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# The merging history of dark matter haloes in a hierarchical universe

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## ABSTRACT

We have developed an algorithm to investigate the merging history of present-day dark matter haloes in a universe where structure is built up hierarchically. The algorithm constructs merging history trees which can be used to trace the merging path of every halo mass element through all the progenitor haloes from which the present object formed. These trees follow merging histories from high redshift until $z=0$ and so can be used to study the formation and merging of galaxies within the material which constitutes a single present-day group or cluster of galaxies. We have tested our results against those derived from numerical simulations of gravitational clustering and find that our algorithm correctly reproduces the mass distribution of the two largest halo progenitors at a series of redshifts. Applying our methods to the question of the survival of galaxy discs, we confirm that merging rates in an $\Omega=1$ cold dark matter universe *may* be too high to allow the observed predominance of spiral galaxies in the field. Low-density models have no such problem. An analysis of the frequency of substructure in galaxy clusters is unable to produce a reliable estimate of $\Omega$ because of the uncertainty in how long such substructure can last.

**Key words:** methods: numerical – galaxies: clustering – galaxies: formation – dark matter – large-scale structure of Universe.

## 1 INTRODUCTION

In hierarchical theories of structure formation, large objects such as galaxies, groups and clusters form through the continuous aggregation of non-linear objects into larger and larger units. One way to study the build-up of structure in such a theory is via numerical simulations of gravitational clustering. However, numerical simulations have definite drawbacks. As well as being computationally expensive, they are restricted in terms of the resolution that can be achieved and give results which may suffer from statistical uncertainty because of the small number of objects that can be simulated.

An analytic theory for the development of structure in a hierarchical universe was first presented by Press & Schechter (1974). In this theory, structure was assumed to grow from Gaussian random phase initial perturbations. Non-linear clumps could then be identified as overdensities in the linear density field. Press & Schechter argued that, if the overdensity at any point in the density field exceeded a critical threshold $\delta_c$ when smoothed with a top-hat filter of radius $R$, then the mass inside $R$ would be incorporated into a non-linear object of mass $M=(4/3)\pi\bar{\rho}R^3$, where $\bar{\rho}$ is the mean density of the universe at the time of collapse. For Gaussian initial conditions, the probability that a volume of the universe will have given overdensity can be determined directly from the power spectrum of linear density fluctuations. Using these assumptions, Press & Schechter were able to derive the multiplicity function of non-linear objects at any given redshift, i.e. the number of objects per unit volume lying in the mass range $(M, M+\mathrm{d}M)$ at redshift $z$.

The Press–Schechter theory for an $\Omega=1$ universe has been tested against $N$-body simulations and has been found to be in very good agreement (Efstathiou et al. 1988). A straightforward adaptation of the theory to the case of a low-density universe has also been demonstrated to match $N$-body results (Kauffmann, in preparation). Further progress has recently been made by an extension of the Press–Schechter theory by Bower (1991) which has facilitated the treatment of halo mergers. An independent derivation based on a completely different formalism has also been given by Bond et al. (1991). In these papers, expressions were derived from the conditional probability that material in an object of mass $M_1$ at redshift $z_1$ would end up in an object of mass $M_0$ at redshift $z_0$. The Press–Schechter formalism has thus been developed into a powerful tool for investigating the evolution of galaxies, groups and clusters in models of large-scale structure formation, enabling comparisons between theory and observations to be made.

The Press–Schechter formalism has been applied to a number of problems, for example the evolution of rich clusters of galaxies (Kaiser 1986), the formation of rare objects at high redshift (Efstathiou & Rees 1988) and gravitational lensing (Narayan & White 1988). It has also formed the basis of a pseudo-analytic study of galaxy formation in a cold dark matter (CDM) universe by White & Frenk (1991). In this study, gas is allowed to cool and form stars within dark matter haloes whose distribution at some initial redshift $z_1$ is given by the Press–Schechter theory. The conditional probabilities derived by Bower (1991) are then used to distribute the stars amonst the haloes present at a subsequent redshift $z_1 - \mathrm{d}z$. Integrating forward step by step from high redshift until $z=0$, one obtains the present-day abundance of stars in dark matter haloes as a function of their mass. To obtain the *galaxy* luminosity function, an additional 'no-merger' hypothesis must be made. According to this hypothesis, star formation takes place in each halo for one halo collapse time, and the resulting galaxy then survives intact until the present day.

The hypothesis that galaxies never merge is no doubt overly simplistic and there must in practice be a large dispersion in the effective gas accumulation and star formation times between different haloes of the same mass. Many examples of interacting and merging galaxies are known observationally. Indeed, the faint-end slopes of the galaxy luminosity functions derived by White & Frenk (1991) are considerably too steep, although it is not clear whether mergers alone can solve this problem. A high merging rate has also been suggested as an explanation for the unexpected behaviour of the faint galaxy counts in the $B$ and $K$ bands (Guiderdoni & Rocca-Volmerange 1991; Broadhurst, Ellis & Glazebrook 1992). On the other hand, arguments have been made against too high a merging rate by Toth & Ostriker (1992), who point out that mergers may destroy thin galactic discs. The fact that 80–90 per cent of galaxies in the field show spiral structure puts strong constraints on how much merging could have been taking place over the past 5 Gyr or so. Clearly it would be advantageous if we had a method for addressing the difficult question of galaxy mergers.

In this paper we present a new method of determining the typical merging history of dark matter haloes in a hierarchical universe. Our algorithm constructs a set of histories for a present-day halo of given mass, using a Monte Carlo technique and merging probabilities drawn from the conditional probabilities derived by Bower. The merging histories are stored in the form of a *tree*. The trunk of the tree corresponds to the dark matter halo at $z=0$. Each successive layer represents a step backwards in redshift and branches indicate the merging paths of the dark matter halo's progenitors (see Fig. 1). This new algorithm lays the foundation for a more detailed treatment of galaxy merging. Such a scheme, which can specify the detailed formation history of representative haloes, will also provide a way to search for explanations of galaxy colours, luminosities and morphologies and their observed dependence on environment. The merging history of haloes has also recently been discussed by Lacey & Cole (1993).

Here we give a detailed description of our methods and report on comparisons between the results of our algorithm, which is based on the Press–Schechter formalism, and results using numerical simulations carried out by Frenk et al. (1988) and Kauffmann & White (1992). Finally, we present two applications of our method – an investigation of the survival probabilities of galactic discs, and a study of the frequency of occurrence of substructure in galaxy clusters. Although these applications are simple, they are of considerable astrophysical interest. A description of more complex applications to the theory of galaxy formation and evolution is left to a future paper.

## 2 CONSTRUCTION OF MERGING HISTORIES FOR DARK MATTER HALOES

We adopt the notation used in White & Frenk (1991). Haloes are modelled as truncated singular isothermal spheres. The

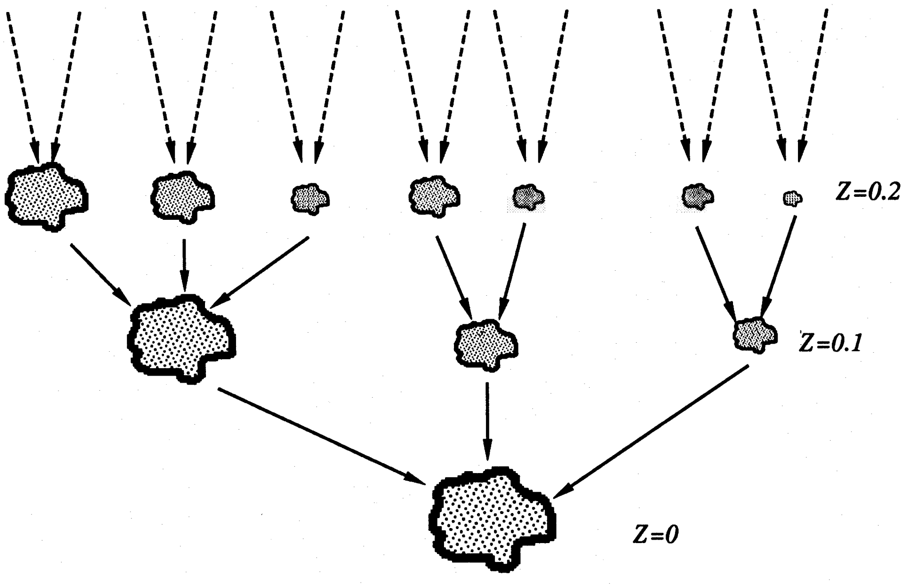


**Figure 1.** A schematic representation of a halo merging history 'tree'.

mass and circular velocity of a halo are related to its initial size, $r_0$, expressed in current units, and redshift, $z$, by

$$M=(4\pi/3)\rho_0 r_0^3, \qquad V_c=1.67(1+z)^{1/2}H_0 r_0. \tag{1}$$

The Press–Schechter formula for the fraction of matter in the universe which is in haloes with circular velocities between $V_c$ and $V_c+dV_c$ at redshift $z$ is

$$f(V_c,z)\,dV_c=-\left(\frac{2}{\pi}\right)^{1/2}\frac{1.68(1+z)}{\Delta^2}\frac{d\Delta}{dV_c} \times \exp\left[\frac{-1.68^2(1+z)^2}{2\Delta^2}\right]dV_c, \tag{2}$$

where $\Delta\equiv\Delta[r_0(V_c)]$ is the rms linear overdensity in a sphere of radius $r_0$, extrapolated to the present day. The value of $r_0$ is determined from $V_c$ and $z$ using equation (1). For a CDM power spectrum, $\Delta(r_0)$ can be approximated to within 10 per cent over the range $0.05<r_0<40$ Mpc by

$$\Delta(r_0)=16.3\ b^{-1}(1-0.3909\ r_0^{0.1}+0.4814\ r_0^{0.2})^{-10}, \tag{3}$$

where $b$ is the biasing parameter and the normalization is chosen so that $\Delta(16\text{ Mpc})=1$ for $b=1$. We have adopted $H_0=50$ km s$^{-1}$ Mpc$^{-1}$.

The reader is referred to White & Frenk (1991) for a detailed explanation of how the above expressions are derived. As given above they are valid for an Einstein–de Sitter universe. Straightforward extension of the formalism to other cosmological models is described and tested by Kauffmann (in preparation).

Using the results of Bower (1991) and Bond et al. (1991), one can write down an expression for the fraction of matter which is in haloes of circular velocity $V_1$ at redshift $z_1$, and later in haloes of circular velocity $V_0$ at $z_0<z_1$:

$$f(V_1,V_0,z_1,z_0)\,dV_1\,dV_0=\frac{2\times1.68^2\Delta_1(z_1-z_0)(1+z_0)}{\pi(\Delta_1^2-\Delta_0^2)^{3/2}\Delta_0^2} \times\frac{d\Delta_1}{dV_1}\frac{d\Delta_0}{dV_0}\exp\left\{\frac{-1.68^2}{2}\left[\frac{(z_1-z_0)^2}{\Delta_1^2-\Delta_0^2}+\frac{(1+z_0)^2}{\Delta_0^2}\right]\right\}dV_1\,dV_0. \tag{4}$$

One can then obtain the conditional probability that material which is in a halo of circular velocity $V_0$ at $z_0$ had previously been in a halo with circular velocity $V_1$ at $z_1$,

$$f(V_1,z_1\,|\,V_0,z_0)=f(V_1,V_0,z_1,z_0)/f(V_0,z_0). \tag{5}$$

So, on average, the number of $(V_1;z_1)$ progenitors of a *single* $(V_0;z_0)$ halo is

$$N_{\rm proj}(V_1;z_1)=f(V_1,z_1\,|\,V_0,z_0)\frac{M(V_0,z_0)}{M(V_1,z_1)}, \tag{6}$$

where $M(V_0,z_0)$ and $M(V_1,z_1)$ are the masses of a $(V_0;z_0)$ and $(V_1;z_1)$ halo respectively. These numbers are valid for ensemble statistics only. To construct a tree, i.e. to determine how a *single* dark matter halo will break up into a set of progenitors at some earlier redshift, we need an algorithm that will generate random realizations of this process. At the same time, the algorithm must preserve the merging probabilities of equations (5) and (6) when a large number of haloes are considered as an ensemble. It is clearly not appropriate to select halo progenitors randomly according to the probability distribution of equation (5). This is because we must impose the constraint that the mass of a halo be equal to the sum of masses of its progenitors at every stage. This constraint causes the selection of halo progenitors to differ markedly from an ordinary Poisson process. In effect, we have to 'throw out' all random realizations which do not conserve mass and we have to find some way to do this efficiently.

The algorithm we have adopted proceeds as follows. We consider the breakup of a $(V_0;z_0)$ halo into a set of progenitors $(V_1, V_2,\ldots, V_i;\ z_0+dz_0)$, where $dz_0$ represents a small time-step (typically 0.1 at low redshift). We wish to obtain merger histories for a large number of haloes, typically around 100. Then it is reasonable to impose the constraint that the combined mass distribution of the progenitors of all 100 haloes obeys equations (5) and (6). To obtain the total number of $(V_i;\ z_0+dz_0)$ progenitors of 100 $(V_0;z_0)$ haloes, we use $N_{\rm proj}(V_i,\ z_0+dz_0)$ from equation (6), multiply this number by 100 and round to the nearest integer. In practice, we adopt a circular-velocity grid and divide circular velocity into intervals $(V_i,\ V_i+dV_i)$. The number of progenitors in each such bin is evaluated to give a set of progenitors for the 100 $(V_0;z_0)$ haloes with a distribution given by equations (5) and (6).

Our next task is to distribute the progenitors among the original set of haloes. There are clearly a number of ways to do this. We have adopted the following method. We start with the largest progenitors and work our way down to the smallest. We assign progenitors to haloes randomly, but with probability proportional to the amount of mass that is still 'free', i.e. the amount of mass that has not been taken up by progenitors which have already been assigned. At every state we are also careful not to violate the mass conservation constraint for each individual halo. This procedure is designed to distribute progenitors among haloes more or less evenly. No halo will end up with more than its 'share' of progenitors of any given mass. In this way we hope to minimize the overall deviation between the mass distribution of each progenitor set and the ensemble mass distribution given by equations (5) and (6).

Using this procedure we generate a set of single-time-step halo histories on a circular-velocity–redshift grid. These histories are then stored. It now becomes a simple matter to construct a merging history tree by working backward from $z=0$ to high redshift, randomly selecting single-time-step histories from the appropriate grid positions at each stage. It should be noted that we have imposed a minimum progenitor circular velocity of 30 km s$^{-1}$, but have kept track of the fraction of the halo mass which comes from progenitors smaller than this.

## 3 TESTS OF THE MERGING HISTORY ALGORITHM

By construction, our algorithm selects halo progenitors which have correct ensemble properties. However, our procedure for distributing progenitors has no rigorous mathematical basis, although it may seem intuitively reasonable. Therefore it is important to test our results against the merging histories of haloes determined from numerical simulation of gravitational clustering.

Frenk et al. (1988) have investigated the formation of dark haloes in a flat universe dominated by cold dark matter using $N$-body simulations. In particular, they analysed the history of material that ended up in the large systems present at the end of the simulations. These systems or groups of particles may be identified with the dark matter haloes which are the subject of this paper. Each large group was selected from the final output file and its $N_t$ particles were partitioned into subsets according to the groups they belonged to at some earlier redshift. $N_1$ and $N_2$ were taken to refer to the numbers of particles in the two largest subsets. The ratio $N_t/N_1$ can then be regarded as an estimate of the factor by which the mass of the group has grown since redshift $z$, and the ratio $N_1/N_2$ measures whether growth occurred primarily by accretion on to a dominant core or by merging of comparable systems. Scatter plots of $N_t/N_1$ against $N_1/N_2$ for three different redshifts are given in fig. 5 of Frenk et al.

We have carried out a similar analysis using the 100-Mpc ($h=0.5$) simulation of an $\Omega=0.2$ CDM universe described in Kauffmann & White (1992). Using our algorithm, we are able to construct analogous $N_t/N_1$ versus $N_1/N_2$ scatter plots (Fig. 2) and compare them to the results derived from the $N$-body simulations (Fig. 3). We divide the haloes into two groups – those with masses less than $10^{13}$ $M_\odot$ (denoted by crosses), and those with masses greater than this value (denoted by circles). The median circular velocities of the haloes in the two groups are 200 and 350 km s$^{-1}$ respectively. The simulation plots show rather more irregularity and scatter than our 'tree' plots, particularly at large values of $N_1/N_2$. This is partially the result of the fact that our algorithm constructs merging history trees on a circular-velocity *grid*. In addition, the haloes in Fig. 3 span a range of present-day masses, whereas we have chosen only two specific circular velocities to derive the histories shown in Fig. 2. Nevertheless, the overall trends with redshift agree exceptionally well.

At low redshift a large fraction of haloes survive relatively intact and the distribution is concentrated at low values of $N_t/N_1$. Haloes which grow by accreting very much smaller lumps appear at the right-hand side of the plot, while haloes which are formed by the merging of progenitors with comparable masses appear towards the left. The characteristic shape of the distribution, including the upwards 'bend' of the distribution at small values of $N_1/N_2$, indicates that haloes do not tend to grow by accreting a large amount of mass in small lumps. If this were the case, we ought to see points in the scatter plot with simultaneously high values of $N_t/N_1$ and $N_1/N_2$. Thus, if a significant change in mass of the original halo has taken place, the mechanism has probably been a merger between almost equal-sized objects. As the redshift increases, the distribution shifts increasingly upwards and to the left as more and more haloes break up into equal-sized pieces. It is also interesting to note that both Figs 2 and 3 show that low-mass and high-mass haloes appear to occupy separate 'tracks' in the distribution, particularly at higher redshift. This shows that high-mass haloes break up more readily than smaller haloes and are also more likely to have been formed from equal-mass mergers.

We have made similar scatter plots for an $\Omega=1$, $b=2.5$ universe and show these in Fig. 4. In this case circles and crosses correspond to circular velocities of 200 and 100 km s$^{-1}$ respectively. The reader is encouraged to compare our plots with fig. 5 of Frenk et al. (1988) in order to see the excellent agreement that is obtained. In this model merging takes place at a much more rapid rate. The mass distribution of halo precursors at a redshift of 2.5 is already comparable with the distribution at a redshift of 4 in the low-density model.

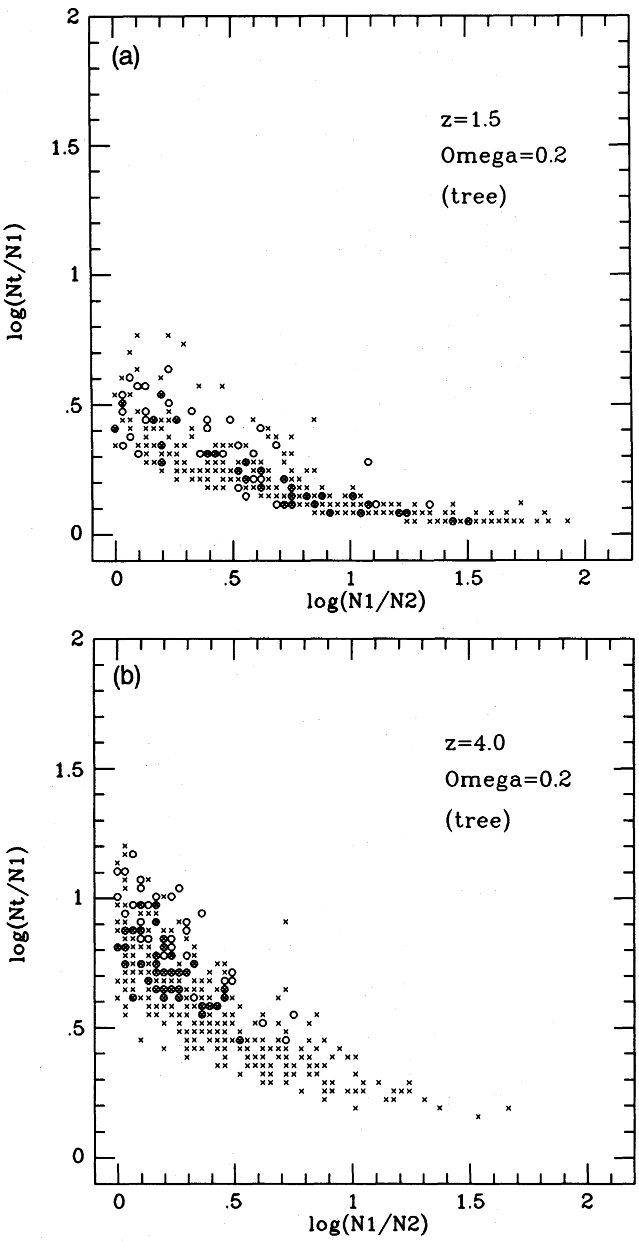


**Figure 2.** Merging histories of haloes present at $z=0$ as determined by the merging tree algorithm. For each halo, $N_t$ is the mass of the halo at the final time. $N_1$ and $N_2$ are the masses of its two largest progenitors at each epoch shown. Open symbols and crosses refer to haloes with circular velocities of 350 and 200 km s$^{-1}$ respectively.

It is also important to study the behaviour of the largest halo precursor mass ($N_1$) as a *function* of redshift if we are to be confident about applying our algorithm to questions of galaxy, group and cluster evolution. For example, in a galaxy-sized halo the evolution of the largest precursor will guide our treatment of how the central galaxy forms stars and

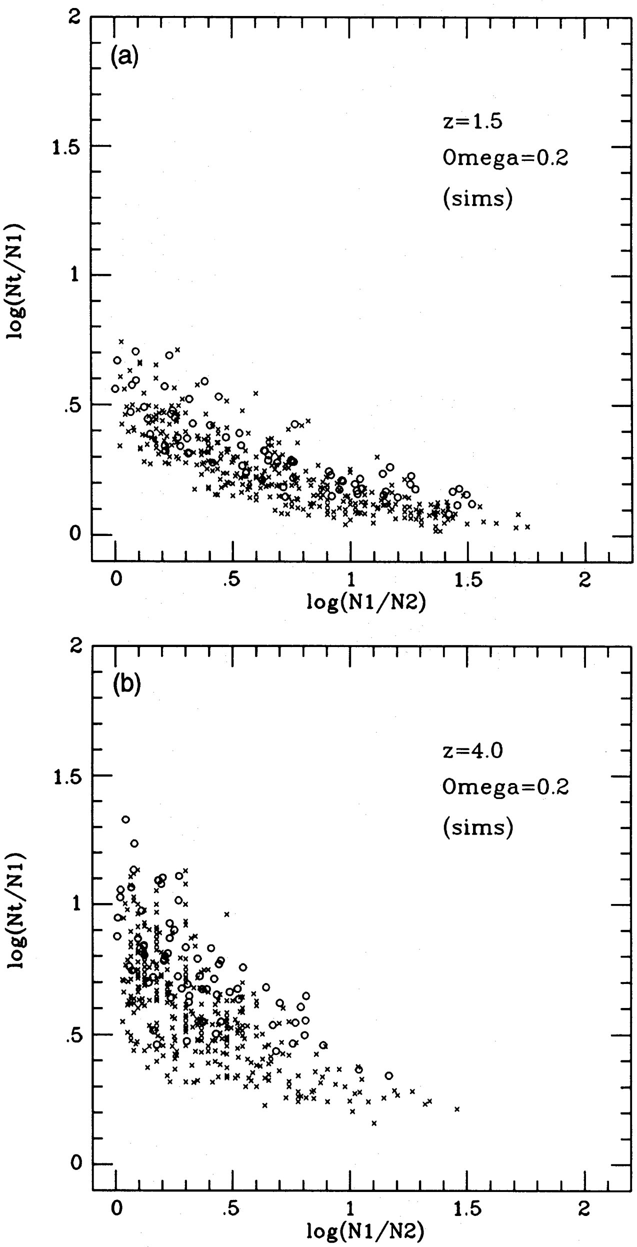


**Figure 3.** As in Fig. 2, except that the merging histories are from $N$-body simulations and groups are for masses greater than and less than $10^{13}\,\mathrm{M}_{\odot}$. These two mass groups have the median circular velocities used in Fig. 2.

evolves. We must therefore be sure that haloes accrete and merge in roughly the same way in the tree algorithm and in the simulations. In Figs 5(a) and (b) we plot the evolution of $N_1/N_t$ as a function of redshift in an $\Omega=0.2$ CDM universe using the tree algorithm. Fig. 5(a) shows the evolution of $N_1/N_t$ for representative present-day haloes of galactic mass ($V_c=220\ \mathrm{km\ s^{-1}}$), while Fig. 5(b) is relevant to the evolution of cluster-sized haloes ($V_c=100\ \mathrm{km\ s^{-1}}$). We show 10 curves in each plot in order to illustrate the scatter between different merging histories. In Figs 6(a) and (b) we show the evolution of $N_1/N_t$ using haloes of roughly the same masses identified in the low-density CDM simulation. To construct Fig. 6(b), we selected the 10 most massive groups in the simulation with circular velocities in the range 750 to 1100 km s$^{-1}$. It should be noted that there were a total of only nine output times in the simulation between redshifts of 5.4 and 0, so the resolution of the simulation plots is a good deal lower than that of the tree plots.

Looking at individual curves in Figs 5 and 6, one can identify intervals where $N_1/N_t$ decreases very slowly with redshift, corresponding to periods where the halo was growing by slow accretion. There are also many cases where precipitous drops in $N_1/N_t$ occur over a single time-step. These correspond to the equal-mass merging events discussed above, during which the mass of the largest halo precursor can jump by almost a factor of 2. These merging events no doubt have an important impact on the evolution of the galaxy which is forming within the halo, as will be discussed in the next section. The overall agreement between Figs 5 and 6 is very good. The curves show the same trend with redshift, have the same scatter and even have roughly the same slope during periods of slow growth by accretion. It is again evident from these plots that $N_1/N_t$ evolves more rapidly with redshift for large haloes.

We have obtained excellent agreement between the results of our tree algorithm and those of $N$-body simulations. Because of the many simplifying assumptions that go into the derivation of the Press–Schechter-based merging probabilities, and ambiguities related to the way we partition halo progenitors, we regard this agreement as a significant success for our method.

## 4 TWO SIMPLE APPLICATIONS OF THE ALGORITHM

### 4.1 Survival of galaxy discs

As pointed out by Ostriker (1990) and then in more detail by Toth & Ostriker (1992), the thinness and coldness of galactic discs can be used to set limits on the current rate of infall of satellite systems on to spiral galaxies. Toth & Ostriker (1992) argue that not more than 4 per cent of the mass inside the solar radius could have accreted in the last 5 billion years, or else the scale height and Toomre $Q$ parameter of our Galaxy would exceed observed values.

In Fig. 7 we plot, as a function of redshift, $z$, the fraction of present-day $V_c=200\ \mathrm{km\ s^{-1}}$ haloes that have merging events of varying degrees of severity between redshifts $z$ and 0. We have classified the merging events according to the ratio $N_1/N_2$ of the halo's two largest progenitors. For example, the 20:1 curve shows the percentage of haloes whose second largest precursor has 5 per cent or more of the mass of the largest precursor. Fig. 7(a) shows results for an $\Omega=1$ CDM universe with a bias factor $b=2.5$. Fig. 7(b) is for a low-density ($\Omega=0.2$) CDM universe in which galaxies are assumed to trace the dark matter distribution.

As can be seen from Fig. 7(a), the merging rates in an $\Omega=1$ universe are very high. 80 per cent of haloes will have had a merging event more severe than $N_1/N_2=10$, since a redshift of 0.4 which corresponds to a time of 5.1 Gyr. This number is relatively insensitive to the value of the bias parameter $b$. For the low-density universe, the situation is much better. Only 25 per cent of haloes have merging events more severe than 10:1 over this period, and the fraction does

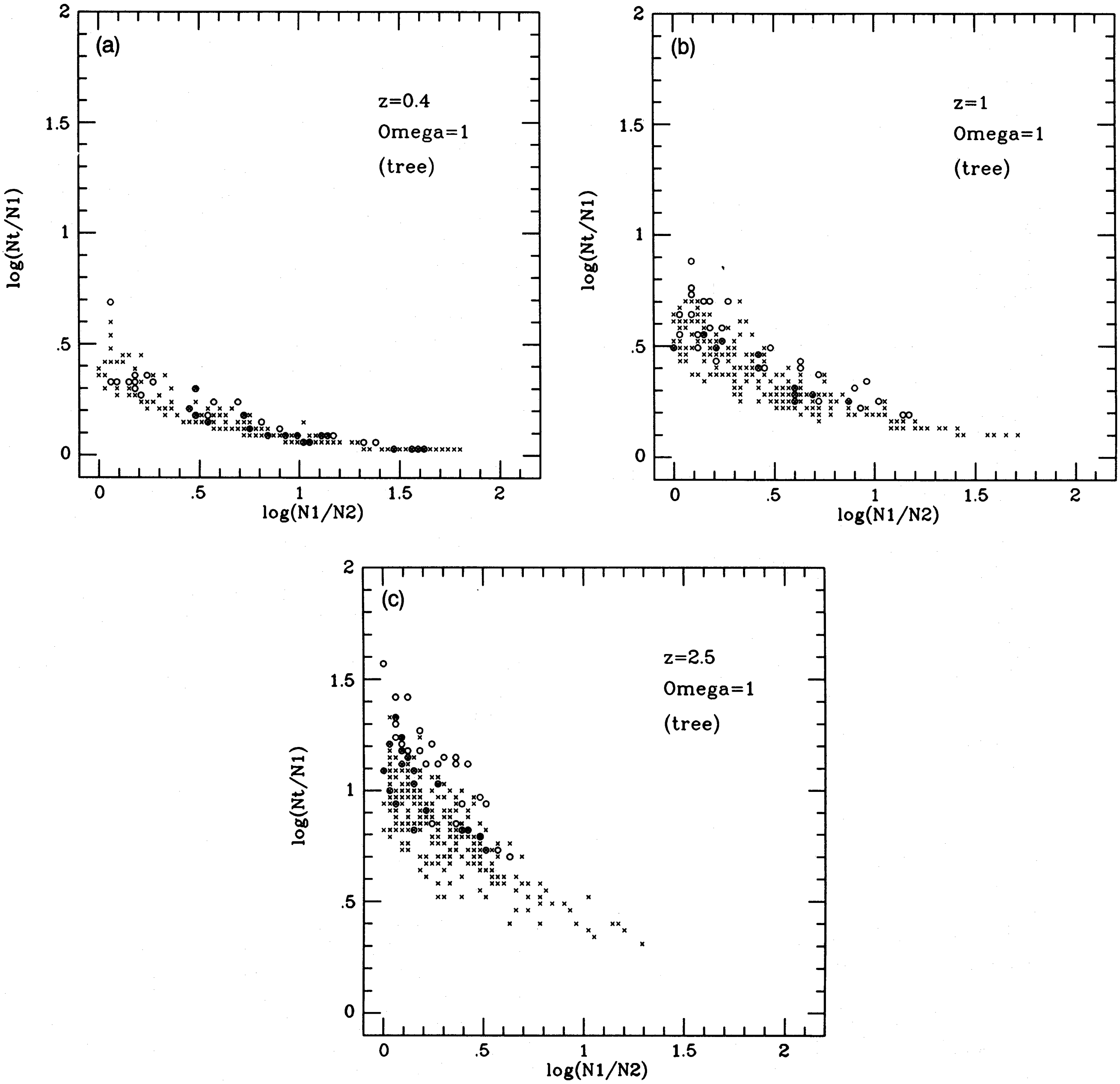


**Figure 4.** As in Fig. 2, but for a flat CDM universe with $b = 2.5$. This figure should be compared to fig. 5 of Frenk et al. (1988). In this case, the median circular velocities are 200 and 100 km $s^{-1}$.



not reach 0.5 until a redshift of nearly 1. The low merging rates of the low-density model are the result of a considerable slowdown in the growth of perturbations at low redshift in an open universe. Our merging rates are in agreement with those of Toth & Ostriker (1992). These authors claim that the high merger rates in an $\Omega = 1$ CDM universe are in conflict with the fact that 90 per cent of field galaxies have pronounced disc components. This conclusion depends on additional assumptions about how small galaxies are disrupted and how they affect an existing disc during a merger event. The conflict may also be alleviated if galaxies merge with very much lower frequency than their surrounding haloes.

### 4.2 Substructure in galaxy clusters

In hierarchical clustering scenarios such as CDM, considerable merging activity is also expected to occur in very much larger haloes with masses corresponding to those of rich clusters. One of the indicators of the current dynamical state of a cluster is the degree of substructure exhibited by either the galaxy or the gas distribution. X-ray observations have proved particularly suited to studies of substructure in the cluster potential.

Jones & Foreman (1992) have analysed *Einstein* observations of 400 clusters with redshifts less than 0.2. Of these 400 clusters, 208 were sufficiently X-ray bright to facilitate

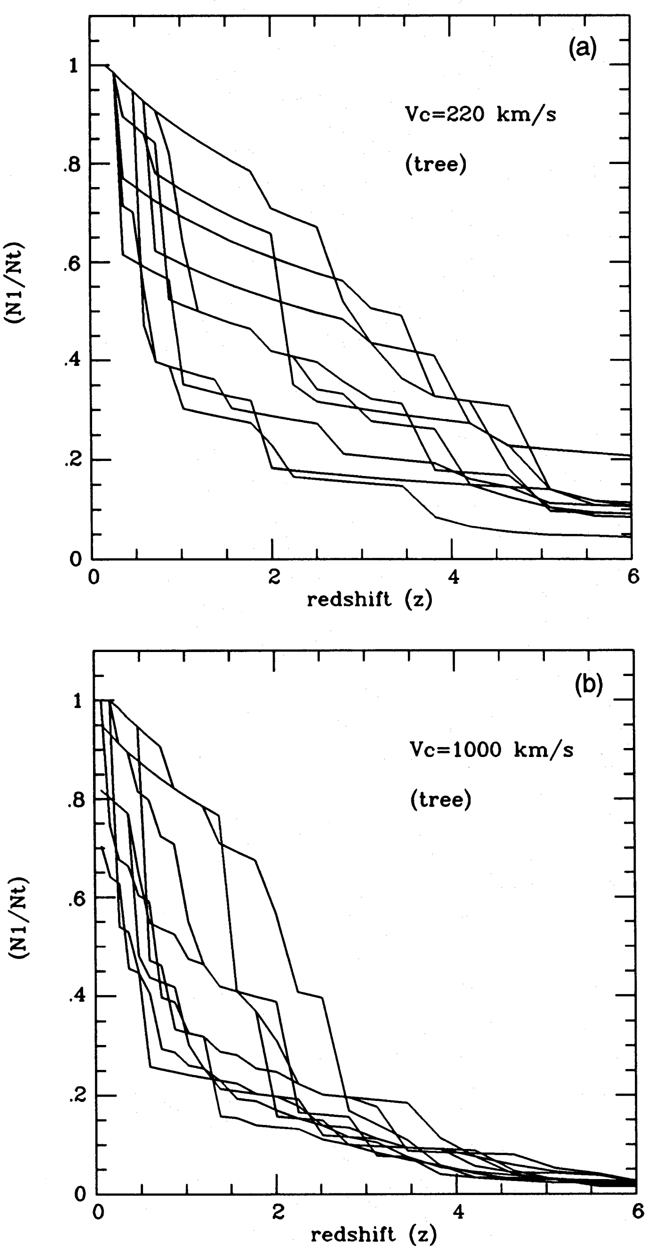


**Figure 5.** The evolution of the ratio of the largest progenitor mass $N_1$ to the mass of the original halo $N_t$ as a function of redshift in an $\Omega=0.2$ CDM universe as determined by the tree algorithm: (a) for galaxy-sized haloes, (b) for cluster-sized haloes.

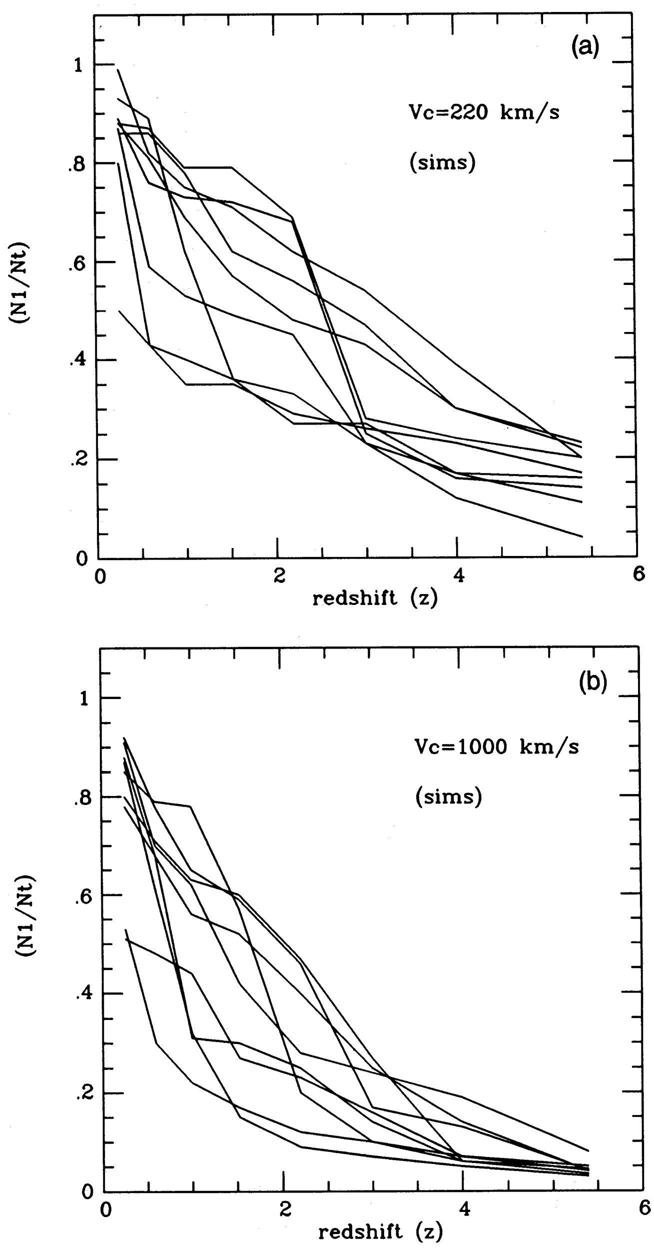


**Figure 6.** As in Fig. 5, but using results from *N*-body simulations.

the detection of subclusters containing 10 per cent of the total cluster luminosity. Jones & Foreman divide their bright cluster sample into a set of different morphological classes and find that 6 per cent of the sample can be considered 'double', i.e. made up of two roughly equal components. A further 16 per cent of the sample clearly exhibits substructure, either in the form of a primary cluster with a clearly identifiable secondary component, or in the form of complex, multiple structures.

If we are to compare these figures with the predictions of theoretical models, we must have an estimate of how long substructure would be expected to persist, as this will determine the percentage of clusters which will still exhibit evidence of a recent merging event. We can make a very rough estimate by assuming that substructure will last for a single crossing time. The projected separations of the subclusters identified by Jones & Foreman (1992) are typically less than 1.5 Mpc. If we consider the case of the merger of two equal-mass clusters having typical line-of-sight velocity dispersions of 700 km $s^{-1}$, we obtain an infall velocity of roughly 2000 km $s^{-1}$. This leads to an estimate for the substructure survival time of about 1 Gyr.

The prediction for the percentage of clusters undergoing merging events with roughly equal-mass progenitors over a 1-Gyr period at low redshift is 3 per cent for the open model and 15 per cent for the $\Omega=1$ model. For 10:1 merging events, the figures are 15 and 50 per cent respectively. Neither model appears to fit the observations particularly

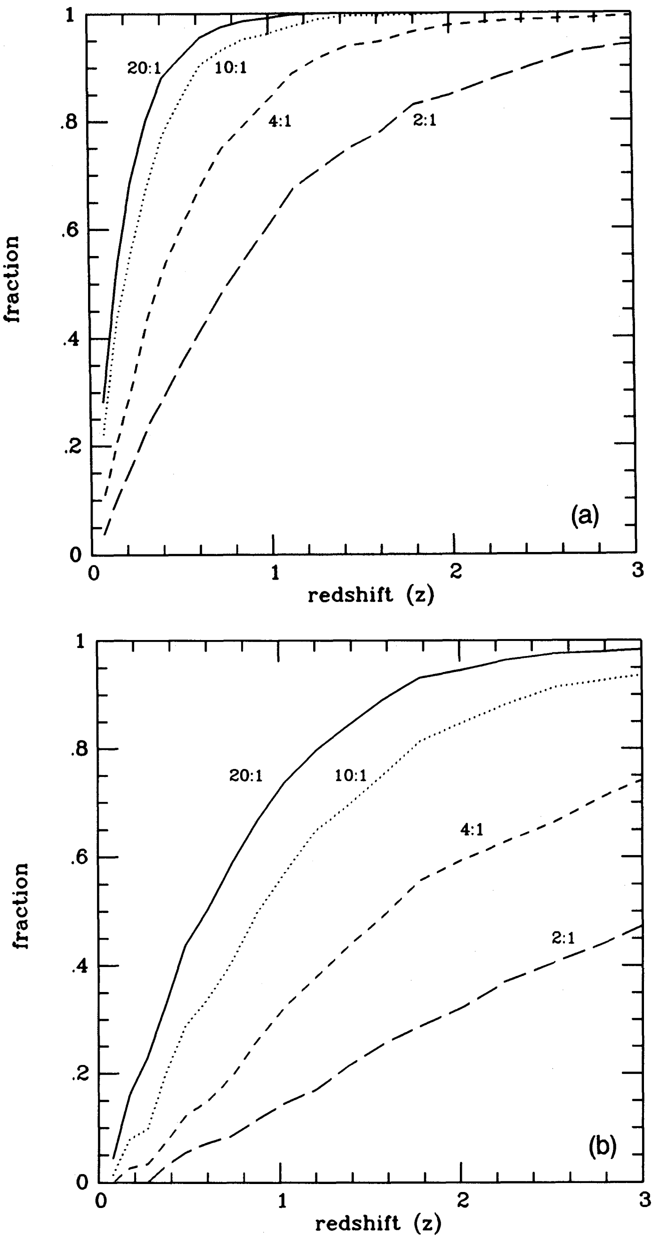


**Figure 7.** The fraction of present-day haloes of circular velocity $V_c = 220$ km s$^{-1}$ which have undergone merging events since redshift $z$. The ratio used to label each curve is the ratio $M_1$:$M_2$, where $M_1$ and $M_2$ are the masses of the largest and second largest halo precursor. (a) shows results for an $\Omega = 1$ CDM universe with bias factor $b = 2.5$. (b) is for an $\Omega = 0.2$ universe in which light traces mass.

well, although an open universe with slightly higher density would undoubtedly be acceptable.

Richstone, Loeb & Turner (1992) have used spherical collapse models to estimate the fractional rate of cluster formation and have concluded that the observed degree of subclustering implies that the density parameter $\Omega$ is greater than 0.5. Their analysis did not distinguish between mergers of nearly equal systems and accretion of smaller clumps. In addition, their estimate for the fraction of systems exhibiting substructure is 30–40 per cent rather than the 22 per cent quoted by Jones & Foreman (1992) for systems which show *unambiguous* substructure. This higher estimate of the frequency of substructure leads to a higher estimate for $\Omega$. However, Richstone et al. comment that selection effects ought to favour regular clusters, so 22 per cent may indeed be an underestimate.

The uncertainties in the interpretation of the observations and in our estimate of substructure survival time are too large to allow any reliable measurement of $\Omega$ at this stage. The survival of substructure is clearly an issue which deserves further study using simulations.

## 5 DISCUSSION

We have described a method for the construction of merging history trees for present-day dark matter haloes and have investigated two simple applications of our techniques. The main advantage of studying halo histories is that one can then keep track of the formation and merging path of every halo progenitor. This opens many possibilities for the study of the formation and evolution of galaxies, groups of galaxies and clusters. For example, a dark matter halo with a present-day circular velocity of 1000 km s$^{-1}$ can be identified with a galaxy cluster. By constructing the merging tree of such a halo, one can address questions such as the time of formation of the galaxies within the cluster, the incidence of mergers between them, and, by using the results of stellar population models, obtain estimates of the colours and luminosities of galaxies. If elliptical galaxies are assumed to result from mergers of spirals, issues such as the galaxy morphology–density relation can be addressed. We intend to tackle these and other questions in future work.


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