0. The slope of this tangent line is $-(w_1^0/w_2^0)$ and, by construction, the lines ending in F, G, and H have this same slope. The line ending at H passes through x^1. The line ending at G passes through $\beta^1 x^0 \equiv [\beta^1 x_1^0, \beta^1 x_2^0]$, and the line ending at F is tangent to the period 1 input possibilities curve. It can be shown that $\delta_L = \mathrm{OF}/\mathrm{OD} < \mathrm{OG}/\mathrm{OD} = \beta^1$ and $\delta_P = \mathrm{OE}/\mathrm{OA} > \mathrm{OE}/\mathrm{OB} = \beta^0$.[55]

[53] Note that the y_1 intercept of a line with the slope of the relevant price ratio – i.e., the y_1 intercept of a line with the slope of the tangent to the designated production possibilities frontier – equals the revenue from the designated output vector denominated in equivalent amounts of good 1.

[54] If there is only one output and if $c^s = c^t$, then δ_L and δ_P reduce to indexes proposed by Allen (1949, p. 199).

[55] The tangency relation follows using Shephard's (1953, p. 11) Lemma: $x_1^0 = \partial c^0(y^0, w_1^0, w_2^0)/\partial w_1$ and $x_2^0 = \partial c^0(y^0, w_1^0, w_2^0)/\partial w_2$. Similarly, the fact that the tangent line ending at E has slope equal to w_1^1/w_2^1

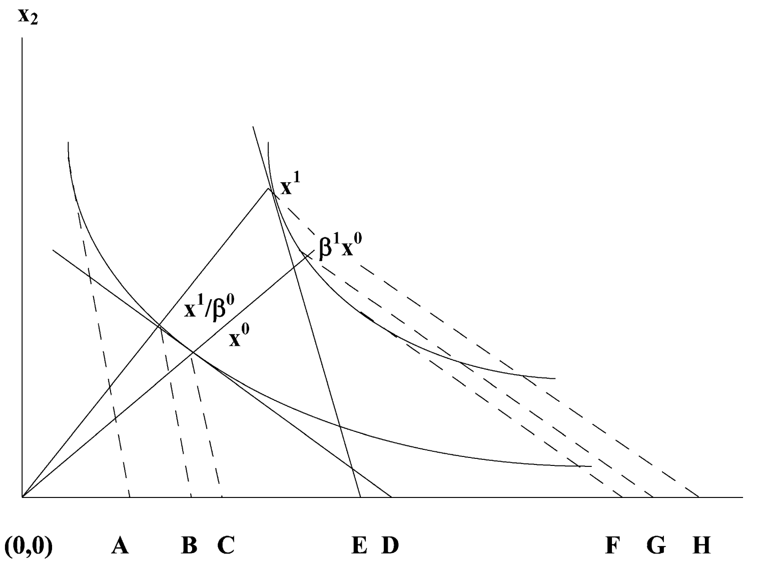

Figure 5. Alternative price based economic input indexes.

8. The Divisia approach

In discrete time approaches to productivity measurement, the price and volume data are defined only for integer values of t, which denotes discrete unit time periods. In contrast, in Divisia's (1926, p. 40) approach, the price and volume variables are defined as functions of continuous time.[56] To emphasize the continuous time feature of the Divisia approach, here the price and volume of output m at time t are denoted by $p_m(t)$ and $y_m(t)$ and the price and volume of input n at time t are denoted by $w_n(t)$ and $x_n(t)$. The price and volume functions are assumed to be differentiable with respect to time over an interval of $0 \leqslant t \leqslant 1$.

Revenue and cost can be represented as

$$R(t) \equiv \sum_{m=1}^{M} p_m(t) y_m(t) \tag{8-1}$$

follows from $x_1^1 = \partial c^1(y^1, w_1^1, w_2^1)/\partial w_1$ and $x_2^1 = \partial c^1(y^1, w_1^1, w_2^1)/\partial w_2$. Note that the x_1 intercept of a line with the slope of $-(w_1^0/w_2^0)$, as is the case for the lines ending in D, F, G or H, or of a line with the slope of $-(w_1^1/w_2^1)$, as is the case for the lines ending in A, B, C or D, is equal to the cost of the stated input vector denominated in units of input factor 1.

[56] For more on the Divisia approach see Hulten (1973) and also Balk (2000).

and

$$C(t) \equiv \sum_{n=1}^{N} w_n(t)x_n(t). \tag{8-2}$$

Differentiating both sides of (8-1) with respect to time and dividing by $R(t)$, we obtain

$$\begin{aligned} R'(t)/R(t) &= \left[\sum_{m=1}^{M} p'_m(t)y_m(t) + \sum_{m=1}^{M} p_m(t)y'_m(t)\right] \Big/ R(t) && (8\text{-}3)\\ &= \sum_{m=1}^{M} [p'_m(t)/p_m(t)][p_m(t)y_m(t)/R(t)] \\ &\quad + \sum_{m=1}^{M} [y'_m(t)/y_m(t)][p_m(t)y_m(t)/R(t)] \\ &= \sum_{m=1}^{M} [p'_m(t)/p_m(t)]s_m^R(t) + \sum_{m=1}^{M} [y'_m(t)/y_m(t)]s_m^R(t), && (8\text{-}4) \end{aligned}$$

where a prime denotes the time derivative of a function and $s_m^R(t) \equiv [p_m(t)y_m(t)]/R(t)$ is the revenue share of output m at time t. $R'(t)/R(t)$ represents the (percentage) rate of change in revenue at time t.

The first set of terms on the right-hand side of (8-4) is a revenue share weighted sum of the rates of growth in the prices. Divisia (1926, p. 40) defined the aggregate output price growth rate to be[57]

$$P'(t)/P(t) \equiv \sum_{m=1}^{M} [p'_m(t)/p_m(t)]s_m^R(t). \tag{8-5}$$

The second set of terms on the right-hand side of (8-4) is a revenue share weighted sum of the rates of growth for the volumes of the individual output products. Divisia defined the aggregate output volume growth rate to be

$$Y'(t)/Y(t) \equiv \sum_{m=1}^{M} [y'_m(t)/y_m(t)]s_m^R(t). \tag{8-6}$$

Substituting (8-5) and (8-6) into (8-4) yields:

$$R'(t)/R(t) = P'(t)/P(t) + Y'(t)/Y(t). \tag{8-7}$$

[57] This is much like declaring the Törnqvist output index to be a measure of output price growth, since it is a weighted aggregate of the growth rates for the prices of the individual output goods.

In words, (8-7) says that the revenue growth at time t is equal to aggregate output price growth plus aggregate output volume growth at time t. Equation (8-7) is the Divisia index counterpart to the output side product test decomposition.

A decomposition similar to (8-7) can be derived in the same way for the (percentage) rate of growth in cost at time t, $C'(t)/C(t)$. Differentiating both sides of (8-2) with respect to t and dividing both sides by $C(t)$ yields

$$C'(t)/C(t) = \left[\sum_{n=1}^{N} w'_n(t)x_n(t) + \sum_{n=1}^{N} w_n(t)x'_n(t)\right]\Big/ C(t)$$
$$= \sum_{n=1}^{N}\big[w'_n(t)/w_n(t)\big]s_n^C(t) + \sum_{n=1}^{N}\big[x'_n(t)/x_n(t)\big]s_n^C(t). \tag{8-8}$$

Here $w'_n(t)$ is the rate of change of the nth input price, $x'_n(t)$ is the rate of change of the nth input volume, and $s_n^C(t) \equiv [w_n(t)x_n(t)]/C(t)$ is the input n share of total cost at time t.

Let $W(t)$ and $X(t)$ denote the Divisia input price and input volume aggregates evaluated at time t, where their proportional rates of change are defined by the two cost share weighted sums of the rates of growth of the individual microeconomic input prices and volumes:

$$W'(t)/W(t) \equiv \sum_{n=1}^{N}\big[w'_n(t)/w_n(t)\big]s_n^C(t) \quad \text{and} \tag{8-9}$$

$$X'(t)/X(t) \equiv \sum_{n=1}^{N}\big[x'_n(t)/x_n(t)\big]s_n^C(t). \tag{8-10}$$

Substituting (8-9) and (8-10) into (8-8) yields the following input side version of (8-7):

$$C'(t)/C(t) = W'(t)/W(t) + X'(t)/X(t). \tag{8-11}$$

In words, (8-11) says that the rate of growth in cost is equal to aggregate input price growth plus aggregate input volume growth at time t. Equation (8-11) is the Divisia index counterpart to the input side product test decomposition in the axiomatic approach to index number theory.

The Divisia TFPG index can be defined as the Divisia measure for the aggregate output volume growth rate, as given in (8-6), minus the Divisia measure for the aggregate input volume growth rate, as given in (8-10)[58]:

$$\mathrm{TFPG}(t) \equiv \big[Y'(t)/Y(t)\big] - \big[X'(t)/X(t)\big], \tag{8-12}$$

[58] See Jorgenson and Griliches (1967, p. 252). Note that the Divisia productivity measure is defined as a difference in rates of growth whereas our previous productivity definitions all involved taking a ratio of growth rates. (Note that the log of a ratio equals the difference of the logs.)

where $Y'(t)/Y(t)$ is given by (8-6) and $X'(t)/X(t)$ is given by (8-10).[59]

A dual expression for TFPG can be derived under the additional assumption that costs equal revenue at each point in time.[60] In this case we have

$$R'(t)/R(t) = C'(t)/C(t), \tag{8-13}$$

and hence the right-hand sides of (8-7) and (8-11) can be equated. Rearranging the resulting equation and applying (8-12) yields:

$$\begin{aligned} \mathrm{TFPG}(t) &\equiv \left[W'(t)/W(t)\right] - \left[P'(t)/P(t)\right] \\ &= \left[Y'(t)/Y(t)\right] - \left[X'(t)/X(t)\right]. \end{aligned} \tag{8-14}$$

Thus, under assumption (8-13), the Divisia TFPG measure equals the Divisia input price growth rate minus the Divisia output price growth rate.

Continuous time formulations can be analytically convenient. Of course, to make them operational for the production of index values, it is necessary to replace derivatives by finite differences. The apparent precision of the Divisia approach vanishes when we do this.[61]

9. Growth accounting

We begin in Section 9 by showing how the growth accounting framework is constructed and its relationship to productivity growth measures and to the exact index number approach. Productivity measures involve comparisons of output and input volume measures, where the volume data are usually derived (as is appropriate) by using price information to transform value data. This same information can be reformulated in a growth accounting framework.

Solow's famous 1957 paper lays out the basics of the growth accounting approach. We take this up in the following Subsection 9.1. We do not attempt to survey the vast growth accounting literature;[62] we seek only to establish the close relationship between growth accounting and productivity measurement for nations.

We complete our brief treatment of growth accounting in Subsection 9.2 with an introduction to the KLEMS approach and the EU KLEMS and World KLEMS initiatives.

[59] For the one output, one input case when $t = 0$, we let $Y(t) = y_1(t) = y(t)$ and $X(t) = x_1(t) = x(t)$. In order to operationalize the continuous time approach, we approximate the derivatives with finite differences as $Y'(0) = y'(0) \cong y(1) - y(0) = y^1 - y^0$ and $X'(0) = x'(0) \cong x(1) - x(0) = x^1 - x^0$. Substituting into (8-12) yields $\mathrm{TFPG}(0) = [y'(0)/y(0)] - [x'(0)/x(0)]$, which is the Divisia approach counterpart to (2.1-3).

[60] See Jorgenson and Griliches (1967, p. 252).

[61] Diewert (1980a, pp. 444–446) shows that there are a wide variety of discrete time approximations to the continuous time Divisia indexes. More recently, Balk (2000) shows how almost any bilateral index number formula can be derived using some discrete approximation to the Divisia continuous time index. Also, as we make the period of time shorter, price and volume data for purchases and sales become "lumpy" and it is necessary to smooth out these lumps. There is no unique way of doing this smoothing.

[62] Virtually all developments in growth accounting are relevant for productivity measurement, and vice versa.

9.1. Solow's 1957 paper

Solow begins with a production function:

$$Y = F(K, L; t), \tag{9.1-1}$$

where Y denotes an output volume aggregate, K and L are aggregate measures for the capital and labor inputs, and t denotes time. A host of index number and aggregation issues are subsumed in the construction of the Y, K and L data series.[63] Solow states that the variable t "for time" appears in the production function F "to allow for technical change". Having introduced t in this way, he goes on to state that this operational definition in no way singles out the adoption of new production technologies. He notes that "slowdowns, speed-ups, improvements in the education of the labor force, and all sorts of things will appear as 'technical change' ".

Solow suggests that we measure technical change by shifts in output associated with the passage of time that are unexplained by increases in expenditures on factor inputs (capital and labor) with all marginal rates of substitution unchanged. This definition of technical change has obvious deficiencies. New technologies are often incorporated into new machinery and new business processes. Solow and others recognized this issue, and a large literature has developed on embodied technical change. However, here we use the original 1957 Solow model because it is a convenient framework for introducing growth accounting, and also for showing how productivity measurement for nations and growth accounting are related. Since Solow assumes that technological change is Hicks neutral in his 1957 paper, the production function in (9.1-1) can be rewritten as

$$Y = A(t) \cdot f(K, L). \tag{9.1-2}$$

That is, the production function can be decomposed into a time varying multiplicative technical change term and an atemporal production function.[64] The multiplicative factor, $A(t)$, represents the effects of shifts over time after controlling for the growth of K and L.

Solow's 1957 study represented a reconciliation of the forecasting results for early estimated aggregate production functions with direct measures of the growth of aggregate product. Abramovitz (1956) had previously compared a weighted sum of labor and capital inputs with a measure of total output and had concluded that to reconcile these,

[63] Some studies such as Hall (1990) essentially treat the economy of a nation as though it produced the single output of income or GDP. However, from one time period to another (or one nation to another) the product mix that makes up national output can shift. See Diewert, Nakajima, A. Nakamura, E. Nakamura and M. Nakamura (2007) – DN4 for short.

[64] Solow's recommendations in his 1957 paper encouraged other researchers to be interested in measuring efficiency improvement in their econometric studies by the ratio of period t and period s efficiency parameters, with the production function for each period specified as the product of a time varying efficiency parameter and an atemporal production function f.

it was necessary to invoke a positive role for technical progress over time. He recommended using time itself as a proxy for productivity improvements. Still earlier, in a 1942 German article, Tinbergen made use of an aggregate production function that incorporated a time trend. His stated purpose in doing this was to capture changes over time in productive efficiency.

In his 1957 paper, Solow re-formulates the output and capital input variables as $(Y/L) = y$ and $(K/L) = k$. Notice that y is output per unit of labor input: a labor productivity index.[65] He specifies that the production function is homogeneous of degree one (thereby assuming constant returns to scale), and that capital and labor are paid their marginal products so that total revenue equals the sum of all factor costs.

Making use of the Divisia methodology, Solow arrives at the following growth accounting equation[66]:

$$\dot{y}/y = (\dot{A}/A) + s_K(\dot{k}/k), \tag{9.1-3}$$

where the dots over variables denote time derivatives, and s_K stands for the national income share of capital.[67] Solow approximates the term $(\dot{A}/A)$ in (9.1-3) by $(\Delta A/A)$. He uses similar discrete approximations for the other variables, and rearranges terms to obtain

$$(\Delta A/A) = (\Delta y/y) - s_k(\Delta k/k). \tag{9.1-4}$$

Solow then produces values for $A(t)$ for the years of 1910 through 1949 by setting $A(1909) = 1$ and using the formula $A(t+1) = A(t)[1 + \Delta A(t)/A(t)]$.

Solow computes his productivity growth values – the values for $(\Delta A/A)$ – using index number rather than econometric methods. The correspondence he establishes between the functional form he assumes for the production function and this productivity growth measure is an application of the exact approach to index numbers (outlined in Section 5).

The growth accounting literature grew phenomenally from 1957 on. The methodology was extended and applied in large scale empirical studies by Griliches (1960, 1963), Denison (1967) and Kendrick (1973, 1976, 1977) and by Dale W. Jorgenson and his colleagues. In his Presidential Address delivered at the one-hundred tenth meeting of the American Economic Association, Harberger (1998, p. 1) describes growth accounting as an important success story for the economics profession, and asserts that the work of Jorgenson and Griliches (1967), Jorgenson, Gollop and Fraumeni (1987), and Jorgenson (1995a, 1995b) has carried growth accounting to the level of a "high art".

65 If L is measured as an aggregate of hours for different types of labor weighted by their respective average wages, then this is a wage weighted hours labor productivity measure, as defined in (2.3-4). If L is total (unweighted) hours of work, then y is hours labor productivity, as defined in (2.3-5) in Section 2, whereas if L is measured as the number of workers, then y is worker based labor productivity defined in (2.3-6).

66 The Divisia productivity index, defined by (8-12) in Section 8 of our paper, was related to measures of production function shift by Solow (1957) for the two input, one output case, and by Jorgenson and Griliches (1967) for the general N input, M output case.

67 Solow assumes that all factor inputs can be classified as capital or labor; hence $s_L = 1 - s_K$ is the national income share of labor.

9.2. Intermediate goods and the KLEMS approach

We come now to the question of how intermediate goods should be treated. Not all current period production in a nation is for final demand. Many firms sell some or all of their output to other firms as intermediate inputs. For example, increasing numbers of firms are outsourcing business services such as call center and accounting operations. Some of the outsourcing takes place with other firms in the same nation, but increasing amounts are with firms in other nations (the so-called "off shoring").

Output can be measured as value added, or as gross output. GNP and GDP are both value added measures, despite the fact that these terms begin with the word "gross". GNP and GDP are value added measures because they exclude intermediate inputs (i.e., they exclude produced and purchased energy and goods and services used in the production of final demand products). In contrast, a gross output measure includes the intermediate products. Either a value added or a gross output measure can be used in a growth accounting study and in specifying any of the productivity measures that have been discussed in previous sections, but the results will differ depending on this choice.

The difference between the two output concepts is less pronounced at the national level than it is at the sectoral or industry level. At the aggregate level, gross output and value added measures differ only to the extent that intermediate inputs are part of international trade.[68]

However, for the economy of a nation as a whole, changes in intermediate input usage can have productivity impacts (using either a gross or value added output measure). Research efforts to understand productivity impacts with their origin in intermediate product usage will be hampered if we do not have data on these inputs. For example, there can still be ongoing substitution effects between factor inputs such as labor and intermediate inputs, especially including business services through outsourcing and off shoring.[69] Also, modern productivity improvement techniques are aimed at improving the efficiency with which both intermediate and primary inputs are used. For example, in the manufacturing sector, just-in-time production, statistical process control, computer-aided design and manufacturing, and other such processes reduce error rates and cut down on sub-standard rejected production. In so doing, they reduce the wastage of materials as well as workers' time. Such efficiencies should probably be taken into account in measuring productivity growth.

An advantage of gross output measures is that they acknowledge and allow for intermediate inputs as a source of industry growth. In this sense, they provide a more complete picture of the production process [Sichel (2001, p. 7)]. It is true that the net productivity measures based on value added reflect savings in intermediate inputs

[68] At the industry or sector level, intermediate usage tends to be a much higher proportion of gross output. See Hulten (1978).

[69] This is demonstrated, for instance, by Gullickson and Harper (1999). Price and output measurement in many areas of business services are problematical, including core banking services. See Wang and Basu (2007).

because real value added per unit of primary input rises when unit requirements for intermediate inputs are reduced, but the effect is not explicit. Gross output-based measures explicitly indicate the contribution of savings in intermediate inputs. The deflation of gross output is conceptually straightforward too. An index of the nominal value of output is divided by an output price index to derive a volume index of gross output.

The deflation of value added output is complex. It involves double deflation because the volume change for value added combines the volume change of gross output and intermediate inputs. The term 'double' indicates that both production and intermediate inputs must be deflated in order to measure changes in the real output attributable to the factors of production in an industry.

Since value added is defined as the difference between separately deflated gross output and intermediate inputs, the use of value added as a measure of output in productivity studies imposes restrictions on the generality of the model of producer behaviour and on the role of technological change [see Diewert (1980b)]. The implied model of sectoral production does not allow for substitution possibilities between the elements of the value added function (capital and labor) and intermediate inputs. For example, it assumes that price changes in intermediate inputs do not influence the relative use of capital and labor. It restricts the role of technological change by assuming that such change only affects the usage of capital and labor.

With appropriate treatment of intermediate inputs, a mutually consistent set of estimates can be obtained at each level of economic activity. This is one objective of the KLEMS (capital, labor, energy, materials and services) approach. This approach is important because consistent aggregation is necessary to answer questions about the contribution of individual industries to overall national economic growth and productivity growth.

Jorgenson, Gollop and Fraumeni (1987) were the first scholars to work out and apply the basic KLEM methodology for a detailed industry analysis of productivity growth in the post-war US economy.[70]

The primary aim of the European KLEMS (EU KLEMS) project is to arrive at an internationally comparable dataset for a KLEMS-type analysis of productivity growth for European countries. Originally there were eight participating nations – Denmark, Finland, France, Germany, Italy, Netherlands, Spain and the United Kingdom – but the list soon grew to more than 30.[71] The World KLEM project, of which EU KLEM is the first component, represents an international platform for national level research and data collection efforts with a clear emphasis on the need for international comparability.

[70] For more on the development of the KLEMS approach in the United States, see Dean and Harper (2000), Gullickson (1995), and Gullickson and Harper (1999) and also Jorgenson (2001), Gollop (1979), and Gollop and Jorgenson (1980, 1983).

[71] In addition, the dataset, which includes the development of purchasing power parities, can be used for other purposes such as the analysis of international competitiveness and investment opportunities. It can serve as a base for further research into for example the impact of high-tech industries or human capital building on economic growth and productivity change. For more information, and for free use of the EU KLEMS database go to http://www.euklems.org/.

All of the productivity measures introduced in this paper can be recast in a KLEMS formulation. TFP or MFP growth as measured by the value added method will systematically exceed the index values based on gross output by a factor equal to the ratio of gross output to value added.[72] Productivity in the gross output formulation is $Y/(E + M + L + K)$ where Y is gross output, E is energy, M is materials, L is labor input and K is capital input. Productivity in the real value added framework is roughly $(Y - E - M)/(L + K)$. Given a productivity improvement of ΔY with all inputs remaining constant, the gross output productivity growth rate is

$$\begin{aligned}&\big((Y + \Delta Y)/(K + L + E + M)\big)\big/\big(Y/(K + L + E + M)\big)\\&\quad = (Y + \Delta Y)/Y = 1 + (\Delta Y/Y),\end{aligned} \tag{9.2-1}$$

which is less than the real value added productivity growth rate of

$$\begin{aligned}&\big((Y + \Delta Y - E - M)/(K + L)\big)\big/\big((Y - E - M)/(K + L)\big)\\&\quad = 1 + \big(\Delta Y/(Y - E - M)\big).\end{aligned} \tag{9.2-2}$$

Thus, the smaller denominator in the value added productivity measure translates into a larger productivity growth measure.[73] Several studies have found that productivity growth measured according to a value added model is greater than that derived from a model that also takes intermediate inputs into account.[74]

Diewert (2002a) notes that industry estimates of output and intermediate input are fragile in all countries due to the lack of adequate surveys on *intermediate input flows* and in particular, of *service flows* between industries.

10. Improving the model

The basic framework for productivity measurement and growth accounting for nations continues to be improved. Here we consider two of the areas of development: the specification of the measure of national output (Subsection 10.1), and efforts to relax the assumption of constant returns to scale that has been a central feature of the conventional productivity measurement and growth accounting framework (Subsection 10.2).

72 See Diewert (2002a, p. 46, endnote 21).

73 See also Schreyer (2001, p. 26).

74 For example, Oulton and O'Mahony (1994) show that the value added method produces estimates of MFP growth for manufacturing in the United Kingdom that are roughly twice those given by the gross output method. It is to be expected, of course, the sub-national level studies will be more affected by the choice of a value added or gross output measure. For example, van der Wiel (1999) shows that MFP estimates for various Dutch industries are much larger for the value added than for the gross output method.

10.1. Different concepts of national product and income

Economists have long argued that net domestic product (NDP) is the proper measure of national output for welfare analyses.[75] Yet most studies of the economic strength of a country use gross domestic product (or sometimes gross national product, GNP, as in Solow's 1957 paper) as "the" measure of output, as we did too in the previous sections of this paper. The difficulty of devising satisfactory measures of depreciation is a key reason for the dominance of the GDP and GNP measures.[76] However, by deducting even a very imperfect measure of depreciation (and obsolescence) from gross investment, we could probably come closer to a measure of output that could be consumed period after period without impairing future production possibilities.[77]

Each definition of net product gives rise to a corresponding definition of "income". In the economics literature, most of the discussion of alternative measures of net output has been conducted in terms of alternative "income" measures, so here we follow the literature and discuss alternative "income" measures rather than alternative measures of "net product". The key ideas can be understood by considering alternative income concepts in a very simple two period ($t = 0, 1$) economy with only two goods: consumption C^t with unit price p_C^t and a durable capital input K^t. Net investment I^t during period t is defined as the end of the period capital stock, K^t, less the beginning of the period capital stock, K^{t-1}: i.e., $I^t \equiv K^t - K^{t-1}$.

Samuelson (1961, p. 45) used the Marshall (1890)–Haig (1921/1959) definition of income as consumption plus the consumption equivalent of the increase in net wealth over the period, and we follow his example in this regard. Nominal income in period 1 can be represented as $p_C^1 C^1 + p_I^1 I^1$ where I^1 can be defined as net investment in period 1.

Net investment can be redefined in terms of the difference between the beginning and end of period 1 capital stocks. If we substitute this representation of net investment into Samuelson's definition of period 1 nominal income, we obtain the following definition for *period 1 nominal income*:

$$\begin{aligned}\text{Income } A &\equiv p_C^1 C^1 + p_I^1 I^1 = p_C^1 C^1 + p_I^1 \left(K^1 - K^0\right) \\ &= p_C^1 C^1 + p_I^1 K^1 - p_I^1 K^0. \qquad (10.1\text{-}1)\end{aligned}$$

Here, the beginning and end of period capital stocks are valued at the same price, p_I^1.

On conceptual grounds, it might be more reasonable to value the beginning of the period capital stock at the beginning of the period opportunity cost of capital, p_K^0, and

[75] For a closed economy, there is no distinction between net domestic product (NDP) and net national product (NNP), but the economies of countries like the United States, Canada and Japan are not closed, and the term globalization that is often used in conjunction with commentaries on the way the world economic situation is changing describes a condition of increasing openness.

[76] On the treatment of depreciation effects in the US statistics, see Fraumeni (1997). See also Hulten and Wykoff (1981a, 1981b). For a more current and international perspective and references, see T.P. Hill (2005). This topic has long occupied economists. See, for example, Hotelling (1925).

[77] This material is developed more fully in Diewert (2006d) and Diewert and Schreyer (2006b).

the end of the period capital stock at the end of the period expected opportunity cost of capital, p_K^1. That is, perhaps we should replace p_I^1 in (10.1-1) by p_K^1 for the K^1 portion of $I^1 = K^1 - K^0$, and by p_K^0, adjusted for the effects of inflation over the duration of period 1, for the K^0 portion.[78] To adjust p_K^0 for inflation we could use either a capital *specific price index*, denoted here by $1 + i^0$, or a *general price index* that is based on the movement of consumer prices, denoted by $1 + \rho^0$:

$$1 + i^0 \equiv p_K^1 / p_K^0 \quad \text{or} \tag{10.1-2}$$

$$1 + \rho^0 \equiv p_C^1 / p_C^0. \tag{10.1-3}$$

These alternative adjustment factors lead to different *measures of income from the perspective of the level of prices prevailing at the end of period 1*:

$$\text{Income B} \equiv p_C^1 C^1 + p_K^1 K^1 - \left(1 + i^0\right) p_K^0 K^0, \tag{10.1-4}$$

$$\text{Income C} \equiv p_C^1 C^1 + p_K^1 K^1 - \left(1 + \rho^0\right) p_K^0 K^0. \tag{10.1-5}$$

Comparing (10.1-4) and (10.1-1), it is easily seen that Income B equals Income A. Thus, for a measure of output, we are left with the options of choosing between Income A, which is adjusted for (i.e., net of) wear and tear,[79] and Income C, which is adjusted for wear and tear and also anticipated revaluation,[80] or of sticking with a gross output measure.

The "traditional" user cost of capital (which approximates a market rental rate for the services of a capital input for the accounting period), u^1, consists of three additive terms:

$$u^1 = U^1 + D^1 + R^1, \tag{10.1-6}$$

where U^1 denotes the reward for waiting (an interest rate term), D^1 denotes the cross sectional depreciation term (the wear and tear depreciation term), and R^1 is the anticipated revaluation term which can be interpreted as an obsolescence charge if the asset is anticipated to fall in price over the accounting period. The gross output income concept corresponds to the traditional user cost term u^1. This gross income measure can be used as an approximate indicator of short run production potential, but it is not suitable for use as an indicator of sustainable consumption. For an indicator of sustainable consumption, income concept A or C is more appropriate.

Expressed in words, for Income A, we take the wear and tear component of the traditional user cost, D^1, times the beginning of period corresponding capital stock, K^0, out of the primary input category and treat this as a negative offset to the period's gross investment. Diewert (2006d) suggests that the Income A

[78] In order to simplify our algebra, we will assume that it is not necessary to adjust p_C^1 into an end of period 1 price.

[79] We can associate this income concept with Marshall (1890), Haig (1921/1959), Pigou (1941) and Samuelson (1961). On machine replacement issues, see, for example, Cooper and Haltiwanger (1993).

[80] We can associate this income concept with Hayek (1941), Sterling (1975) and T.P. Hill (2000).

concept can be interpreted as a *maintenance of physical capital approach* to income measurement. In terms of the Austrian production model favored by Hicks (1939, 1940, 1942, 1946, 1961, 1973) and by Edwards and Bell (1961), capital at the beginning and end of the period ($K^0$and K^1, respectively) should both be valued at the end of period stock price for a unit of capital, p_K^1, and the contribution of capital accumulation to current period income is simply the difference between the end of period value of the capital stock and the beginning of the period value (at end of period prices), $p_K^1 K^1 - p_K^1 K^0$. This difference between end and beginning of period values for the capital stock can be converted into consumption equivalents and then can be added to actual period 1 consumption in order to obtain Income A.

Income C can be computed by subtracting from gross output both wear and tear depreciation, $D^1 K^0$, and the revaluation term, $R^1 K^0$, and treating both of these terms as negative offsets to the period's gross investment.[81] Diewert (2006d) terms this a *maintenance of real financial capital* approach to income measurement.

In the Austrian production model tradition followed by Hicks (1961) and Edwards and Bell (1961), capital stocks at the beginning and end of the period should be valued at the prices prevailing at the beginning and the end of the period,[82] p_K^0 and p_K^1 respectively, and then these beginning and end of period values of the capital stock should be converted into consumption equivalents (at the prices prevailing at the beginning and end of the period). Thus the end of the period value of the capital stock is $p_K^1 K^1$ and this value can be converted into consumption equivalents at the consumption prices prevailing at the end of the period. The beginning of the period value of the capital stock is $p_K^0 K^0$. To convert this value into consumption equivalents at end of period prices, we must multiply this value by $(1 + \rho^0)$, which is one plus the rate of consumer price inflation over the period. This price level adjusted difference between end and beginning of period values for the capital stock, $p_K^1 K^1 - (1 + \rho^0) p_K^0 K^0$, can be converted into consumption equivalents and then can be added to actual period 1 consumption in order to obtain Income C.

The difference between Income A and Income C can be viewed as follows. Income A (asymmetrically) uses the end of period stock price of capital to value both the beginning and end of period capital stocks and then converts the resulting difference in values into consumption equivalents at the prices prevailing at the end of the period. In contrast, Income C symmetrically values beginning and end of period capital stocks at the stock prices prevailing at the beginning and end of the period and *directly* converts these values into consumption equivalents and then adds the difference in these consumption equivalents to actual consumption.

[81] The resulting Income 3 can be interpreted to be consistent with the position of Hayek (1941), Sterling (1975) and T.P. Hill (2000).

[82] Strictly speaking, the end of period price is an expected end of period price.

In symbols, the difference between income concepts A and C is as follows:

$$\begin{aligned}
&\text{Income A} - \text{Income C}\\
&\quad = p_C^1 C^1 + p_I^1 K^1 - p_I^1 K^0 - \left[p_C^1 C^1 + p_K^1 K^1 - \left(1+\rho^0\right) p_K^0 K^0\right]\\
&\quad = \left(\rho^0 - i^0\right) p_K^0 K^0. \qquad (10.1\text{-}7)
\end{aligned}$$

If ρ^0 (the general consumer price inflation rate) is greater than i^0 (the asset inflation rate) over the course of the period, then there is a negative real revaluation effect (so that obsolescence effects dominate). In this case, Income C will be less than Income A, reflecting the fact that capital stocks have become less valuable (in terms of consumption equivalents) over the course of the period. If ρ^0 is less than i^0 over the course of the period, then the real revaluation effect is positive (so that capital stocks have become more valuable over the period). In this case, Income C exceeds Income A, reflecting the fact that capital stocks have become more valuable over the course of the period and this real increase in value contributes to an increase in the period's income which is not reflected in Income A.

Both Income A and Income C have reasonable justifications. Choosing between them is not a straightforward matter. Income A is easier to justify to national income accountants because it relies on the standard production function model. However, we lean towards Income C over Income A for three reasons: (i) It seems to us that (expected) obsolescence charges are entirely similar to normal depreciation charges and Income C reflects this similarity. (ii) In contrast to Income A, Income C does not value the beginning and end of period value of the capital stock in an asymmetric manner. And (iii) it seems to us that waiting services ($U^1 K^0$) along with labor services and land rents are natural primary inputs whereas depreciation and revaluation services ($D^1 K^0$ and $R^1 K^0$, respectively) are more naturally regarded as intermediate input charges.[83]

10.2. Relaxing the constant returns to scale assumption

There has also been strong and persistent interest in finding theoretically palatable and empirically feasible ways to relax the assumption of constant returns to scale in the growth accounting and productivity measurement literatures. Denny, Fuss and Waverman (1981, pp. 196–199) relate the Divisia TFP measure, given in Section 8 by (8-12), to shifts in the cost function without making the assumption of constant returns to scale. Here we summarize the analysis of Denny, Fuss and Waverman using slightly different notation than they did.

Our discussion of Divisia indexes in Section 8 made no mention of cost minimizing behavior. In contrast, the approach of Denny, Fuss and Waverman requires us to assume

[83] Income C is based on the Austrian model of production which has its roots in the work of Böhm-Bawerk (1891), von Neumann (1937) and Malinvaud (1953) but these authors did not develop the user cost implications of the model. On the user cost implications of the Austrian model, see Hicks (1973, pp. 27–35) and Diewert (1977, pp. 108–111, 1980a, pp. 472–474).

that the productive unit continuously minimizes costs over the time period of interest: $0 \leqslant t \leqslant 1$. The production unit's cost function will be written here as $c(y, w, t)$ to emphasize the treatment of time as continuous, where $y(t) \equiv [y_1(t), \ldots, y_M(t)]$ denotes the vector of outputs and $w(t) \equiv [w_1(t), \ldots, w_N(t)]$ denotes the vector of input prices.[84] (The t variable in $c(y, w, t)$ is viewed as representing the fact that the cost function is continuously changing due to technical progress.) Under the assumption of cost minimizing behavior, for $0 \leqslant t \leqslant 1$, we have

$$C(t) \equiv \sum_{n=1}^{N} w_n(t)x_n(t) = c\big[y(t), w(t), t\big]. \tag{10.2-1}$$

We define the continuous time technical progress measure as minus the (percentage) rate of increase in cost at time t:

$$\mathrm{TP}(t) \equiv -\big\{\partial c\big[y(t), w(t), t\big]/\partial t\big\}/c\big[y(t), w(t), t\big]. \tag{10.2-2}$$

Shephard's (1953, p. 11) Lemma implies that the partial derivative of the cost function with respect to the nth input price equals the cost minimizing demand for input n, given by

$$x_n(t) = \partial c\big[y(t), w(t), t\big]/\partial w_n, \quad n = 1, 2, \ldots, N. \tag{10.2-3}$$

Differentiating both sides of (10.2-1) with respect to t, dividing both sides of the resulting equation by $C(t)$, and using (10.2-2) and (10.2-3), we obtain

$$\begin{aligned} C'(t)/C(t) &\equiv \sum_{m=1}^{M} \big\{\partial c\big[y(t), w(t), t\big]/\partial y_m\big\}\big[y'_m(t)/C(t)\big] \\ &\quad + \sum_{n=1}^{N} x_n(t)\big[w'_n(t)/C(t)\big] - \mathrm{TP}(t) \\ &= \sum_{m=1}^{M} \varepsilon_m(t)\big[y'_m(t)/y_m(t)\big] + \sum_{n=1}^{N} s_n^C(t)\big[w'_n(t)/w_n(t)\big] - \mathrm{TP}(t), \end{aligned} \tag{10.2-4}$$

where

$$\varepsilon_m(t) \equiv \big\{\partial c\big[y(t), w(t), t\big]/\partial y_m\big\}\big/\big\{c\big[y(t), w(t), t\big]/y_m(t)\big\}$$

is the elasticity of cost with respect to the mth output volume and

$$s_n^C(t) \equiv \big[w_n(t)x_n(t)\big]/C(t)$$

is the nth input cost share.

[84] To reconcile the notation used here with the notation used in Sections 2–8, note that

$$c^0\big(y^0, w^0\big) = c\big[y(0), w(0), 0\big] \quad \text{and} \quad c^1\big(y^1, w^1\big) = c\big[y(1), w(1), 1\big]$$

with $y(t) \equiv y^t$ and $w(t) \equiv w^t$ for $t = 0, 1$.

Denny, Fuss and Waverman (1981, p. 196) define the rate of change of the continuous time output aggregate, $Q(t)$, as follows:

$$Q'(t)/Q(t) \equiv \sum_{m=1}^{M} \varepsilon_m(t)\big[y'_m(t)/y_m(t)\big] \Big/ \sum_{i=1}^{M} \varepsilon_i(t). \tag{10.2-5}$$

Recall that the Divisia expression for the output growth rate given in (8-6) weights the individual output growth rates, $y'_m(t)/y_m(t)$, by the revenue shares, $s_m^R(t)$. Alternatively, in (10.2-5), $y'_m(t)/y_m(t)$ is weighted by the mth cost elasticity share, $\varepsilon_m(t)/\sum_{i=1}^{M}\varepsilon_i(t)$. It can be shown that $\sum_{i=1}^{M}\varepsilon_i(t)$ is the percentage increase in cost due to a one percent increase in scale for each output.[85] We define the reciprocal of this sum to be a measure of (local) returns to scale:

$$\mathrm{RS}(t) \equiv \left[\sum_{i=1}^{M} \varepsilon_i(t)\right]^{-1}. \tag{10.2-6}$$

Now equate the right-hand side of (8-11) to the right-hand side of (10.2-4). Using (8-9), (10.2-5), and (10.2-6), we obtain the following decomposition of the technical progress measure in terms of returns to scale, output growth and input growth:

$$\mathrm{TP}(t) = \big[\mathrm{RS}(t)\big]^{-1}\big[Q'(t)/Q(t)\big] - \big[X'(t)/X(t)\big]. \tag{10.2-7}$$

In order to relate the technical progress measure $\mathrm{TP}(t)$ defined by (10.2-7) to the Divisia productivity measure $\mathrm{TFPG}(t)$ defined by (8-12), we use Equation (8-12) to solve for $X'(t)/X(t) = [Y'(t)/Y(t)] - \mathrm{TFPG}(t)$ and then solve for $X'(t)/X(t)$. Equating these two expressions for $X'(t)/X(t)$ and rearranging terms yields

$$\mathrm{TFPG}(t) = \big[Y'(t)/Y(t)\big] - \big[\mathrm{RS}(t)\big]^{-1}\big[Q'(t)/Q(t)\big] + \mathrm{TP}(t) \tag{10.2-8}$$

$$= \mathrm{TP}(t) + \big\{Q'(t)/Q(t)\big\}\big\{1 - \big[\mathrm{RS}(t)\big]^{-1}\big\} + \big\{\big[Y'(t)/Y(t)\big] - \big[Q'(t)/Q(t)\big]\big\}. \tag{10.2-9}$$

Equation (10.2-8) is due to Denny, Fuss, and Waverman (1981, p. 197). This equation says that the Divisia productivity index equals the technical progress measure $\mathrm{TP}(t)$ plus

[85] The elasticity of cost with respect to a scale variable k is defined as $\{1/c[y(t), w(t), t]\}$ times the following derivative evaluated at $k = 1$:

$$\partial c\big[ky(t), w(t), t\big]/\partial k = \sum_{m=1}^{M} y_m(t)\partial c\big(y(t), w(t), t\big)/\partial y_m = c\big[y(t), w(t), t\big] \sum_{m=1}^{M} \varepsilon_m(t),$$

where the last equality follows from the definition of $\varepsilon_m(t)$ below (10.2-4). Therefore, the elasticity of cost with respect to scale equals

$$\big\{1/c\big[y(t), w(t), t\big]\big\}\big\{c\big[y(t), w(t), y\big]\big\} \sum_{m=1}^{M} \varepsilon_m(t) = \sum_{m=1}^{M} \varepsilon_m(t).$$

the marginal cost weighted output growth index, $Q'(t)/Q(t)$, times a term that depends on the returns to scale term, $\{1 - [\mathrm{RS}(t)]^{-1}\}$, and that will be positive if and only if the local returns to scale measure $\mathrm{RS}(t)$ is greater than 1, plus the difference between the Divisia output growth index, $Y'(t)/Y(t)$, and the marginal cost weighted output growth index, $Q'(t)/Q(t)$.

Denny, Fuss, and Waverman (1981, p. 197) interpret the term $Y'(t)/Y(t) - Q'(t)/Q(t)$ as the effect on TFPG of nonmarginal cost pricing of a nonproportional variety. Their argument goes like this. Suppose that the mth marginal cost is proportional to the period t selling price $p_m(t)$ for $m = 1, 2, \ldots, M$. Let the common factor of proportionality be $\lambda(t)$. Then we have:

$$\partial c\big[y(t), w(t), t\big]/\partial y_m = \lambda(t) p_m(t), \quad m = 1, 2, \ldots, M. \tag{10.2-10}$$

Using (10.2-10) together with the definitions of $\varepsilon_m(t)$ and $s_m^R(t)$, we find that

$$\varepsilon_m(t) = s_m^R(t)\lambda(t)R(t)/C(t), \quad m = 1, 2, \ldots, M. \tag{10.2-11}$$

Substituting (10.2-11) into (10.2-4) and using (8-6) yields

$$Y'(t)/Y(t) = Q'(t)/Q(t). \tag{10.2-12}$$

If marginal costs are proportional to output prices[86] so that (10.2-10) holds, then the term $Y'(t)/Y(t) - Q'(t)/Q(t)$ vanishes from (10.2-9).[87] This approach provides a continuous time counterpart to the economic approaches to productivity measurement developed in previous sections.

Since the 1981 Denny–Fuss–Waverman paper was published, many others have worked on finding empirically tractable ways of treating nonconstant returns to scale in growth accounting and productivity analysis, and on dealing with the associated issue of imperfect markets and markups.

The traditional approach to estimating returns to scale is to define the elasticity of scale in the context of a producer behavioral relationship, and then estimate that parameter along with all the others for the behavioral relationship. This approach tends to be plagued by degrees of freedom and multicollinearity problems. Building on the original results of Yoshioka, Nakajima and M. Nakamura (1994) in a 2007 paper, Diewert, Nakajima, A. Nakamura, E. Nakamura and M. Nakamura (DN4 for short) extend

[86] It can be shown that if the firm (i) maximizes revenues holding constant its utilization of inputs and (ii) minimizes costs holding constant its production of outputs, then marginal costs will be proportional to output prices; i.e., we obtain $p^t/p^t \cdot y^t = \mathrm{mc}^t/\mathrm{mc}^t \cdot y^t$. Hence prices in period t, p^t, are proportional to marginal costs, mc^t. Note that assumptions (i) and (ii) above are weaker than the assumption of overall profit maximizing behavior.

[87] Note also that if there is only one output good, then this will automatically hold. In this case, (10.2-9) can be rewritten as $\mathrm{TFPG}(t) = \mathrm{TP}(t) + [1 - (1/\mathrm{RS}(t))] + [Y'(t)/Y(t)]$. This expression is analogous to Equation (6.1-11) where, for the one input, one output case, we decomposed TFPG into the product of a technical progress term and a returns to scale term. In both of these equations, if output growth is positive and returns to scale are greater than one, then productivity will exceed technical progress.

and apply what they term a *semi exact estimation approach*.[88] In this approach, exact index number methods are used to greatly reduce the number of other parameters that must be estimated along with the elasticity of scale. This stream of work can be viewed as a generalization of the basic theoretical results of Diewert (1976, Lemma 2.2, equations (2.11) and Theorem 2.16), the material on noncompetitive approaches in Diewert (1978b) and additional results in Diewert (1981a, including Section 7 results on the treatment of mark-ups).

The technology of a production unit can be represented by a production, revenue or cost function. Technical progress can be conceptualized as a shift in the specified producer behavioral relationship, and returns to scale can be defined as a change in scale with the technology held fixed. Building on the work of Panzer (1989), Hall (1990) and Klette and Griliches (1996), DN4 draw attention to the fact that production, cost and revenue function based definitions of the elasticity of scale differ conceptually and are suitable for different sorts of production situations. These issues must be faced whether a traditional or a semi exact econometric approach is adopted.[89]

In the production function framework, returns to scale are defined as the percentage change in the output quantity in response to a one percent increase in each of the N input quantities. A production function framework is suitable when there is just one output, or with multiple outputs produced in fixed proportions. However, when there are multiple outputs that can be produced in varying proportions, a revenue or cost function framework may be more suitable.

When a revenue function is used to characterize the technology of the designated production unit, a measure of the elasticity of returns to scale for a multiple output, multiple input production unit can be defined conceptually as the percentage change in revenue due to a one percent increase in *each* of the input quantities. This definition of returns to scale seems problematic because most of the sources of what is referred to as returns to scale in the business and public policy literatures involve *changes* in input mix as the scale of production increases. This is the same reason why the definition of the elasticity of scale used in the data envelopment literature is problematic. According to that approach, the returns to scale measure is defined as the equiproportionate change in outputs resulting from an equiproportionate change in inputs. There is virtually no real life change in scale that does not involve changes in the input or in the output

[88] In Yoshioka, Nakajima and M. Nakamura (1994), Nakajima, M. Nakamura and Yoshioka (1998, 2001) and in a 2006 Nakajima, A. Nakamura, E. Nakamura and M. Nakamura (N4 for short) present an estimator for the elasticity of scale for a production process with multiple inputs but only one output. DN4 extend this approach to allow for multiple outputs, but with the assumption of competitive output markets and price taking behavior in these markets. Diewert and Fox (2004) generalize the approach to allow for limited types of imperfect competition and markups in output markets, building as well on Berndt and Fuss (1986), Hall (1990) and Basu and Fernald (1997). Imperfect competition in output markets is allowed for in the Hall (1990), Bartelsman (1995), and Basu–Fernald studies, but with only a single output.

[89] Other related work includes Fox (2007), Schreyer (2007), N4, Diewert and Lawrence (2005), Inklaar (2006), Balk (1998, 2001, 2003), Bartelsman (1995), Basu and Fernald (1997), Hall (1990), Morrison and Siegel (1997), and M.I. Nadiri and B. Nadiri (1999).

mix: indeed, anticipated mix changes are typically a reason for a production unit (like a nation) to strive to grow.

A cost function, like a revenue function, can be used to characterize a multi input, multi output production unit's technology. A cost function based measure of returns to scale implies a conceptually more appealing definition of returns to scale: the percentage change in cost due to a one percent increase in all output quantities. Furthermore, Diewert and Fox (2004) show that a cost function based measure of returns to scale can accommodate certain (albeit restrictive) departures from the assumption of perfectly competitive output markets. Using a cost function framework, a reciprocal form cost function based measure of the elasticity of scale is defined as the percentage change in cost due to a one percent increase in each of the output quantities, controlling for price changes.

11. Diewert–Kohli–Morrison (DKM) revenue function based productivity measures

Decompositions of a volume index number measure of overall growth into individual component sources of growth are not new; what is new are decompositions that have explicit economic interpretations. Diewert and Morrison (1986) obtain this type of economic decomposition for the Törnqvist volume index.[90] The full potential of these decompositions has only lately begun to be recognized by economists and statisticians.

In Section 5, we used the period t production function f^t to define the period t cost function, c^t. The period t production function can also be used to define the period t (net) revenue function:

$$r^t(p,x) \equiv \max_y \{p \cdot y\colon\ y \equiv (y_1, y_2, \ldots, y_M);\ y_1 = f^t(y_2, \ldots, y_M; x)\}, \quad (11\text{-}1)$$

where $p \equiv (p_1, \ldots, p_M)$ is the output price vector that the producer faces and $x \equiv (x_1, \ldots, x_N)$ is the input vector.[91] Diewert and Morrison (1986) use revenue functions for period t and the comparison period s to define a family of theoretical productivity growth indexes:

$$\mathrm{RG}(p,x) \equiv r^t(p,x)/r^s(p,x). \quad (11\text{-}2)$$

[90] The same decomposition was independently derived by Kohli (1990). Diewert (2002c) obtained an analogous economic decomposition for the Fisher formula. Related material on decompositions can be found in Balk and Hoogenboom-Spijker (2003) and Diewert and Nakamura (2003).

[91] If y_m is positive (negative), then the net volume m is for an output (input). We assume that all prices p_m are positive. We assume that all input volumes x_n are positive and if the net input volume for product n is an input (output) volume, then w_n is positive (negative).

This index is the ratio of the net value of the output that can be produced using the period t versus the period s technology with input volumes held constant at some reference net input volume vector x and with prices held constant at some reference unit price vector, p. This is a different approach to the problem of controlling for total factor input utilization in judging the success of the period t versus the period s production outcomes.

Two special cases of (11-2) are of interest:

$$\mathrm{RG}^s \equiv \mathrm{RG}(p^s, x^s) = r^t(p^s, x^s)/r^s(p^s, x^s) \quad \text{and}$$
$$\mathrm{RG}^t \equiv \mathrm{RG}(p^t, x^t) = r^t(p^t, x^t)/r^s(p^t, x^t). \tag{11-3}$$

The first of these, RG^s, is the theoretical productivity index obtained by letting the reference vectors p and x take on the observed period s values. The second of these, RG^t, is the theoretical productivity index obtained by letting the reference vectors be the observed period t output price vector p^t and input volume vector x^t.[92]

Under the assumption of revenue maximizing behavior in both periods, we have:

$$p^t \cdot y^t = r^t(p^t, x^t) \quad \text{and} \quad p^s \cdot y^s = r^s(p^s, x^s). \tag{11-4}$$

If these equalities hold, this means we observe values for the denominator of RG^s and the numerator of RG^t. However, we cannot directly observe the hypothetical terms, $r^t(p^s, x^s)$ and $r^s(p^t, x^t)$. The first of these is the revenue that would result from using the period t technology with the period s input volumes and output prices. The second is the revenue that would result from using the period s technology with the period t input volumes and output prices.

These hypothetical revenue figures can be inferred from observable data if we know the functional form for the period t revenue function and it is associated with an index number formula that can be evaluated with the observable data. Suppose, for example, that the revenue function has the following translog functional form:

$$\ln r^t(p, x) \equiv \alpha_s^t + \sum_{m=1}^{M} \alpha_m^t \ln p_m + \sum_{n=1}^{N} \beta_n^t \ln x_n + (1/2) \sum_{m=1}^{M} \sum_{j=1}^{M} \alpha_{mj} \ln p_m \ln p_j$$
$$+ (1/2) \sum_{n=1}^{N} \sum_{j=1}^{N} \beta_{nj} \ln x_n \ln x_j + \sum_{m=1}^{M} \sum_{n=1}^{N} \gamma_{mn} \ln p_m \ln x_n, \tag{11-5}$$

where $\alpha_{mj} = \alpha_{jm}$ and $\beta_{nj} = \beta_{jn}$ and the parameters satisfy various other restrictions to ensure that $r^t(p, x)$ is linearly homogeneous in the components of the price vector p.[93]

[92] This approach can be viewed as an extension to the general N–M case of the methodology used in defining the output based measures of technical progress given in (6.1-7) and (6.1-8).

[93] These conditions can be found in Diewert (1974a, p. 139). The derivation of (6.3-1) and (6.3-2) also required the assumption of a translog technology.

Note that the coefficient vectors α_0^t, α_m^t and β_n^t can be different in each time period but that the quadratic coefficients are assumed to be constant over time.

Diewert and Morrison (1986, p. 663) show that under the above assumptions, the geometric mean of the two theoretical productivity indexes defined in (11-3) can be identified using the observable price and volume data that pertain to the two periods; i.e., we have

$$\left[\mathrm{RG}^s\,\mathrm{RG}^t\right]^{1/2} = a/(bc), \tag{11-6}$$

where a, b and c are given by

$$a \equiv p^t \cdot y^t / p^s \cdot y^s, \tag{11-7}$$

$$\ln b \equiv \sum_{m=1}^{M} (1/2)\left[\left(p_m^s y_m^s / p^s \cdot y^s\right) + \left(p_m^t y_m^t / p^t \cdot y^t\right)\right] \ln\left(p_m^t / p_m^s\right), \quad \text{and} \tag{11-8}$$

$$\ln c = \sum_{n=1}^{N} (1/2)\left[\left(w_n^s x_n^s / p^s \cdot y^s\right) + \left(w_n^t x_n^t / p^t \cdot y^t\right)\right] \ln\left(x_n^t / x_n^s\right). \tag{11-9}$$

If we have constant returns to scale production functions f^s and f^t, then the value of outputs will equal the value of inputs in each period and we have

$$p^t \cdot y^t = w^t \cdot x^t. \tag{11-10}$$

Note that the same result can be derived without the constant returns to scale assumption if we have a fixed factor that absorbs any pure profits or losses, with this fixed factor defined as in (5-18) in Section 5.

Substituting (11-10) into (11-9), we see that expression c becomes the Törnqvist input index Q_T^*. By comparing (11-8) and (3.5-2), we see also that b is the Törnqvist output price index P_T. Thus a/b is an implicit Törnqvist output volume index.

If (11-10) holds, then we have the following decomposition for the geometric mean of the product of the theoretical productivity growth indexes defined in (11-3):

$$\left[\mathrm{RG}^s\,\mathrm{RG}^t\right]^{1/2} = \left[p^t \cdot y^t / p^s \cdot y^s\right] / \left[P_T Q_T^*\right], \tag{11-11}$$

where P_T is the Törnqvist output price index defined in (3.5-2) and Q_T^* is the Törnqvist input volume index defined analogously to the way in which the Törnqvist output volume index is defined in (3.5-1). Diewert and Morrison (1986) use the period t and s revenue functions to define two theoretical output price effects which show how revenues would change in response to a change in a single output price:

$$P_m^s \equiv r^s\left(p_1^s, \ldots, p_{m-1}^s, p_m^t, p_{m+1}^s, \ldots, p_M^s, x^s\right) / r^s\left(p^s, x^s\right), \quad m = 1, \ldots, M, \quad \text{and} \tag{11-12}$$

$$P_m^t \equiv r^t\left(p^t, x^t\right) / r^t\left(p_1^t, \ldots, p_{m-1}^t, p_m^s, p_{m+1}^t, \ldots, p_M^t, x^t\right), \quad m = 1, \ldots, M. \tag{11-13}$$

More specifically, these theoretical indexes give the proportional changes in the value of output that would result if we changed the price of the mth output from its period s level p_m^s to its period t level p_m^t holding constant all other output prices and the input volumes at reference levels and using the same technology in both situations. For the theoretical index defined in (11-12), the reference output prices and input volumes and technology are the period s ones, whereas for the index defined in (11-13) they are the period t ones. Now define the theoretical output price effect b_m as the geometric mean of the two effects defined by (11-12) and (11-13):

$$b_m \equiv \left[P_m^s P_m^t\right]^{1/2}, \quad m = 1, \ldots, M. \tag{11-14}$$

Diewert and Morrison (1986) and Kohli (1990) show that the b_m given by (11-14) can be evaluated by the following observable expression, provided that conditions (11-4), (11-5) and (11-10) hold:

$$\ln b_m = (1/2)\left[\left(p_m^s y_m^s / p^s \cdot y^s\right) + \left(p_m^t y_m^t / p^t \cdot y^t\right)\right] \ln\left(p_m^t / p_m^s\right)$$
$$m = 1, \ldots, M. \tag{11-15}$$

Comparing (11-8) with (11-15), it can be seen that we have the following decomposition for b:

$$b = \prod_{m=1}^{M} b_m = P_T. \tag{11-16}$$

Thus the overall Törnqvist output price index, P_T, can be decomposed into a product of the individual output price effects, b_m.

Diewert and Morrison (1986) also use the period t and s revenue functions in order to define two theoretical input volume effects as follows:

$$Q_n^{*s} \equiv r^s\left(p^s, x_1^s, \ldots, x_{n-1}^s, x_n^t, x_{n+1}^s, \ldots, x_N^s\right) / r^s\left(p^s, x^s\right)$$
$$n = 1, \ldots, N, \quad \text{and} \tag{11-17}$$
$$Q_n^{*t} \equiv r^t\left(p^t, x^t\right) / r^t\left(p^t, x_1^t, \ldots, x_{n-1}^t, x_n^s, x_{n+1}^t, \ldots, x_N^t\right),$$
$$n = 1, \ldots, N. \tag{11-18}$$

These theoretical indexes give the proportional change in the value of net output that would result from changing input n from its period s level x_n^s to its period t level x_n^t, holding constant all output prices and other input volumes at reference levels and using the same technology in both situations. For the theoretical index (11-17), the reference output prices and input volumes and the technology are the period s ones, whereas for the index in (11-18) they are the period t ones.

Now define the theoretical input volume effect c_n as the geometric mean of the two effects defined by (11-17) and (11-18):

$$c_n \equiv \left[Q_n^{*s} Q_n^{*t}\right]^{1/2}, \quad n = 1, \ldots, N. \tag{11-19}$$

Diewert and Morrison (1986) show that the c_n defined by (11-19) can be evaluated by the following empirically observable expression provided that assumptions (11-4) and (11-5) hold:

$$\ln c_n = (1/2)\big[\big(w_n^s x_n^s / p^s \cdot y^s\big) + \big(w_n^t x_n^t / p^t \cdot y^t\big)\big] \ln\big(x_n^t / x_n^s\big) \tag{11-20}$$

$$= (1/2)\big[\big(w_n^s x_n^s / w^s \cdot x^s\big) + \big(w_n^t x_n^t / w^t \cdot x^t\big)\big] \ln\big(x_n^t / x_n^s\big). \tag{11-21}$$

The expression (11-21) follows from (11-20) provided that the assumptions (11-10) also hold. Comparing (11-20) with (11-9), it can be seen that we have the following decomposition for c:

$$c = \prod_{n=1}^{N} c_n \tag{11-22}$$

$$= Q_T^*, \tag{11-23}$$

where (11-23) follows from (11-22) provided that the assumptions (11-10) also hold. Thus if assumptions (11-4), (11-5) and (11-10) hold, the overall Törnqvist input volume index can be decomposed into a product of the individual input volume effects, the c_n for $n = 1, \ldots, N$.

Having derived (11-16) and (11-22), we can substitute these decompositions into (11-6) and rearrange the terms to obtain the following decomposition:

$$p^t \cdot y^t / p^s \cdot y^s = \big[\mathrm{RG}^s\, \mathrm{RG}^t\big]^{1/2} \prod_{m=1}^{M} b_m \prod_{n=1}^{N} c_n. \tag{11-24}$$

This is a decomposition of the growth in the nominal value of output into the productivity growth term $[\mathrm{RG}^s\, \mathrm{RG}^t]^{1/2}$ times the product of the output price growth effects, the b_m, times the product of the input volume growth effects, the c_n.[94] All of the effects on the right-hand side of (11-24) can be calculated using only the observable price and volume data for the two periods.[95]

12. Concluding remarks

This paper surveys the index number methods and theory behind the national productivity numbers. We close with some remarks on six aspects of the current state of

[94] An interesting case of (11-24) results when there is only one fixed input in the x vector. Then the input growth effect c_1 is unity and variable inputs appear in the y vector with negative components. The left-hand side of (11-24) becomes the pure profits ratio that is decomposed into a productivity effect times the various price effects (the b_m).

[95] See Morrison and Diewert (1990a, 1990b), Diewert (2002c), and Reinsdorf, Diewert and Ehemann (2002) for decompositions for other functional forms besides the translog. Kohli (1990), Fox and Kohli (1998), and Diewert, Lawrence and Fox (2006) use this approach to examine the factors behind the growth in the nominal GDP of several countries.

productivity measurement for nations and directions for future research. Both methodological and data challenges remain, and the two are interrelated. Better data can ease the methodology challenges.

12.1. Choice of measure effects

One goal of this paper has been to draw attention to, and help users distinguish among, different types of productivity measures for nations. We show how different ones relate to each other and to GDP per capita which is a commonly used measure of national economic well being. It is important for the differences among the measures to be kept in mind when it comes to interpreting empirical findings. Some authors make a point of helping their readers to be aware of these differences. For example, in a recent paper with important public policy implications for Canada and the US, Rao, Tang and Wang (2007) note that, unlike their earlier studies, the labor productivity measure used is for the number of persons employed rather than hours of work because they did not have comparable hours of work data by industry for the two countries. Rao, Tang and Wang note that they expect the resulting measured productivity gap with the United States to be about 10 percent higher than in their earlier studies because Canadians, on average, put in about 10 percent less hours on the job than their US counterparts. The continuing development of harmonized data for widening numbers of nations will hopefully make it possible in years to come for researchers to choose the productivity measures that best fit their applied needs rather than having to bend their analysis needs to the available data. But it will still be important for readers to be aware of how the choice of measure affects reported results.

12.2. Better price measurement = better productivity measures

The traditional index number definition of a productivity growth index is an output volume index divided by an input volume index. National statistical agencies (appropriately) collect information on output and input values and prices; not volumes and prices. Volume measures are then produced by applying price indexes to the value information about the outputs and inputs. This means that having good price statistics is critical for the quality of productivity measurement.

The new international CPI and PPI manuals provide an in-depth treatment of the theoretical, methodological, and data advances of recent decades in official consumer and producer price level measurement.[96] Nevertheless, some important problem areas remain.

The treatment of new products in price measurement remains on the critical list, and is of special relevance for productivity measurement.[97] New machinery and equipment,

[96] We are referring here to the new international Consumer Price Index Manual [T.P. Hill (2004)] and Producer Price Index Manual [Armknecht (2004)].

[97] See A. Baldwin et al. (1997), Basu et al. (2004), Greenstein (1997), R.J. Hill (1999a, 1999b, 2001, 2004), Nordhaus (1997), and Wolfson (1999). The treatment of quality change is also important. Hedonic methods are increasingly being used in this regard; see Diewert (2002d, 2003a, 2003b).

new business processes, new material inputs, and new consumer products are key ways in which technological progress is manifested. Price setting mechanisms appear to be evolving because marketing behavior is evolving to take advantage of new IT technologies, and this has potential implications for productivity measurement, along with other aspects of the treatment of new goods.[98]

The proper measurement of the prices for machinery and equipment and other capital services – that is, the proper measurement of user costs for capital – is a second important area of active debate for price measurement. In Subsection 10.1 we argued that, ideally, the measure of the user cost of capital should allow for depreciation and obsolescence effects.

For a KLEMS (capital, labor, energy, materials and services) approach, price indexes are needed as well for the intermediate inputs. The major classes of intermediate inputs at the industry level are: materials, business services and leased capital. In practice, period by period information on costs paid for a list of intermediate input categories is required along with either an intermediate input volume index or a price index for each category. There is a lack of price survey data for intermediate inputs. Price indexes for outputs are often used as proxies for the missing price indexes for intermediate inputs. Also, the intermediate input prices should, in principle, include any commodity taxes imposed on these inputs, since the tax costs are paid by producers.

Of course, many intermediate products are produced by different divisions of the same firms that use these products for producing final demand products. This intra-firm trade can be important for national productivity measurement and growth accounting when the different divisions are in different nations, in which case these movements of product will be counted as part of foreign trade.

Intermediate product transactions *among* firms can be observed and price and value statistics can be collected for these transactions as for other sorts of product transactions. This is not the case, however, for the *intra* firm transactions. As Diewert, Alterman and Eden (2007) and Mann (2007) explain, the transfer prices that firms report for intra firm transactions may not be a very satisfactory basis for the construction of price indexes for the associated flows of goods and services. Even for measuring the productivity of a single nation over time, or making inter-sector or inter-industry comparisons for productivity levels or growth within a single nation, international trade issues must be considered and dealt with. That is, international trade complicates even the choice of a measure for national level output.[99]

Finally, on the subject of relevant price measurement problem areas, there is interest not only in productivity comparisons over time, but also in inter-nation comparisons.

[98] See Hausman (2003), Hausman and Leibtag (2006, 2007), Leibtag et al. (2006), E. Nakamura and Steinsson (2006a, 2006b), Silver and Heravi (2003), Triplett (2002), Triplett and Bosworth (2004), and Timmer, Inklaar and van Ark (2005). On hedonic pricing for new goods, see Diewert, Heravi and Silver (2007).

[99] On the importance of allowing for trade in measuring productivity see, for example, Bernstein (1998), Bernstein and Mohnen (1998), Bernstein and Nadiri (1989), Diewert and Woodland (2004), Woodland and Turunen-Red (2004), Diewert (2007d), Trefler (2004), and Mann (2007).

Thus, inter-nation price statistics are needed for making inter-nation productivity comparisons: international purchasing power parity statistics (PPPs). Methodological as well as data challenges abound in the area of international price comparisons.[100]

12.3. The measurement of capital services

The OECD productivity database[101] distinguishes seven types of assets: hardware, communications equipment, other machinery, transport equipment, nonresidential buildings, structures and software. Diewert, Harrison and Schreyer (2004) state that, conceptually, there are many facets of capital input that bear a direct analogy to labor input. Capital goods are seen as carriers of capital services that constitute the actual input in the production process just as workers are seen as carriers of labor services. When rentals and the cost of capital services cannot be observed directly, methods must be adopted to approximate the costs of capital services. Much progress has been made on the measurement of capital services and user costs, but much still remains to be done.

For example, Brynjolfsson and Hitt (2000)[102] and Corrado, Hulten and Sichel (2006) note that both firm level and national income accounting practice has historically treated expenditure on intangible inputs such as software and R&D as an intermediate expense and not as an investment that is part of GDP. Corrado, Hulten and Sichel (2006) find that the inclusion of intangibles makes a significant difference in the measured pattern of economic growth: the growth rates of output and of output per worker are found to increase at a faster rate when intangibles are included than under the status quo case in which intangible capital is ignored.

Brynjolfsson and Hitt (2000) argue there are important interaction effects between the intangible business practice and process capital investments that are going unmeasured. Brynjolfsson and Hitt (1995), Atrostic et al. (2004), Atrostic and Nguyen (2007), and Dufour, Nakamura and Tang (2007) provide empirical evidence for multiple countries suggesting that IT-users that also invested in organizational capital had higher gains in productivity compared to firms that invested only in IT capital or only in adopting

[100] See, for instance, Armstrong (2001, 2003, 2007), Balk (1996), Diewert (2000, 2005d, 2006b), R.J. Hill (1999a, 1999b, 2001, 2004, 2007), R.J. Hill and Timmer (2004), and D.S.P Rao (2007).

[101] See OECD (2005).

[102] In a 2000 *Journal of Economic Perspectives* article, Brynjolfsson and Hitt write: "Changes in multifactor productivity growth, in turn, depend on accurate measures of final output. However, nominal output is affected by whether firm expenditures are expensed, and therefore deducted from value added, or capitalized and treated as investment. As emphasized throughout this paper, information technology is only a small fraction of a much larger complementary system of tangible and intangible assets. However, current statistics typically treat the accumulation of intangible capital assets, such as new business processes, new production systems and new skills, as expenses rather than as investments. This leads to a lower level of measured output in periods of net capital accumulation. Second, current output statistics disproportionately miss many of the gains that information technology has brought to consumers such as variety, speed, and convenience . . .". See also Berndt and Morrison (1995), Brynjolfsson and Hitt (1995), Colecchia and Schreyer (2001, 2002), and Prud'homme, Sanga and Yu (2005).

new business practices. The idea is that investments in tangible and intangible assets reinforce one another.

Findings of complementarities between business practices and high tech processes lend support to the proposition that there are also important complementarities between the largely intangible unmeasured assets of firms and the (mostly tangible) assets that are being measured by national statistics agencies.

Regarding the measured assets, Jorgenson argues that rental values should be imputed on the basis of estimates of capital stocks and of property compensation rates, with the capital stock at each point of time represented as a weighted sum of past investment. The weights are viewed as measures of the relative efficiencies of capital goods of different ages and of the compensation received by the owners.[103] While agreeing with the objective of adopting a user cost approach for asset pricing, nevertheless it is important to note that the theoretical and empirical basis is slim for many of the practical choices that must be made in doing this. Substantial differences in the productivity measurement results can result from different choices about things such as physical depreciation rates for which empirical or other scientific evidence is largely lacking. Nomura and Futakamiz (2005) report on an initiative in Japan to use the complete records of assets in place in some companies and also the registration data for particular assets to determine the service life of individual assets. We strongly support this sort of data collection initiative.

Yet another capital measurement issue is that the System of National Accounts (SNA) does not regard the placing of nonproduced assets at the disposal of a producer as production in itself but as an action giving rise to property income. Nomura (2004, Chapter 4) shows that neglecting land and inventories leads to a reduction in the average TFP growth rate.

12.4. Labor services of workers and service products

Diewert, Harrison and Schreyer (2004) state that, conceptually, there are many facets of capital input that bear a direct analogy to labor input. They also state that when rentals and the cost of capital services cannot be observed directly, methods must be adopted to approximate the costs of capital services. With hourly workers, we have data on their "rental rates". But is it the *right* price information?

The issue of what sorts of labor to count must be agreed on first. For productivity measurement purposes, there is agreement that the labor to be counted is what is used for the production within the boundary of the System of National Accounts (SNA). However, this still leaves three alternative definitions of the hours of work that have been the subject of international debate on the proper measurement of labor input:

(H1) *Active production time*.

[103] See for example Jorgenson (1963, 1980, 1995a), and Christensen and Jorgenson (1969, 1970).

(H2) *Paid hours*, including agreed on allocations of employer financed personal time, some of which may be taken away from the work site like paid vacation or sick days.

(H3) *Hours at work* whether or not they are paid hours or used for production.

Each of these measurement concepts implies a different factorization of the dollar amount spent on labor into quantity and price components.

The productivity programs in Canada and the United States use the concept of hours at work: the H3 concept. H3 is also the concept that is agreed on for the 1993 System of National Accounts as the most appropriate measure for determining the volume of work.

For many workers, time at work includes some hours in addition to what are stipulated in formal employment agreements. These are volunteered or informally coerced unpaid overtime hours. Hours at work (H3) could rise (or fall) with or without changes in paid hours (H2), but it seems unlikely there would be changes in hours at work without corresponding changes in the same direction in active production time (H1). Conceptually at least, the H3 concept includes the increasing amounts of work done at home by those tele-commuting, from other nations.[104]

A different sort of labor input measurement issue is that none of the measures discussed take account of differences in the knowledge and skills and innate abilities vested in workers.

Economic development history could be written as the progressive substitution of machine for human services. Farmer laborers tilling the fields were replaced by machines that do the tilling, pulled by farmers riding tractors. Phone operators were replaced by electronic routing systems. Type setters were replaced by automated printing processes. Secretarial typing of research papers was replaced by word processors on home or portable computers and the typing of the researchers themselves. To properly account for these substitutions of these sorts, we would need data on the respective volumes and prices, or the value figures and prices, for workers with different types and levels of specific skills.

12.5. A need for official statistics and business world harmonization

The models that economists use to interpret TFPG estimates typically rule out many of the ways in which business and government leaders attempt to raise productivity. The dominant economic index number approach is built, to date, on a neoclassical foundation assuming perfect competition, perfect information and, in most studies, constant returns to scale.

[104] Note too that for materials, inventories represent the difference between the paid quantity for a given time period (concept H2) and the quantity used in that time period (concept H1). In contrast, for material inputs, time "at work" (quantity concept H3) is the same as the counterpart of "paid time" because the materials are owned continuously, once paid for. Diewert and Nakamura (1999) are silent on the issue of inventories for diesel and lube oil.

In a world where all factor inputs are paid their marginal products and there is no potential for reaping increasing returns to scale, then the only way in which growth in output could occur would be through increased input use or through changes in external circumstances, including spillovers from the R&D of others.[105] This is the world assumed by Solow (1957), for example. For such a world, after removing all factor costs in evaluating productivity growth, we would be left with only revenue growth due to *purely external factors*. Thus, Jorgenson (1995a) writes:

> "The defining characteristic of productivity as a source of economic growth is that the incomes generated by higher productivity are external to the economic activities that generate growth" (p. xvii).

However, this definition of productivity growth seems unlikely to satisfy Harberger's (1998, p. 1) recommendation that we should approach the measurement of productivity by trying to "think like an entrepreneur or a CEO, or a production manager". What CEO would announce a productivity improvement plan for their company, and then add that it depends entirely on external happenings including spillovers from the R&D of competitors?

The private business sector is the engine of productivity growth in capitalist economies. Business and government leaders need to be able to communicate effectively about economic policy issues. National productivity matters are of concern to both business and government leaders. When asked, business leaders mostly report that they make little or no use of the productivity measures of economists, preferring instead to use single factor input–output type performance measures. However, businesses make ubiquitous use of real revenue/cost ratios, and we have shown that this is one way of writing a TFPG index. We suggest that the main differences between the way that economists and business leaders have traditionally thought about productivity lie in defining technical change as disembodied, the assumption of constant returns to scale, and the definition of productivity change as due to externalities. Business leaders see technological change as largely embodied in machines, business practices and people working for them. They are obsessed with finding ways to profit from various sorts of returns to scale. They see increasing productivity as a core function of business managers and productivity gains as being achieved by economic activities that also generate growth. And they definitely intend to capture as much as possible of the incomes generated by the productivity gains of their companies.

At present, there is a serious conceptual gulf between the economic approach to the interpretation of TFPG measures and the business world perception of what productivity growth is. This is unfortunate since it is the private business sector on which nations must mostly rely for their economic growth. The challenge for index number theorists

[105] Studies of TFPG focusing explicitly on externalities such as R&D spillovers include Bernstein and Nadiri (1989), Bernstein (1996), and Jaffe (1986). Bernstein (1998) and Bernstein and Mohnen (1998) extend the theory and empirical treatment of spillover effects on productivity growth to an international context.

is to develop models that incorporate rather than assume away what economic practitioners view as some of the main means by which total factor productivity improvement is accomplished. One key to making headway on this goal may be to notice that the productivity measures themselves can be computed without making any of the restrictive assumptions that have been used in showing that certain of these measures can also be derived from economic optimizing models.

12.6. The role of official statistics for globally united nations

Masahiro Kuroda (2006), in his capacity as Director of the Economic and Social Research Institute of the Cabinet Office of the Government of Japan, calls attention to the need in Japan and elsewhere, to review and change the legal framework for official statistics. He links this need to, for example, an emerging need for new rules to enable wider use of administrative records as well as the survey data collected by governments.[106] We heartily support the sorts of goals enunciated by Kuroda. There is an urgent need for initiatives that will allow statistical agencies to continue to produce more and better data, and this situation will inevitably bring into play cost pressures. In addition to the long established importance of official statistics in the management of national economies, official statistics have been evolving into an important medium for communication both within and among nations. Increasingly, the national choices that affect all of us as global inhabitants are being made in the context, and with the aid, of official statistics, including measures of the productivity of nations.

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[106] Halliwell (2005) provides thought provoking perspectives and possible ways of proceeding. For other related issues see Diewert (2006a), Nakamura and Diewert (1996) and McMahon (1995).

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