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The metallicities, velocity dispersions and true shapes of elliptical galaxies

Roberto Terlevich and Roger L. Davies *Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*

S. M. Faber *Lick Observatory, Board of Studies in Astronomy and Astrophysics, University of California, Santa Cruz, California, USA*

David Burstein *National Radio Astronomy Observatory*, Charlottesville, Virginia, USA*

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Summary. We have re-analysed the relationships between velocity dispersion, line strength and absolute magnitude for normal elliptical galaxies. We find that, at fixed absolute magnitude, galaxies with high velocity-dispersions tend to have high line-strengths and those with low velocity-dispersions have low line-strengths. This implies that elliptical galaxies are at least a two-parameter family. This conclusion is based on a preliminary sample of 24 galaxies for which both types of measurements are available. Well-known apparent scale differences in velocity dispersion between various authors are shown to be due to mean line-strength differences between galaxy samples. After correction for this effect, all sources considered here agree well to within a few per cent.

The present data also suggest that the velocity-dispersion and line-strength residuals may in turn be correlated with intrinsic axial ratio, with flatter ellipticals having smaller velocity dispersions and metallicities. The correlation between residual metallicity and axial ratio is of special importance since measured metallicity is independent of aspect angle. If this relation holds up on tests with larger samples, it could offer a number of new ways to determine the true figures and axial ratios of elliptical galaxies.

These new results are discussed in the context of dissipational and dissipationless collapse models for elliptical galaxy formation. They indicate a strong link between the dynamical and chemical evolutions of elliptical galaxies which seems quite compatible with the dissipational picture, where stars formed during the collapse phase at the same time as the final dynamical characteristics of the galaxy were determined. Merger models involving largely

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gaseous fragments also appear viable. By contrast, purely stellar collapses or mergers would seem to require additional, somewhat *ad hoc* assumptions.

The interpretation of the systematically bluer colours of cD galaxies may be complicated by their tendency to be flatter than normal ellipticals.

1 Introduction

Because of their relatively simple structure, elliptical galaxies have always been interesting and useful objects of investigation for students of galaxy formation. Their utility has increased still further of late as a result of several studies (e.g. Sandage 1972; Faber 1973; Faber & Jackson 1976; Kormendy 1977; Sandage & Visvanathan 1978) which have indicated striking correlations among the small number of global parameters needed to characterize elliptical structure. It has even been suggested (Faber 1973) that elliptical galaxies are basically a one-parameter family, the fundamental governing variable being the total luminosity.

The two correlations with luminosity which have been studied most intensively and for which the most observational data exist involve the spectrophotometric properties (colour and line strength) and the velocity dispersion. Although their precise theoretical interpretation is still unclear, these two trends appear to be in qualitative agreement with dissipative collapse models of galaxy formation (e.g. Larson 1975; Tinsley & Larson 1979), which are able to account for both trends in plausible fashion. Regardless of whether this particular picture is correct or not, it is generally acknowledged that these two luminosity correlations, together with their kindred relations involving luminosity density, length-scale, and mass-to-light ratio, are among the most significant clues we have to the process of elliptical galaxy formation.

Because of the theoretical importance of such correlations, it is exceedingly disappointing that recent measurements have tended to show more scatter in these trends than did the earliest studies. The scatter in the spectral properties is now well documented (e.g. Faber 1977; Sandage & Visvanathan 1978). Scatter in the velocity dispersion–luminosity correlation is also quite clear but has usually been dismissed as a sign of systematic measurement errors among various authors (e.g. Sargent *et al.* 1977; Whitmore, Kirshner & Schechter 1979).

In this paper we reconsider these two relations using the small sample of 24 ellipticals for which both velocity dispersions and line strengths have been measured. We are able to show that deviations from the two relations are correlated in the sense that galaxies with high velocity-dispersions also have high line-strengths. It is clear therefore that the scatter in velocity dispersions is real and not caused just by measurement errors. In fact, we show that a fairly large subset of the published velocity dispersion studies agree among themselves to within a few per cent, better even than the authors themselves have believed.

The discovery of this additional correlation between line-strength and velocity-dispersion residuals strongly hints at the existence of a second fundamental parameter aside from luminosity which plays a determining role in elliptical galaxy structure. Further considerations suggest that this second parameter may be associated with the intrinsic axial ratio of the galaxy. This second finding could have potentially important implications for dissipative and stellar collapse models of elliptical formation, and may shed new light on the problem of whether ellipticals are oblate or prolate. For this reason it is important to test these hypotheses on a larger sample of galaxies.

2 Sources of data

The primary sample of galaxies in this study consists of all normal ellipticals brighter than −19.5 for which line-strength measurements exist and for which velocity dispersions have

Table 1.

GALAXY (1)	TYPE (2)	V (3)	$-M_B$ (4)	σ (5)	SOURCES (6)	$\log\sigma$ (7)	Mg_2 (8)	$\delta\log\sigma$ (9)	δMg_2 (10)	$\log\sigma$ corrected (11)	Mg_2 corrected (12)	$\log a/b$ (13)	$\Delta\log\sigma$ (14)	ΔMg_2 (15)
584	E3	1968	21.80	225	1.	2.352	.266	-.067	-.057	2.443	.297	.20	-.081	-.048
596	E0	1968	21.31	174	2,5.	2.241	.242	-.131	-.074	2.363	.303	.20	-.148	-.063
720	E5	1820	21.66	218	2,5.	2.338	.355	-.067	+.034	2.260	.386	.20	-.082	+.043
741	E0	5645	23.07	328	2,5.	2.516	.329	-.024	-.010	2.520	.340	.01	-.030	-.009
1052	E3	1458	20.83	218	2,3,5.	2.299	.355	-.027	+.045	2.201	.368	.16	-.047	+.059
1172	E1	1621	19.74	199	2.	1.991	.232	-.230	-.064	2.094	.339	.04	-.258	-.043
1395	E2	1574	21.41	245	2.	2.389	.307	+.008	-.011	2.394	.303	.11	-.009	.000
1426	E4	1574	20.22	151	2.	2.179	.264	-.088	-.038	2.234	.305	.15	-.113	-.020
1600	E4	4765	22.80	332	5.	2.521	.335	+.006	-.001	2.508	.332	.13	-.001	+.002
1700	E3	3911	22.67	260	5.	2.415	.267	-.087	-.067	2.524	.308	.15	-.095	-.064
3377	E6	741	19.87	176	7.	2.246	.287	+.012	-.011	2.251	.281	.21	-.015	+.009
3379	E0	741	20.85	241	1,4,6,7.	2.382	.329	+.054	+.019	2.332	.304	.05	+.034	+.033
3608	E1	945	19.48	225	1.	2.352	.322	+.156	+.030	2.271	.247	.09	+.126	+.052
4278	E1	944	20.40	243	3,5.	2.386	.327	+.102	+.023	2.328	.279	.02	+.078	+.039
4374	E1	1150	21.62	290	1,6,7.	2.462	.336	+.061	+.016	2.417	.308	.06	+.045	+.025
4406	E2	1150	21.80	261	1,6.	2.417	.323	-.002	+.000	2.402	.324	.13	-.016	+.009
4472	E2	1150	22.50	311	1,6,7.	2.493	.341	+.007	+.009	2.461	.338	.08	-.002	+.014
4552	E0	1150	21.00	290	1.	2.462	.353	+.120	+.041	2.371	.297	.00	+.101	+.054
4649	E2	1150	22.01	360	1.	2.556	.369	+.117	+.044	2.460	.314	.07	+.105	+.051
4889	E4	6866	23.48	400	1.	2.602	.362	+.022	+.018	2.554	.352	.16	+.020	+.016
5846	E1	1733	21.58	255	1.	2.406	.347	+.008	+.027	2.341	.343	.02	-.007	+.037
6166	cD	9360	23.40	350	1.	2.544	.323	-.028	-.020	2.566	.336	.12	-.031	-.022
7619	E3	3799	22.47	356	5.	2.551	.363	+.068	+.032	2.477	.331	.04	+.059	+.036
7626	E1	3799	22.32	308	2,5.	2.488	.323	+.020	-.006	2.484	.314	.08	+.009	-.001

SOURCES:

1. Faber & Jackson (1976)
2. Sargent et al. (1977)
3. Knapp et al. (1978)
4. Sargent et al. (1978)
5. Schechter & Gunn (1979)
6. Davies (1980)
7. Whitmore et al. (1979)

been measured using the Fourier technique (Sargent *et al.* 1977 (SSBS); Sargent *et al* 1978; Schechter & Gunn 1979 (SG); Davies 1981 (D); Whitmore *et al.* 1979 (WKS)). Galaxies in the sample of Faber & Jackson (1976, FJ) and Knapp, Gallagher & Faber (1978), which were measured using the direct comparison technique, were also included. Where more than one measurement was available, the unweighted average value was adopted. These values together with the sources appear in Table 1.

As a line-strength indicator, we have used the Mg_2 index at 5178 Å as defined by Faber, Burstein & Dressler (1977) and measured on the unpublished scans of Faber & Burstein Scans were taken on the nuclei through a 2 × 4 arcsec aperture. Since there are no true continuum regions free of absorption lines in elliptical galaxy spectra, the side-bands of the Mg feature are located in quasi-continuum regions where the blanketing is lower but not absent entirely. At high velocity-dispersions, adjacent spectral features tend to merge, thus depressing the side-bands and filling in the Mg feature itself. As a result, the empirically defined Mg line-strength index is reduced slightly. Experiments on broadened standard stars indicate that this reduction amounts to at most 0.005 mag for galaxies with the highest dispersions and for the majority of galaxies in the sample is much smaller still. For the present, no corrections to the data have been made for this effect. Adopted values of Mg_2 appear in Table 1.

The Mg_2 index is a good metallicity indicator, correlating well with other abundance indicators for galaxies (Faber 1973, 1977; Cohen 1978) and globular clusters (Burstein 1979). Using the method of spectral synthesis, Mould (1978) modelled the behaviour of this parameter as a function of metallicity. When transformed to observable parameters (Burstein 1979), the relationship becomes

$$[\mathrm{Fe/H}] = 3.9\ \mathrm{Mg}_2 - 0.9, \tag{1}$$

where [Fe/H] is the usual differential logarithmic abundance relative to the sun, and Mg_2 is in magnitudes.

Absolute magnitudes for all galaxies in Table 1 were based on the integrated B_T magnitudes from the *Second Reference Catalogue of Bright Galaxies* (RC2: de Vaucouleurs, de Vaucouleurs & Corwin 1976) or on the corrected Harvard magnitudes (m_c) when B_T is not available. In estimating distances, each galaxy was checked for group or cluster membership using de Vaucouleurs (1975), Sandage & Tammann (1975), Turner & Gott (1976), or our own inspection of the position and velocities of nearby galaxies. Adopted radial velocities appear in Table 1. These data, plus galactic absorptions calculated using the method of Burstein & Heiles (1978) and $H_0 = 50\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$, yielded final absolute magnitudes.

Morphological types came principally from Sandage & Visvanathan (1978), with a few classifications from the RC2. There is some disagreement among types, some authors claiming that one or more of these galaxies are S0's. However, these classification uncertainties do not affect our major conclusions.

3 Relationships between velocity dispersion, line strength and luminosity

For our primary sample of 24 galaxies, the relations between velocity dispersion, line strength, and luminosity are illustrated in Figs 1 and 2. The scatter in these diagrams is large compared to the nominal measurement uncertainty, which is generally ±10 per cent in σ and ±0.008 mag in Mg_2. Examination of the distribution of individual points in each diagram suggests that the residuals are correlated, in the sense that at a given luminosity galaxies which have higher-than-average velocity dispersions also have higher-than-average line indices.

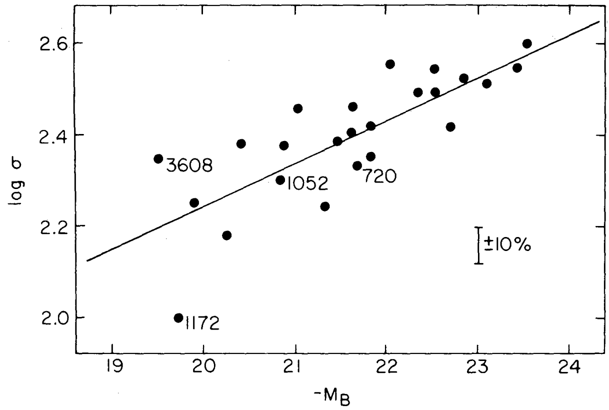

Figure 1. The logarithm of velocity dispersion, log σ, versus absolute magnitude, M_B, for the primary sample of elliptical galaxies.

In order to investigate this effect quantitatively, we computed linear least-square regressions for both relationships:

$$\log \sigma = -0.0959\ (\pm 0.016) M_B + 0.328; \quad (2)$$

$$Mg_2 = -0.0130\ (\pm 0.0068) M_B + 0.039. \quad (3)$$

The values within parenthesis are the standard errors of the parameters. These regressions are indicated by the solid lines in Figs 1 and 2. We then calculated residuals with respect to these relations: $\delta \log \sigma$ and δ Mg_2. These are listed for each galaxy in Table 1 and are plotted versus one another in Fig. 3. The residuals are indeed strongly correlated, although four galaxies, NGC 720, 1052, 1172 and 3608, do deviate noticeably from the otherwise remarkably tight relationship.

It is important to establish at the outset that this correlation between residuals cannot be caused by spurious systematic errors of measurement. The most obvious culprit here would be large errors in the absolute magnitudes, M_B.

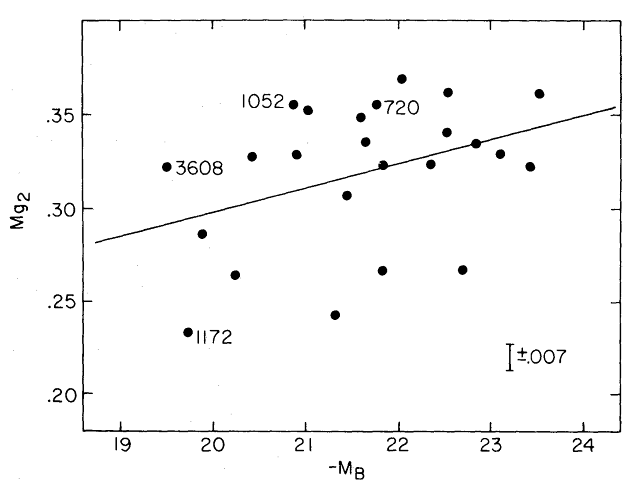

Figure 2. The magnesium absorption line index, Mg_2, versus absolute magnitude, M_B, for the primary sample of elliptical galaxies.

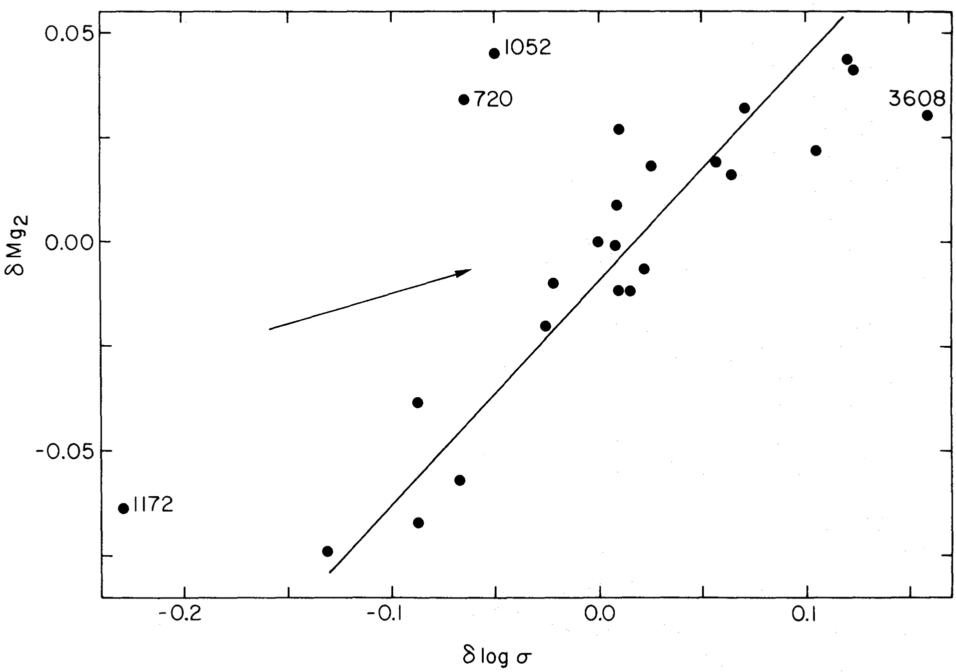

Figure 3. The residual line index, δ Mg_2, with respect to equation (3), versus the residual velocity dispersion, δ log σ, with respect to equation 2.

The arrow in Fig. 3 indicates the shift in the position of a galaxy due to an error of 1 mag in M_B. Magnitude errors would clearly yield a correlation with a slope quite different from the one observed. Furthermore, the sheer size of the effect seems to be much too large to be accounted for in this way since enormous errors of 2 mag would be required.

Could there be some much more subtle error in the measurements of σ and Mg_2 themselves? For example, with the Fourier technique SG noted a correlation between velocity dispersion and normalizing line strength γ. Could this dependence, which might be an artefact of the Fourier method, lead to the correlation we observe by biasing σ in some mysterious fashion? For a variety of reasons, we feel sure that the answer is no. SG tested the dependence of the velocity dispersion on the spectral type of the comparison spectrum and found only a small effect. Illingworth (private communication) has derived velocity dispersions of M81 using comparison spectra that yielded values of γ differing by a factor of 2, yet σ changed by only a few per cent. Furthermore, there is the fact that many of the velocity dispersions in our sample were measured using the direct comparison technique, which weights spectral features quite differently. Agreement with the Fourier method in cases of overlap is good. We also note that as a rule, velocity dispersions are measured in one of two totally separate spectral regions, 4100–4500 Å and 4800–6000 Å, in which the behaviour of spectral lines versus metallicity is quite different. Again, independent measurements from these two regions agree well. Finally, the sheer size of the scatter is hard to explain in terms of errors. The scatter amounts to nearly a factor of 2 in σ, much larger than the error estimates of any author.

Most important, we stress that the Mg_2 measurement used here is completely independent of the γ parameter, which is a normalizing factor given by the Fourier method. Mg_2 is a precisely defined quantity which is measured directly from the spectral scan in a manner which is completely independent of σ. As we discussed above, Mg_2 is, if anything, slightly *underestimated* for galaxies with large velocity dispersions. Thus, the present data actually minimize the correlation between δ Mg_2 and δ log σ. Finally, there is the obvious fact that Mg_2 is in many cases measured from spectral scans completely different from those used to determine σ and γ, often in an entirely different wavelength interval. In summary, it is hard to imagine how some subtle error could affect our entire body of data, gathered from such a variety of sources and analysed in heterogeneous ways. On balance, we believe that the evidence is strong that the correlated scatter between $\delta \log \sigma$ and δ Mg$_2$ is real and not merely an artefact of measurement errors.

Expressed in geometrical terms, the tight correlation between $\delta \log \sigma$ and δ Mg$_2$ implies that the distribution of galaxies in (M_B, log σ, Mg$_2$) space is a surface. This surface is canted with respect to each of the three coordinate axes so that the observed scatter in any two-coordinate diagram is produced by the projection of the surface on to that plane. The relation between M_B and log σ, for example, may be described in terms of a common slope with a zero-point which depends on Mg$_2$. The existence of a surface implies that a second parameter in addition to luminosity is required to specify the position of a galaxy in (M_B, log σ, Mg$_2$) space. We return to the physical nature of this second parameter in Section 6.

4 The reduced velocity-dispersion and line-strength diagrams

Using the tight correlation between $\delta \log \sigma$ and δ Mg$_2$, we can apply an empirical correction to the observed values of log σ in order to reduce them to the mean relation between Mg$_2$ and luminosity. In this analysis and in what follows, we omit the deviant galaxies NGC 720, 1052, 1172 and 3608 from all fits. A least-squares fit to the residuals in Fig. 3 gives

$$\delta \log \sigma = 1.852\ (\delta\ \mathrm{Mg}_2) + 0.015. \tag{4a}$$

Using this relation, a corrected log σ was defined as follows:

$$\log \sigma_{\mathrm{corr}} \equiv \log \sigma - 1.852\ (\delta\ \mathrm{Mg}_2) - 0.015. \tag{4b}$$

Fig. 4(a) shows the new diagram of log σ_{corr} versus M_B. Compared to the original Fig. 1, the reduction in the scatter is dramatic, although the four anomalous members in the sample still fall well off the line. With these four excluded, the rms scatter of the 20 remaining points has been reduced from ± 16 to only ± 5 per cent.

A new least-squares regression was computed for log σ_{corr} versus M_B. We find

$$\log \sigma_{\mathrm{corr}} = +0.0891\ (-M_B) + 0.4754. \tag{5a}$$

This relation is the solid line in Fig. 4(a). The inverse regression of M_B upon log σ_{corr} is

$$-M_B = 10.51 \log \sigma_{\mathrm{corr}} - 3.78. \tag{5b}$$

Within the errors, this relation agrees well with the standard relation $L \propto \sigma^4$ originally suggested by Faber & Jackson (1976).

Equation (5a) represents the best estimate for the mean relation between log σ and luminosity based on the present data. Accordingly, a new set of velocity residuals, denoted by $\Delta \log \sigma$, were calculated from the original data and equation (5a). These also appear in Table 1.

Similarly, we calculated corrected values of Mg$_2$, denoted (Mg$_2$)$_{\mathrm{corr}}$, based on the values of $\delta \log \sigma$. These are plotted versus luminosity in Fig. 4(b). Again, the scatter has been much reduced compared to the original Fig. 2. A linear regression on the 20 normal galaxies gives

$$(\mathrm{Mg}_2)_{\mathrm{corr}} = -0.0190\, M_B - 0.100. \tag{5c}$$

New residuals, Δ Mg$_2$, were determined relative to this line and are given in Table 1. Henceforth we refer to these new sets of residuals, $\Delta \log \sigma$ and Δ Mg$_2$.

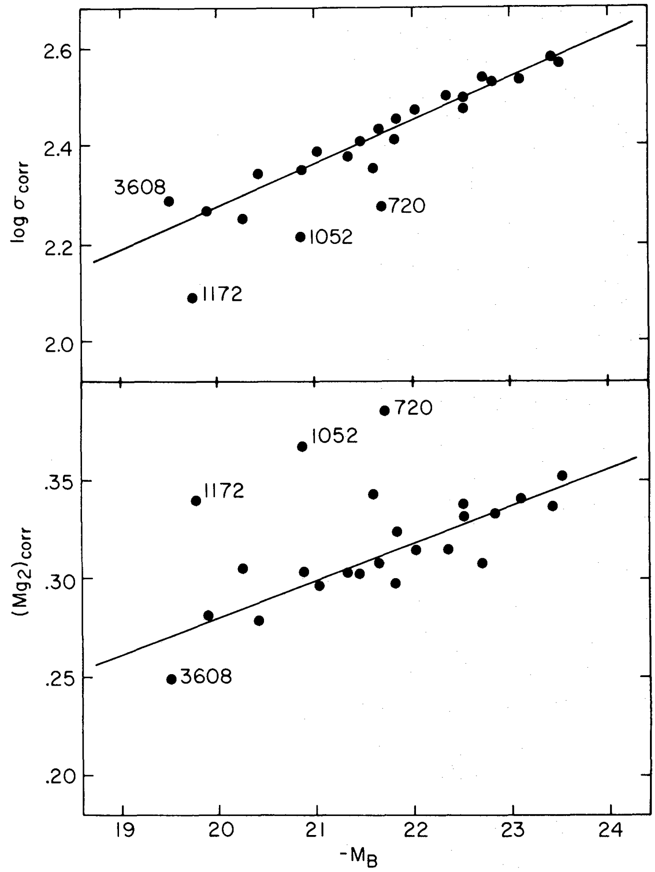

Figure 4. (a) the velocity dispersion corrected to a mean (line index, absolute magnitude) relation, log $\sigma_{\rm corr}$, plotted against absolute magnitude, $M_{\rm B}$. (b) The magnesium line index corrected to the mean (velocity dispersion, absolute magnitude) relation, $\mathrm{Mg}_{2\,\rm corr}$, plotted against absolute magnitude, $M_{\rm B}$.

As a more complete analysis of the geometric interpretation of Section 3, we have computed the least-squares regression plane for the set of 20 points in the three-dimensional space of $M_{\rm B}$, Mg_2 and log σ. The solution of the normal equations gives:

$$M_{\rm B} = 3.25\ (\pm 2.0) + 23.6\ (\pm 3.5)\ \mathrm{Mg}_2 - 13.4\ (\pm 1.1) \log \sigma \qquad (6)$$

where the values within parentheses are the estimated standard errors of the coefficients. The 95 per cent confidence intervals for the Mg_2 and log σ coefficients are

$$16.4 \leqslant C_{\mathrm{Mg}_2} \leqslant 30.8,$$

$$11.1 \leqslant C_{\log \sigma} \leqslant 15.7.$$

Both are significantly different from zero.

The scatter in luminosity in relationship (6) is 0.33 mag, half of that about the 2-D fit in equation (2). Further this scatter is almost exactly equal to the scatter in luminosity in equation (5b) indicating that the 3-D and corrected 2-D formulations are equivalent.

We also tested the null hypothesis that the true value of C_{Mg_2} is zero, or that the inclusion of Mg_2 does not reduce the scatter in luminosity. For the sample of 20 galaxies the null hypothesis is rejected at more than the 99.95 per cent confidence level. This indicates that the second parameter is overwhelmingly significant.

5 Systematic errors in existing velocity-dispersion measurements

The history of velocity-dispersion measurements has been marked by the manifest inability of various authors to agree on dispersions for individual galaxies. However, most of this disagreement lies in the early measurements; recent values have tended to be more consistent, and the agreement between authors has sometimes been surprisingly good (compare the values of FJ, WKS and D). Nevertheless, a few serious disagreements do persist (e.g. NGC 3115, where the recent values quoted differ by up to 100 km s^{-1}). Even the introduction of the more quantitative Fourier technique has not completely resolved individual discrepant cases like NGC 3115.

Apart from noting these few problematical galaxies, however, most authors have intercompared their velocity scales not on an object-by-object basis but rather by comparing the zero-point of the (L, σ^4) relation. WKS have shown, for example, that σ_{21} (the value of σ at $M_B = -21$) ranges from 184 km s^{-1} for SSBS to 244 km s^{-1} for FJ. Despite the fact that this

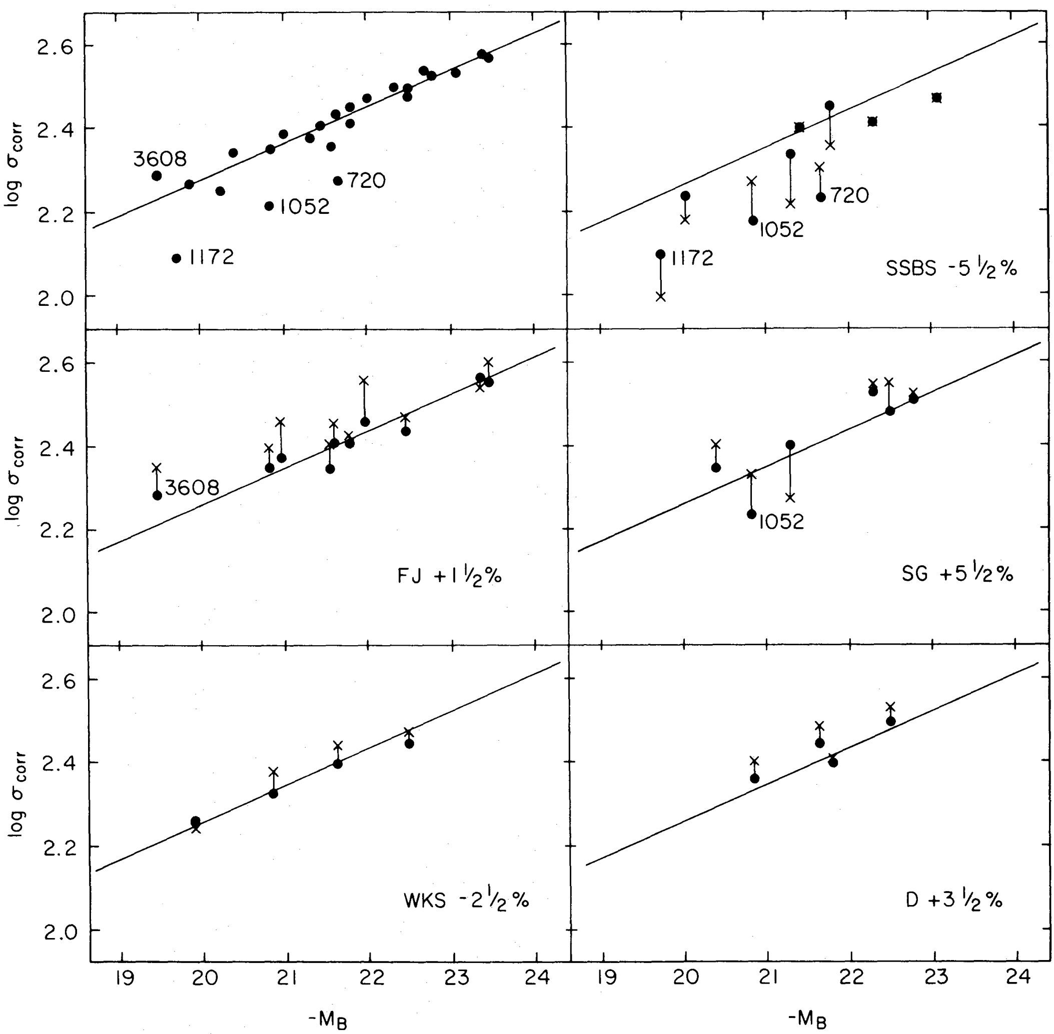

Figure 5. Velocity dispersion plotted against absolute magnitude for the primary sample, separated into the individual measurements by author. The original data from each author are plotted as crosses; those values which have been corrected using the mean (δ Mg$_2$, δ log σ) relation in equation 4(b) are plotted as filled circles. Fig. 4(a) is reproduced in the upper left hand box for comparison and the regression line from equation 5(a) is plotted in each diagram. The authorships are: SSBS – Sargent *et al.* (1977); FJ – Faber & Jackson (1976); SG – Schechter & Gunn (1979); WKS – Whitmore *et al.* (1979); D – Davies (1981).

variation is larger than the authors' own estimates of systematic errors, most workers have nevertheless assumed that the unbiased σ_{21} comparison is the more reliable and have concluded that sizeable systematic scale errors do indeed exist.

Our preceding analysis has shown that the scatter in velocity-dispersion measurements can be greatly reduced if the dependence on line strength is allowed for. This result in turn suggests that the variations in σ_{21} among various studies could be due not to scale errors but instead to mean line-strength differences among galaxy samples. In the following, we assess the magnitude of the remaining scale differences between studies after this line-index effect has been removed. In Fig. 5(b)–(f), the primary sample is divided up according to authorship, and each individual dispersion measurement is shown. The original published values are indicated as crosses, values after correction for δ Mg_2 via equation (4b) as solid circles. Fig. 5(a) reproduces the reduced relation between log σ_{corr} and absolute magnitude which was derived in Fig. 4(a) and the reduced mean relation (equation 5a) is repeated in each frame as a heavy straight line.

The corrected values of velocity dispersion plainly agree much better with the mean line than do the original values. Sample selection effects are clearly evident. For example, the sample of SSBS is particularly unfortunate in containing three of the four anomalous galaxies, plus a sizeable number of objects with below-average Mg_2. This combination resulted in an extremely low value of σ_{21}. Conversely, FJ, often thought to have measured systematically high values of σ, in fact observed an excess of high-metallicity galaxies.

If the four anomalous objects are disregarded, it is possible to compute the residual systematic deviation of each individual study from the mean line. These values are shown in each panel of Fig. 5. The residual systematic deviations now range from only +6 to −6 per cent, well within the authors' own error estimates of ± 10 per cent. Indeed, it appears that these various sets of velocity measurements are more consistent with one another than even the most optimistic authors had dared to hope.

It should be clearly stated, however, that we have shown only that his body of velocity dispersion data is self-consistent in the mean. Sizeable individual discrepancies still remain (e.g. NGC 3115). Furthermore, the serious and systematic disagreement with earlier direct measurements remains undiminished. In particular, a comparison of dispersions from de Vaucouleurs (1974), Morton & Cheavlier (1972, 1973) and Williams (1977) with those analysed here still reveals significant discrepancies.

6 Correlations with axial ratio

In searching for a physical interpretation of the second parameter, we have found suggestive evidence that it may be related to intrinsic axial ratio. The data supporting this idea are shown in Fig. 6(a) and (b), where we plot $\Delta \log \sigma$ and ΔMg_2 against the logarithm of apparent axial ratio taken from the RC2. Our primary sample of 24 galaxies is indicated by filled circles. To maximize the sample, we have added several more galaxies for which only σ or Mg_2 is known. Values of σ were taken only from the sources listed in Table 1. Additional measurements of Mg_2 were derived from the unpublished observations of Faber & Burstein. All galaxies outside the primary sample are plotted as open circles, and the four anomalous objects are labelled.

A correlation between $\Delta \log \sigma$ and apparent flattening is quite evident in Fig. 6(b). Although Fig. 6(a) looks more random, a trend apparently exists here also. We have used a variety of statistical tests to determine the strength of the relationship between ΔMg_2 and log a/b. The sample was split into high and low residual line strengths at $Mg_2 = 0.0065$. A Kolmogorov–Smirnov test (Siegel 1956) was used to determine the probability that these two halves were drawn from the same parent population. The K–S test indicated that the

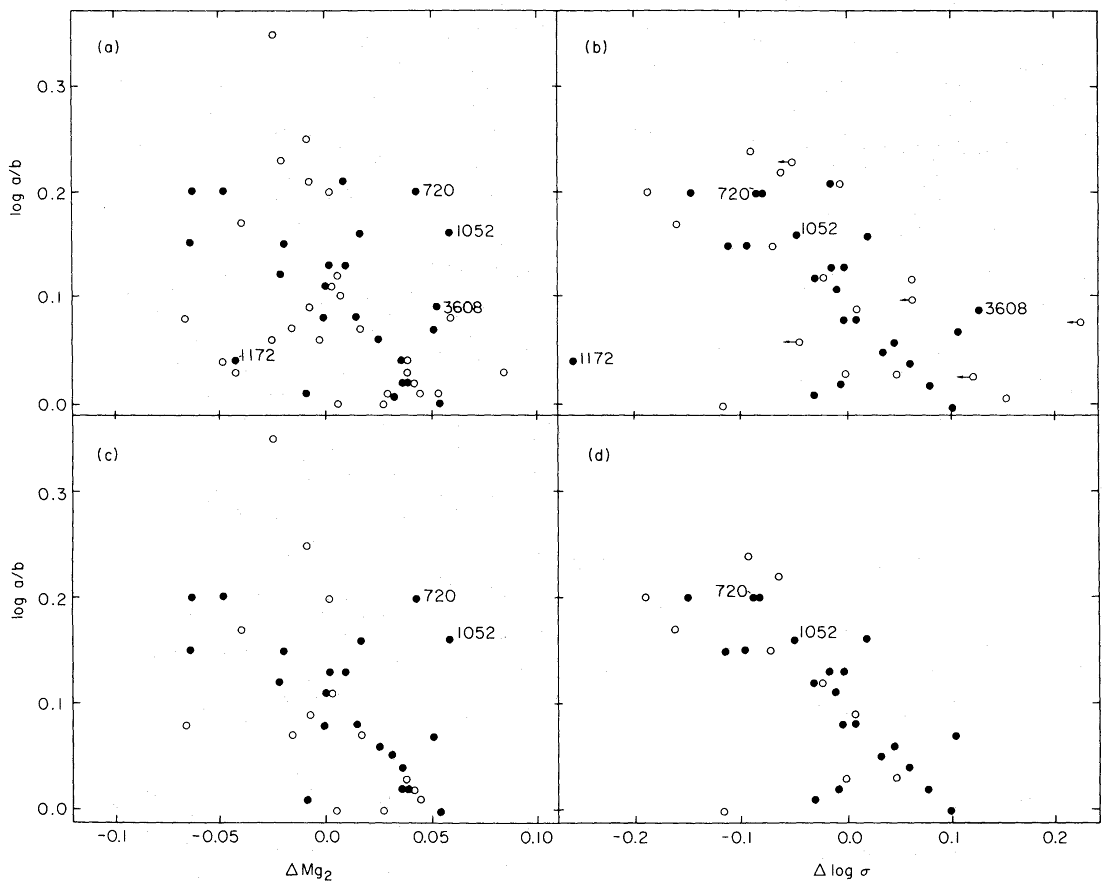

Figure 6. (a) Δ Mg$_2$, the residual line index with respect to equation (5b), plotted versus the logarithm of the axial ratio, log (a/b). The primary sample of galaxies are indicated by the filled circles open circles are from unpublished line strengths by Faber & Burstein. (b) Δ log σ, the residual velocity-dispersion with respect to equation (5a) versus log (a/b). Filled circles are the primary sample, open circles are values of the velocity dispersion from the sources of Table 1 for which there are no corresponding line indices. (c) As in (a), except those galaxies with M_B fainter than -20 have been removed. (d) As in (b), except those galaxies with M_B fainter than -20 have been removed.

two distributions in log a/b are different at 99 per cent confidence level. A Mann–Whitney test (Siegel 1956) shows that the two distributions are drawn from populations with different mean values also at the 99 per cent confidence level.

The scatter in Fig. 6(a) is in part due to the inclusion of galaxies of intrinsically low luminosity. Some of this scatter perhaps reflects the wide variation in the colours and line strengths of fainter elliptical galaxies (cf. Faber 1977). In Fig. 6(c) and (d) all galaxies fainter than $M_B = -20$ have been eliminated and the relationships between Δ Mg$_2$, Δ log σ and log a/b are more evident. Again splitting this brighter sample into two halves according to Δ Mg$_2$, the K–S and M–W tests indicate differences in the distribution of values of log a/b for the two samples at greater than the 99 per cent confidence level.

Finally, a correlation analysis assuming a bivariate normal distribution for Δ Mg$_2$ and log a/b (Mendenhall & Scheaffer 1973) demonstrated that these variables have a 99 per cent chance of being correlated. The correlation coefficient was larger for the sample of brighter galaxies, but the statistical significance of the correlation remained the same because of the smaller number of galaxies. The result that line strength is related to apparent flattening extends the analysis of van der Bergh (1979) and Knapp & Cardelli (private communication), who find that flatter ellipticals are also bluer.

An important source of scatter in Fig. 6 is the effect of projection, since only the apparent axial ratio rather than the intrinsic axial ratio can be observed. The effect on the relationship between flattening and metallicity is especially easy to unravel because $\Delta\,Mg_2$ should be independent of aspect angle. (Since σ may be anisotropic, $\Delta \log \sigma$ may *not* be aspect-independent.) As a result of projection, some galaxies appear rounder than their true figures and thus are shifted vertically downward in the diagrams. Hence, even if the correlation between intrinsic axial ratio and $\Delta\,Mg_2$ were perfect, galaxies would be expected to populate a triangularly shaped region below the line and to the left.

If a correlation between $\Delta\,Mg_2$ and axial ratio is supported by future observations, $\Delta\,Mg_2$ could be a most useful indicator of intrinsic axial ratio for elliptical galaxies. As an example of the potential power of this approach, consider the round galaxies in the luminous sample with $\log a/b < 0.05$ in Fig. 6(c). Taken at face value, their distribution in $\Delta\,Mg_2$ suggests that most are intrinsically spherical rather than flattened objects seen in projection. We are led to this conclusion because nearly all these apparently round galaxies have high line-strengths, yet there are very few flattened ellipticals with high line-strengths from which these objects could be derived via projection. This result is unexpected in view of previous analyses of apparent axial ratios for E galaxies (e.g. Sandage, Freeman & Stokes 1970; Binggeli 1980), which indicate that most round Es are actually rather flattened objects seen in projection. A larger sample of galaxies might allow us to determine whether the distribution of points in Fig. 6 (c) is merely a statistical accident.

7 Discussion

So far we have presented strong evidence that elliptical galaxies are at least a two-parameter family. We have also tentatively suggested that the second parameter may be related to intrinsic axial ratio. (Henceforth the use of the term axial ratio will signify the intrinsic axial ratio unless otherwise specified.) It is crucial to test these ideas on a larger sample of objects because of their potentially important implications for the structure and formation of elliptical galaxies. For example, correlations between luminosity, metallicity, velocity dispersion and axial ratio would most easily be explained if the stellar population formed during the collapse phase, when the dynamical parameters of the galaxy were laid down. This class of models is usually termed dissipational (e.g. Larson 1975). In this picture, the final metallicity might plausibly depend on how long gas is retained within the protogalaxy for further processing (Tinsley & Larson 1979). Protogalaxies with higher binding energy could thus attain higher metallicities, consistent with our observation that, at a fixed luminosity, ellipticals with higher velocity dispersions have higher Mg_2.

By contrast, the currently available dissipationless models (e.g. Gott 1975) offer no easy explanation for these trends. Since star formation is completed before collapse, there seems to be no obvious physical connection between the stellar metallicity and the dynamical parameters. In the context of these models, additional assumptions would have to be invoked. For example, a fraction of the star formation could be delayed until after the collapse, so that recycled gas could play a significant role in forming the observed stellar population. Alternatively, the metallicity and dynamical parameters could jointly be derived from some prior existing condition, perhaps the properties of a massive, non-luminous component.

A correlation between metallicity and axial ratio would be especially important in testing merger models for elliptical formation (e.g. Toomre 1977; Aarseth and Fall 1980). The axial ratio of a merger product ought to depend strongly on the orbital parameters of the encounter and thus should be largely uncoupled from the metallicity of stars in the colliding objects, most of which formed before the collision. So, as with dissipationless models, the problem arises mainly because in the original merger models star formation has already been completed at the time of merging. On the other hand, merger models are attractive from other points of view, particularly because they may be able to reproduce the low values of v/σ observed in elliptical galaxies, Aarseth & Fall (1980). It will be of great interest to see if future merger models incorporating a gaseous component predict a natural correlation between axial ratio and metal enrichment.

Mergers have also been invoked as a possible formation mechanism for cD galaxies (Gunn & Tinsley 1976; Hausman & Ostriker 1978). This view has been supported by recent data on the colours of cD galaxies which have been shown to be bluer than expected by extrapolation of the colour–magnitude relation for normal galaxies (Lugger 1979). The interpretation of this in support of merger models is complicated by considering the axial ratios of cD galaxies. Leir & van den Bergh (1977) demonstrated that cD galaxies are significantly flatter than normal ellipticals. Our results suggest that flatter galaxies may be intrinsically bluer and thus the interpretation proposed by Lugger is somewhat weakened.

Relationships between luminosity, axial ratio, velocity dispersion and metallicity also have an obvious application in determining the true shapes of elliptical galaxies. The well-established correlation between axial ratio and velocity dispersion (Fig. 6b and d) doubtless carries important information, but its interpretation is tricky, since aspect effects may be bound up with intrinsic variations in a complex way. For example, after noting that round galaxies have higher velocity dispersions, one might be tempted rashly to conclude that most ellipticals are prolate. However, it might actually be that velocity dispersions are intrinsically larger in round objects, and the observed trend in Fig. 6(b) and (d) may be dominated by this intrinsic variation rather than the hoped-for aspect effect.

In contrast to σ, the metallicity index should be independent of aspect. As noted above, if the correlation between ΔMg_2 and flattening holds true, we would have an aspect-independent estimate of intrinsic axial ratio. This possibility could in turn open the door to several new statistical methods of studying the true figures of elliptical galaxies. However, a much larger sample size would be required.

In conclusion we offer a word of caution. From the present data it seems probable that velocity dispersion and perhaps metallicity as well vary systematically from round ellipticals to flattened ones. Other properties may vary too, including luminosity density, scale-length, and mass-to-light ratio. If round ellipticals really differ from flattened ones, many of the methods which have been used to discriminate between prolate and oblate forms are potentially invalidated. Such tests have usually assumed the constancy of one physical parameter independent of axial ratio (e.g. luminosity density (Marchant & Olsen 1979) or mass-to-light ratio (Lake 1979)) and looked for a dependence in aspect angle. The preceding arguments suggest that these apparently reasonable assumptions may in fact be quite dangerous and that alternative approaches must be found.

8 Concluding remarks

Of the several hypotheses put forward in this paper, we feel the following are relatively well-established:

(1) The scatter in the (L, σ) and (L, Mg_2) diagrams is real and is related to the intrinsic properties of elliptical galaxies.

(2) Elliptical galaxies are at least a two-parameter family.

(3) For those sources of velocity dispersion considered here, systematic differences among dispersion measured by different authors primarily reflect differences among the mean line-strength of the galaxy samples selected, not measurement errors.

(4) In the final corrected (L, σ) and (L, Mg_2) diagrams, the scatter is consistent with the internal errors of measurement.

The following hypotheses we think are less well-established:

(5) The second parameter for elliptical galaxies may be related to the true axial ratio.

(6) At least some of the measured properties of E galaxies, such as $\Delta \log \sigma$ and ΔMg_2, are intrinsically different for flat and round galaxies.

(7) Correlations between luminosity, axial ratio, metallicity, and velocity dispersion may offer new ways of determining the true shapes of elliptical galaxies.

(8) These correlations also provide a strong test of galaxy formation models and at the moment seem to support models with some degree of dissipation in which the dominant stellar population forms *during* the collapse or merger phase.

(9) The interpretation of the bluer colours of cD galaxies relative to normal galaxies of the same luminosity may be complicated by the excess flattening of cDs as a class.

Several major questions remain which should be addressed in future studies. The present analysis is based on nuclear measurements of velocity dispersion and line-strength through very small apertures. Theoretically, it is of great importance to know whether our conclusions refer to a major fraction of the luminous portion of the galaxy or to the cores only. This point can be answered satisfactorily only when accurate gradients in line-strength and dispersion have been measured for a large number of ellipticals.

It is also obvious that if the correlations among M_B, $\log \sigma$ and Mg_2 hold true, there could be a real improvement in the use of velocity dispersions and line-strengths as distance indicators for elliptical galaxies. The former technique is already being applied successfully to spiral galaxies (Tully & Fisher 1977; Aaronson, Huchra & Mould 1979). The latter has been used by Sandage & Visvanathan (1978) for ellipticals, but without correction for the second parameter effect. Distances based on elliptical galaxies provide a useful and important comparison to those based on spirals, especially for groups such as the Virgo cluster which are rich in both morphological types.

Finally, as is clear from Figs 4 and 5, it is possible to reduce the scatter in the (L, σ) and (L, Mg_2) correlations to a level where observational errors become important. High-quality data are thus essential. For example, if the errors in the line-strength and velocity dispersions were as little as a factor of 2 larger, we would not be able to distinguish deviant objects like NGC 720, 1052 and 1172 from the bulk of the data. With errors four times larger, the correlation between $\delta \log \sigma$ and δMg_2 would itself disappear. Future studies must thus take care to ensure high accuracy in both velocity dispersions and line-strengths.

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