



**Asymptotic Wave Theory.** By Maurice Roseau. Publisher North-Holland Publishing Co., Amsterdam & Oxford; American Elsevier Publishing Co., Inc.—New York. Vol. 20. North-Holland Series in Applied Mathematics and Mechanics. 1976. 349 Pages. \$29.50.

**REVIEWED BY T. K. CAUGHEY<sup>1</sup>**

This text should serve very nicely for a first graduate level course in wave-phenomena for students in applied mathematics and mechanics. The book provides a grounding in the mathematical tools required to treat numerous applied problems from the asymptotic point of view.

The first four chapters are devoted to detailed discussion of such mathematical tools as Fourier-Laplace transforms, operational calculus, special functions, and asymptotic methods. The remaining four chapters are devoted to the application of asymptotic methods to wave propagation phenomena such as open channel flow, seismic waves, and water waves. Chapter 5 develops the elements of scattering matrix theory and contains a discussion of the inverse problem and the Gelfand-Levitan integral equation. Considerable space is devoted in Chapter 6 to some investigations of problems of flow in open channels which, through suitable formulation, is developed into a basic discussion of linear and nonlinear wave theory. Chapter 7 is devoted to propagation of elastic waves in a layered spherical body and their relationship to seismic phenomena in the earth. The eighth and last chapter contains a discussion of problems associated with gravity waves in an incompressible fluid having a free surface.

The book is well written and can be read by any student acquainted with the elements of ordinary differential equations and the theory of functions of a complex variable.

**The Mechanics of the Contact Between Deformable Bodies.** A. D. de Pater and J. J. Kalker, Editors. Delft University Press. 1975. 414 Pages.

**REVIEWED BY L. E. GOODMAN<sup>2</sup>**

The cognate fields generally included under the terms contact stress analysis or "tribology" have flowered in the last decade. They form today one of the most active and fruitful branches of applied mechanics. Therefore, the decision of the governing body of the International Union of Theoretical and Applied Mechanics to convene a symposium on the subject was most appropriate. The present volume contains the complete text of the 26 papers presented by invitation at that symposium, held at Enschede, Netherlands, August 20–23, 1974. With a few exceptions they are presentations of new research results. Although the quality of the papers is not uniform, many of them are of outstanding technical value and will undoubtedly become standard references. Viewed collectively the articles provide what is, except for the deliberate omission of problems involving elastohydrodynamic lubrication, the best overview of research activity in contact stress analysis currently available in print. The volume should find a place in every university library.

In the limited space available in a brief book review, it is hardly possible to do justice to all of these contributions. The complete list below is followed by a regrettably brief comment.

- 1 Aspects of Contact Mechanics—J. J. Kalker.
- 2 Non-Hertzian Contact of Elastic Spheres—K. L. Johnson.
- 3 Signorini's Problem in Viscoelasticity—M. Boucher.
- 4 Properties of Elastic Bodies in Contact—J. Dundurs.
- 5 Similarity Considerations for Contact Between Dissimilar Elastic Bodies—D. A. Spence.
- 6 Consideration of the Theory of Cracks From the Point of View of Contact Problems of the Theory of Elasticity—A. Ju. Islinskij.
- 7 Certain Asymmetrical Contact Problems for a Half Space—B. L. Abramjan.
- 8 Unbonded Contact Between a Circular Plate and an Elastic Foundation—G. M. L. Gladwell.
- 9 On the Two-Dimensional Contact Problem of a Rigid Cylinder Pressed Between Two Elastic Layers—J. B. Alblas.
- 10 Influence of an Elastic Layer on the Tangential Compliance of Bodies in Contact—L. E. Goodman and L. M. Keer.
- 11 Small Scale Plastic Flow Associated With Rolling—J. Christoffersen.
- 12 Heat Effects in Rolling Contact—F. F. Ling.
- 13 Thermoelastic Contact Problems—J. R. Barber.
- 14 An Axisymmetric Contact Patch Configuration for Two Slabs in Frictionally Heated Contact—R. A. Burton.
- 15 Dynamic Contact Stresses Produced by Impact in Elastic Plates of Finite Thickness—Y. M. Tsai.
- 16 Transition of Collision Contact Force Between a Viscoelastic Half Space and a Flat-Headed Rigid Body—K. Kawatake.
- 17 Impact on a Worn Surface—P. A. Engel.
- 18 The Normal Contact of Arbitrarily Shaped Multilayered Elastic Bodies—T. G. Johns and A. W. Leissa.
- 19 Contact Stresses for Multiply-Connected Regions—The Case of Pitted Spheres—K. P. Singh, B. Paul, and W. S. Woodward.
- 20 Stylus Profilometry and the Analysis of the Contact of Rough Surfaces—J. F. Archard, R. T. Hunt, and R. A. Onions.
- 21 The Interaction and Lubrication of Rough Surfaces—F. T. Barwell, M. H. Jones, and S. D. Probert.
- 22 Contact of Rough Surfaces of Work-Hardening Materials—J. Halling and K. A. Nuri.
- 23 Factors Influencing the Real Trend of the Coefficient of Friction of Two Elastic Bodies Rolling Over Each Other in the Presence of Dry Friction—H. Krause and A. Halim Demirci.
- 24 The Frictional Contact of Rubber—A. Schallamach.
- 25 Applications for Contact Theories in Nuclear Reactor Technology—L. A. Mitchell.
- 26 Linearized Contact Vibration Analysis—P. R. Nayak.

Broadly speaking, the papers fall into two groups of which the first is concerned with solutions of the field equations of thermoelasticity and plasticity that satisfy mixed boundary conditions and the second is concerned with experimentally based tribomechanical research of direct application in technology. The introductory paper by Kalker [1, above] presents a valuable review and classification of solved and (as yet) unsolved problems of the first group. Kalker also includes a welcome introduction to the variational theory of contact mechanics associated with the name of Signorini. This development of the Franco-Italian school is used by Boucher in his paper [3]. Other mixed boundary-value methods such as the elegant self-similar solution technique of Spence [5], integral equation, and integral transform techniques [7–10, 12, 15], and sophisticated computer-based approaches [18, 19] are exemplified. The relation of the complementary problem to the theory of hydraulic fracture developed in the USSR

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by Hristianovic, Zeltov, and Barenblatt is developed by Islinskij [6]. Also included in this first group are a number of interesting general contact theorems which are analogous to Michell's theorem in plane elasticity [4], proven by Dundurs.

In the second group mentioned in the foregoing, Johnson's work [2] combines theory and laboratory experiment to achieve an entirely satisfactory explanation of two hitherto troublesome questions: the effect of friction on the strength of brittle materials as measured in the spherical indenter test and the influence of adhesion on the normal contact of smooth and rough nominally flat surfaces. Archard, et al. [20], and Barwell, et al. [21], describe the measurement and characterization of surface texture, so important in practical engineering problems. Among other papers in this group [22, 23, 25] may be included Schallamach's work on the rolling and sliding friction of rubber [24]. Presentations of fundamental dynamic and thermal contact problems appear [12-16, 17, 26]. Plastic effects in rolling contact are treated by Christoffersen [11].

It is a pleasure here to record that several contributors to these *Proceedings* have been at pains to include extensive bibliographical references. These will be of aid to others doing research in the field. The general format and typography reflect credit upon the editors who, in addition to other onerous duties, have in some cases had to undertake the English translation. There are a few minor typographical errors, but these are not irritating. The work reflects credit also upon the Delft University Press.

At the risk of injecting a personal note, the reviewer would wish to express his regret at the necessity of omitting the stimulating discussion that followed the presentation of these papers, especially the remarks of M. Frémond on the practical application of variational techniques, of J. Willis on the Radon transform, of D. Tabor and, indeed, of many other participants. This element, probably the best excuse for international symposia, must, like the Delft-blue skies of Enschede, remain a privileged recollection of the participants.

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**Small Elastic Deformations of Thin Shells.** By Paul Seide. Noordhoff International Publishing. Leyden. 1975. Pages XIII-654. Price 190 Dfl.

REVIEWED BY R. P. NORDGREN<sup>3</sup>

The book makes an important contribution to shell theory by collecting and uniformly rederiving numerous analytical solutions for shells. About two thirds of the text is devoted to static solutions for shells of revolution under symmetric and asymmetric loads. Solutions are obtained in terms of special functions for cylindrical, conical, and spherical shells. The method of asymptotic integration is applied to general shells of revolution. The derivations are sufficiently detailed and a reasonable quantity of numerical results is presented.

Numerical solution by the finite-element method is briefly discussed for shells of revolution and general shells approximated by triangular plates. However, the mixed-type element with constant stress couples does not seem adequate for the degree of the differential equations involved. The widely used finite-difference method for shells of revolution is only mentioned in passing.

Limited results are presented for shallow shells, thermal loading, anisotropic shells, layered shells, stiffened shells, and shells of non-

uniform thickness. The book closes with two chapters on free and forced vibrations of cylindrical, spherical, and conical shells. Reviewer believes that static and dynamic problems of shallow shells deserve more attention.

In the opening chapter of the book the differential geometry of surfaces is reviewed. Then the equations of Love-type shell theory are derived from the theory of elasticity by the usual approximate methods and by minimizing potential energy. The derivations are rather awkward mainly due to the use of expanded notation in lines-of-curvature coordinates. A more serious difficulty arises from the author's hypothesis that the shell has infinite resistance to transverse extension and transverse shear. This hypothesis leads to the same constitutive equations as the Love-Kirchhoff hypothesis and the author considers his hypothesis more plausible. The reviewer disagrees and suggests that an appreciation of the Love-Kirchhoff hypothesis is essential to understanding and having confidence in the theory of shells. Such an appreciation can be gained by examining a few three-dimensional elasticity solutions for plates and shells. Also, recent research on error estimates gives some indication of the accuracy of shell theory under the Love-Kirchhoff hypothesis.

Seide's book can be recommended as a reference on analytical solution techniques. It also could be used as a teaching text in combination with supplementary material on the foundations of shell theory.

<sup>3</sup>Shell Development Company, Bellaire Research Center, Houston, Texas.